Lec 11/8

Wednesday, November 8, 2017 10:50

"Functional Calculus"

$$L(y) = \sum a_k y^{(k)} = 0$$

find roots of

$$\sum a_k t^k = 0$$

to get basis $\{\phi_i\}$ in terms of roots

Nonhomogeneous equations

$$L(y) = \sum a_k y^{(k)} = b(x)$$

Solve homogeneous eqn, find particular soln ψ_p via V.o.C.

Ch 3.6:

$$L(y) = \sum a_k(x)\, y^{(k)} = b(x)$$

Variation of constants works here too.

A particular solution has to be

$$\psi_p = u_1\phi_1 + \cdots + u_n\phi_n, \qquad u_i : I \to \mathbb{C}.$$

$$u_i' = \frac{W_k(x)}{W(\phi_1, \ldots, \phi_n)(x)}$$ where $W(\phi_1, \ldots, \phi)$ is wronskian,

W_k is wronskian matrix determinant w/ kth col $\mapsto \begin{pmatrix} 0 \\ \vdots \\ 0 \\ b(x) \end{pmatrix}$

<u>Ex 5</u> When $u_k(x) = \int_{x_0}^{x} \frac{W_k(t)}{W(\phi_1,\dots,\phi_n)(t)} b(t)\, dt$

$$\psi_p(x_0) = \psi_p'(x_0) = \cdots = \psi_p^{(n-1)}(x_0) = 0.$$

$\downarrow$

plug in x_0 for x in u_k

Example 6.3

<u>Ex 7</u>:

$$y'' + y = b(t)$$

where $\int_1^{\infty} |b(x)|\, dx < \infty$

$$\psi_p(x) = \int_1^{x} b(t) \sin(x-t)\, dt$$

Solution 1: Differentiate by using Leibniz rule.

Solution 2: Use variation of constants.