

Monic: left cancellation.

$$C \begin{array}{c} \xrightarrow{g_1} \\ \xrightarrow{g_2} \end{array} A \xrightarrow{f} B$$

$$f g_1 = f g_2 \Rightarrow g_1 = g_2$$

Epic: Right Cancellation

$$A \xrightarrow{f} B \begin{array}{c} \xrightarrow{h_1} \\ \xrightarrow{h_2} \end{array} C$$

$$h_1 f = h_2 f \Rightarrow h_1 = h_2$$

Basic Facts

$$A \xrightarrow{f} B \xrightarrow{g} C$$

$$f, g \text{ monic (epic)} \Rightarrow gf \text{ monic (epic)},$$

$$gf \text{ monic (epic)} \Rightarrow f \text{ monic (g epic)}.$$

$$A \begin{array}{c} \xrightarrow{f} \\ \xleftarrow{g} \end{array} B$$

if $gf = 1$ then f is monic & g is epic.

Example: in Set, R-mod, Grp, monic \Leftrightarrow injective & epic \Leftrightarrow surjective.

pf (\Rightarrow) • suppose $f: A \rightarrow B$ is not injective. let $C = \text{Ker}(f)$. $C \neq 0$.
(R-mod) let $g: C \hookrightarrow A$. Then $fo = fg$ but $0 \neq g$.
...

(R-mod) let $g: C \hookrightarrow A$. Then $fo = fg$ but $o \neq g$.

• Suppose $f: A \rightarrow B$ is not surjective. let $I = \text{Im}(f)$. let

$g: B \rightarrow B/\text{Im}(f)$. Then $gf = 0f$ but $g \neq 0$ since $\text{Im}(f) \neq B$.
!!
 $\text{coker}(f)$.

For groups, the first argument works but the second doesn't (coker is not necessarily in the category).

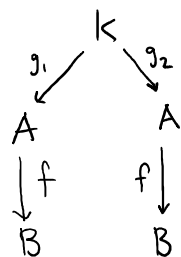
Example: in Ring monic \Leftrightarrow injective and epic \Leftrightarrow surjective but epic \neq surjective.

pf (\Rightarrow) suppose $f: A \rightarrow B$ not injective. we form the product ring

$A \times A$ and consider $K = \{(a_1, a_2) \in A \times A \mid f(a_1) = f(a_2)\}$.

\cup
 $D = \{(a, a) \mid a \in A\}$.

maps $g_1, g_2: K \rightarrow A$ are projections onto 1st & 2nd factors.



Then $fg_1 = fg_2$ but $g_1 \neq g_2$

since $\exists (a_1, a_2) \in K \setminus D$.

$$\begin{array}{cc} g_1 \downarrow & g_2 \downarrow \\ a_1 & a_2 \end{array}$$

Counterexample: $\mathbb{Z} \hookrightarrow \mathbb{Q}$ is epic.

slight generalization: R - any comm ring
 \cup
 S - any mult. closed subset

$R \hookrightarrow S^{-1}R$ is epic but not necessarily surj.