Lec 1/11

Wednesday, January 11, 2017 14:54

Ch 10 point estimation

P.E: use value of statistic to directly estimate parameter of pop. or dist.

eg. $\bar{X} = \dfrac{X_1 + \cdots + X_n}{n} \sim \mu$: mean of pop.

The statistic $\bar{X}$ is a point estimator.

Value of $\bar{X}$ is the point estimate.

Ex: μ : pop. mean (parameter)

$\bar{X}$ is a RV, so think about properties a good estimator would have:

(1) Unbiasedness

(2) Minimum Variance

(3) Efficiency

(4) Consistency

(5) Robustness

§10.2 Unbiased estimators

let p be pop. param, s be statistic (point estimator).

want $E(s) = p$. This is unbiasedness.

Defn: Bias of estimator s is: $\text{Bias}(s) = E(s) - p$

would like $\text{Bias}(s) = 0$.

Ex: Poisson dist. w/ param λ. $X \sim f(x;\lambda) = \dfrac{\lambda^x e^{-\lambda}}{x!}$, $x \in \mathbb{N}$

given a sample $\{X_1, \dots, X_n\}$, want to estimate λ. Note that $E(X) = \lambda$, $Var(X) = \lambda$.

Consider sample mean $\bar{X} = \dfrac{X_1 + \cdots + X_n}{n}$

Q: is $\bar{X}$ unbiased estimator for λ?

$E(\bar{X}) = E(X_i) = \lambda$. yes.

Also consider sample variance $S^2 = \dfrac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$

Q: is S^2 unbiased estimator for λ?

my try $\to$ $E(S^2) = \frac{1}{n-1}\sum_{i=1}^{n} E[(X_i - \bar{X})^2] = \frac{1}{n-1}\sum_{i=1}^{n}\left[E(X_i^2) + E(\bar{X}^2) - 2E(X_i\bar{X})\right] = \frac{1}{n-1}\left[n\lambda^2 + n\lambda^2 + 2\sum_{i=1}^{n} E(X_i\bar{X})\right]$

$$E(S^2) = \frac{1}{n-1} E\left(\sum_{i=1}^{n}\big((X_i - \lambda) + (\lambda - \bar{X})\big)^2\right) = \frac{1}{n-1}\left(E\left[\sum_{i=1}^{n}(X_i - \lambda)^2\right] - \underbrace{2E\left[\sum_{i=1}^{n}(X_i - \lambda)(\lambda - \bar{X})\right]}_{2E\left[(\lambda - \bar{X})\underbrace{\sum_{i=1}^{n}(X_i - \lambda)}_{n\bar{X} - n\lambda = n(\bar{X} - \lambda)}\right]} + \underbrace{E\left[\sum_{i=1}^{n}(\lambda - \bar{X})^2\right]}_{nE[(\bar{X} - \lambda)^2]} \right)$$

$$2E[(\lambda - \bar{X})\, n(\bar{X} - \lambda)] = -2nE[(\bar{X} - \lambda)^2]$$

$$= \frac{1}{n-1}\sum_{i=1}^{n}\Big[\underbrace{E[(X_i - \lambda)^2]}_{\lambda \text{ (Var. of } X_i)}\Big] - n\,\underbrace{E[(\bar{X} - \lambda)^2]}_{\frac{\lambda}{n} \text{ (Var of } \bar{X})}$$

$\hookrightarrow$ since $E(\bar{X}) = \lambda$

$= \frac{1}{n-1}\left[n\lambda - \lambda\right] = \lambda.$ A: Yes.

In this example, both $\bar{X}$ and S^2 are unbiased estimators for λ. so which to choose?

there can be more than 1 unbiased estimator in general for a parameter.

e.g. $\frac{\bar{X}+S^2}{2}$.

Q: Which estimator is the best? (later)

Ex: Consider RS $\{X_1,\dots,X_n\}$ from pdf $f(x;\theta)$ $\frac{1}{2}(1+\theta x)$ $x\in(-1,1)$, $\theta\in(-1,1)$.

try using $\bar{X}$. is $\bar{X}$ unbiased for θ?