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$x^4 - 2 \in \mathbb{Z}_3[x]$

$\parallel$

$x^4 + 1 = (x^2 - x - 1)(x^2 + x - 1)$

$G \cong \mathbb{Z}_2$. $\mathbb{F}_{3^2}$ – the only quadratic extn of $\mathbb{F}_3$.

So this is spl. field of f.

<u>Note</u>: all galois groups of finite fields are cyclic.

5) have a comp series which leads to a tower:

$$\begin{array}{c} K \\ \| p \\ F_{r-1} \\ \| p \\ \vdots \\ \shortparallel \end{array}$$

$\forall i, \quad F_i = F_{i-1}(\sqrt[p]{a_i})$

$$\begin{array}{c} \vdots \\ \| \\ F_1 \\ \| \\ F_0 = F \end{array}$$

So we get r nested radicals.

<u>Theorem</u>: Cyclic extn $\Longleftrightarrow$ radical extn (modulo including roots of unity).

14) $F \subseteq K \subseteq E$, $G = Gal(E/F)$, K/F is normal iff $H = Gal(K/F) \trianglelefteq G$.

$$\forall \varphi \in G, \quad Gal(E/\varphi(K)) = \varphi H \varphi^{-1},$$

so $\varphi(K) = K$ iff $\varphi H \varphi^{-1} = H$.

7) $f(x) = x^4 + x^3 + x^2 + x + 1 \in \mathbb{F}_2[x]$.

Galois gp. is cyclic

f is irreducible over $\mathbb{F}_2$ of deg 4, so $\mathrm{Gal} = \mathbb{Z}_4$.

12) powers of 2.

8)
$$\begin{tikzcd}
 & \bar{E} = \bar{F} \\
L \arrow[ur, "nm"] \arrow[d, dash, "m", "\text{sep}"'] & \\
K \arrow[uur, "n \text{ embeddings}"'] \arrow[d, dash, "n", "\text{sep}"'] & \\
F &
\end{tikzcd}$$

(b is true)

$$\begin{tikzcd}
 & \mathbb{Q}(\sqrt[4]{2}) \arrow[d, dash] \\
m_{\sqrt[4]{2}, \mathbb{Q}(\sqrt{2})}(x) = (x^2 - \sqrt{2}) \arrow[r] & {} \\
 & \mathbb{Q}(\sqrt{2}) \arrow[d, dash, "2"] \\
 & \mathbb{Q}
\end{tikzcd}$$

but $\mathbb{Q}(\sqrt[4]{2})/\mathbb{Q}$ is <u>not</u> normal

(doesn't contain $\pm i\sqrt[4]{2}$)