

Review Problems

something similar will be on in-class midterm.

1) $f(x) = \frac{1}{x^2 + 2x - 3}$. Graph f

3) In class

Remark: not true for closed interval

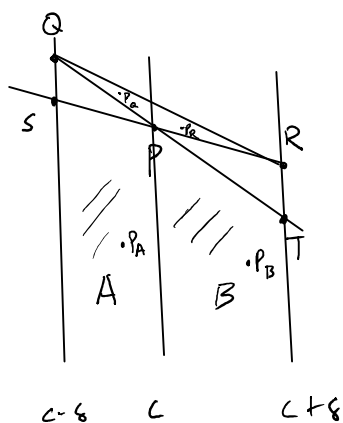
Suppose $c \in (a, b)$. Then for some $\delta > 0$, $c \pm \delta \in (a, b)$.

Let $p = f(c)$, $q = f(c - \delta)$, $r = f(c + \delta)$.

We can have $f(c - \delta) > f(c) > f(c + \delta)$

or $f(c - \delta) < f(c) < f(c + \delta)$

or $f(c) < f(c - \delta)$, $f(c) < f(c + \delta)$.



Claim: graph of f must lie inside the triangles ΔPQS and ΔPRT :

- 1) graph must lie below \overline{QR} (convexity)
- 2) " " below \overline{QP} (P_A above \overline{QP})
- 3) " " below \overline{PR} (P_B above \overline{PR})
- 4) graph cannot be in A (P would be above $\overline{R P_A}$)
- 5) cannot be in B (P would be above $\overline{Q P_B}$)

So let ψ be the function whose graph is $\overline{PQ} \cup \overline{PR}$
 " φ " " $\overline{PS} \cup \overline{PT}$

Then $\varphi(x) \leq f(x) \leq \psi(x) \quad \forall x \in [c-\delta, c+\delta]$

so by squeeze theorem, $\lim_{x \rightarrow c} f(x) = f(c)$. •

Note: for #5, $\frac{\partial y}{\partial x} = - \frac{\partial f / \partial x}{\partial f / \partial y}$

$$f(x,y) = (xy - 1)(2y^2 + x^2 - 3) = 0$$

= union of graphs of $xy = 0$

$$\text{and } 2y^2 + x^2 - 3 = 0$$

and $(1,1)$ is an intersection point.

