Rec 11/5

Tuesday, November 5, 2019 10:23

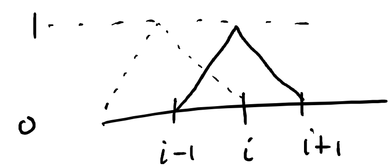
#58: X infinite-dim'l, normed

(1) find a seq (x_n) s.t. $\|x_n\| = 1 \ \forall n$, $\|x_n - x_m\| \geq \frac{1}{2} \ \forall m \neq n$

(2) Deduce X is not locally cpt.

Example: for $x \in C_0(\mathbb{R})$, $\|x\| := \sup_t |x(t)|$.

x_i = tent fn at integer i:

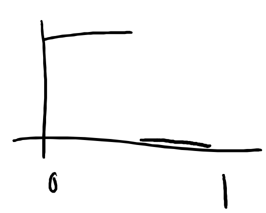


$\|x_i - x_{i-1}\| = 1$

Ex2

$[0,1]$

$$f_x(t) = \begin{cases} 1 & 0 \leq t \leq x \\ 0 & t > x \end{cases}$$



$\|f\| := \sup_t |f(t)|$

$\|f_x - f_y\| = 1$ if $x \neq y$.

General:

1. Take any $x_1 \neq 0$, $\|x_1\| = 1$

$M_1 = \mathbb{C} x_1$

$\exists f : X \longrightarrow \mathbb{C}$ s.t. $f(x_1) = 1$ and $\|f\| = 1$

i.e. $f(x_1) = \|x_1\|$.

M_1 is a closed subspace of X;

$\exists \varphi \in X^*$ w/ $\varphi(x) = \inf_{y \in M_1} \|x - y\|$ $\forall x \in X \setminus M_1$ and $\|\varphi\| = 1$.

[Cor 2 of HB]

Since $\|\varphi\| = 1$, $\exists$ $x_2 \in X \setminus M_1$ s.t. $\|x_2\| = 1$ & $\|\varphi(x_2)\| > \frac{1}{2}$

not sure about all this

$\exists \varphi \in X \setminus M$ s.t. $\varphi_x(x) = \inf_{m \in M} \|x - m\|$, $\|\varphi_x\| = 1$

Claim: $\exists y$ w/ $\dfrac{\|\varphi_x(y)\|}{\|y\|} \geq \frac{1}{2}$.

. . .

$\inf \|x - y\| = d$

$y \in Y$

$\exists\, y_0 \in Y,\ \underbrace{\|x - y_0\|}_{z'} < 2d$

$\|z'\| < 2d$

$\forall\, y \in Y,\ \|z' - y\| \geq d,\quad$ let $z = \dfrac{z'}{\|z'\|}$.

$\|z - y\| \geq \frac{1}{2}$

(2) Locally cpt $\rightsquigarrow$ $\forall x \in X$, $\exists$ cpt nhd of x.

suff to show $\nexists$ cpt nhd of 0.

pf $\forall$ nhd of 0, pick $\delta > 0$ s.t. $\overline{B_\delta(0)} \subset$ nhd.

$\rightsquigarrow$ $\{\delta x_i\} \in$ nhd, but $\{\delta x_i\}$ has no cvgt subseq.

<u>#59</u> Sequence spaces C^∞

ℓ^1 norm $\longrightarrow$ $\ell' = \{(x_n) \mid \sum |x_n| < \infty\}$

$\ell' \subset c_0 \subset c \subset \ell^\infty$

sup $\begin{cases} c_0 = \{(x_n) \mid x_n \to 0\} \\ c = \{(x_n) \mid \ldots \end{cases}$

$\|(x_n)\|_1 = \sum |x_n|$

$$\begin{array}{l} \text{sup} \\ \text{norm} \\ (\infty) \end{array} \begin{cases} c_0 = \{(x_n) \mid x_n \to 0\} \\ c = \{(x_n) \mid \lim x_n \text{ exists}\} \\ \ell^\infty = \{(x_n) \mid \sup|x_n| < \infty\} \end{cases}$$

$$\|(x_n)\|_{\ell_1} = \sum |x_n|$$

$$\|(x_n)\|_\infty = \sup |x_n|$$

Show each one is a Banach Space.