Lec 11/28

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$SL(2,\mathbb{Z})$ is non-amenable (it contains F_2)

there are non-comm non-amenable groups s.t. (1): $x^p = e \ \forall x \in G$

(2): if $xy \neq yx$ then $\langle x, y \rangle = G$.

Look for ramsey-theory result on free semigroup (infinite tree).

Convexity Packet:

Convex body: compact convex $B \subseteq \mathbb{R}^n$ w/ non-empty interior.

a line thru an interior point intersects bdry at 2 points

extreme points. Krein-Milman

Ex: T/F the set of extreme points of a convex body is closed.

Ex: $\forall$ polytope $P \subseteq \mathbb{R}^n$, $x \in P$, there is a face of P where the altitude from x intersects the face.

Theorem: Let $n_k \nearrow \infty$, $n_k \in \mathbb{N}$. Then for a.e. $x \in \mathbb{R}$,

$(n_k x)$ is u.d. mod 1.

The following sequences are dense mod 1 $\forall \alpha \notin \mathbb{Q}$.

$n\alpha$, $n^2\alpha$, $p_n\alpha$, $p_n^3\alpha$

$\overline{\{2^n 3^m \alpha : n, m \in \mathbb{N}\}} = [0,1]$.

↑

Fürstenbürg

$(\log n)\cdot\sqrt{2}$ is dense but not u.d.

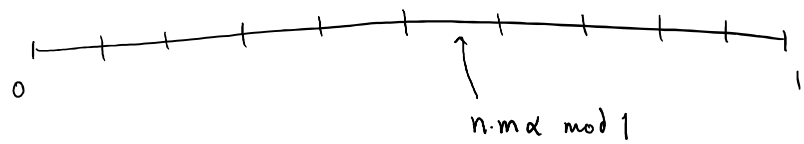
for $n^2\alpha$: use VdW in $\mathbb{Z}^2$.

denseness

Remark: it is enough to show that $\forall \varepsilon > 0$, $\exists n$ s.t. $\|n^2\alpha\| < \varepsilon$.

$\left(\|n_0^2\alpha\| < \varepsilon \Rightarrow \{n_0^2\alpha, 4n_0^2\alpha, 9n_0^2\alpha, \dots\} \text{ is } \sqrt{\varepsilon}\text{-dense in } [0,1] \right)$

$\forall \varepsilon > 0$ (say $\varepsilon = \frac{1}{10}$ for convenience) create a coloring of (n,m) as follows:



by vdw, $\exists$ $(n, m+d)$ $(n+d, m+d)$ in one color

(n, m) $(n+d, m)$

$$(n+d)(m+d)\alpha - (n+d)m\alpha - n(m+d)\alpha + nm\alpha = d^2\alpha$$

So $\|d^2\alpha\| < \varepsilon$

Same pf works for $(n^3\alpha)$.