Rec 8/22

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X any non-empty set made into a metric space?

$$\rho(x,y) = \begin{cases} 0 & \text{if } x=y \\ 1 & \text{if } x \neq y \end{cases}$$ yes.

$$X = \{ x = (x_1, \dots, x_N) \mid x_i \in \{0,1\} \}.$$

Hamming distance: $\rho(x,y) = \#\{ i : x_i \neq y_i \}$

$$B_r(x) = \{ y \in X \mid \rho(x,y) < r \}$$

$$\mathring{E} = \bigcup \{ G \subset E \mid G \text{ open} \}$$

$$\overline{E} = \bigcap \{ F \supset E \mid F \text{ closed} \}$$

$x_n \longrightarrow x \iff \rho(x_n, x) \longrightarrow 0$ $\{x_n\}$ is a convergent sequence

$f : X_1 \longrightarrow X_2$ is cts if

(X_1, ρ_1) (X_2, ρ_2)

$f^{-1}[U]$ is open in X_1 $\forall$ open $U \subset X_2$

$[U]$ is open in ...

$$\rho(E,F) = \inf\{\rho(x,y) : x \in E, y \in F\}$$

$$\operatorname{diam}(E) = \sup_{x,y \in E} \rho(x,y)$$

Cauchy sequence: $\{x_n\}$ s.t. $\rho(x_n, x_m) < \varepsilon$ if n,m large enough.

<u>Cover</u> of a set E is $\{V_\alpha\}$ s.t. $\bigcup V_\alpha \supset E$.

E is compact if $\forall$ open cover $\{V_\alpha\}$, $\exists$ a finite subcover

E is totally bounded if $\forall\, \varepsilon > 0$, $\exists$ a finite cover by balls of radius $< \varepsilon$.

A set $E \subset X = (X, \rho)$

<u>Bolzano-Weierstrass property</u> = every sequence in E has a convergent subsequence.

<u>Heine-Borel property</u> = Every open cover has a finite subcover.

Claim: These are equivalent and equivalent to:

E is closed & totally bounded

All 3 mean "E is compact".

Special Case: in $\mathbb{R}^n$ w/ euclidean metric,

E is cpct iff E is closed & bounded.

$C^0(I) = \{ f \text{ cts on } I \}$

metric $\rho(f,g) = \sup_{x \in I} |f(x) - g(x)|$