Lec 4/3

Monday, April 3, 2017 14:57

test paired data distribution of differences

$H_0: \mu = 0$ $H_1: \mu > 0$ using sign test.

X = # of positive results = 14, $n = 19$ (discard 0).

$H_0: \theta = \frac{1}{2}$ vs $H_1: \theta > \frac{1}{2}$ $\theta = \mathbb{P}(\text{positive})$

$X \sim Bin(19, \frac{1}{2})$ $\mathbb{P}(X \geq 14) =$

use normal approximation (usually only use when $n \geq 30$)

$Z = \dfrac{14 - 0.5(19)}{\sqrt{19 \cdot 0.5 \cdot 0.5}} - 2.06$ p-val = $\mathbb{P}(Z \geq 2.06) \overset{0.0199}{\leq} 0.05$, so conclude the drug works.

for one-sample t test (if we assume differences are normally dist)

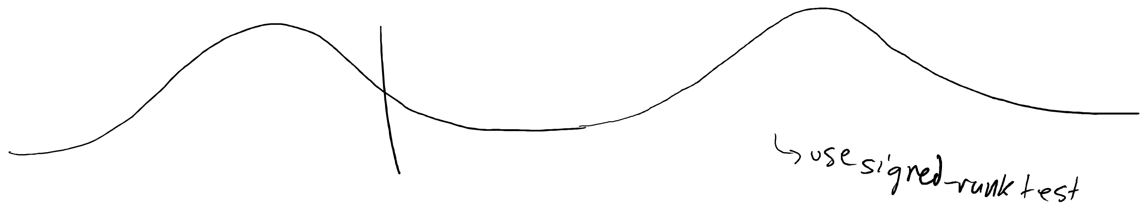
$\bar{X} = 12.9$, $s^2 = 199.42$ $\Rightarrow$ $t = 3.95$ so p-value $\leq 0.01 \Rightarrow$ reject H_0

p-val in NP test is higher so power of NP test is lower

which is better??????????????? !!!!

If X is about $\frac{n}{2}$

Data histogram

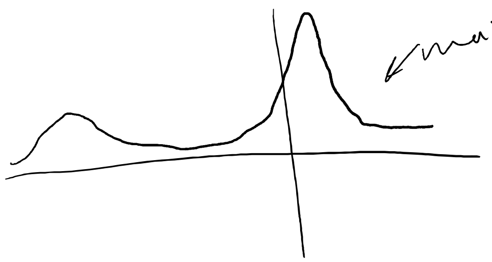


↳ use signed-rank test

0

$\mu > 0$ but $X \approx \frac{n}{2}$ soparametric test is better.

but if data not normal, sign test is better.



makes $\mu < 0$ but sign test says $\mu > 0$.

§16.3 Signed-rank test.

Like sign test but considers magnitude.

Assume population is symmetric about μ.

test $H_0: \mu = \mu_0$ vs $H_1: \mu \neq \mu_0$ or $\gtrless \mu_0$

1: Compute $x_i - \mu_0$ for each x_i

2: rank $|x_i - \mu_0|$ $\forall x_i$ from 1 to n

- Discard diffs of 0
- If there are ties, take the average:

e.g. $|x_i - \mu_0| = $ 2.1 1.7 3.8 2.1
2.5 1 4 2.5

3: Compute T^+ and T^- (sum of ranks for which $x_i - \mu_0 > 0$ and < 0 resp).

$T = \min\{T^+, T^-\}$

Note ①: $T^+ + T^- = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

So if we know one we know the other (and T).

larger $T^+ \Rightarrow$ smaller T etc.

(2) T, T^+, T^- are discrete RVs.

When assuming pop is cts, there are no ties.

Dist of T, T^+, T^- depends only on n.

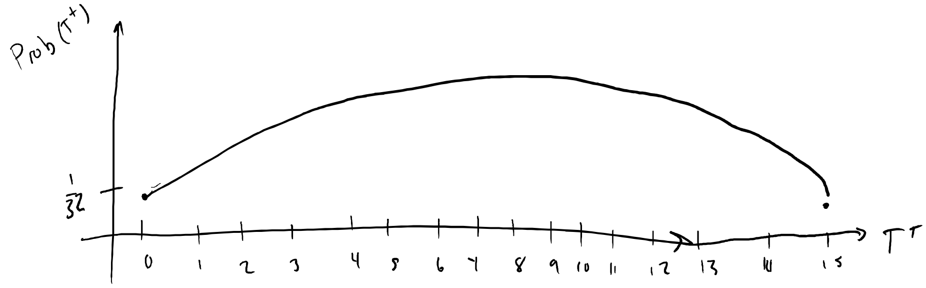
Thus we can precompute the distributions.

$H_0: \mu = \mu_0$ $\quad H_1: \mu \neq \mu_0$ $\quad$ reject H_0 if $T \leq T_\alpha$

$\mu > \mu_0$ $\quad$ reject H_0 if $T^- \leq T_\alpha^- = T_{2\alpha}$ $\quad \Leftarrow \mathbb{P}(T \leq k) = 2\mathbb{P}(T^+ \leq k)$

$\mu < \mu_0$ $\quad$ reject H_0 if $T^+ \leq T_\alpha^+ = T_{2\alpha}$

ex: $n=5$



$\mathbb{P}(T^+ = 0)$