Lec 8/30

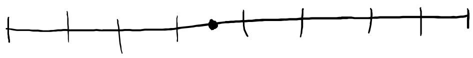
Wednesday, August 30, 2017 14:10

function from HW Ch4

$$f(x) = \frac{x^n (1-x)^n}{n!} = \frac{1}{n!} \sum_{i=n}^{2n} c_i x^i$$

Diophantine approximation

$\left|x - \frac{p}{q}\right| < \varepsilon$ is okay, $\left|x - \frac{p}{q}\right| < \frac{1}{cq^n}$ larger $c, n \to$ better approximation



Dirichlet: $\exists$ inf. many $\frac{p}{q}$ s.t. $\left|x - \frac{p}{q}\right| < \frac{1}{2p^2}$ (for any $x \notin \mathbb{Q}$)

remember

A. Hurwitz

Pell's eqn: $x^2 - Dy^2 = 1$ where $D \in \mathbb{N}$ not a square

remember

(*)

$\forall x \notin \mathbb{Q}$, $\exists$ inf. many $\frac{p}{q}$ s.t. $\left|x - \frac{p}{q}\right| < \frac{1}{\sqrt{5}\, q^2}$

Exercise: this doesn't always work for $x \in \mathbb{Q}$ (see if any such inequality would even work)

maybe next toughest class is quadratic irrationals.

Exercise: this can't be improved for $x = \varphi$

Hint: continued fraction for golden mean φ.

Exercise: What other x for which (*) can't be improved

What's really bad is the tail of continued fraction

If we don't consider such numbers, how good can we get?

$$\left|x - \frac{p}{q}\right| < \frac{1}{q^3}$$

$$\sum \frac{1}{10^{n!}} \notin \mathbb{A}$$

Algebraic #s are countable. I know this.

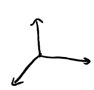
(bonus) <u>Exercise</u>: Are normal numbers transcendental?

Legendre polynomials

$$f(x) = \frac{x^n (1-x)^n}{n!} = \frac{1}{n!} \sum_{i=n}^{2n} c_i x^i$$

$$\bar{a}, \bar{b} \in \mathbb{R}^n, \quad \langle \bar{a}, \bar{b} \rangle = \sum a_i b_i$$

Lin alg: Like orthonormal bases



$$\|\bar{a}\| = \sqrt{\sum a_i^2}$$

$$\det \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix} \begin{matrix} \leftarrow \bar{a}_1 \\ \\ \leftarrow \bar{a}_n \end{matrix}$$

$$\leq$$

$$\prod_{i=1}^{n} \|\bar{a}_i\|$$

[illegible] volume? when a_i orthogonal)

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Hadamard's Inequality (what is maximal volume? when a_i orthogonal)

$C[0,1]$: $\langle f,g\rangle = \int_0^1 fg$

$1, x, x^2, \dots$

orthonormalize polynomials to get Legendre's polynomials.

$\zeta(2) \notin \mathbb{Q}$, $\quad \zeta(s) = \sum \frac{1}{n^s}$

$\zeta(4) \notin \mathbb{Q}$

$\zeta(3) \notin \mathbb{Q}$ (Apéry)

↳ unknown value

Proofs of irrationality of $\sqrt{2}$

1. $\sqrt{2} = \frac{p}{q} \Rightarrow p^2 = 2q^2 \Rightarrow \dots$

2. $\underbrace{(\sqrt{2}-1)^n}_{= a_n} \longrightarrow 0$

$\{a+b\sqrt{2} \,;\, a,b \in \mathbb{Z}\} = R$ is a ring,
so $(\sqrt{2}-1)^n \in R$. if $\sqrt{2}$ ratl, $\sqrt{2} = \frac{p}{q}$
$(\sqrt{2}-1)^n = c + d\frac{p}{q} = \frac{cq}{q} + \frac{dp}{q} = \frac{cq+dp}{q} > \frac{1}{q} \not\to 0$

$a_{2^n} = (\sqrt{2}-1)^{2^n}$

$a_2 = 3-2\sqrt{2}$

$a_{2^n} = x_n\sqrt{2} + y_n \quad , x_i, y_i \in \mathbb{Z}$.

3. $\sqrt{2} = 1 + \cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cdots}}}$

4. Via criterion for irreducibility of polynomials

5. Possible last non-zero digits of n^2 in base 10 $1, 4, 9, 6, 5$

$2n^2$ $2, 8, 8, 2, 0$

in base 3: n^2 : 1

$2n^2$: 2

So p^2 ends in a diff digit than $2q^2$.

Exercise Show that n^2 ends in 1 (before 0's) in base 3.

$4n+1$ $4n+3$

$an+b$

$6n+1$ $6n+5$

$5n+1$ $5n+2$ $5n+3$ $5n+4$

Iff $p = 4n+1$, $p = x^2 + y^2$ (Fermat)

$\forall n \in \mathbb{N}$, $n = x_1^2 + x_2^2 + x_3^2 + x_4^2$ (Lagrange)