Lec 11/25

Monday, November 25, 2019 11:31

Symplectic Geometry

$\dim V$ even, B nondegen alternate form

let $Sp(V, B) = \{\eta : V \xrightarrow{\sim} V$ s.t. $B(\eta x, \eta y) = B(x,y)\ \forall\, x, y\}$, group structure

called the Symplectic Group. In fact this doesn't depend on B, only the dimension of V (and the field F).

So denote it $Sp_n(F)$, where $n = \dim V$ is even.

Symplectic transformations are those that take symplectic bases to symplectic bases

(symplectic base $\{u_1, v_1, \ldots, u_r, v_r\}$ satisfies $B(u_i, v_j) = \delta_{ij} = -B(v_i, u_j)$
$B(u,u) = 0 = B(v,v)$.)

Exterior Algebras I

Def An associative algebra over F is a pair consisting of a ring $(A, +, \cdot, 0, 1)$ and a vector space A over F s.t. the underlying set A and addition & 0 coincide, and

$$a(xy) = (ax)y = x(ay)$$

$\forall\, a \in F$, $x, y \in A$. If A is f-d v.s. over F then we say the algebra A is f-d.

Suppose $a \in F$, $1 \in A$, so $a1 \in A$. by $(*)$, $(a1)x = 1(ax) = ax$.

So A contains a copy of F.

Conversely, any ring containing F in the center is an F-algebra.