Lec 10/4

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Def If S' is a comm. ring, S is a subring and U is a subset of S', $S[U]$ is the subring of S' gen'd by S and U.

Ex If $U, V \subset S'$, $S[U][V] = S[U \cup V] = S[V][U]$

Thm any $\sigma \in S_n$ induces an automorphism of $R[x_1, \dots, x_n]$ permuting the x_i's.

Thm let $\eta : R \to S$, let $u \in S$. Then $\exists!$ η_u s.t.

$$\begin{tikzcd} R \arrow[r, "\eta"] \arrow[d, hook] & S \\ R[x] \arrow[ur, "\eta_u"'] & \end{tikzcd}$$

commutes & $\eta_u(x) = u$.

Cor if $R \subset S$ & $u \in S$, $R[x]/_{\mathrm{Ker}(id_u)} \cong R[u]$.

Note that $\mathrm{Ker}(id_u) \cap R = 0$.

Conversely, if $I \subset R[x]$ is an ideal w/ $I \cap R = 0$,

Then $R[x]/_I = R[u]$

where $u = x + I$.

$\Big\}$ viewing R as a subring of $R[x]/_I$.

Def If id_u is the ext of the id map $R \hookrightarrow S$ to $R[x] \to S$, then $u \in S$ is called transcendental if $\mathrm{Ker}(id_u) = 0$.

If $\mathrm{Ker}(id_u) \neq 0$, u is called <u>algebraic</u> (over R).