Rec 9/3

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$F : \mathbb{R} \to \mathbb{R}$ increasing, right cts

$\mu_0 : \mathcal{A}(\mathcal{H}) \to \mathbb{R}$

$\mu_0((a,b]) = F(b) - F(a)$ is a premeasure.

σ-finite

semifinite

<u>Q1</u> on a nonempty set X,

$\mu(\emptyset) = 0$

$\mu(A) = \infty \quad \forall A \neq \emptyset.$ $\quad (\mu : P(X) \to [0,\infty])$.

Not semifinite.

Add a point w/ measure 1 to get another one.

Another one:

$$\left\{ \begin{array}{l} F : \mathbb{N} \to [0,\infty] \\ A \subseteq \mathbb{N}, \ \mu_F(A) = \sum_{n \in A} F(n) \end{array} \right.$$

not semifinite if $F(n) = \infty$ for some n.

$n \in A$ for some n.

semifinite but not σ-finite:

Counting measure on an uncountable set.

what is semifinite part?

replace F by $\tilde{F}(n) = \begin{cases} F(n) & \text{if } F(n) \neq \infty \\ 0 & \text{if } F(n) = \infty \end{cases}$

$\mu_F = \mu_{\tilde{F}} + \rho$

where $\rho = \mu_g$ where $g(j) = \begin{cases} 0 & F(j) < \infty \\ \infty & F(j) = \infty \end{cases}$

$F = 1 + \infty \cdot 1_n$

$\tilde{F} = 1 - 1_n$

$\rho_1(A) = \begin{cases} \infty & \text{if } n \in A \\ 0 & \text{o.w.} \end{cases}$

~~$\rho_2(A) = \{ \infty$ if $n \in A$ or $|A| = \infty$~~

μ ~ $\mu_0((a,b]) = F(b) - F(a)$.

eg $F(x) = x$

Construction gives μ on $\mathcal{B}_{\mathbb{R}}$.

Complete measure: if $F \subset E$, $\mu(E) = 0 \Rightarrow \mu(F) = 0$.

Lebesgue measure.

N: Nonmeasurable sets (Vitali).

$m(E) = 0$

$E \cap N = F$ is not Borel.

Complete m by adding all such F:

$$\mathcal{L} \supsetneq \mathcal{B}_{\mathbb{R}}$$

Sometimes $F : \mathbb{R} \to \mathbb{R}$ gives μ on $P(X)$:

eg