Lec 9/18

Monday, September 18, 2017 14:20

$P_3 = \{ P \cap 4\mathbb{N}+3 \}$ $S_3 = \{ \prod_{p \in P_3} p^\alpha \}$ $S_1 S_3 = \mathbb{N}$,

$P_1 = \{ P \cap 4\mathbb{N}+1 \} \cup \{2\}$ $S_1 = \{ \prod_{p \in P_1} p^\alpha \}$ what is $d(S_1)$, $d(S_3)$?

$d(S_1) = 0$ since it's a subset of $d(n_1^2 + n_2^2)$.

$$E = \{ n : n = p_{i_1}^{c_1} \cdots p_{i_k}^{c_k}, \ \sum c_i \in 2\mathbb{N} \}$$

$$O = \{ n : n = p_{i_1}^{c_1} \cdots p_{i_k}^{c_k}, \ \sum c_i \in 2\mathbb{N}+1 \}$$

like multiplicative evens & odds.

$$d(E) = d(O) = \frac{1}{2}$$

$P \subset O$, $P-1$?

Midterm: Think: why Sarközy only works for $P \pm 1$, not P

$(P-1) \cap E$?

$(P-1) \cap O$?

Unknown which is infinite

$E - E = \mathbb{N}$, $O - O = \mathbb{N}$, $E + O \stackrel{?}{=} \mathbb{N}$

Let $x \in (0,1)$ be a normal # in base 2.

0-1 seqs $\rightleftarrows$ subsets of $\mathbb{N}$, so define "normal" for sequences/sets.

Almost all $x \in (0,1)$ is normal, so "Almost all" $E \in P(\mathbb{N})$ is normal.

So the typical set $A \subseteq \mathbb{N}$ is Thick.

Champernowne's constant is normal proof:

Niven : Numbers rational and Irrational

Classical:

Cantor Set: $C = \left\{ x \in (0,1),\ x = \sum \frac{t_i}{3^i},\ t \in \{0, 2\} \right\}$

Measure 0, uncountable.

A General Cantor set is any set which is homeomorphic to C.

$\{0,1\}^{\mathbb{N}} \approx C$

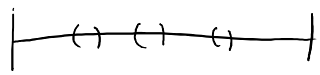
$$\underset{\underset{(x_i)}{\parallel}}{x}, \underset{\underset{(y_i)}{\parallel}}{y} \in \{0,1\}^{\mathbb{N}}$$

$$d(x,y) := \sum_{i=1}^{\infty} \frac{|x_i - y_i|}{2^i}$$

(exercise: show this is a metric on $\{0,1\}^{\mathbb{N}}$)

(Exercise: show that $\{0,1\}^{\mathbb{N}}$ w/ this general cantor set:

metric is homeomorphic to C)



take out some arbitrary # of intervals each step, only requiring that intervals don't touch.

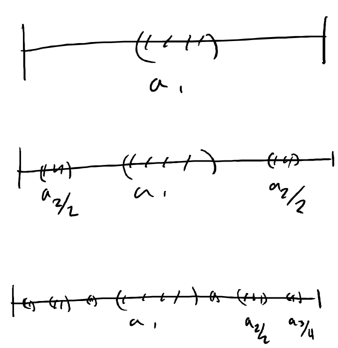
'Cantor set' is intersection of these remaining and maximal length of remaining intervals goes to 0.

And this is homeomorphic to C (Exercise)

$C \not\cong [0,1]$ (exercise)

$a_i > 0$

$0 < \sum a_i < 1$



K_1

K_2

K_3

$A_{du} = \bigcap K_i$,

exercise: A_{du} does not have measure 0.

exercise: $[0,1]$ is not of measure 0.

Reading: Ch 10.