Lec 2/8

Wednesday, February 8, 2017 15:00

Thm 11.8: $X_1 \sim Bin(n_1, \theta_1)$ $X_2 \sim Bin(n_2, \theta_2)$, $\underset{\vee}{X_1, X_2 \text{ independent.}}$ n_1, n_2 large, $\hat{\theta}_1 = \frac{X_1}{n_1}$ $\hat{\theta}_2 = \frac{X_2}{n_2}$.

$$\hat{\theta}_1 - \hat{\theta}_2 \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{\theta}_1(1-\hat{\theta}_1)}{n_1} + \frac{\hat{\theta}_2(1-\hat{\theta}_2)}{n_2}}$$ is a $(1-\alpha)\times 100\%$ CI for $\theta_1 - \theta_2$

Ex: Wicking. Day 1: Observe 10 wicking boxes in 100 (X_1)

Day 2: observe 6 wicking boxes in 50 (X_2)

Find a 95% CI for $\theta_2 - \theta_1$:

$$0.12 - 0.1 \pm 1.96 \sqrt{\frac{0.1(0.9)}{100} + \frac{0.12(0.88)}{50}}$$

$$= (-0.0876, 0.1276)$$

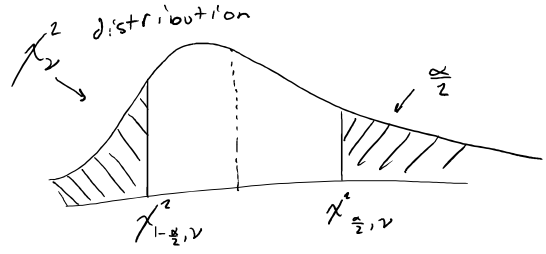
Note: in example from last time, $\hat{\theta}$ in getting number n required is a new $\hat{\theta}$.

§11.6 estimation of variances

Recall: S^2 sample variance; $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$ (chi-squared)

depends on σ^2, S^2 → doesn't depend on any parameter

use pivot idea:



$$P\left(\chi^2_{1-\frac{\alpha}{2},n-1} < \frac{(n-1)S^2}{\sigma^2} < \chi^2_{\frac{\alpha}{2},n-1}\right) = 1-\alpha.$$

$$= P\left(\frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2},n-1}} < \sigma^2 < \frac{(n-1)S^2}{\chi^2_{1-\frac{\alpha}{2},n-1}}\right) = 1-\alpha$$

So a $(1-\alpha)\times 100\%$ CI for σ^2 is $\left(\frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2},n-1}}, \frac{(n-1)S^2}{\chi^2_{1-\frac{\alpha}{2},n-1}}\right)$

to get a CI for σ, just take $\sqrt{\ }$ of endpoints

Ex: (usually assumption is that samples are iid from normal dist).

find a 95% CI for σ^2 if a RS of 20 boxes gives $S = 0.25$.

from table, $\chi^2_{\frac{0.05}{2},19} = 32.852$ $\quad \chi^2_{1-\frac{\alpha}{2},19} = 8.907$

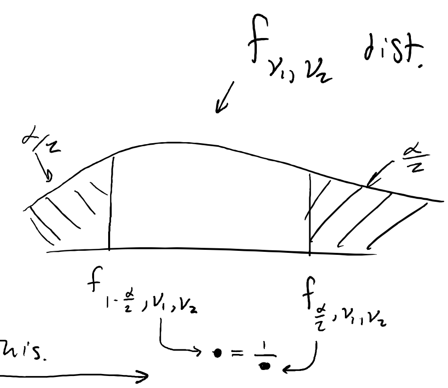
$$\Rightarrow \quad 95\%\ CI: \left(\frac{19(0.25)^2}{32.852}, \frac{19(0.25)^2}{8.907} \right) = (0.0361, 0.1333).$$

§11.7 Estimation of Ratio of two Variances.

Recall F distribution: $U \sim \chi^2_{\nu_1}$ $\quad V \sim \chi^2_{\nu_2}$

$$F = \frac{U/\nu_1}{V/\nu_2}$$ has an F distribution w/ dfs ν_1 and ν_2.

Thm 8.15: $$F = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} = \frac{S_1^2\,\sigma_2^2}{S_2^2\,\sigma_1^2} \sim f_{n_1-1,\, n_2-1}.$$



$$\mathbb{P}\left(f_{1-\frac{\alpha}{2},n_1-1,n_2-1} < \frac{\sigma_1^2\, S_2^2}{\sigma_2^2\, S_1^2} < f_{\frac{\alpha}{2},n_1-1,n_2-1} \right) = 1-\alpha$$

$$\Rightarrow \mathbb{P}\left(\frac{S_1^2}{S_2^2}\frac{1}{f_{\frac{\alpha}{2},n_1-1,n_2-1}} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{S_1^2}{S_2^2} f_{\frac{\alpha}{2},n_1-1,n_2-1} \right)$$

using this.

So a $(1-\alpha)\times 100\%$ CI for $\frac{\sigma_1^2}{\sigma_2^2}$ is $\left(\frac{S_1^2}{S_2^2}\frac{1}{f_{\frac{\alpha}{2},n_1-1,n_2-1}}, \frac{S_1^2}{S_2^2} f_{\frac{\alpha}{2},n_1-1,n_2-1} \right)$.

if you want a CI for $\frac{\sigma_1}{\sigma_2}$, take $\sqrt{\ }$ of endpoints.

Ex: Variation in weights of dog food in two days.

Day 1: $n_1 = 100$ $\quad S_1 = 0.25$ $\quad$ find a 90% for $\frac{\sigma_1^2}{\sigma_2^2}$.

Day 2: [illegible]

Day 1: $n_1 = 100$ $S_1 = 0.25$

Day 2: $n_2 = 50$ $S_2 = 0.18$

find a 90% for $\frac{\sigma_1^2}{\sigma_2^2}$.

Note $f_{\frac{0.05}{2}, 99, 49} = 1.58$.

So $CI = \left(\frac{0.25^2}{0.18^2} \frac{1}{1.58}, \frac{0.25^2}{0.18^2} 1.58 \right) = (1.221, 2.894)$.