Lec 4/9

Monday, April 9, 2018 12:40

$$K' = \{e : \Phi_e^K(e)\downarrow\}$$

$$\Phi_e^K(e)\downarrow \iff \exists \sigma\ (\sigma \subset \chi_K \text{ and } \Phi_e^\sigma(e)\downarrow)$$

$$\exists \sigma\ (\exists s \sim \wedge \forall s' \sim \wedge \exists s''\ \Phi_{e,s}^\sigma(e)\downarrow)$$

$$\exists \exists \exists \forall$$

$\therefore K' \in \Sigma_2$

$$\text{INCOMP} = \{\langle a,b\rangle : W_a \text{ and } W_b \text{ are incomparable}\}$$

$$= \{\langle a,b\rangle : \forall e,\ \Phi_e^A \neq \chi_B \text{ and } \Phi_e^B \neq \chi_A\}$$

$$= \{\langle a,b\rangle : \forall e, \exists \langle x_1, x_2\rangle,\ \Phi_e^A(x_1) \neq \chi_B(x_1) \text{ and } \Phi_e^B(x_2) \neq \chi_A(x_2)\}$$

$$= \{\langle a,b\rangle : \forall e, \exists \langle x_1, x_2\rangle, \forall s, \exists \ldots$$

at most in Π_4.

<u>Trade-off Lemma</u> $\forall A$ and n, $\Sigma_{n+1}^A = \Sigma_n^{A'}$

<u>DFN</u> For each $n \geq 0$, let

$$\text{poly}\,\Sigma_n = \{\{x : \exists y_1 \forall y_2 \cdots Q_n y_n\ P(x, y_1, \ldots, y_n)\} : P \text{ is computable in poly-time wrt } |x|\}.$$

$$\text{poly}\,\Pi_n = \{\bar{A} : A \in \text{poly}\,\Sigma_n\}$$

$\text{poly}\,\Sigma_0 = P, \quad \text{poly}\,\Sigma_1 = NP.$

Clique $\in$ poly Σ_1

MAX-CLIQUE = {<G, k> : the largest clique in G has size k}

= {<G, k> : $\exists$ guess^0 $\forall$ ~~guesses~~^{1} > ~~guess~~^{0}, 1 is not a clique, 0 is, and $|0| = k$}.

$\in$ poly Σ_2

CRAZY-CLIQUE = {<G, k> : each $V' \subseteq V$ s.t. $|V'| = \frac{|V|}{2}$ induces a subgraph in G whose largest clique has size k}

final exam

$\in$ poly Π_3

<u>Theorem1</u>: if $\text{poly}\,\Sigma_n = \text{poly}\,\Pi_n$ then $\text{poly}\,\Sigma_n = \text{poly}\,\Sigma_{n+1} = \cdots$

<u>Theorem2</u>: $\forall n \geq 1$, $A \in \text{poly}\,\Sigma_n \implies A$ can be computed in poly-space

[illegible]

<u>Theorem3</u>: $\forall n \geq 1$, $\exists$ a poly Σ_n-complete language & a poly Π_n-complete language (under $\leq_p$ reduction).

<u>Sacks Splitting Theorem</u>: Let A be r.e. but not recursive. Then A can be partitioned into incomparable r.e. sets B & C.