Lec 9/20

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Hint for 1.14 on u.d. handout.

$$\frac{1}{N}\sum_{n=1}^{N} f(x_n) \xrightarrow[N\to\infty]{} \int_0^1 f \qquad \forall f \in C[0,1].$$

Ergodic principle: time average = space average.

$\exists$ cts $f: [0,1] \to [0,1]^2$ s.t. $f([0,1]) = [0,1]^2$. Peano curve.

Brownian motion ...

Monte carlo method is basis, but u.d. sequence works as well as random.

Benford Law:

Problem Consider the sequence $1, 2, 4, 8, 1, 3, 6, 1, 2, 5, 1, 2, \ldots$

(first digit of 2^n.)

Which symbol is more frequent? 7 or 8.

answer: 7 (exercise) (Note: show these limiting frequencies exist).

Problem: Let $(x_n) \subset [0,1]$ be a dense sequence. Then $\exists$ a permutation (Exercise) of (x_n), (y_n) which is u.d. mod 1.

Important Theorem/§ in Hardy/Wright:

ch1: 1, 2, 3, 4 (a few proofs), 5, 6, 7,
↕ ↕
8 9
(know equivalences)

Note: proofs of infinitude of diff kinds of primes:

11, 12, 14, 15

Handouts: weyl criterion.

Ch 2: 10, 13, 16, 18, 19, 21, §2.8, 25, §2.10

Green-Tao: P is AP-rich

Szemerédi, Sárközy

Ch 4: know many proofs of irrationality of $\sqrt{2}$, $\sqrt{3}$, etc.

44, 45, 46, 47, 48

47: method; 48: known proof

Ch 5: ϕ, 58, What are Gauss Sums,

Ch 6: two proofs: 70, 71, 72, 74, §6.5, 80, 82, 84-88, 89 (formulation), 92, 93, 94, 95, 96, 97 (derivation), §6.12, §6.13, §6.19 (know a proof of QR)

Ch 7: Know good enough things from §7.1–7.6

Ch 8: §8.1–8.4

Ch 9: Cantor set. Normal numbers, etc. 137, §9.5, 143, 144, 146

(Exercise) x is weakly normal (base-2) if every word appears. A.E. $x \in (0,1)$ is weakly normal.

$FS(3^n) = 01011\ldots$ in base 3.

$$\frac{1}{N}\sum f(x_n) + i\,g(x_n) \longrightarrow \int (f + i\,g)$$

$$\frac{1}{N}\sum_{n=1}^{N} e^{2\pi i h x_n} \xrightarrow[n\to\infty]{} 0 \quad \forall h \in \mathbb{Z}\setminus\{0\} \quad \text{iff} \quad (x_n) \text{ u.d mod } 1$$

$e^{2\pi i h x_n}$: dense in space of fns.

Similar thing to below.

Weierstrass:

$$\overline{\mathbb{R}[x]|_{[0,1]}} = C[0,1]$$

or

$$\forall f \in C[0,1],\ \forall \varepsilon > 0,\ \exists g \in \mathbb{R}[x] \text{ s.t. } \max_{x \in [0,1]} |f(x) - g(x)| < \varepsilon$$

So (x_n) is u.d. mod 1 iff $\frac{1}{N} \sum_{n=1}^{N} x_n^p \longrightarrow \int_0^1 x^p \, dp \quad \forall p \in \mathbb{N}$.