Lec 8/25

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V/F V is a vector space over the field F.

Def suppose $\phi \neq S \subset V$. S is a subspace of V if $\forall a,b \in S$, $a+b \in S$ and $\forall \lambda \in F$, $\lambda a \in S$.

Cor. If S is a subspace of V/F, it is a V.S. over F also (wrt $+$ and $\cdot$ inherited from V).

Two conditions of subspace $\Leftrightarrow$ $(*)$: $\forall a,b \in S$, $\forall \alpha,\beta \in F$, $\lambda = \underbrace{\alpha a + \beta b}_{\text{linear Combination}} \in S$.

Subspaces are not arbitrary amorphous. They are thicc.

Let V/F. Let $\{a_1, \dots, a_n\} \subset V$. $\{\alpha_1 a_1 + \dots + \alpha_n a_n \mid \alpha_i \in F \ \forall i\}$ is a subspace.

This subspace is $S(a_1, \dots, a_n)$, the subspace generated by $\{a_1, \dots, a_n\}$.

What is $S(a_1, a_2, \dots)$?. it is $\bigcup_{n=0}^{\infty} \{\alpha_1 a_{i_1} + \dots + \alpha_n a_{i_n} \mid \alpha_j \in F \ \forall j,\ i_1 < i_2 < \dots < i_n \in \mathbb{N}\}$

Example

$$\begin{aligned}
\alpha_{11} x_1 + \dots + \alpha_{1n} x_n &= 0 \\
\alpha_{21} x_1 + \dots + \alpha_{2n} x_n &= 0 \\
\vdots \qquad\qquad & \ \ \vdots \\
\alpha_{m1} x_1 + \dots + \alpha_{mn} x_n &= 0
\end{aligned}$$

$\alpha_{ij} \in \mathbb{R}$

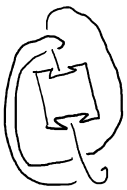
$X = (x_1, \dots, x_n) \in \mathbb{R}^n$

if x, y are solutions, $x+y$ is a soln. and if $\lambda \in \mathbb{R}$, λx is a soln.

Solutions $S \subseteq \mathbb{R}^n$.

$$S = S(v_1, \dots, v_k)$$

$\mathbb{R}^n = S(e_1, \dots, e_n)$ Where $e_i = (0, \dots, 0, \underset{i\text{th position}}{1}, 0, \dots, 0)$



$\alpha_0 y^{(n)} + \alpha_1 y^{(n-1)} + \cdots + \alpha_n y = 0 \qquad y \in \underbrace{C^n(\mathbb{R})}$

Infinite dimensional,
but space of solns has
finitely many generators.

Linear Dependence:

$A = \{a_1, \ldots, a_n\} \subset V$ is a set of linearly dependent Vectors if for some i,

$a_i = \alpha_1 a_1 + \cdots + \alpha_{i-1} a_{i-1} + \alpha_{i+1} a_{i+1} + \cdots + \alpha_n a_n$ for some α_i s.

$\Updownarrow$

$\alpha_1 a_1 + \cdots + \alpha_n a_n = 0$ for some α_i not all 0.

Def $A = \{a_1, \ldots, a_n\}$ is lin. indp. if no a_i is a liner combo of the rest

$\Updownarrow$

$\lambda_1 a_1 + \cdots + \lambda_n a_n = 0 \Rightarrow \lambda_1 = \cdots = \lambda_n = 0$.

Ex: $a = (1,2)$ $b = (2,1)$ are lin. indp in $\mathbb{R}^2$.

$\mu_1 a + \mu_2 b = (\mu_1 + 2\mu_2, 2\mu_1 + \mu_2) = 0 \Rightarrow \mu_1 + 2\mu_2 = 0$

$2\mu_1 + \mu_2 = 0$

$3(\mu_1 + \mu_2) = 0 \Rightarrow \mu_1 + \mu_2 = 0$

subtract from each $\Rightarrow \mu_1 = \mu_2 = 0$.

$\{a, b\}$ is lin. dep. iff $a = \lambda b$.

$\{\sin x, \cos x\} \subset C^\infty(\mathbb{R})$

$\lambda \sin x + \mu \cos x = 0 \qquad \forall x \in \mathbb{R}$.

take $x = 0 \Rightarrow \mu = 0$. take $x = \pi/2 \Rightarrow \lambda = 0$. the set is lin indp.

$\Rightarrow \mu = 0$. take $x = \pi/2 \Rightarrow \lambda = 0$. the set is lin indp.

how about: $\{ \sin x, \cos x, \sin(2x), \cos(2x), \dots \sin(nx), \cos(nx), \dots \}$

exercise: this is lin. indp.