Fox - Free - Differential Calculus

$F_n = \langle s_1, \dots, s_n \rangle \quad , \quad \mathbb{Z}[F_n] \supseteq F_n$

$$\frac{\partial}{\partial s_j} : \mathbb{Z}[F_n] \longrightarrow \mathbb{Z}[F_n]$$

* $\dfrac{\partial}{\partial s_j}(a \cdot b) = \dfrac{\partial}{\partial s_j}(a) + a \dfrac{\partial}{\partial s_j}(b) \qquad (a, b \in F_n)$
* $\dfrac{\partial}{\partial s_j}(s_i) = \delta_{ij}$

Lemma: There is a unique $\frac{\partial}{\partial s_i}$ w/ these properties.

Lemma $\dfrac{\partial}{\partial s_j}(s_j^k) = 1 + s_j + s_j^2 + \dots + s_j^{k-1}$ for $k > 0$

$$\frac{\partial}{\partial s_j}(1) = 0$$

$$\frac{\partial}{\partial s_j}(s_j^{-k}) = -s_j^{-k}(1 + s_j + s_j^2 + \dots + s_j^{k-1})$$

$$\forall B \in \mathbb{Z}[F_n], \quad \sum_j \left(\frac{\partial B}{\partial s_j}\right)\cdot(s_j - 1) = B - 1$$

- Use this to describe stuff in Alexander module.

Torus Knot

$$T = T_{p,q}$$

$$\pi_T = \langle u, v \mid u^q = v^p \rangle$$

$$\mathcal{D} = \mathbb{Z}_{(t)}, \quad \mathbb{Z}[\mathcal{D}] = \mathbb{Z}[t, t^{-1}]$$

$$\pi_t \xrightarrow{\lambda} \mathcal{D} \qquad \begin{aligned} u &\longmapsto t^p \\ v &\longmapsto t^q \end{aligned}$$

$$\vdots$$

Alexander Polynomial: $\Delta = \dfrac{(t-1)(t^{pq}-1)}{(t^p-1)(t^q-1)} \in \mathbb{Z}[t, t^{-1}]$.

<u>In general,</u>

$$H_1(\tilde{S}_K) = \mathbb{Z}[t,t^{-1}] \big/ \Delta\,\mathbb{Z}[t,t^{-1}]$$

Alexander polynomial.