- S can generate a ring (or a field).

<u>Prop</u> if D is a subring of a field,
Let F be the field generated by D.
Then $F = \{ ab^{-1} \mid a, b \in D,\ b \neq 0 \}$.

<u>Pf</u> That set is a field and it's contained in any field containing D.

<u>Remark</u>: $ab^{-1} = cd^{-1}$ in F iff $ad = bc$.

Let D be a commutative domain.
Can we embed D in a field?
(commutativity is necessary).

<u>Thm</u> The field of fractions exists.

<u>Weaker Statement w/ Same proof</u>: Any commutative monoid w/ a

cancellation law can be embedded in an abelian group.

<u>Thm</u> Let D be a comm. domain, F its field of fractions. Any injective hom $\eta : D \longrightarrow F'$ (another field) can be extended uniquely to $\varphi : F \longrightarrow F'$, an injective field hom. s.t.

$$\begin{tikzcd} D \arrow[r, "\eta"] \arrow[d, "\iota"'] & F' \\ F \arrow[ur, dashed, "\exists!\, \varphi"'] & \end{tikzcd}$$

commutes.

<u>Corollary</u> if $F_1 = \langle D \rangle = F_2$, Then

$$\begin{tikzcd} D \arrow[r, hook] \arrow[d] & F_1 \arrow[dl, dotted, shift left] \\ F_1 \arrow[ur, dotted, shift left] & \end{tikzcd}$$

$\exists!$ isomorphism ϕ s.t. $\phi|_D = id_D$.