Master Degree in Artificial Intelligence
Statistical and Mathematical Methods for Artificial Intelligence

2019-2020

Homework 2

Numerical solution of linear systems

1. **Direct methods.**

   - Write a Matlab script that, given a matrix A and the solution vector $x = (1, 1, \dots 1)^T$:
     - Computes the right hand side $b = Ax$.
     - Computes the condition number of the matrix A.
     - Solves the linear system $Ax = b$ with the backslash operator (Gaussian elimination method) or with Cholesky decomposition (Matlab function `chol`) when the matrix is symmetric and positive definite.
     - Computes the relative error bewteen the exact solution and the computed solution.
     - Uses theMatlab functions `tic` and `toc` to measure the computational time and plot a graph of the time as a function of n.
     - Plot a graph with the relative errors as a function of n and a graph with the condition number K(A) as a fucntion of n.
   - Test the program with:
     - A random matrix obtained with the Matlab function `randn` with size varying with $n = 10 : 50 : 1000$.
     - The Vandermonde matrix obtained with the Matlab function `vander` of size varying with $n = 5 : 5 : 30$.
     - (Symmetric positive definite) The matrix obtained as $A^T A$, where A is a random matrix ($n = 10 : 50 : 1000$).
     - (Symmetric positive definite) The Hilbert matrix (Matlab function `hilb`) with n varying with $n = 4 : 1 : 12$.

2. **Iterative methods**

   - In the previous script solve the linear system with the iterative methods: the Jacobi iterative method (function `jacobi.m` on the course webpage) and the Conjugate Gradient method (Matlab function `pcg`).
   - Test the program with:
     - The symmetric matrix obtained as $A^T A$, where A is a random matrix ($n = 10 : 50 : 1000$) and compare the results in terms of relative error and computational time with those obtained with the direct methods. Then increase the dimension until 5000.
     - The tridiagonal matrix

$$T = \begin{pmatrix} 5 & -1 & -1 & 0 & \dots & 0 & \\ -1 & 5 & -1 & -1 & \dots & 0 & \\ -1 & -1 & 5 & -1 & -1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & & \\ 0 & \dots & 0 & 0 & -1 & -1 & 5 \end{pmatrix}$$

       of size $n = 50 : 100 : 5000$.
       Store only the nonzero elements of the matrix.