Recall: Likelihood Ratio Test.

Consider $H_0: \theta \in \omega$, $H_1: \theta \notin \omega$, $\omega \subseteq \Omega$ ← total parameter space

LRT: reject H_0 if $\Lambda = \frac{\max L_0}{\max L} \leq k$ for some $k \in (0,1)$.

where $\max L_0 = \prod_{i=1}^{n} f(x_i; \hat{\theta})$ $\hat{\theta}$ MLE in ω

$\max L = \prod_{i=1}^{n} f(x_i; \hat{\hat{\theta}})$ $\hat{\hat{\theta}}$ MLE in Ω

Problem 12.20 $X \sim Bin(n, \theta)$ find LRT of $H_0: \theta = \frac{1}{2}$ $H_1: \theta \neq \frac{1}{2}$

Sol: $\Omega = (0,1)$, $\omega = \{\frac{1}{2}\}$ $\Rightarrow$ $\hat{\theta} = \frac{1}{2}$, $\hat{\hat{\theta}} = \frac{x}{n}$ $\Rightarrow$ $\max L_0 = \binom{n}{x}(\frac{1}{2})^n$, $\max L = \binom{n}{x}(\frac{x}{n})^x(1-\frac{x}{n})^{n-x}$

So $\Lambda = \frac{\max L_0}{\max L} = \frac{(\frac{1}{2})^n}{(\frac{x}{n})^x(1-\frac{x}{n})^{n-x}} \leq k$

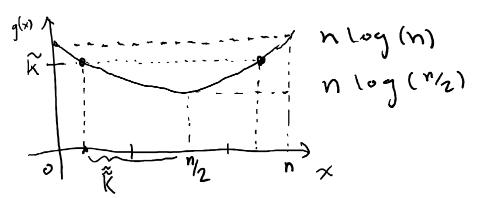
$$\Rightarrow -n\log(2) - x\log(x) + x\log(n) - (n-x)\log(1-\tfrac{x}{n}) \leq \log(k)$$

$$\Rightarrow x\log(x) - x\log(n) + (n-x)\log(1-\tfrac{x}{n}) \geq -\log(k) - n\log(2)$$

$$\Rightarrow x(\log(x) - \log(n)) + (n-x)(\log(n-x) - \log(n)) \geq -\log(k) - n\log(2)$$

$$\Rightarrow x\log(x) + (n-x)\log(n-x) \geq \underbrace{n\log(n) - \log(k) - n\log(2)}_{\tilde{k}}$$

Consider $g(x) = x\log(x) + (n-x)\log(n-x)$. define $0\log(0) = 0$



want $g(x) \geq \tilde{k}$

So $g(x) \geq \tilde{k} \iff |x - \frac{n}{2}| \geq \tilde{\tilde{k}}$

Hence reject H_0 if $|X - \frac{n}{2}| \geq \tilde{\tilde{K}}$ where $\tilde{\tilde{K}}$ is determined by α. (type I error rate)

Ch 13 testing Hypotheses for Mean, Variance, Proportion.

Terminology example:

Consider $X_1, \dots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$, σ^2 is known.

want to test $H_0: \mu = \mu_0$ vs $H_1: \mu \neq \mu_0$.

we showed that

LRT : Reject H_0 if $|\bar{X} - \mu_0| \geq z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}$

or equivalently

$$\text{Reject } H_0 \text{ if } |Z| = \left| \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \right| \geq z_{\frac{\alpha}{2}}$$

Intuition: Reject H_0 if the distance between $\bar{X}$ and μ_0 is too large.

H_1 is a two-sided alternative & leads to a two-sided test. (two-tailed).

We could also formulate a one-sided alternative $H_0: \mu = \mu_0$ and $H_1: \mu > \mu_0$.

Then our test is: Reject H_0 if $\frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \geq z_\alpha$ Note that.

$\Leftrightarrow$ $\bar{X} \geq \mu_0 + z_\alpha \frac{\sigma}{\sqrt{n}}$. this is a "one-sided" or "one-tailed" test.

<u>Example</u>: Consider RS of 100 bags of dog food, we find average weight is 0.955 lbs. suppose we know $\sigma = 0.17$. want to test $H_0: \mu = 1$ lb vs $H_1: \mu \neq 1$ lb, w/ $\alpha = 0.05$.

our test: reject H_0 if $|Z| = \left| \frac{\bar{X} - 1}{0.17/\sqrt{100}} \right| \geq z_{\frac{\alpha}{2}} = 1.96$

$\bar{X} = 0.955$. $|Z| = \left| \frac{0.955 - 1}{1.7} \right| = |-2.65| = 2.65 \geq 1.96$ so reject H_0.

☆ the smallest α at which we still reject H_0 is the p-value.

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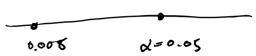
Another way to look at test is to compute p-value:

Idea: is a z-value of -2.65 unusual when H_0 is true?

$$\text{P-value} = \mathbb{P}(Z \le -2.65 \text{ or } Z \ge 2.65) = 0.004 + 0.004 = 0.008.$$



So $\underset{0.008}{\overset{\alpha}{\vee}}$ is large enough to reject H_0. <u>P-hacking</u>.

p-value = probability of observing something as extreme or more extreme than what we observed, assuming that H_0 is true.

("extreme" is something in the direction of H_1)