Lec 8/27

Monday, August 27, 2018 11:30

G group

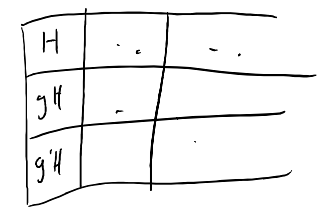
IV

H subgroup

To define: set (left) Coset, denoted by G/H

Break G into disjoint union of subsets:

$\Updownarrow$ find an equivalence reln on G.

G: 

Define a reln $x \sim y$ on G by $x \underset{L}{\sim} y$ if $x^{-1}y \in H$ (L for left)

reflexive: $x \underset{L}{\sim} x$ since $x^{-1}x = 1 \in H$

Symmetric: $x \underset{L}{\sim} y \iff x^{-1}y \in H \underbrace{\iff}_{H \text{ a subgroup}} y^{-1}x \in H \iff y \underset{L}{\sim} x$

transitive: $(x^{-1}y)(y^{-1}z) = x^{-1}z$.

G = disjoint union of eq. classes. $C \subset G$ is an eq. class if it is maximal s.t. $x, y \in C \Rightarrow x \sim y$

Lemma: If $C \subset G$ is an eq. class then $\forall x \in C$ $\begin{array}{ccc} H & \longrightarrow & C \\ \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \\ h & \longmapsto & xh \end{array}$ is a bijection

G/H is the set of eq classes w.r.t. the reln $\underset{L}{\sim}$

$$G = \bigsqcup_{i \in I} C_i$$

Pick $g_i \in C_i \ \forall i \in I$. Then $G = \bigsqcup_{i \in I} g_i H$

eg: $\mathbb{Z}/6\mathbb{Z}$

eg $S_5 \leq S_6$ (elements in S_5 just don't move 6)

$\|$

[illegible]

$\sigma \in S_6 : \sigma(6) = 6 \}$

$$\pi_1 \sim \pi_2 \iff \pi_1^{-1}\pi_2 \in S_5 \iff (\pi_1^{-1}\pi_2)(6) = 6 \iff \pi_1(6) = \pi_2(6)$$

$$S_6/S_5 \longleftrightarrow \{1,2,3,4,5,6\}$$

$$\pi \longrightarrow \pi(6)$$

Consequences: if G is finite, $|G/H| = |G|/|H|$.

eg: How many subgroups are there in a group of prime order? 2: $\{e\}$ & G.

eg: $\forall x \in G$, $\operatorname{ord}(x) \mid |G|$ ($\nexists$ an elt. of order 7 in S_5).

Ex: Let $a, b \in G$ s.t. $ab = ba$. Then $\underbrace{\operatorname{ord}(ab) \mid \operatorname{lcm}(\operatorname{ord}(a), \operatorname{ord}(b))}$

since $(ab)^l = a^l b^l = e$. where $l = \operatorname{lcm}(\operatorname{ord}(a), \operatorname{ord}(b))$ $=$ if $(\operatorname{ord}(a), \operatorname{ord}(b)) = 1$.