

1.13 #2 Show there is no simple gp of order pq (p, q primes)

$p = q \Rightarrow \exists$ p -subgp & its normal (index p from HW problem). (has nontrivial center)

$p < q \Rightarrow n_q = 1 \pmod{q}$ & $n_p \mid p \Rightarrow n_q = 1$ so q is normal.

1.13 #6 Show every nonabelian gp of order 6 is isomorphic to S_3 .

$n_2 = 1$ & $n_3 = 1 \rightarrow$ bad

\vdots

Recall:

$$[x] = {}^G x = \{gxg^{-1} \mid g \in G\}$$

Note: if G is abelian, $[x] = \{x\}$.

Goal: describe conj classes of S_n .

for $\sigma \in S_n$, $\sigma = \overset{\substack{\text{disjoint} \\ \text{cycles}}}{\pi_1 \cdots \pi_r}$

let $k_i =$ length of π_i

σ has cycle type (k_1, \dots, k_r) with $k_1 \geq \dots \geq k_r$ and $\sum_{i=1}^r k_i = n$.

· Conj. each cycle: $\alpha \sigma \alpha^{-1} = \alpha \pi_1 \alpha^{-1} \alpha \pi_2 \alpha^{-1} \dots \alpha \pi_k \alpha^{-1}$.

claim1: σ, τ are conj iff σ, τ have same cycle type

claim2: conj. classes of $S_n \iff \lambda \vdash n$