Rec 9/17

Tuesday, September 17, 2019 10:23

$$C = \bigcap C_n = \lim_{n\to\infty} C_n$$

$$H_p(C_n) = 2^n\left(\frac{1}{3^n}\right)^p = \left(\frac{2}{3^p}\right)^n,$$

only one value of p for which $H_p(C_n) \not\to \infty$, $\not\to 0$

$$\frac{2}{3^p} = 1 \quad , \text{ solve for } p.$$

Product Spaces, Product Measures

$$\mathbb{C} = \mathbb{R}\times\mathbb{R}$$

$X = X_1 \times X_2$, $\quad \pi_\alpha : X \to X_\alpha$, $\mathcal{M}_\alpha$ σ-alg on X_α.

$\mathcal{M} = \mathcal{M}_1 \otimes \mathcal{M}_2$, generated by $\{E_1 \times E_2 \mid E_\alpha \in \mathcal{M}_\alpha\}$.

In particular, $\mathcal{B}_{\mathbb{R}^2} = \mathcal{B}_{\mathbb{R}} \otimes \mathcal{B}_{\mathbb{R}}$.

Prob 22

Suggestion: open sets are generated by open rectangles

$z = x + iy$, $\{a < x < b,\ c < y < d\}$

Also open discs: $|z - z_0| < r$

$\Big\}$ either works.

Is $\mathcal{B}_{\mathbb{C}}$ generated by open rectangles?

Yes, need to show open sets are <u>countable</u> unions.

From the def'ns,

$f: X \longrightarrow \mathbb{C}$ is mble iff $\underbrace{\mathrm{Re} f \ \& \ \mathrm{Im} f}_{\text{as fns } X \longrightarrow \mathbb{R}}$ are.

$(X, \mathcal{M})$

Sufficient to show $f^{-1}(E) \in \mathcal{M}$ whenever E is an open rectangles.

do some stuff, intersect some stuff

$\mathcal{S}$ = set of $\mathcal{M} - \mathcal{B}_{\mathbb{C}}$ mble fns.

$\mathcal{S}$ is a $\mathbb{C}$-vector space

$f \in \mathcal{S} \Rightarrow \alpha f \in \mathcal{S}$, $f, g \in \mathcal{S} \Rightarrow f + g \in \mathcal{S}$.

game sufficient to check on a generating set (eg (a,∞)).

Sufficient to check out $\mathrm{Re}(\alpha f)$ and $\mathrm{Im}(\alpha f)$.

$$\{f+g>a\} = \bigcup_{r\in\mathbb{Q}} \{f>r\} \cap \{g>a-r\}$$

$$fg = \frac{1}{2}\left(f^2 + g^2 - (f-g)^2\right)$$

$f^2(x) \in (a,b)$ (real-valued)

$$f \in \mathcal{S} \Rightarrow |f| \in \mathcal{S}$$

$f = u + iv$, $|f|^2 = f\bar{f} = (u+iv)(u-iv)$

$$f_n \in \mathcal{S} \Rightarrow \lim f_n \in \mathcal{S}.$$