Master Degree in Artificial Intelligence
Statistical and Mathematical Methods for Artificial Intelligence

2019-2020

Homework 1

Finite Numbers

Choose three of the following exercises.

1. Use the Matlab functions `eps`,`realmax`, `realmin` to compute the Floating Point Systems parameters t,L,U assuming that $\beta = 2$. Fill the gap.

```
---------------------------------------------------------
                  MATLAB FLOATING POINT SYSTEM
                         F(2,t,L,U)
----------------------------------------------------------
  UFL   =
  OFL   =
  Precisione Macchina =
  t =
  L =
  U =
---------------------------------------------------------
```

   Tips:

   Given the Floating Point system $\mathcal{F}(\beta, t, L, U)$, you know that:

$$\epsilon_{mach} = \beta^{1-t}, UFL = \beta^{L}, \quad OFL = (\beta - \beta^{-t+1})\beta^{U}.$$

   Can you get t from the expression of ϵ_{mach}?

2. Compute the machine precision ϵ for single and double precision, using its alternative definition:

$$fl(1+\epsilon) > 1.$$

   Tips:

   Use a `while` structure. To set a single precision use the function `single`.

3. Truncation error in π approximation. In mathematics, the Leibnitz formula for π, states that:

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots$$

   It is obtained using Taylor formula for the tan^{-1} function:

$$\arctan(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + o(x^9),$$

   setting $x = 1$ since $\arctan(1) = \frac{\pi}{4}$. Fix $n \in \mathbb{N}$ and compute the approximation of π using Leibnitz formula truncated at n-th term. Compare the result with the true value of π.

4. Let's consider the sequence $a_n = (1 + \frac{1}{n})^n$. It is well known that:

$$\lim_{n \to +\infty} a_n = e,$$

where e is the Euler costant. Choose different values for n, compute a_n and compare it to the real value of the Euler costant. What happens if you choose a large value of n?