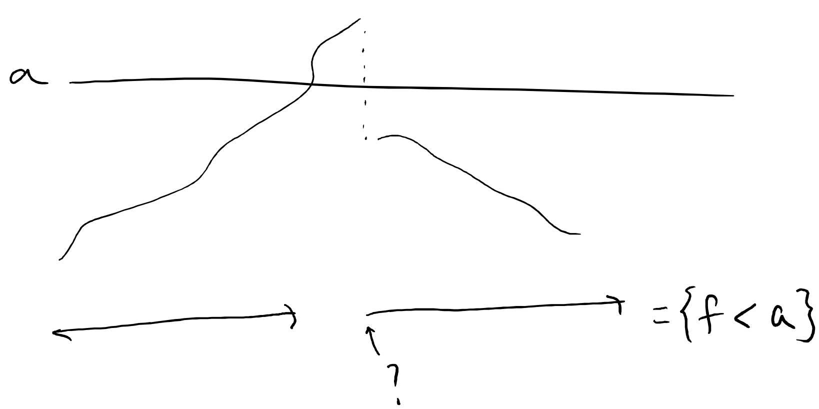
Rec 9/26

Thursday, September 26, 2019 10:23

<u>Defn</u> $f: X \longrightarrow \mathbb{R}$ is upper semicontinuous if

$$\{x \mid f(x) < a\} \text{ is open } \forall a \in \mathbb{R}.$$

(lower semicontinuous: replace $<$ with $>$)



depends on value of f at that point.

<u>Example</u> (Folland)

$$f(x) = \begin{cases} \frac{1}{\sqrt{x}} & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

$\{r_n\}$ an enumeration of $\mathbb{Q}$.

$$g(x) = \sum_{1}^{\infty} \frac{1}{2^n} f(x - r_n)$$

Show

a) $g \in \mathcal{L}^1$, $g < \infty$ a.e.

b) g discontinuous everywhere, unbounded on every interval

c) $g^2 < \infty$ a.e.

g^2 not integrable.

- g discontinuous on $\mathbb{Q}$

- g unbdd on every nh of a rational.

Let $x_0 \notin \mathbb{Q}$.

If g cont at x_0, $\forall \varepsilon > 0$, $\exists \delta > 0$ . . .

if $g(x_0) < \infty$, we are good.