

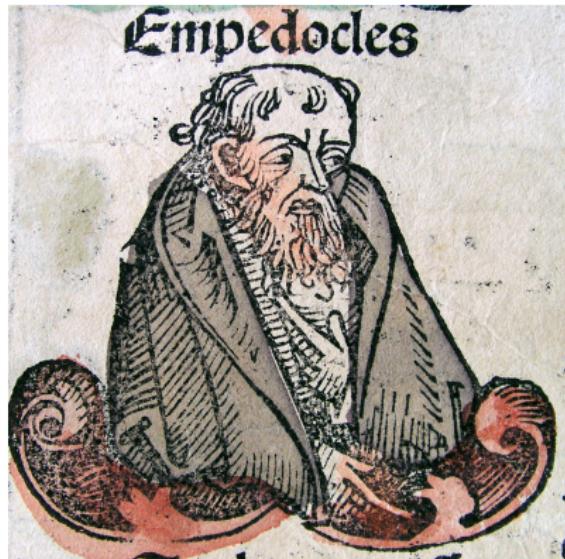
The Mathematics of Light and Vision

Vilas Winstein

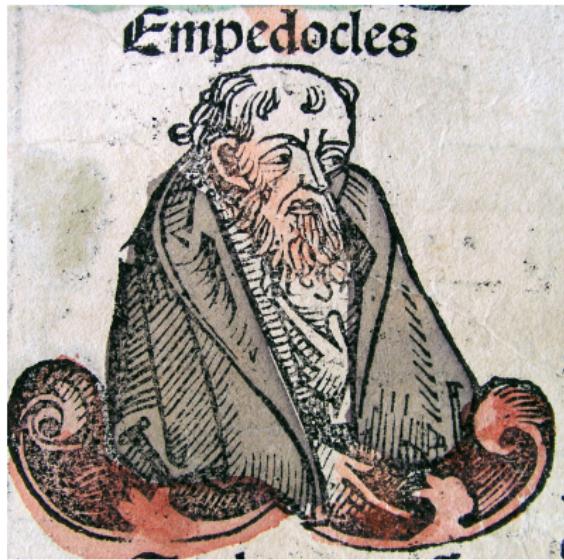
January 25, 2022

Emission Theory of Vision

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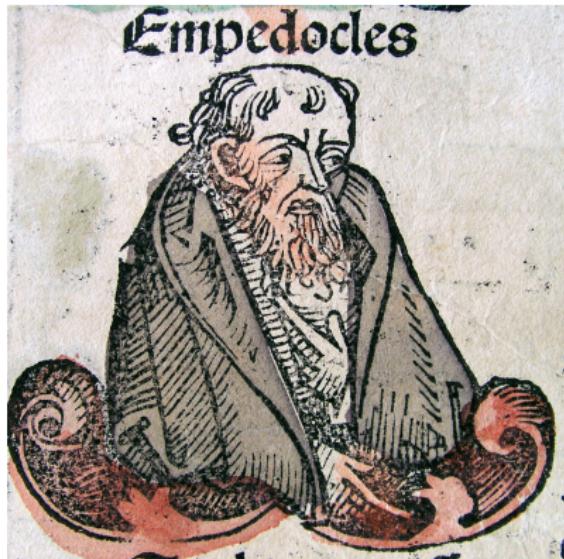


Emission Theory of Vision



494-434 BC (pre-Socratic).

Emission Theory of Vision



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Credited with originating the theory
of the *four elements*

Emission Theory of Vision



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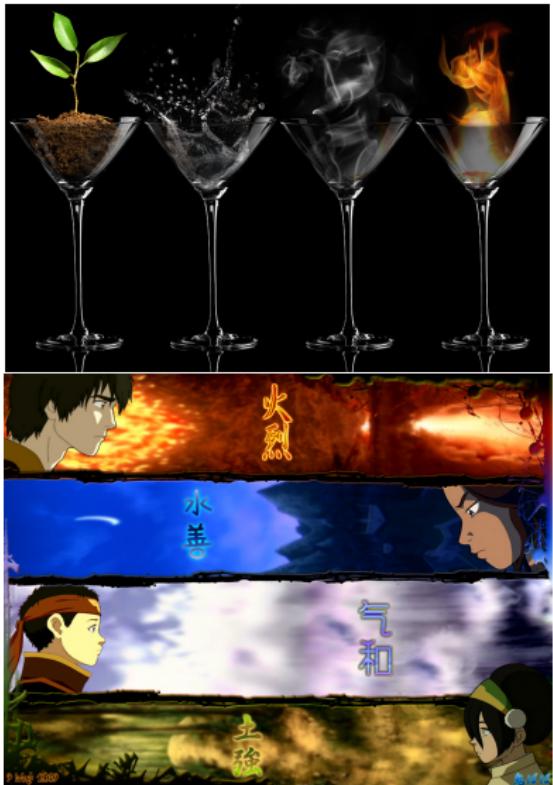
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Emission Theory of Vision



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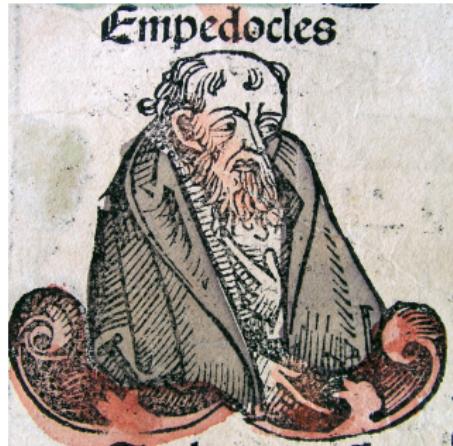
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Emission Theory of Vision



Emission Theory of Vision



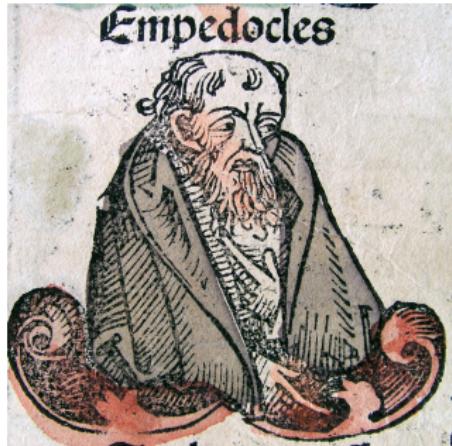
“Aphrodite made human eyes out of
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Emission Theory of Vision



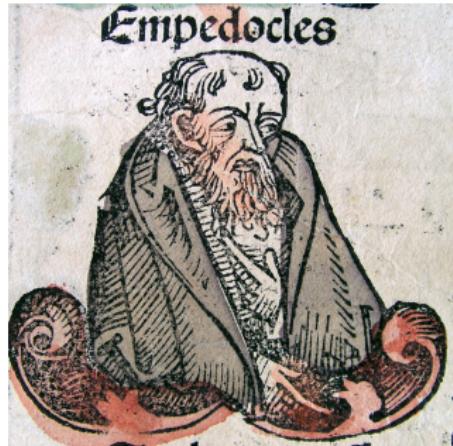
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Emission Theory of Vision



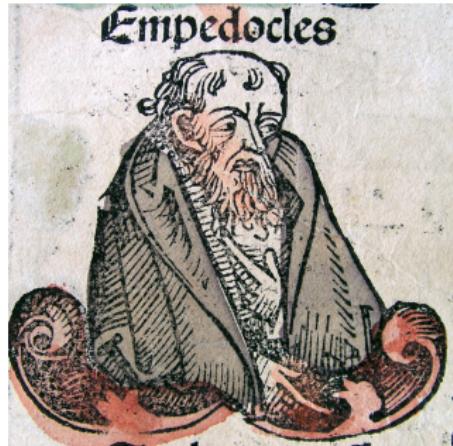
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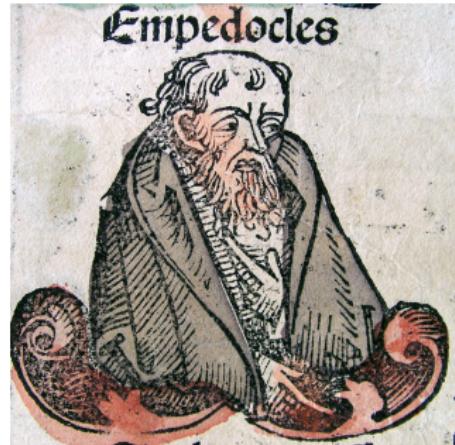
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Emission Theory of Vision (Evidence)

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Emission Theory of Vision (Evidence)



Emission Theory of Vision (Evidence)

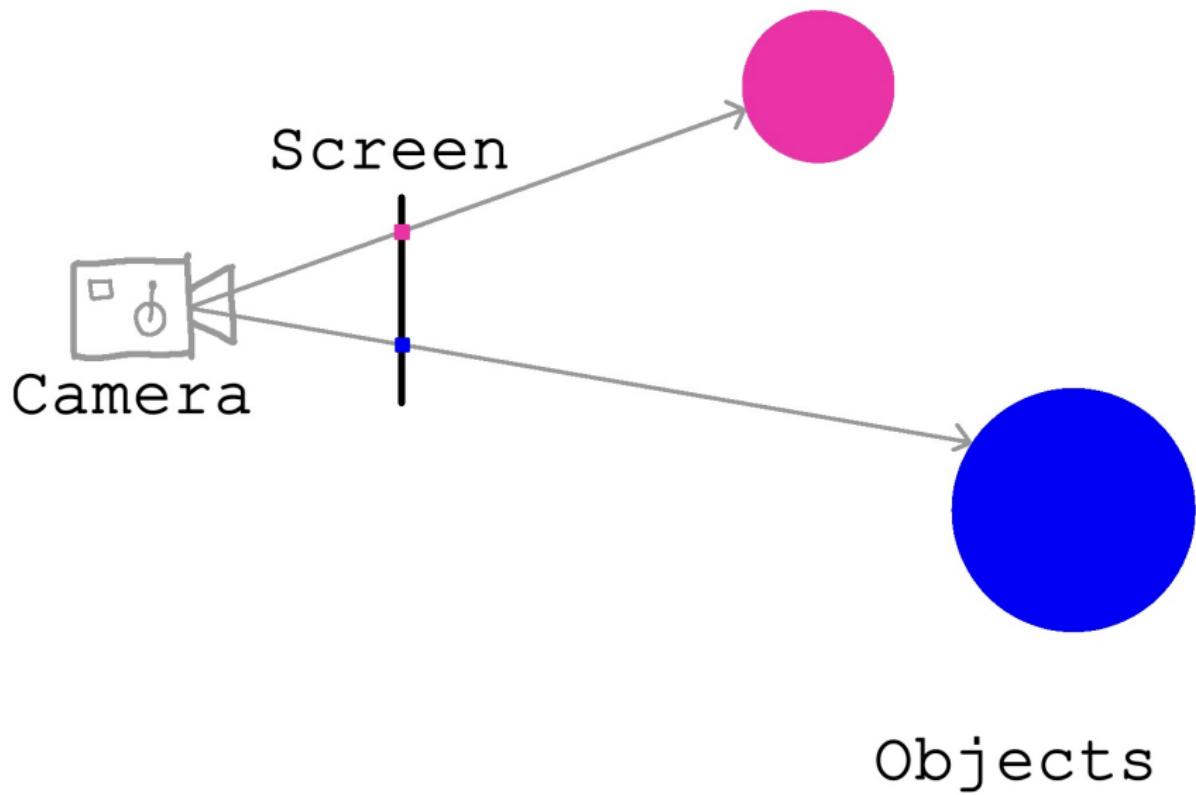


Emission Theory of Vision (Evidence)

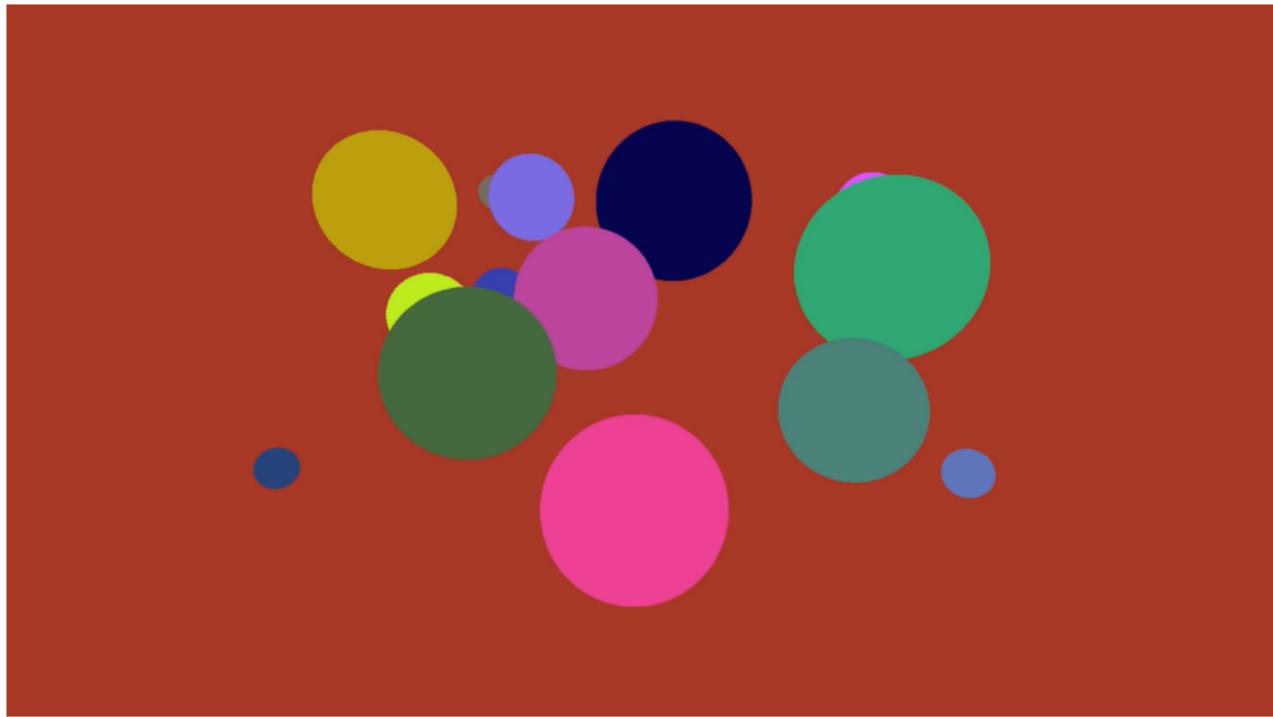


Computer Graphics

Computer Graphics



Computer Graphics



Reflection Theory of Vision

Reflection Theory of Vision

Ibn al-Haytham



Reflection Theory of Vision

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965 - 1040 AD (Islamic Golden Age)

Reflection Theory of Vision

Ibn al-Haytham



“A hypothesis must be supported by experiments and/or mathematics.”

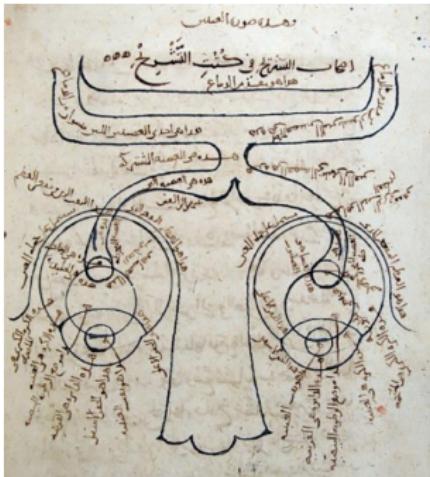
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Reflection Theory of Vision

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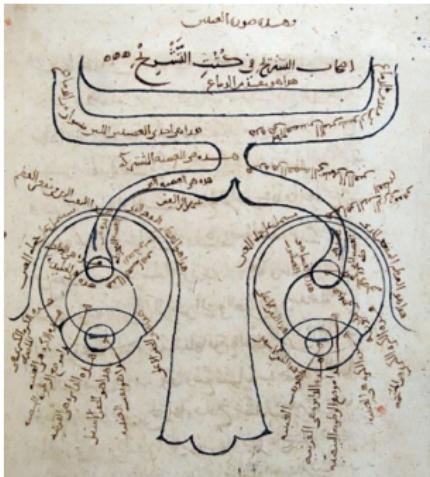
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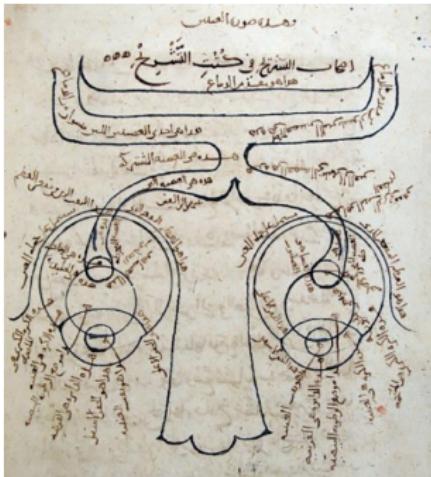
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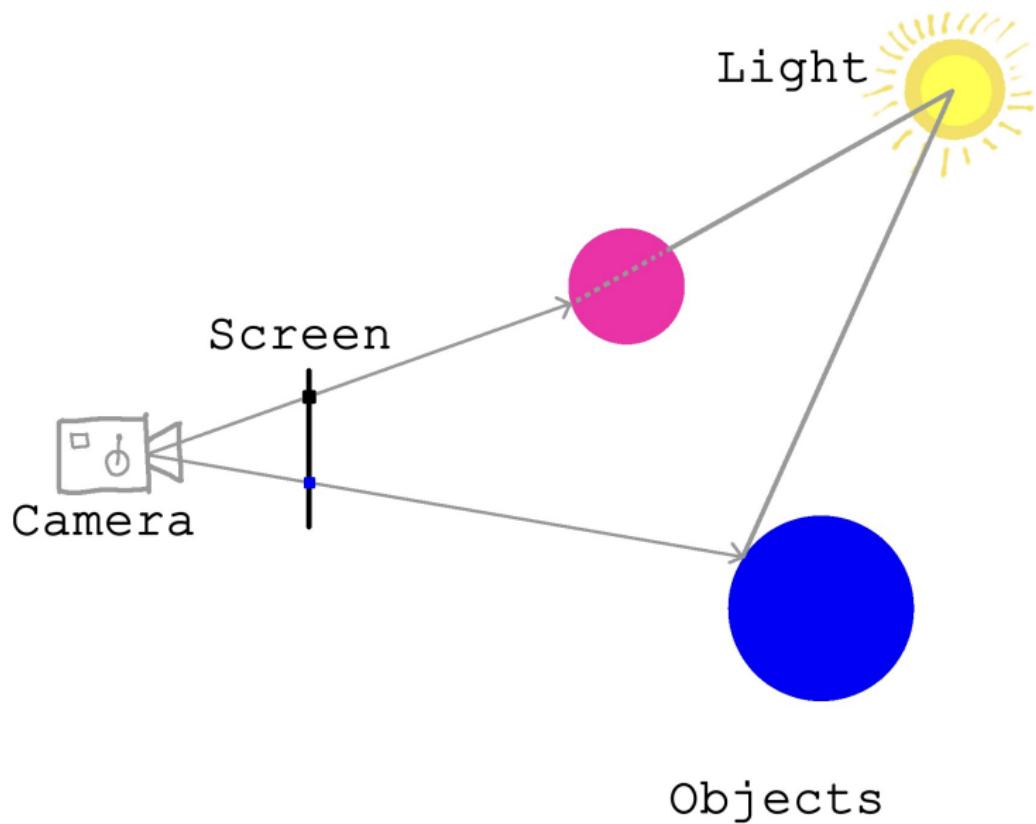
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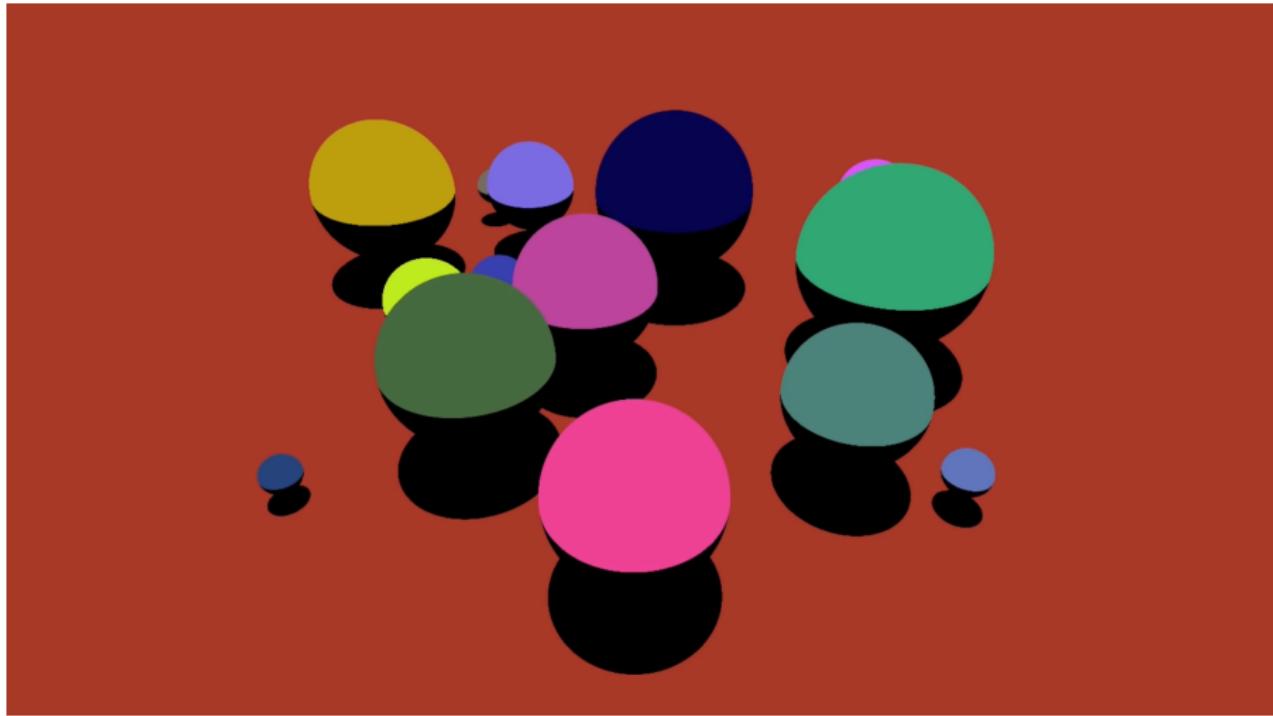
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"The brain processes visual signals from the eyes; this is how we see."

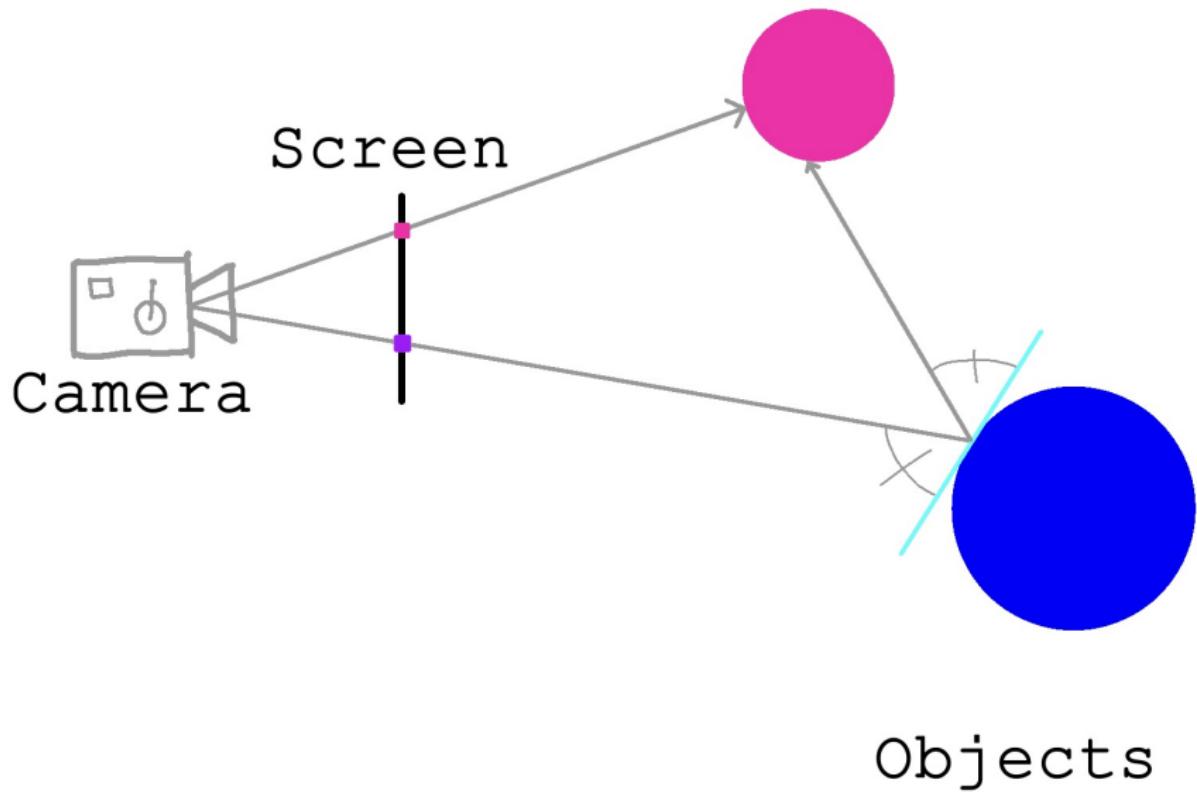
Computer Graphics (Shadows)



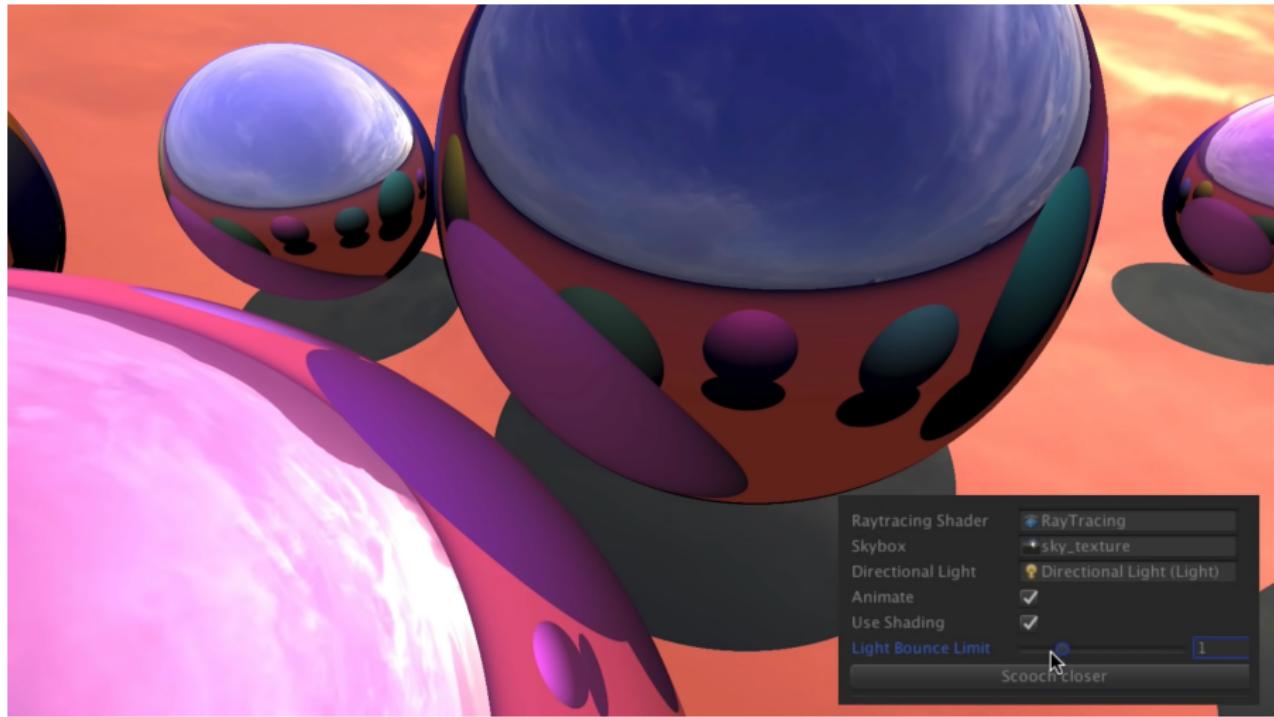
Computer Graphics (Shadows)



Computer Graphics (Reflections)



Computer Graphics (Reflections)



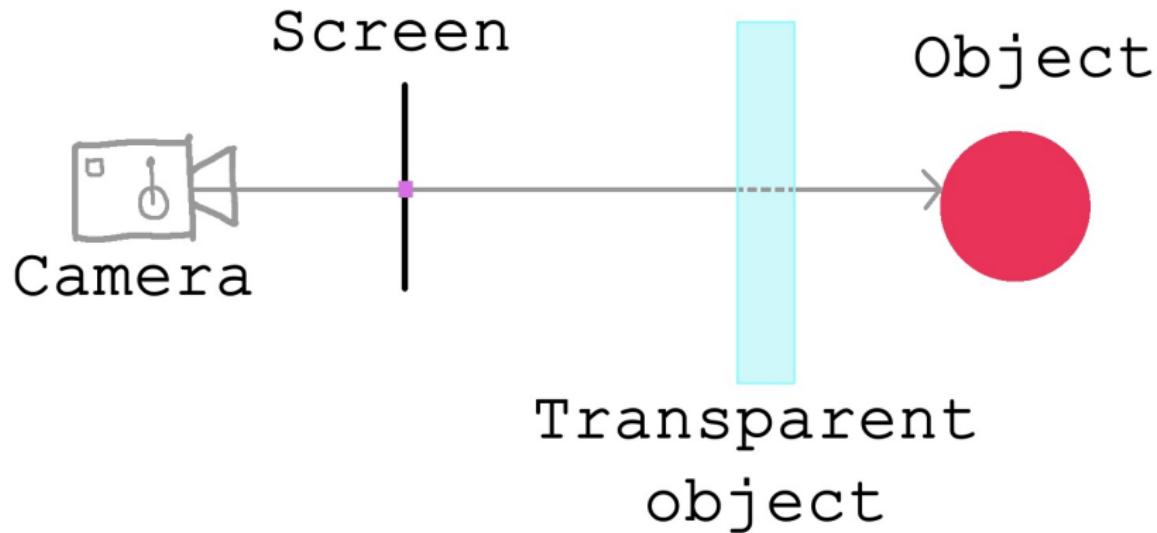
Computer Graphics (Reflections)



Computer Graphics (Reflections)



Computer Graphics (Transparency)



Computer Graphics (Transparency)



Refraction



Refraction

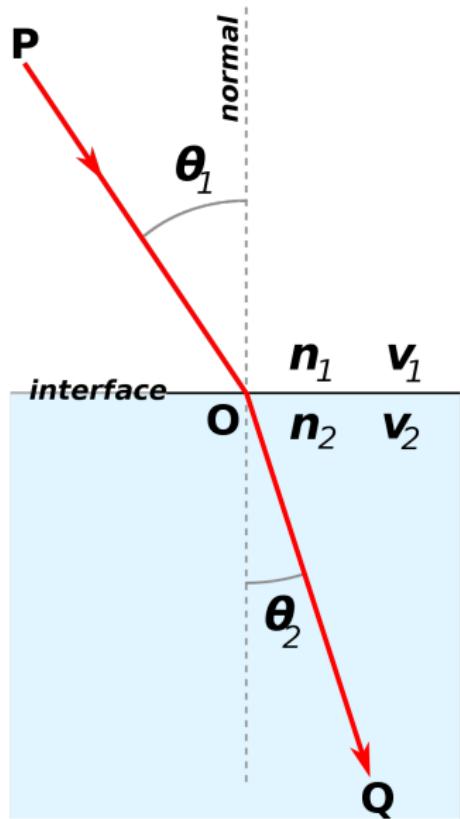


Refraction

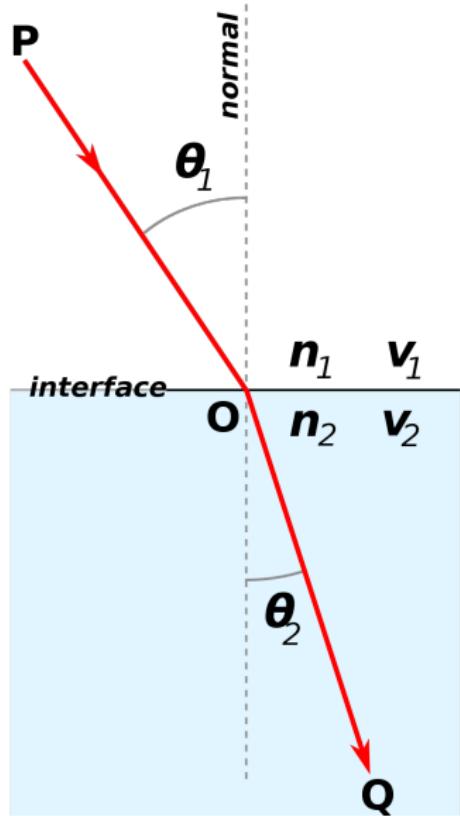


Snell's Law

Snell's Law

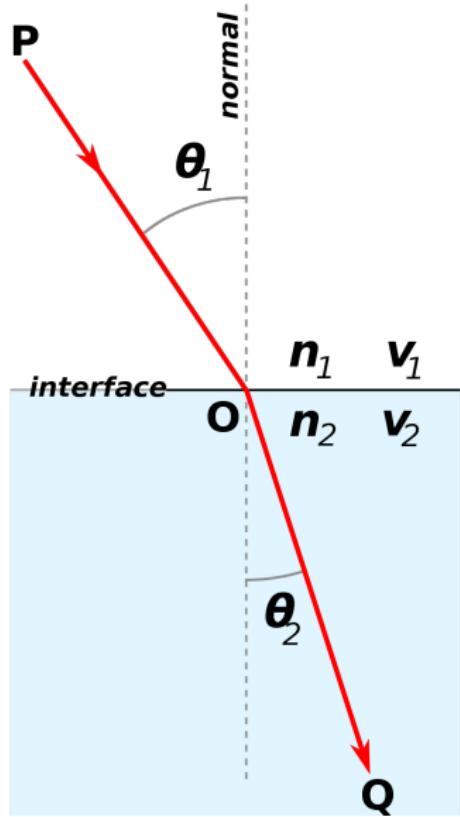


Snell's Law



v_i = velocity of light in medium i .

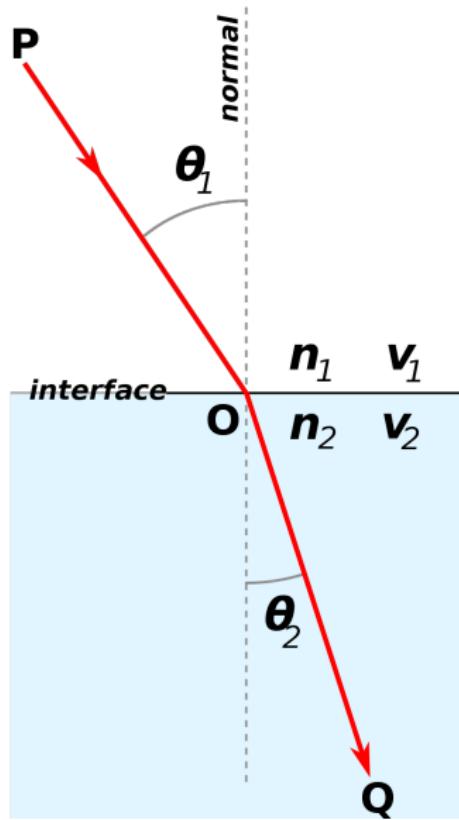
Snell's Law



v_i = velocity of light in medium i .

n_i = refractive index of medium i :

Snell's Law

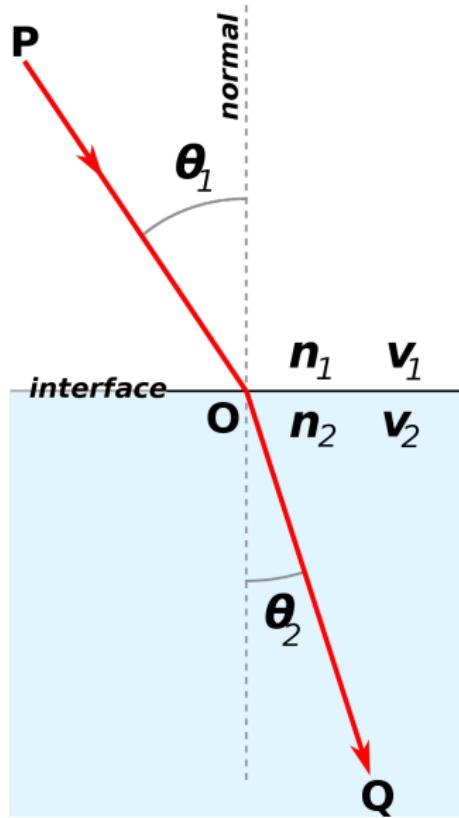


v_i = velocity of light in medium i .

n_i = refractive index of medium i :

$$n_i = \frac{c}{v_i},$$

Snell's Law



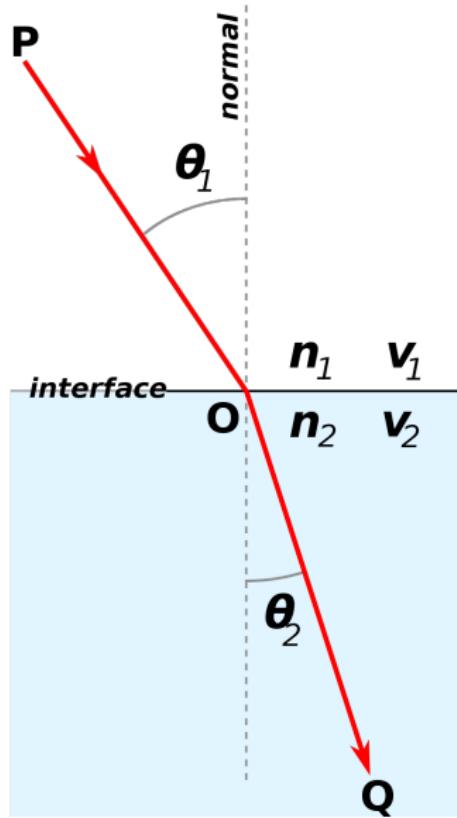
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Snell's Law



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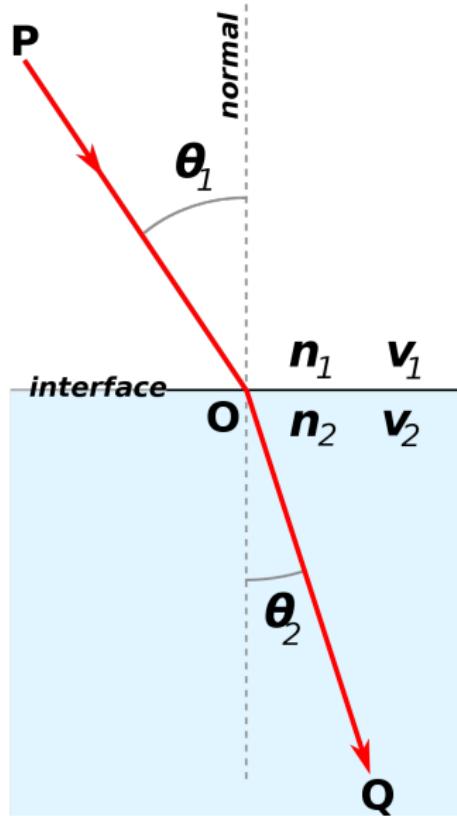
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Snell's Law



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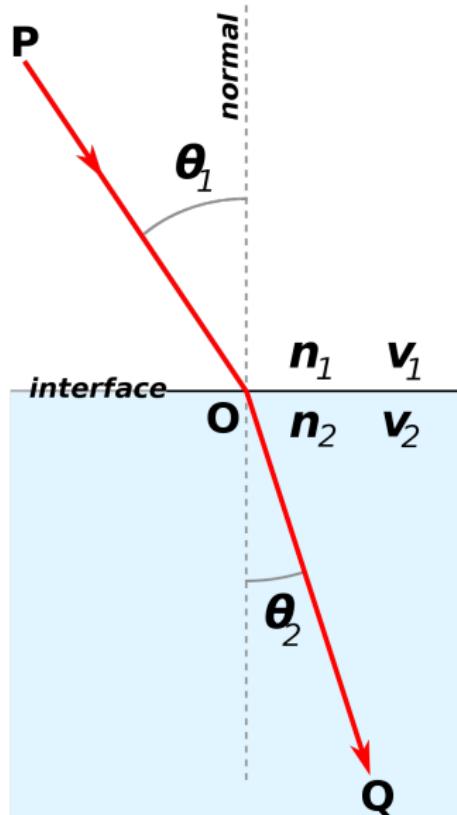
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$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

Snell's Law (History)

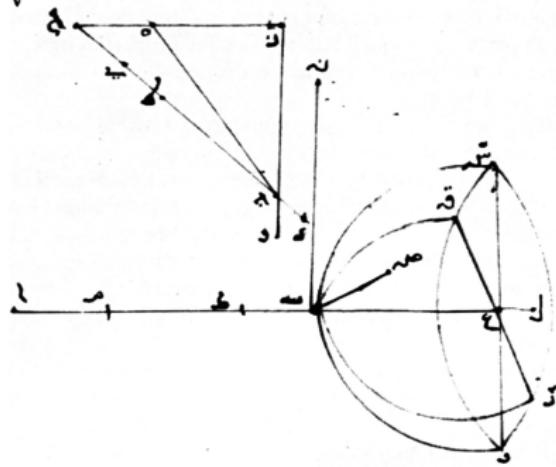
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Snell's Law (History)

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Snell's Law (History)

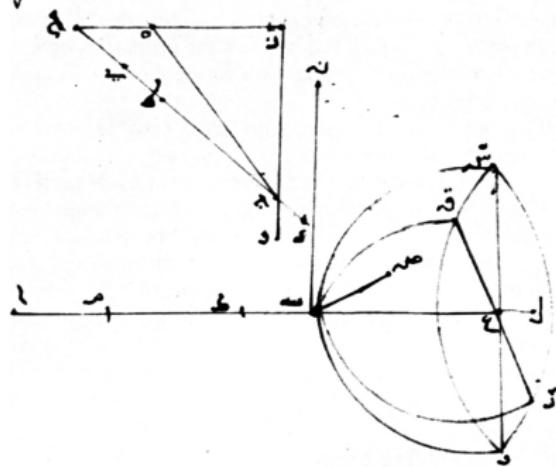
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لأنه إنما تده على سطح متساوٍ غيره فلان هذا السطح يقطع خط زهر
على نقطة بـ فلا بد من أن يقطع أحد خطوط بن هر ولكن في ذلك
الخط يبقى والفضل المشير بين هذا السطح وبين خط قطع زهر
خط بـ فلان هذا السطح ينافي سطـ بـ على نقطـة بـ خط
بـ على سطـ قـ بـ على نقطـة بـ وذلك خط بـ و هنا حالـ
فلا يـا تـ سـ يـطـ بـ على نقطـة بـ سـ يـطـ مـتسـاوـيـغـرـ سـ بـ نـصـ ٥

Snell's Law (History)

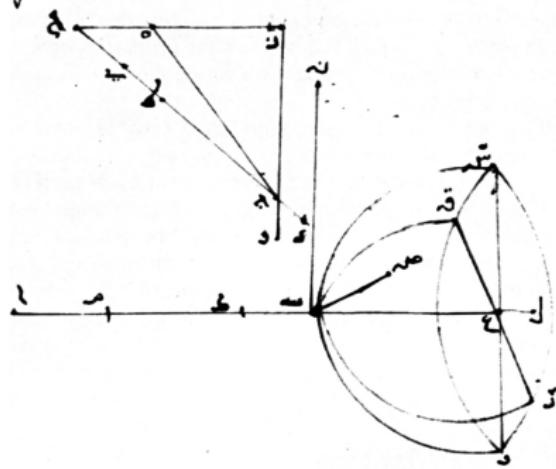
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لأنه إنما تده على سطح متساوٍ غيره فلان هذا السطح يقطع خط زهر
على نقطة بـ فلا بد من أن يقطع أحد خطوط بنـ بمثلكـ فيـ ذـاكـ
الخط بـ قـرـنـ الفـصـلـ الشـكـرـ بـينـ هـذـاـ السـطـحـ وـيـنـ يـقـطـعـ قـرـ
خطـ بـشـرـ فـلـانـ هـذـاـ السـطـحـ يـمـاـتـ بـ سـطـ بـ عـلـىـ نقطـةـ بـ خطـ
بـشـرـ قـطـ قـبـرـ عـلـىـ نقطـةـ بـ سـطـ مـسـيـوـغـرـ سـطـ بـ نـصـ ٥ـ

Snell's Law (History)

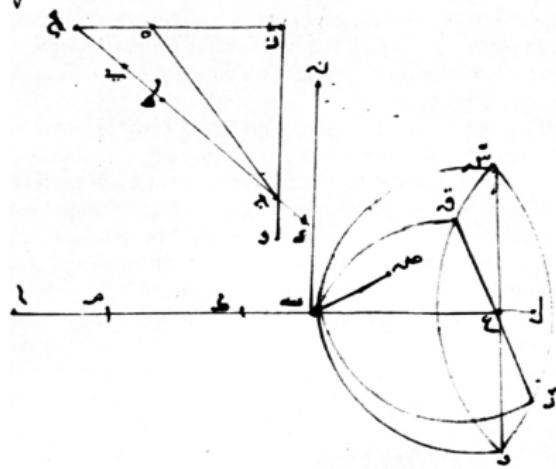
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 - 1621: Dutch astronomer Willebrord Snellius derives a mathematically equivalent law (does not publish it).



لأنه إنما شد عليه سلطنه متوجهه فلأن هذا السلطنه يقطع سلطنه بنصر
عاقبتة بـ فلا بد من أن يقطع احدهن في بنصر ولكن ذلك
الخطيب من الفضل المشركي بين هذا السلطنه وبين سلطنه قرار
خواصه فلأن هذا السلطنه يناس سلطنه على عاقبتة خط
بـ ثم على قرار سلطنه قرار على عاقبتة ولكن خط سلطنه فهذا حال
فلابد سلطنه على عاقبتة بـ سلطنه متوجهه سلطنه بنصر ٥

Snell's Law (History)

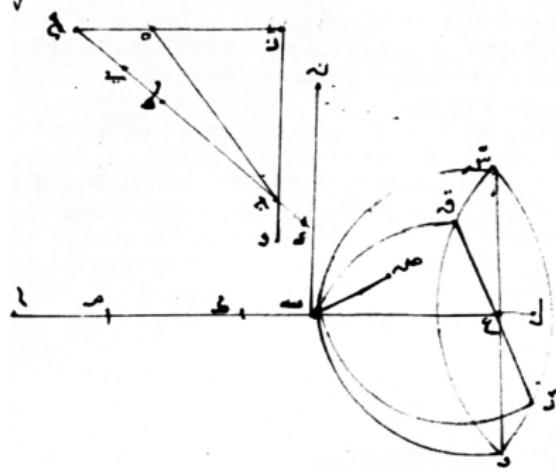
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- 1637: René Descartes independently derives the law and publishes it.



لأنه مات عليه سلطنة متوجة فلا ينطلي على نصر
علاقة بـ فلا بد من أن ينطلي الحظ في بنـ بمـ لكنـ ذـ
الخط بـ وـ الفـ لـ الشـ يـ يـ هـ الـ سـ طـ يـ قـ طـ قـ رـ
خط بـ شـ فـ لـ اـ لـ اـ سـ طـ يـ يـ اـ سـ طـ يـ بـ عـ يـ قـ طـ بـ خـ خطـ
بـ شـ عـ قـ طـ قـ بـ دـ عـ يـ قـ طـ بـ وـ كـ خطـ بـ عـ وـ فـ عـ اـ
فـ لـ يـ اـ سـ طـ يـ بـ عـ يـ قـ طـ بـ مـ سـ طـ مـ سـ طـ يـ بـ نـ صـ ٥

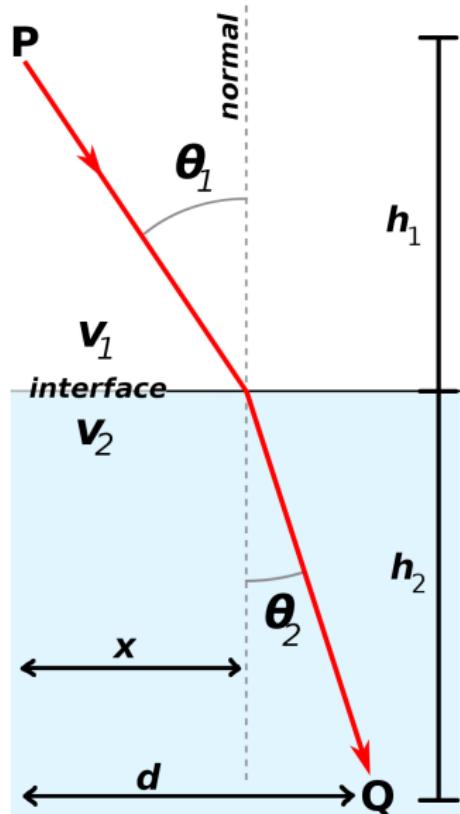
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 - 1637: René Descartes independently derives the law and publishes it.
 - 1662: Pierre de Fermat rejects Descartes's derivation, reproves the law using his *principle of least time*.

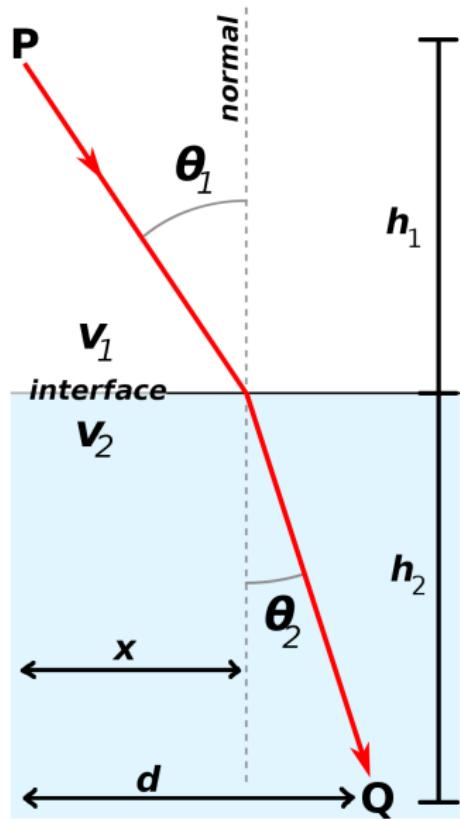


لأنه إنما شد عليه سلطنه متوجهه فلأن هذا السلطنه يقطع سلطنه بنصر
عاقبتة بتغلبها من يقطع احدهما بنصر فيكون ذلك
الخطبته الفعل الشكر بين هذا السلطنه وبين سلطنه قلقاً
خواصه فلأن هذا السلطنه يناس سلطنه على عاقبتة خط
بسنت سلطنه قلقاً بعد على عاقبتة وكل خطبته وفالحال
فلابد سلطنه على عاقبتة سلطنه متوجهه سلطنه بنصر

Snell's Law (Proof)

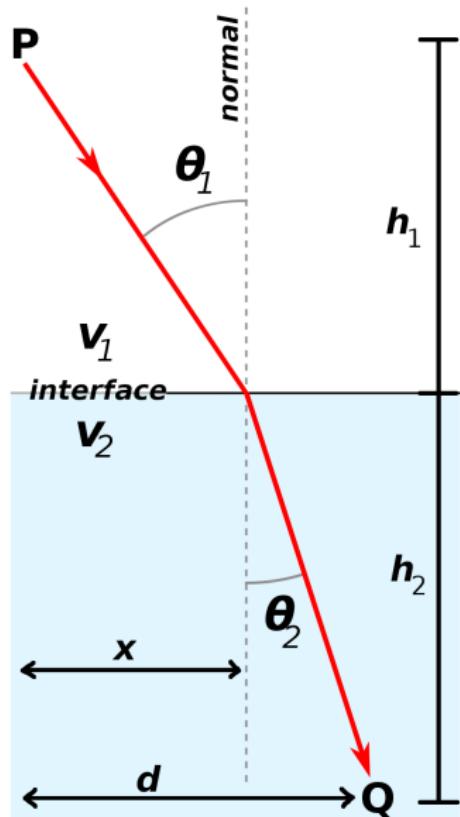


Snell's Law (Proof)



Fermat's Principle: A ray of light follows the path which takes the least time to traverse.

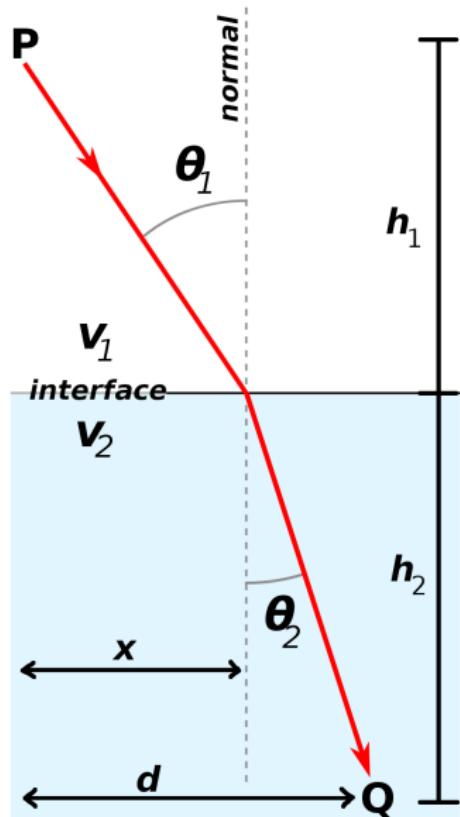
Snell's Law (Proof)



Fermat's Principle: A ray of light follows the path which takes the least time to traverse.

$$T = \frac{\sqrt{x^2 + h_1^2}}{v_1} + \frac{\sqrt{(d-x)^2 + h_2^2}}{v_2}.$$

Snell's Law (Proof)

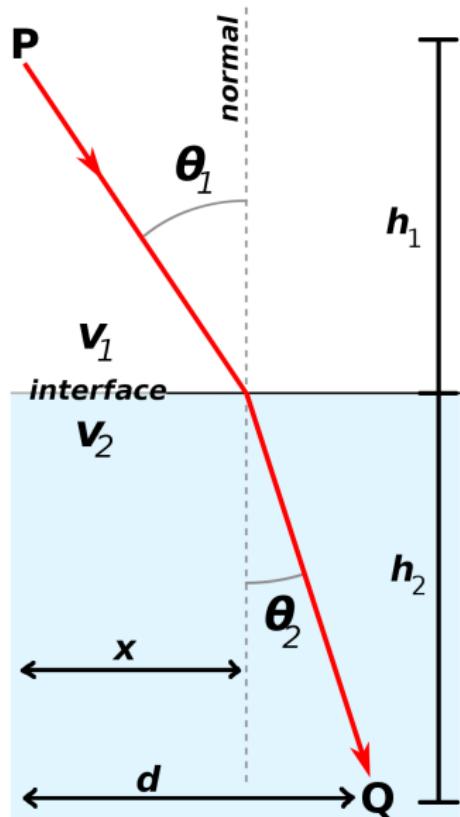


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$$\frac{dT}{dx} = \frac{x}{v_1 \sqrt{x^2 + h_1^2}} - \frac{d-x}{v_2 \sqrt{(d-x)^2 + h_2^2}}.$$

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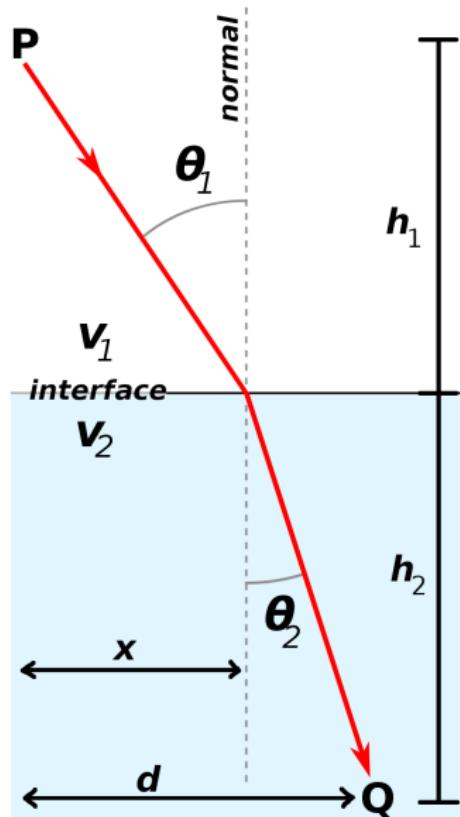
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Notice that

$$\frac{x}{\sqrt{x^2 + h_1^2}} = \sin \theta_1,$$

Snell's Law (Proof)



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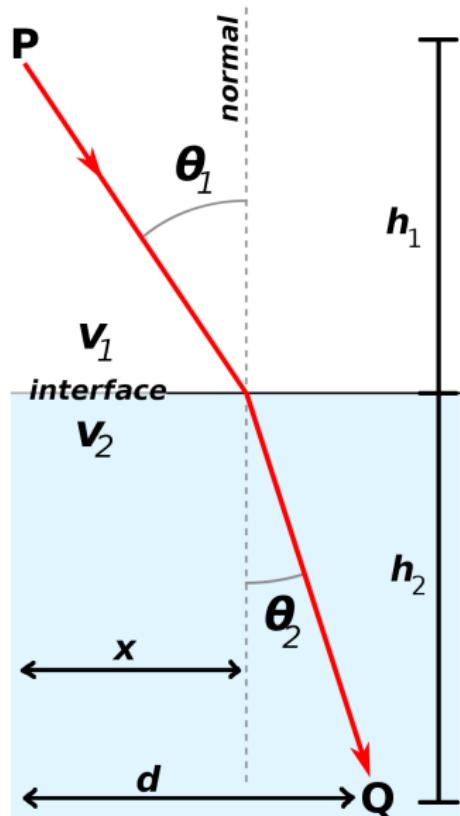
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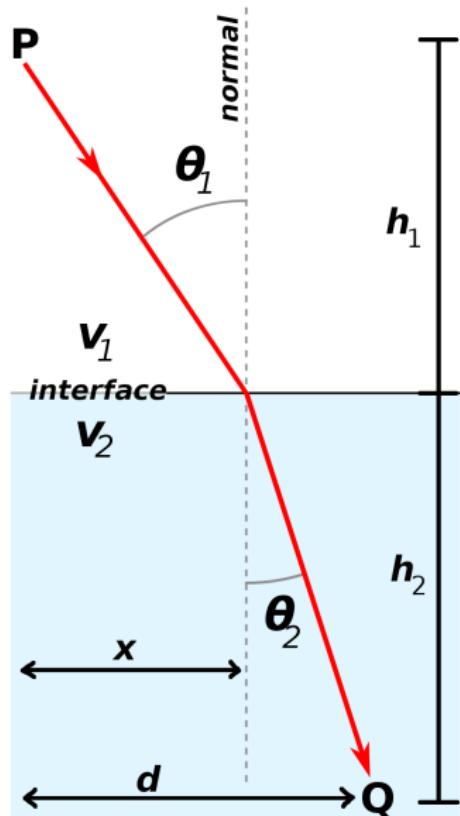
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$$\text{Thus } \frac{dT}{dx} = 0$$

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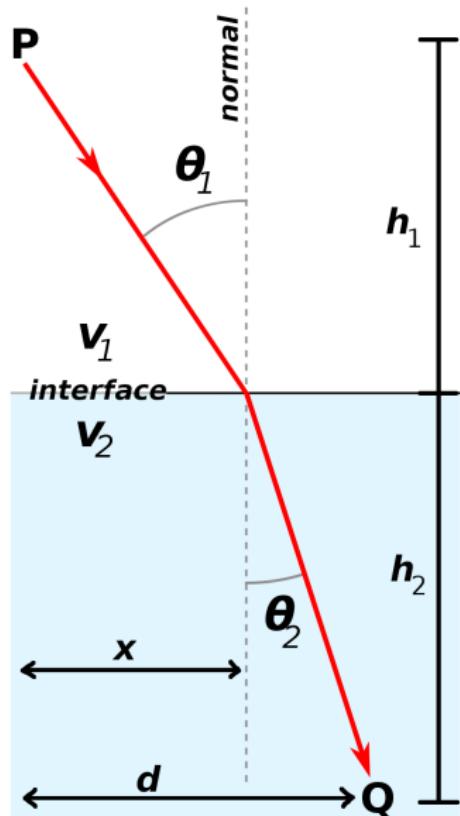
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Notice that

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$$\text{Thus } \frac{dT}{dx} = 0 \Leftrightarrow \frac{\sin \theta_1}{v_1} = \frac{\sin \theta_2}{v_2}$$

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Fermat's Principle: A ray of light follows the path which takes the least time to traverse.

$$T = \frac{\sqrt{x^2 + h_1^2}}{v_1} + \frac{\sqrt{(d-x)^2 + h_2^2}}{v_2}.$$

$$\frac{dT}{dx} = \frac{x}{v_1 \sqrt{x^2 + h_1^2}} - \frac{d-x}{v_2 \sqrt{(d-x)^2 + h_2^2}}.$$

Notice that

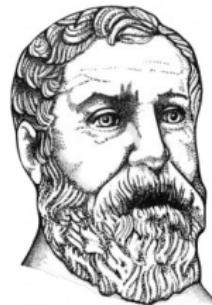
$$\frac{x}{\sqrt{x^2 + h_1^2}} = \sin \theta_1, \quad \frac{d-x}{\sqrt{(d-x)^2 + h_2^2}} = \sin \theta_2.$$

$$\text{Thus } \frac{dT}{dx} = 0 \Leftrightarrow \frac{\sin \theta_1}{v_1} = \frac{\sin \theta_2}{v_2} \Leftrightarrow \frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2}.$$

Fermat's Principle of Least Time (History)

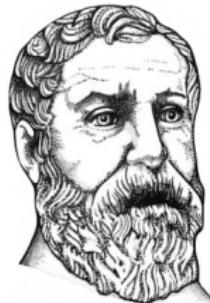
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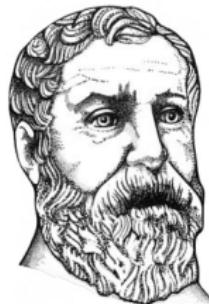
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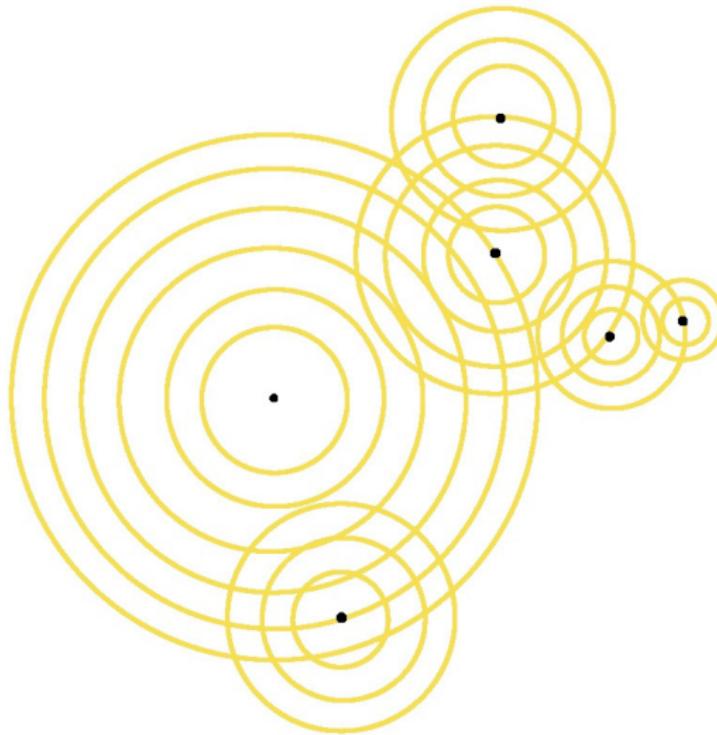
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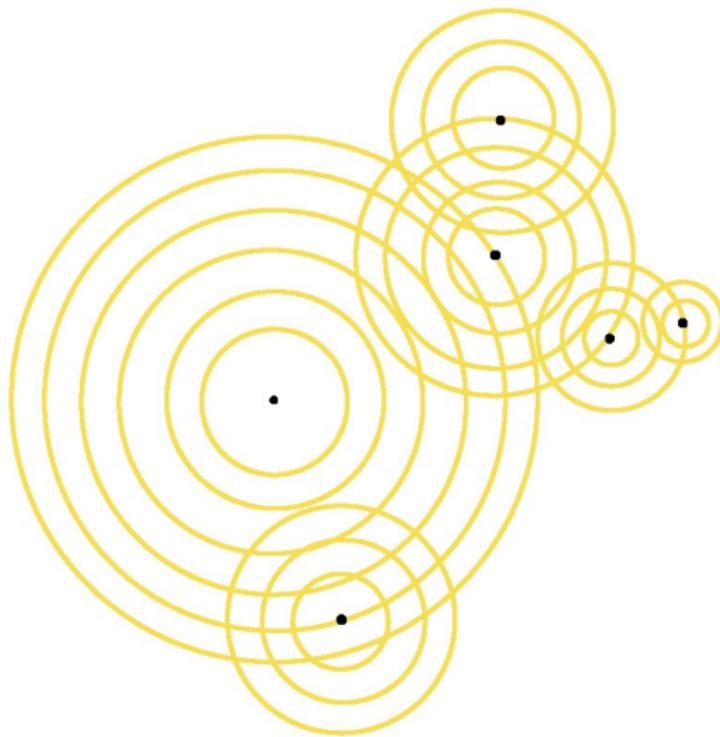


The Wave Theory of Light

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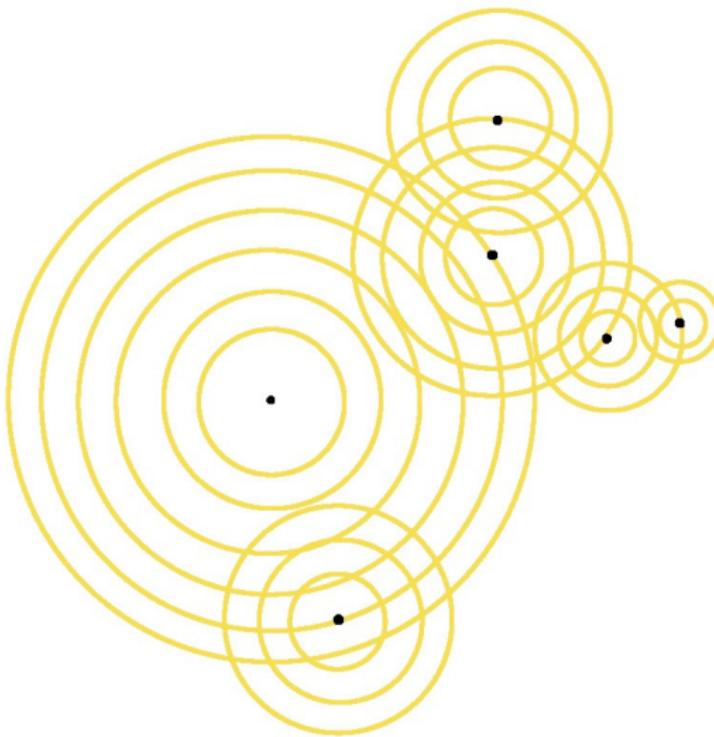


The Wave Theory of Light



Light is a disturbance which propagates in waves through a medium.

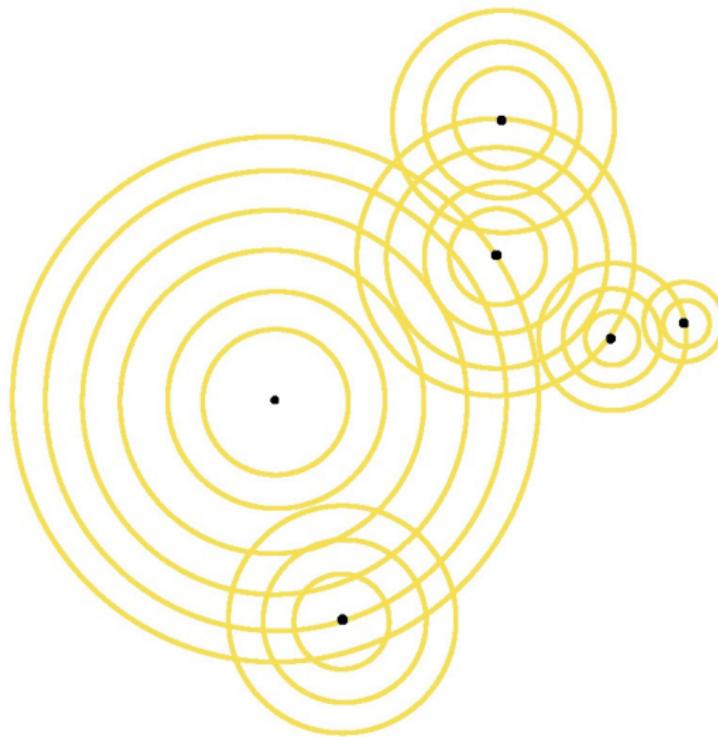
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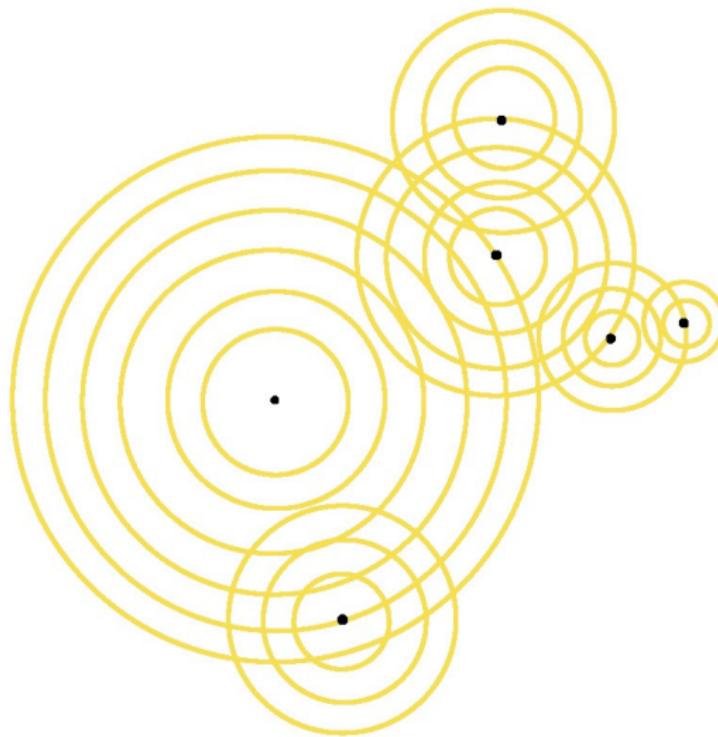


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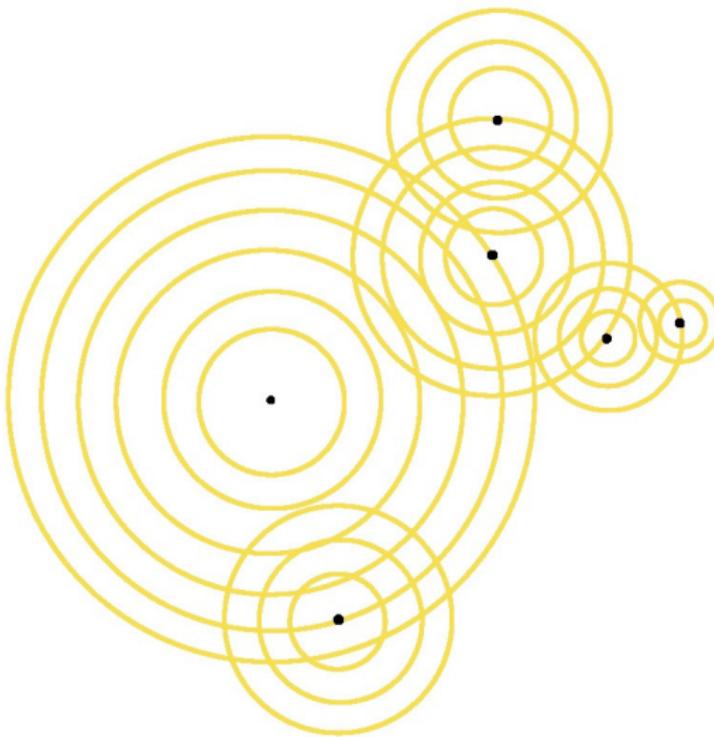
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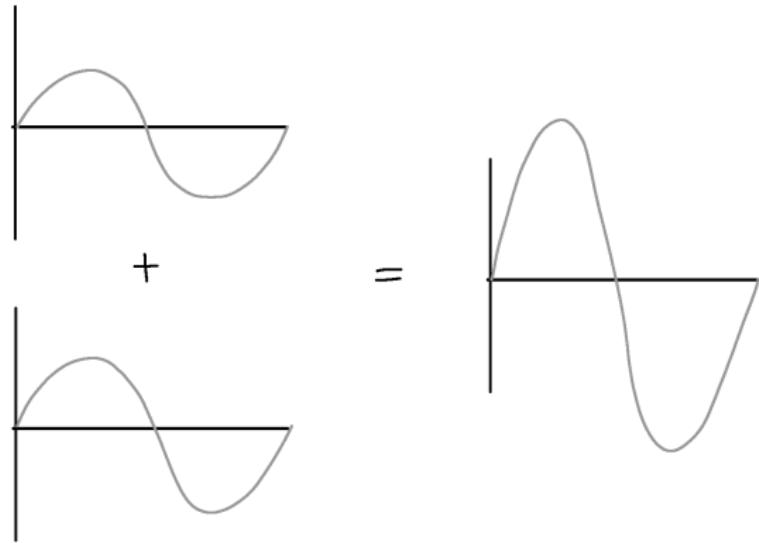
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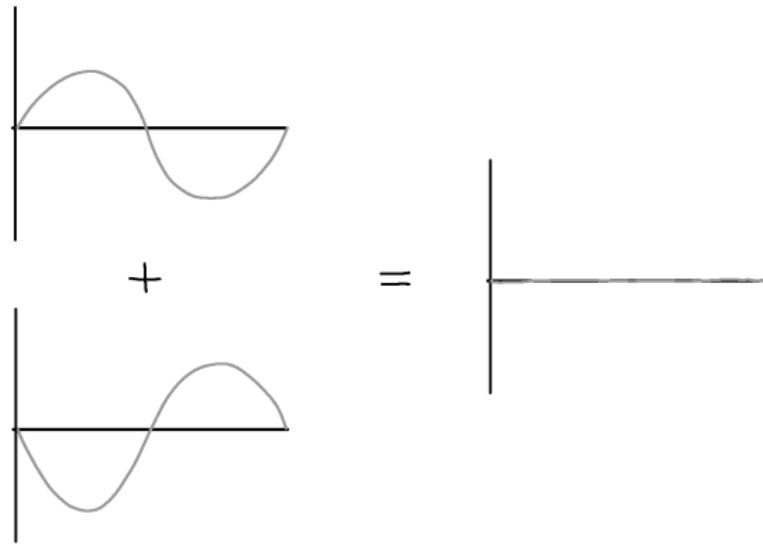
Light takes *all* paths, but some paths cancel out due to interference of waves.

Interference

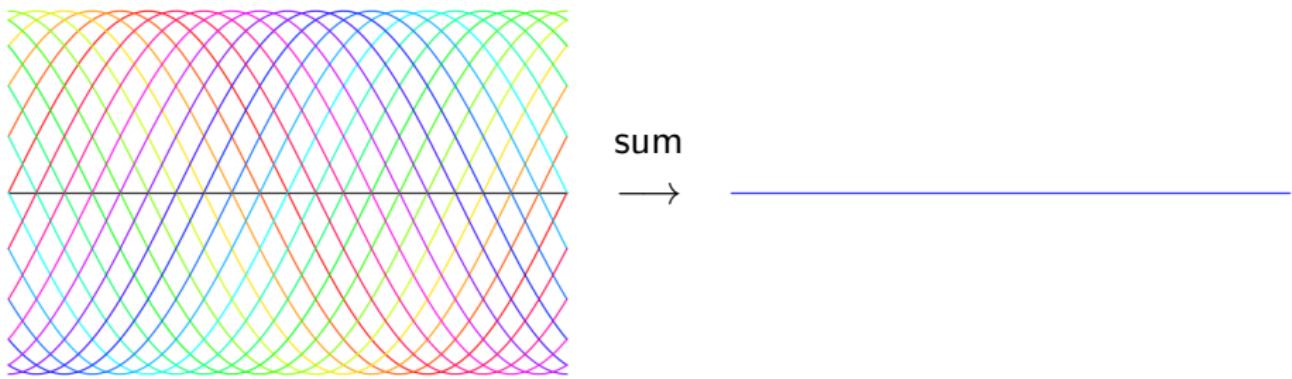
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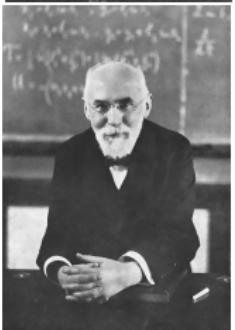
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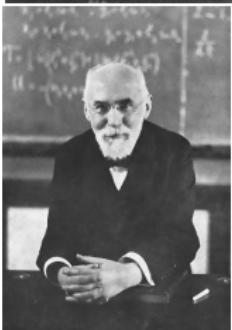
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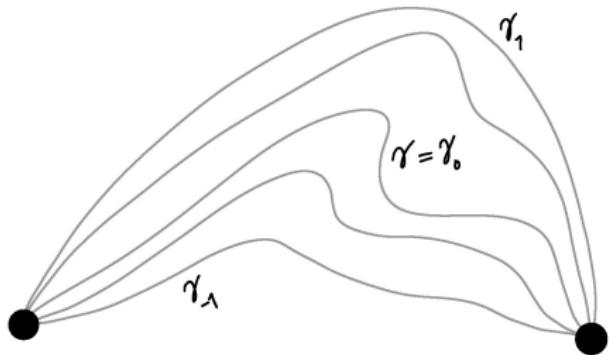
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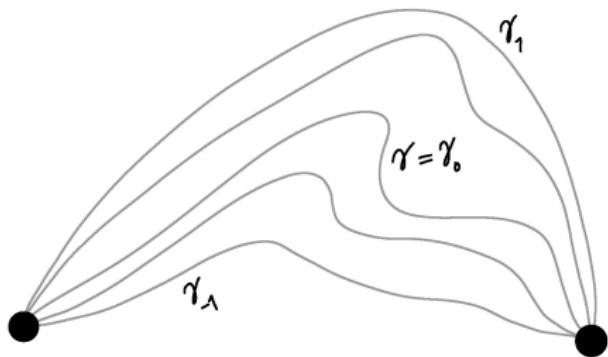
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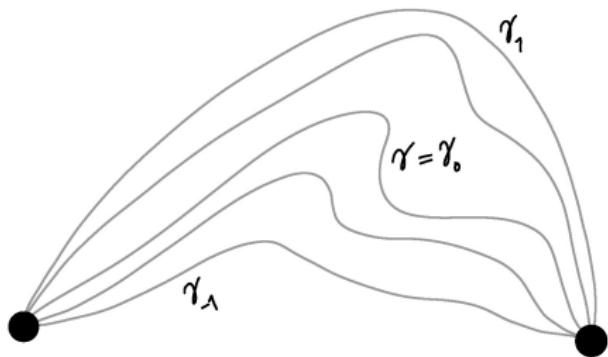


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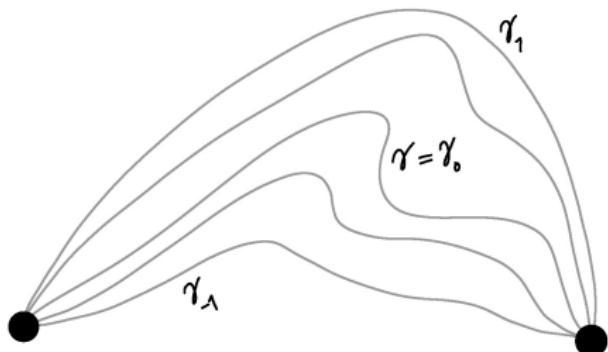
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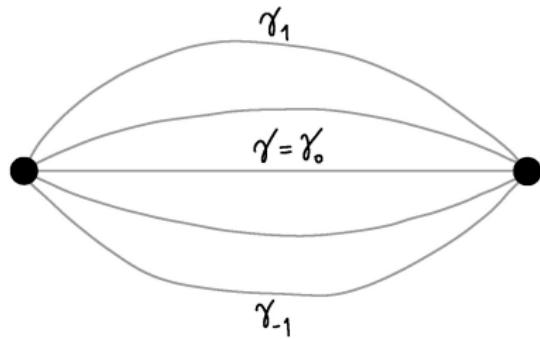


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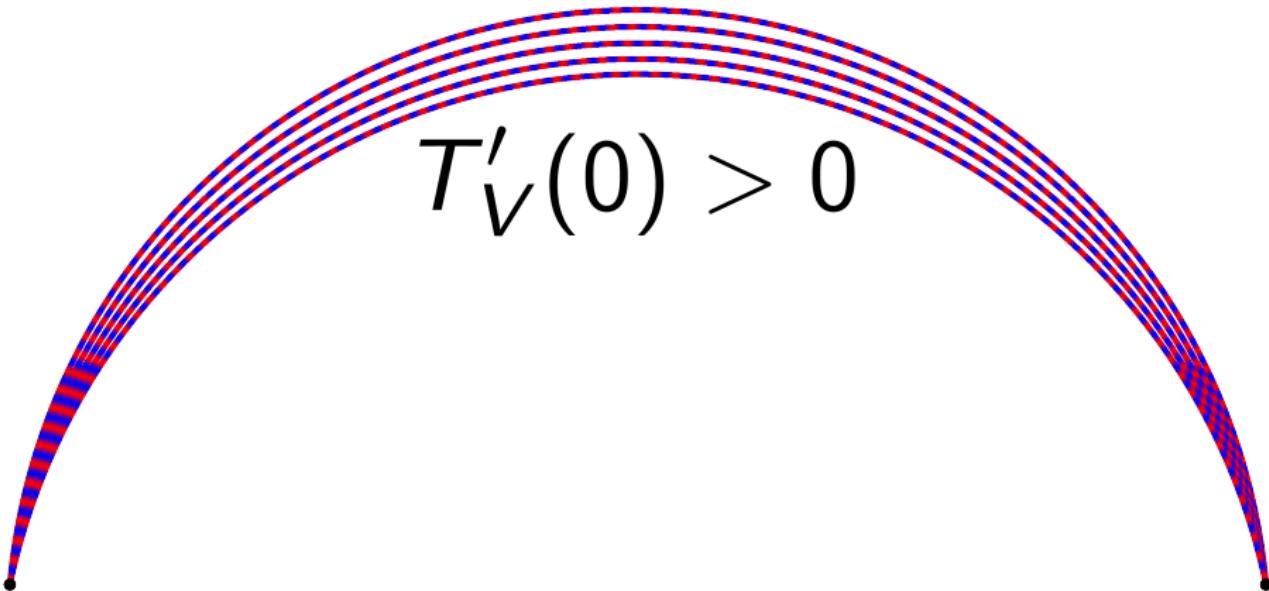
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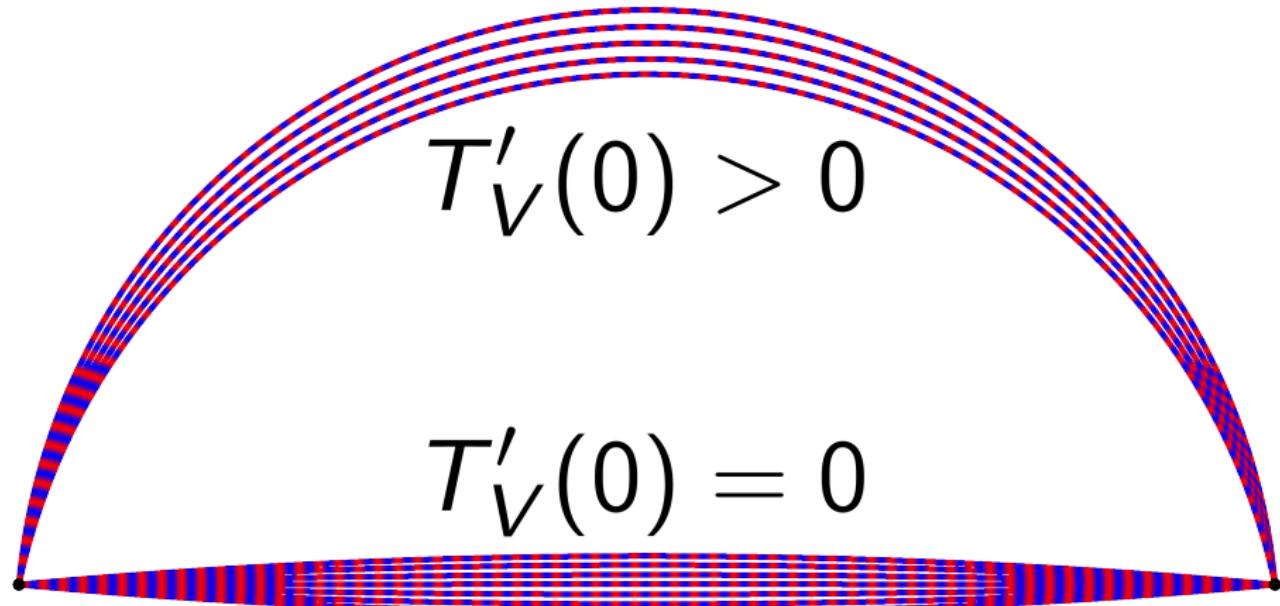


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