Lec 9/6

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$\alpha_{ij} \in \mathbb{F}$

$$(*) \begin{cases} \alpha_{11}x_1 + \cdots \alpha_{1n}x_n = 0 \\ \vdots \qquad\qquad \vdots \\ \alpha_{m1}x_1 + \cdots + \alpha_{mn}x_n = 0 \end{cases}$$

$\mathcal{N}$ = subspace of solutions $\subseteq \mathbb{F}^n$

$(*)_{nh} = (*)$ but replace 0 by β_i.

$x = (x_1, \ldots, x_n) \in \mathbb{F}^n$

$y = (y_1, \ldots, y_n) \in \mathbb{F}^n$

Solns of $(*)_{nh}$ but $x+y$ not.

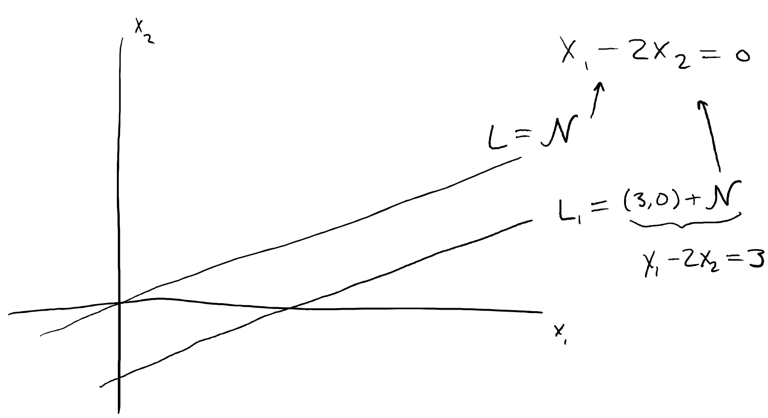
So set of solns of $(*)_{nh}$ not a subspace

Let $v = (\gamma_1, \ldots, \gamma_n)$ solve $(*)_{nh}$.

then any other soln w of $(*)_{nh}$ is $v + u$ when u solves $(*)_{nh}$

since $w - v$ is soln of $(*)$.

$v + \mathcal{N} = \{ v + x : x \in \mathcal{N} \}$ is a <u>linear manifold</u> in $\mathbb{F}^n$.



If W is a lin mfd in $\mathbb{F}^n$. Take $w, a \in W$. then $w - a \in S$.

$W = v + S \quad S = ?$

$S = W - a$ for any $a \in W$.

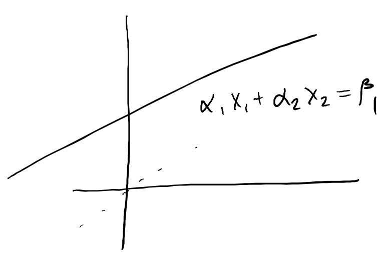
$= \{w - a : w \in W\}$

S is the Directing subspace of W.

$\dim W = \dim S$

Now go backwards from lin mfd to system of eqns:

Q: given W lin mfd s.t. $\dim W = d$. Find a system of non-hom lin. eqns s.t. W is its set of solutions.



First lets assume W is a subspace $S \subseteq \mathbb{F}^n$.

Let $\{b_1, \ldots, b_d\}$ be a basis for W.

$\Rightarrow$ $b_1, \ldots, b_d$ are solns of the system.

$b_1 = (\beta_{11}, \beta_{12}, \ldots, \beta_{1n}), \ldots, b_d = (\beta_{d1}, \ldots, \beta_{dn})$

$r_1 = (\alpha_1, \dots, \alpha_{1n}), \dots, r_m = (\alpha_{m1}, \dots, \alpha_{mn})$ are rows of eqn system.

$r_i \cdot b_j = 0 \quad \forall i, j$ gives system of equations.

Now we can solve for r_i to obtain system of eqns.

solve system $\begin{cases} b_1 \cdot x = 0 \\ b_2 \cdot x = 0 \\ \vdots \\ b_d \cdot x = 0 \end{cases}$ to get system whose solution is $S(\underbrace{b_1, \dots, b_d}_{\text{basis.}})$.

we get $n-d$ equations.