Lec 3/6

Monday, March 6, 2017 15:02

four steps to test a hypothesis.

1: formulate H_0, H_1.

2: (method 1): use samp. dist to determine CR of size α.

(method 2): specify test statistic.

⋮

Ex: RS $n = 20$, $\bar{x} = 4.05$, $s = 0.2$. evidence that $\mu > 4$ at level $\alpha = 0.5$?

Sol. 1: $H_0: \mu = 4$. $H_1: \mu > 4$, $\alpha = 0.05$.

2: test stat $t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$

(1)

(2) t distributed.

CR: $t \geq t_{\alpha, n-1} = t_{0.05, 19} = 1.729$

3: (1) $t = \frac{4.05 - 4}{0.2/\sqrt{20}} = 1.118$

(2) P-value $= 0.13875$

$\hookrightarrow = P(t_{19} > 1.118)$

4: (1) $t \notin CR$ so not evidence to reject H_0.

(2) p-value > 0.05 so " " "

§13.3 tests w/ Diffs of means

Pop. 1: μ_1, σ_1. Pop 2: μ_2, σ_2. assume both Normally distributed.

take independent samples of size n_1, n_2 from 2 populations.

Want to test hypothesis about relation btwn μ_1 & μ_2.

typically consider $H_0: \mu_1 = \mu_2$. $H_1: \mu_1 \overset{\neq}{\underset{>}{<}} \mu_2$.

or more general: $H_0: \mu_1 - \mu_2 = \delta$ $H_1: \mu_1 - \mu_2 \overset{\neq}{\underset{<}{>}} \delta$.

We can show that LRT in this setting is based on the statistic $Z = \dfrac{\bar{X}_1 - \bar{X}_2 - \delta}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$

and the test rejects H_0 in favor of $H_1: \mu_1 - \mu_2 \overset{\neq}{\underset{<}{>}} \delta$ (2-sided, 1-sided, 1-sided)

if $|Z| \geq Z_{\frac{\alpha}{2}}$ or $Z \geq Z_\alpha$ or $Z \leq -Z_\alpha$ depending on H_1.

Q what if σ_1, σ_2 not known or populations not normal.

if n_1, n_2 large then can use $\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$ so

$$Z \quad \frac{\bar{X}_1 - \bar{X}_2 - \delta}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \overset{\text{Approx}}{\sim} N(0,1) \quad \text{still.}$$

Ex: Data: $\alpha = 0.05$.

group	n	$\bar{x}$	s
1 treatment	31	51.48	11.01
2 control	33	41.52	17.15

suff. to state treatment is improving?

1: $H_0: \mu_1 = \mu_2$. $H_1: \mu_1 > \mu_2$

2: statistic $Z = \dfrac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \overset{\text{Approx}}{\sim} N(0,1)$

CR: $Z \geq Z_\alpha = 1.68$

3: $Z = 2.781$

$P = 0.0027$

4: $Z \in CR$, $P > \alpha$ so this is sufficient to conclude treatment has effect.

If n_1, n_2 are small & σ_1^2, σ_2^2 not known.

- If we can assume $\sigma_1 = \sigma_2 = \sigma$, and pops are normal, then use t-test. (see sec 11.3, LRT)

  Stat is $t = \dfrac{\bar{X}_1 - \bar{X}_2 - \delta}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$ where $S_p^2 = \dfrac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}$

  Under H_0, $t \sim t_{\underbrace{n_1+n_2-2}_{d.f.}}$

  so we reject H_0 if of $H_1: \mu_1 - \mu_2 \begin{matrix} \neq \\ < \\ > \end{matrix} \delta$ $\begin{matrix} \text{2-sided} \\ \text{1-sided} \\ \text{1-sided} \end{matrix}$ $\begin{matrix} \text{if } |t| > t_{\alpha/2, n_1+n_2-2} \\ \text{if } t > t_{\alpha, n_1+n_2-2} \\ \text{if } t < -t_{\alpha, n_1+n_2-2} \end{matrix}$

Q: What if we can't assume $\sigma_1 = \sigma_2$? No simple formula.

Methods are available, see references @ end of textbook section.

Q: What about independence assumption?

Another common design involves "paired" data.