Lec 9/19

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$T = (D - I)^2 : P_n(\mathbb{R}) \to P_n(\mathbb{R})$

matrix of T wrt basis $\{1, x, \dots, x^n\}$ is.

$$A = \begin{pmatrix} 1 & -2 & 2 & 0 \\ 0 & 1 & -4 & 6 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$D(A) = 1$ so A (and thus T) is invertible.

$\mathrm{Rank}(A) = 4$

$L(P_3(\mathbb{R}), P_3(\mathbb{R})) \cong M_4(\mathbb{R})$

Can find inverse by row-reducing A (to I) & performing same operations on I.

$$A^{-1} = \begin{pmatrix} 1 & 2 & 6 & 24 \\ 0 & 1 & 4 & 18 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$T^{-1}(\lambda_0 + \lambda_1 x + \lambda_2 x^2 + \lambda_3 x^3)$

$= \lambda_0 + 2\lambda_1 + \lambda_1 x + 6\lambda_2 + 4\lambda_2 x + \lambda_2 x^2 + 24\lambda_3 + 18\lambda_3 x + 6\lambda_3 x^2 + \lambda_3 x^3$

Solve $T(f) = x - 1$, $f = T^{-1}(x-1)$

$f = -1 + 2 + x = x + 1$