Maekawa’s Algorithm: Key Idea

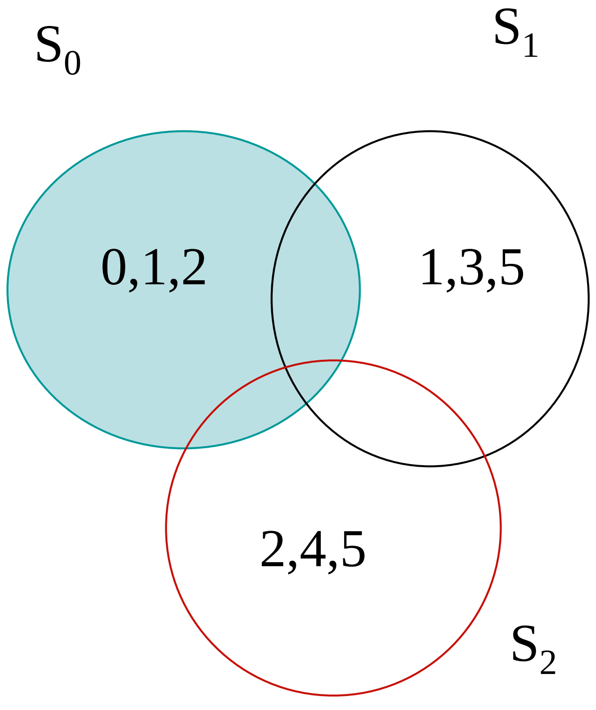
- The algorithm of Ricart-Agrawala requires replies from all processes in group, similar to that of Lamport’s algorithm
- Instead, get replies from only some processes in group
- But ensure that only process one is given access to CS (Critical Section) at a time

Quorum Based Algorithms

- First solution with a sublinear O(sqrt N) message complexity.
- Each process is required to obtain permission from only a subset of peers

Maekawa's Algorithm

- With each process i, associate a subset S_i.
- Divide the set of processes into subsets that satisfy the following two conditions:
  1. $i \in S_i$
  2. $\forall i,j: \; 0 \le i, j \le n-1 \quad S_i \cap S_j \neq \emptyset$



- Main idea. Each process i is required to receive permission from S_i only.
- Correctness requires that multiple processes will never receive permission from all members of their respective subsets.

Maekawa's Algorithm

Example. Let there be seven processes 0, 1, 2, 3, 4, 5, 6

$S_0 = \{0, 1, 2\}$
$S_1 = \{1, 3, 5\}$
$S_2 = \{2, 4, 5\}$
$S_3 = \{0, 3, 4\}$
$S_4 = \{1, 4, 6\}$
$S_5 = \{0, 5, 6\}$
$S_6 = \{2, 3, 6\}$

Maekawa's Algorithm

Version 1 {Life of process I}

1. Send timestamped request to each process in S_i.
2. Request received : send an ack to process with the lowest timestamp. Thereafter, "lock" (i.e. commit) yourself to that process, and keep others waiting.
3. Enter CS if you receive an ack from each member in S_i.
4. To exit CS, send release to every process in S_i.
5. Release received : unlock yourself. Then send an ack to the next process with the lowest timestamp.

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Maekawa's Algorithm - Version 1

- Safety: At most one process can enter its critical section at any time.

- Let i and j attempt to enter their Critical Sections
- $S_i \cap S_j \neq \emptyset$ implies there is a process k $\in S_i \cap S_j$
- Process k will never send ack to both.
- So it will act as the arbitrator and establishes the safety property.

$S_0 = \{0, 1, 2\}$

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Maekawa's Algorithm - Version 1

- No deadlock? Unfortunately, deadlock is possible! Assume 0, 1, 2 want to enter their critical sections.

- From S_0= {0,1,2}, 0,2 send ack to 0, but 1 sends ack to 1;
- From S_1= {1,3,5}, 1,3 send ack to 1, but 5 sends ack to 2;
- From S_2= {2,4,5}, 4,5 send ack to 2, but 2 sends ack to 0;
- Now, 0 waits for 1 (to send a release), 1 waits for 2 (to send a release), and 2 waits for 0 (to send a release), . So deadlock is possible!

S_0 = {0, 1, 2}

S_1 = {1, 3, 5}

S_2 = {2, 4, 5}

S_3 = {0, 3, 4}

S_4 = {1, 4, 6}

S_5 = {0, 5, 6}

S_6 = {2, 3, 6}

Maekawa's Algorithm - Version 2

- Avoiding deadlock
- If processes always receive messages in increasing order of timestamp, then deadlock "could be" avoided. But this is too strong an assumption.

- This can be fixed using three additional messages:
  - failed
  - inquire
  - relinquish

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$S_3 = \{0, 3, 4\}$

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Maekawa’s Algorithm - Version 2

- Send ack and set lock as usual.
- If lock is set and a request with a larger timestamp arrives, send FAIL (you have no chance).
- If the incoming request has a lower timestamp, then send INQUIRE (are you in CS?) to the locked process.
- Receive inquire and at least one failed message send RELINQUISH.
- The recipient resets the lock.

S_0 = {0, 1, 2}
S_1 = {1, 3, 5}
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S_3 = {0, 3, 4}
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Maekawa's Algorithm - Version 2

- There are many more algorithms proposed for mutual exclusion. We will not be able to cover those.
- Read them if you are interested.

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$S_3 = \{0, 3, 4\}$

$S_4 = \{1, 4, 6\}$

$S_5 = \{0, 5, 6\}$

$S_6 = \{2, 3, 6\}$