3/6

Wednesday, March 6, 2019 10:37

Filterd Sp Cpx

$\{K_r = (V_r, \Sigma_r)\}_{r\geq 0}$ and $\{f_{r,s} : V_r \longrightarrow V_s \text{ simplicial map}\}_{r\leq s}$

Such that if $r \leq t \leq s$ then

$$\begin{tikzcd} V_r \arrow[r, "f_{rt}"] \arrow[rr, bend right, "f_{rs}"'] & V_t \arrow[r, "f_{ts}"] & V_s \end{tikzcd}$$

commutes.

FFSC is a finite one.

Eg: Vietoris-rips. Let (X, d) be a finite metric space.

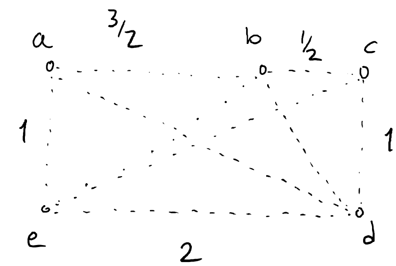
Let $VR(X, r) = (V_r, \Sigma_r)$ where

$V_r = X$, $\Sigma_r = \left\{ \sigma \subseteq X : \begin{matrix} d(x,y) \leq r \\ \forall\, x,y \in \sigma \end{matrix} \right\}$

$VR(X) = \{VR(X,r)\}_{r\geq 0}$, $\{f_{rs} = id_X\}_{r\leq s}$

$VR(X)$ is a FFSC because d takes finitely many values $r_0, r_1, \dots, r_N$

Ex:



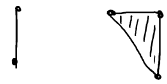
d	a	b	c	d	e
a	0	$3/2$	2	$\sqrt{5}$	1
b		0	$1/2$	$\sqrt{5}/2$	$\sqrt{13}/2$
c			0	1	$\sqrt{5}$
d				0	2
e					0

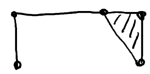
$$0,\ \tfrac{1}{2},\ 1,\ \tfrac{\sqrt{5}}{2},\ \tfrac{3}{2},\ \tfrac{\sqrt{13}}{2},\ 2,\ \sqrt{5}$$

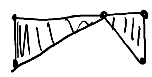
$VR(X, 0) =$ 

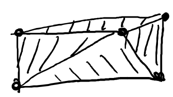
$VR(X, 1/2) =$ 

$VR(X, 1) =$ 

$VR(X, \frac{\sqrt{5}}{2}) =$ 

$VR(X, \frac{3}{2}) =$ 

$VR(X, \frac{\sqrt{13}}{2}) =$ 

$VR(X, 2) =$ 

$VR(X, \sqrt{5}) =$ 4-d simplex.

In general, $VR(X, 2^{2^{2^{10000}}}) = (|X|-1)$-d simplex.

How to quantify topological features? Homology.

The k^{th} persistent homology of a filtered sp. cpx.

$\mathbb{K} = \{K_r\}_{\geq 0}$ is $PH_k(\mathbb{K}) := \{H_k(K_r)\}_{r \geq 0}$

Notice: $\forall r \leq s$, we have an induced map

$$H_k(f_{rs}) : H_k(K_r) \longrightarrow H_k(K_s)$$

by functoriality