Lec 8/23

Tuesday, August 23, 2016 9:02 AM

Syllabus

Zbigniew (Zig) Fiedorowicz

MW 504 : 292-0724

Mentors: Michael Crawford, Nik Henderson

mentoring sessions: wed 6-8 ? Sun 3-5 ?

Grades: HW 30%

2 Mid 20% each

Final 30%

Schedule (tentative)

Week 1: § 1 of Spivak

2: § 2

3: § 3-4

4: § 5-6

5 (Sept 19-23): midterm, § 7

6: § 8, 27

7: § 9, 10

....

---

§ 1: What are the real numbers $\mathbb{R}$

$0.99\ldots \stackrel{?}{=} 1.00\ldots$

Newton, Leibniz: does $\mathbb{R}$ contain infinitesimals

infinitesimal: number > 0 greater than $\frac{1}{n}$ $\forall n \in \mathbb{Z}_+$

- no, $\mathbb{R}$ does not contain such numbers

Properties of real numbers (axioms) (modern approach)

9 Field axioms: addition, multiplication, subtraction, division

3 Order axioms: ordering, positive, negative → rationals

1 Completion axiom: distinguishes between $\mathbb{Q}$ and $\mathbb{R}$

$\mathbb{R}$ has 2 binary operations, $\underbrace{\mathbb{R}\times\mathbb{R} \xrightarrow{+} \mathbb{R} \quad (a,b) \mapsto a+b}_{\text{addition}}$ and $\underbrace{\mathbb{R}\times\mathbb{R} \xrightarrow{\cdot} \mathbb{R} \quad (a,b) \mapsto ab}_{\text{multiplication}}$

Field axioms:

P1 – associativity of addition: $\forall a,b,c \in \mathbb{R}$, $a + (b + c) = (a+b) + c$

P2 – additive identity: $\exists 0 \in \mathbb{R}$ st. $\forall a \in \mathbb{R}$, $a + 0 = 0 + a = a$

P3 – additive inverses: $\forall a \in \mathbb{R}, \exists (-a) \in \mathbb{R}$ st $a + (-a) = (-a) + a = 0$

P4 – commutativity of addition: $\forall a, b \in \mathbb{R}$ $a + b = b + a$

P5 – associativity of multiplication: $\forall a, b, c \in \mathbb{R}$, $a(bc) = (ab)c$

P6 – multiplicative identity: $\exists 1 \in \mathbb{R}$ st $\forall a \in \mathbb{R}$ $a \cdot 1 = 1 \cdot a = a$ $(1 \neq 0)$

P7 – multiplicative inverse: $\forall a \in \mathbb{R}\setminus\{0\}$, $\exists a^{-1}$ st $aa^{-1} = a^{-1}a = 1$

P8 – commutativity of multiplication: $\forall a, b \in \mathbb{R}$, $ab = ba$

P9 – distributive property: $\forall a, b, c \in \mathbb{R}$, $a(b+c) = ab + ac$

Definitions: $a - b \equiv a + (-b)$

$a/b \equiv ab^{-1}$

Proposition: $0 \cdot a = 0 \quad \forall a \in \mathbb{R}$

Proof:	Justification
$a(1+0) = a \cdot 1$	P2
$a \cdot 1 + a \cdot 0 = a \cdot 1$	P9
$a + a \cdot 0 = a$	P6
$(-a) + (a + a \cdot 0) = (-a) + a$	P3
$((-a) + a) + a \cdot 0 = (-a + a)$	P1
$0 + a \cdot 0 = 0$	P3
$a \cdot 0 = 0$	P2
$0 \cdot a = 0$	P8

mistake in Spivak: page 7

Assignment: Find the mistake in the proof that $a - b = b - a \Rightarrow b = a$

Work:

$$a - b = b - a$$

$$(a-b) + b = (b - a) + b = b + (b - a) \qquad \text{P4}$$

$$a + ((-b) + b) = (b + b) + (-a) \qquad \text{P1, P3}$$

$a + 0 = (b+b) + (-a)$ P3

$a + a = (b+b) + (-a) + a$ P2

$a + a = (b+b) + ((-a) + a)$ P1

$a + a = b + b$ P3, P2

$1 \cdot a + 1 \cdot a = 1 \cdot b + 1 \cdot b$ P6

$a(1+1) = b \cdot (1+1)$ P9

maybe that $1+1 \neq 0$ is not apparent. $\longrightarrow$ $a \cdot (1+1) \cdot (1+1)^{-1} = b \cdot (1+1) \cdot (1+1)^{-1}$ P7

$a \cdot 1 = b \cdot 1$ P7

$a = b$ P6

This is where we need ordering.