

This note contains an example of a nowhere continuous weakly Picard operator. Such function contradicts Lemma 1 in [1].

Definition 1 ([1, Definition 2]). Let X be a metric space. We say that a mapping $f: X \rightarrow X$ is a *weakly Picard operator* if for each $x \in X$ the sequence $(f^n(x))_{n \in \mathbb{N}}$ is convergent to $b_x \in X$, which is a fixed point of f .

Lemma 2 ([1, Lemma 1]). *Let $f: X \rightarrow X$ be a weakly Picard operator and let $b \in X$ be a fixed point of f . Then f is continuous at point b .*

Example 3. For $q \in \mathbb{Q}$ let $n_q \in \mathbb{Z}$ and $d_q \in \mathbb{N}$ be such that $q = \frac{n_q}{d_q}$ and $\gcd(n_q, d_q) = 1$. Consider a function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined as

$$f(x) = \begin{cases} 0, & x \in \mathbb{R} \setminus \mathbb{Q}, \\ d_x, & x \in \mathbb{Q}. \end{cases}$$

Notice that $f(\mathbb{R}) = \mathbb{N}_0$ and $f(\mathbb{N}_0) = \{1\}$, hence for all $x \in \mathbb{R}$ we get $f(f(x)) = 1 = f(1)$. Furthermore f is discontinuous at each point of \mathbb{R} .

References

- [1] Vasile Berinde. On the solution of steinhaus functional equation using weakly picard operators. *Filomat*, 1, 04 2011.