Def Suppose $N \hookrightarrow M$ is a submfld of codim $=1$. Call N 2-sided if it has a trivial tubular n.h.

$\Leftrightarrow$ embedding extends to $N \times (-\varepsilon, \varepsilon) \hookrightarrow M$.

eg not line inside Möbius strip

Lemma if N and M are oriented then N is 2-sided.

Corollary Seifert surfaces are 2-sided.

Def $\varphi: A \to B$ mfds, $x \in A$ is a regular point if $d\varphi_x : T_xA \to T_{\varphi(x)}B$ has max'l rank

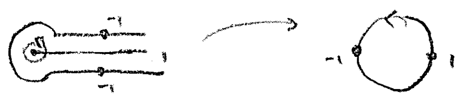
$p \in B$ is a regular value if $\varphi^{-1}(p)$ consists of regular points.

Thm $\varphi^{-1}(p) \hookrightarrow A$ is a submanifold if p is a regular value

Lemma $\exists\, \varphi : M - K \xrightarrow{\text{smooth}} S^1 \subset \mathbb{C}$ s.t. $1 \in S^1$ is a regular value so $\varphi^{-1}(1)$ is a seifert surface,

and $\varphi_* : H_1(M-K) \longrightarrow H_1(S^1)$ is an isomorphism.

generated by meridian $\leftarrow \mathbb{Z} \longrightarrow \mathbb{Z}$



$\Sigma \times (a,a) \longrightarrow S^1$

$(x, s) \longmapsto e^{\frac{\pi i}{a} s}$

$\Sigma \times \{0\} \longmapsto \{1\}$

φ is a submersion surjective

Fiber bundle: $U \subset B$, $\exists \psi$

$$\begin{tikzcd} \varphi^{-1}(U) \arrow[rr, "\cong", "\psi"'] \arrow[dr, "\varphi"'] & & U \times F \arrow[dl, "p"] \\ & U & \end{tikzcd}$$

Knots which give such φ are called fibered knots.

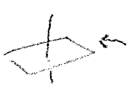
Suppose all $p \in S^1$ are regular pts. $\Sigma_p = \varphi^{-1}(p) \hookrightarrow M - K$ so $M - K = \bigsqcup_p \Sigma_p$

Ehresman's Lemma if $\varphi: A \to B$ is a surjective submersion & A is compact, then $\varphi =$ fiber bundle / locally trivial fibration.

eg 3_1 is fibered, Alexander Polynomial excludes $5_2, 6_1, \ldots$ from being fibered.

Lemma • $\varphi : M - K \to S^1$ smooth, $\varphi_* : H_1(M-K) \xrightarrow{\cong} H_1(S^1)$ $([\mu] \mapsto [c])$. Suppose $\gamma : S^1 \longrightarrow M - K$ is a smooth curve.

Then $[\gamma] \in H_1(M-K) = \mathbb{Z}$. $\varphi_*([\gamma]) \in H_1(S^1)$ $[\varphi \circ \gamma]$, $\varphi \circ \gamma : S^1 \to S^1$.



Assume $\gamma \pitchfork \Sigma$: $\frac{d\gamma}{dt} \notin T_x\Sigma$. So $\varphi \circ \gamma$ passes thru 1 w/ non-zero velocity.

so $[\varphi \circ \gamma] = \#\uparrow - \#\downarrow$. $\longleftarrow$ this is the linking # of K and γ.

Def if $A, B \subseteq M$ (oriented) s.t. $\overbrace{\dim A}^{a} + \overbrace{\dim B}^{b} = \dim M$, $A \pitchfork B$, and $A \cap B$ is finite,

if $x \in A \cap B$, $(p_1 \cdots p_a)$ $(q_1 \cdots q_b)$ $\leftarrow$ oriented, give same or opposite orientation to $(p_1 \cdots p_a q_1 \cdots q_b)$ for M.

Accordingly, $\varepsilon(A, B, x) = \pm 1$. Let $I(A,B) = \sum_{x \in A \cap B} \varepsilon(A, B, x)$. Note $I(A,B) = (-1)^{ab} I(B,A)$

Corollary if γ parameterizes curve $R \subset M - K$, $[R] = I(\Sigma, R)[c]$.