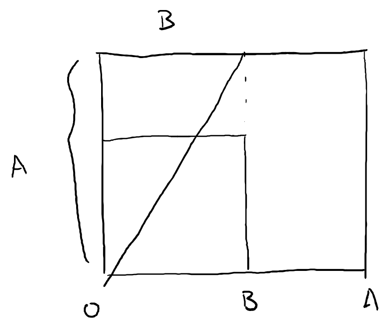
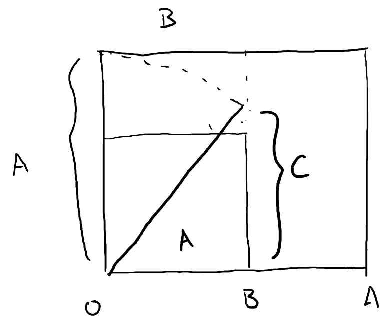
Vatsa

Friday, October 20, 2017 14:18



diagonal gives square which is sum of A^2 and B^2



$C^2 = A^2 - B^2$

Exercise: using these two rope tricks, given a rectangle construct a square of equal area.

$ax - by = \pm c$ kuttaka (pulverizer)

$\downarrow$

reduced to $ax' - by' = \pm 1$; Then work backwards from euclidean algorithm.

$$b_1 y - a_1 x_1 = c_1$$
$$\vdots$$
$$b_r y - a_r x_r = c_r$$

Exercise

Using chinese remainder thm,

Show this has a solution

where $(b_i, a_i) = 1$ and a_i relatively prime.

The Process

$$y_i^2 = D x_i^2 + m_i \quad \text{for } i = 1, 2,$$

$$\left\{ \begin{aligned} y &= y_1 y_2 + D x_1 x_2 \\ x &= x_1 y_2 + y_1 x_2 \\ m &= m_1 m_2 \end{aligned} \right.$$

$$y^2 = D x^2 + m.$$

bhavana (process)

$$(x_1, y_1, m_1) \odot (x_2, y_2, m_2) = (x, y, m)$$

This bhavana operator can be used to construct a soln to any pell eqn.

$$(p_n, q_n, m_n) \xrightarrow{\odot\,(1, y, y^2 - D)} \underbrace{\left(\frac{p_n y - q_n}{m_n}, \frac{D p_n - q_n y}{m_n}, \frac{y^2 - D}{m_n} \right)}_{\text{choose } y \text{ s.t. integers, } |y^2 - D| \text{ minimal}} = (p_{n+1}, q_{n+1}, m_{n+1})$$

$m_n > m_{n+1} > \cdots \geq 1$ so this is a solution to original pell eqn.

And this is faster than the CF approach.

$a_r a_{r-1} \cdots a_0$ digits of a number, with r+1 digits $1 \leq a_i \leq 9$

$$a_r + \cdots + a_0 = S \leq r + 10$$

Exercise Prove that the number of numbers which do this is $\binom{S-1}{r}$

Book: Studies in the history of indian mathematics

C.S. Seshadri.