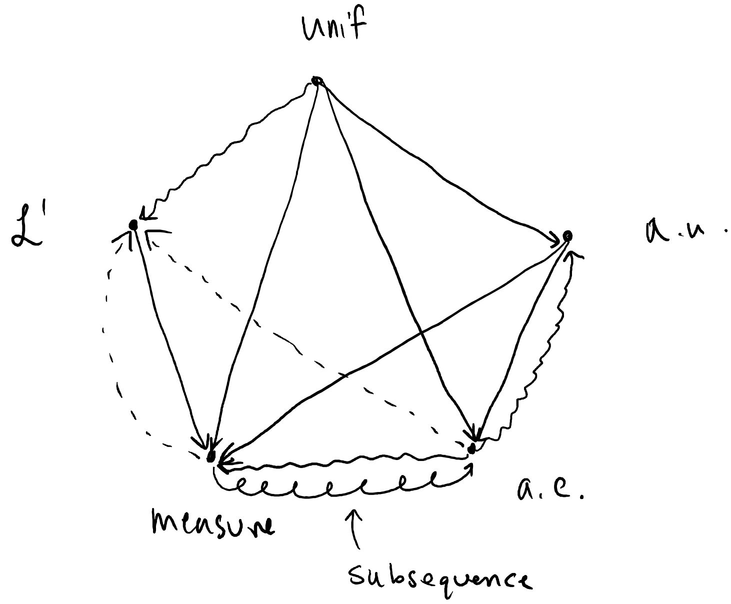
Rec 9/24

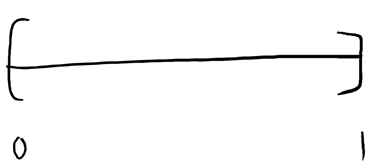
Tuesday, September 24, 2019 10:19

Munroe



$\mu(X) < \infty$ $\rightsquigarrow$

domination $|f_n| \leq g_0 \in \mathcal{L}^1$ $\dashrightarrow$



$E_1 = [0,1]$

$E_2 = [0, \frac{1}{2})$

$E_3 = [\frac{1}{2}, \frac{1}{2} + \frac{1}{3})$

$E_4 = [\frac{1}{2}+\frac{1}{3}, \frac{1}{2}+\frac{1}{3}+\frac{1}{4})$ mod 1

$\vdots$

$f_j = \chi_{E_j}$

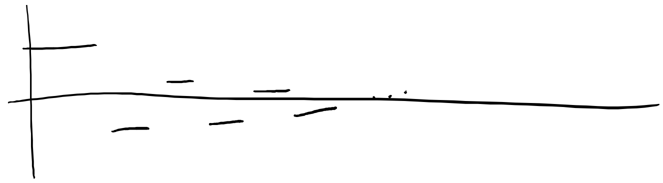
(f_j) converges in [measure] & [$\mathcal{L}^1$], (to 0).

not pointwise or a.e.

Cauchy [measure]

Riemann + Lebesgue

$f(x) = \frac{(-1)^n}{n}$ for $x \in (n-1, n]$



Riemann integral $\int_0^\infty f = \lim_{M\to\infty}\int_0^M f$ makes sense,

but $f \notin \mathcal{L}^1$.

N non-measurable

if $f = \begin{cases} 1 & N \cap [0,1] \\ -1 & N^c \cap [0,1] \end{cases}$

then $f \notin \mathcal{L}^1$ even though $|f| \in \mathcal{L}^1$.

since f is not mble.

but f mble $\Rightarrow$

$$|f| \in \mathcal{L}^1 \iff f \in \mathcal{L}^1.$$

riemann integrable

$f = \chi_{\mathbb{Q}} \notin \mathcal{R}[a,b]$

$\overline{\int} f$ and $\underline{\int} f$ always exist.

If $f \in \mathcal{L}^1[a,b]$, when is $f \in \mathcal{R}[a,b]$.

"clearly" f cts is sufficient.

Piecewise cts is sufficient.

n.a.s.c: $\{x \mid f \text{ discontinuous at } x\}$ is Lebesgue-null.