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Monday, November 18, 2019 15:00

Lens Spaces

$\mathbb{Z}/p \circlearrowright S^3 \qquad p \in \mathbb{N}$

$S^3_{\sqrt{2}} = \{ (z,w) : |z|^2 + |w|^2 = 2 \}$

primitive p^{th} root of 1 $\zeta_p = e^{\frac{2\pi i}{p}}$

$g : S^3 \longrightarrow S^3, \quad g(z,w) = (\zeta_p z, \zeta_p^q w)$ where p,q coprime.

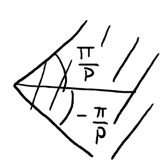
Check: * $g^p = id.$

* g^j is fixed-pt free if $0 < j < p$.

fixed-pt free action $\mathbb{Z}/p \circlearrowright S^3$.

$$L(p,q) := \underbrace{S^3/(\mathbb{Z}/p)}_{\text{manifold}} \xleftarrow{\text{cover}} S^3$$

Lens space $L(p,q)$. $\quad \pi_1(L(p,q)) = \mathbb{Z}/p$.

Another View: $\quad W_p =$ 

$\subset \mathbb{C}$.

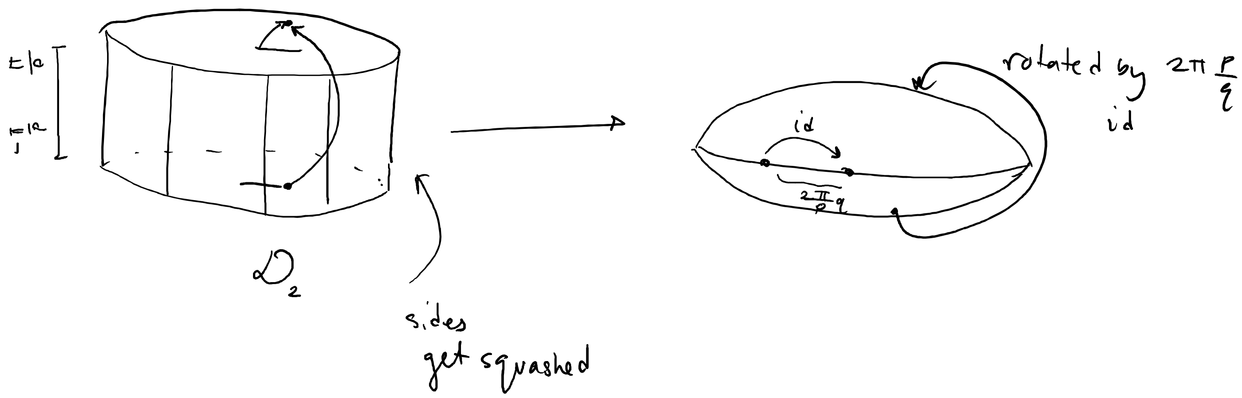
$$F_p = S_3 \cap W_p$$

$$\mathbb{Z}/p\,(F_p) = S_3\,,$$

$$L(p,q) = F_p/_\sim$$

$$[-\tfrac{\pi}{p}, \tfrac{\pi}{p}] \times D_2 \twoheadrightarrow F_p$$

$$(\theta, w) \longmapsto (e^{i\theta}\sqrt{2-|w|^2},\ w),$$ mod by this.



Some id of top & bottom & edge of lens

<u>Theorem</u>: $L(p',q') \approx L(p,q)$ iff $p = p'$ and $q' = \pm q \mod p$.

homeomorphism

$L(p',q') \sim L(p,q)$ iff $p = p'$ and $q' = \pm q^{\pm 1} r^2 \mod p$ $(r \in \mathbb{Z})$.

same homotopy type