Rec 10/17

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Given a net $\langle x_\alpha \rangle_{\alpha \in I}$ we say $\langle y_\beta \rangle_{\beta \in J}$ is a subnet of $\langle x_\alpha \rangle$ if $\exists f : J \longrightarrow I$ s.t. $y_j = x_{f(j)}$ <u>and</u>

$\forall i_0 \in I$, $\exists j_0 \in J$ with $f(j) \geq i_0$ $\forall j \geq j_0$

"clearly" if $\langle x_\alpha \rangle \longrightarrow x$ then any subnet $\langle y_\beta \rangle \longrightarrow x$

<u>Prop 4</u> if $\langle x_\alpha \rangle \subset X$ then TFAE:

1. x is a cluster pt of $\langle x_\alpha \rangle$ (x_α frequently in any nhd of x)
2. $\exists$ subnet $\langle y_i \rangle$ converging to x.

<u>Compactness</u>

<u>Thm</u> TFAE (any top sp X)

1. X is compact
2. If $\{F_\alpha\}$ are closed & have the finite intersection property ($\bigcap_{\text{finite}} F_\alpha \neq \emptyset$) then $\bigcap_\alpha F_\alpha \neq \emptyset$.
3. Every net in X has a cluster pt
4. Every net has a convergent subnet

Pf ① ⇔ ② HW, ③ ⇔ ④ above.

② ⇒ ③ given a net $\langle x_\alpha \rangle$, define sets $F_\alpha = \{x_\beta \mid \beta \geq \alpha\}$.

$\forall$ finite collection $\{F_{\alpha_i}\}_1^n$, $\bigcap_1^n F_{\alpha_i} \neq \emptyset$

since $\exists\ \gamma \geq$ all α_j, so $\overset{\in}{x_\gamma}$.

$\{\overline{F}_\alpha\}$ are closed & satisfy FIP, $\bigcap_\alpha \overline{F}_\alpha \neq \emptyset$,

Claim: $x \in \bigcap_\alpha \overline{F}_\alpha$ is a cluster pt of $\langle x_\alpha \rangle$

③ ⇒ ②