Lec 10/23

Monday, October 23, 2017 14:14

Problem Bovinum Archimedes (google)

D fixed, nonsquare.

1. using easy enough diophantine knowledge, prove there is $M \in \mathbb{N}$ s.t. for infinitely many x, y, $x^2 - Dy^2 = C_{x,y} \leq M$, $C_{x,y} \geq 0$.

2. using pigeonhole principle, $\exists\, c \leq M$ s.t. $x^2 - Dy^2 = c \leq M$ for infinitely many x, y.

3. (Algebra) Actually one can show (2) works for $c = 1$.

4. from any soln x, y one can get as many more by using

$$\begin{vmatrix} x & Dy \\ y & x \end{vmatrix} = 1$$

5. $\exists$ "minimal" soln s.t. all solns can be generated by (4).

(this stuff could be on midterm, organize a proof as above that there are inf. many solns to $x^2 - Dy^2 = 1$)

What about $x^3 - Dy^3 = 1$? Hopeless,

$x^3 + y^3 + z^3 - Dxyz = 1$? Maybe

$$\mathbb{Z}[\sqrt[3]{D}] = \{x + y\sqrt[3]{D} + z\sqrt[3]{D^2} : x, y, z \in \mathbb{Z}\}.$$

closed Domain

$$\begin{vmatrix} x & Dz & Dy \\ y & x & Dz \\ z & y & x \end{vmatrix} = x^3 + Dy^3 + D^2z^3 - 3Dxyz = 1$$

$$x^3 + 2y^3 + 4z^3 - 6xyz = 1$$

$x = y = z = 1$ is soln, have ∞ many by Det trick

Suppose D is a cube. can you factor it?

$$x^3 + y^3 + z^3 - Dxyz = \underbrace{(x + a_1 y + a_2 z)(x + b_1 y + b_2 z)(x + c_1 y + c_2 z)}_{\text{ex: find this factorization (all } a_i, b_i, c_i)} = 1$$

Can get a 4-d version via:

Exercise Give matrix representation of $\mathbb{Z}[\sqrt{2}, \sqrt{3}]$ and $\mathbb{Z}[\sqrt[4]{2}]$

Read: first 7 pgs of ch 15 for friday (skip 15.5)

Read 15.6 onwards. (or just read all of 15)