Lec 2/16

Friday, February 16, 2018 14:22

$\{e^{i\theta} : \theta \in [0,2\pi]\}$

<u>Defns</u> Let X be a top. sp. A loop in X is a continuous map from S to X

To say J is a Jordan curve in X means J is the image of a simple (1-1) loop in X.

Jordan Curve Theorem.

Let J be a Jordan curve in $\mathbb{C}$. Then:

a) $\mathbb{C} \setminus J$ has exactly two components, one bounded & one unbounded

b) J is the boundary of each component of $\mathbb{C} \setminus J$.

<u>Theorem</u> Let γ be a simple loop in $\mathbb{C}$ and let J be the range of γ. For every a in the bounded component of $\mathbb{C} \setminus J$, the winding number of γ wrt a is 1 or -1 (and is the same for all such a).

<u>Theorem</u> Let J be a Jordan curve in $\mathbb{C}$. Then each point of J is accessible from each component of $\mathbb{C} \setminus J$ (meaning if $a \in J$, $b \in \mathbb{C} \setminus J$, then $\exists \gamma : [0,1] \longrightarrow \mathbb{C}$ s.t. $\gamma(0) = a$, $\gamma(t) \in \mathbb{C} \setminus J$ $\forall t > 0$, and $\gamma(1) = b$).

The Schoenflies Extension Theorem

Let γ be a simple closed loop in $\mathbb{C}$. Then there is a homeomorphism from $\mathbb{C}$ onto $\mathbb{C}$ whose restriction to S^1 is γ.

Schoenflies's Converse to the Jordan Curve theorem

Let J be a cpt $\subseteq \mathbb{C}$ s.t. $\mathbb{C} \setminus J$ has exactly two components (one bdd, one unbdd) and every point of J is accessible from each component of $\mathbb{C} \setminus J$.

then J is a Jordan curve.

<u>Thm</u> (Brouwer, 1911)

Let J be the range of a 1-1 cts map $f: S^{n-1} \to \mathbb{R}^n$. Then $\mathbb{R}^n \setminus J$ has exactly two components (one bdd, one unbdd), and each point of J is accessible from each component of $\mathbb{R}^n \setminus J$. Furthermore, $\forall a \in$ the bdd component of $\mathbb{R}^n \setminus J$, the map $f_a: S^{n-1} \to S^{n-1}$ defined by $f_a(x) = \dfrac{f(x)-a}{\|f(x)-a\|}$ has degree equal to 1 or -1 (the degree is the same $\forall$ such a).

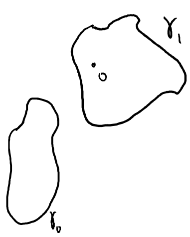
<u>Defns</u> Let X and Y be top. sps. Let $f_0, f_1 : X \to Y$ be continuous

a) to say H is a homotopy in Y from f_0 to f_1 means H is a cts map from $X \times [0,1]$ to Y such that $\forall x \in X$, $H(x,0) = f_0(x)$, $H(x,1) = f_1(x)$

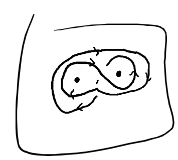
b) To say f_0 is homotopic to f_1 in Y means $\exists$ a homotopy in Y from f_0 to f_1

$$H: f_0 \underset{\text{in } Y}{\simeq} f_1$$

eg



$Y = \mathbb{C} \setminus \{0\}$. γ_0 is not homotopic to γ_1.



bad

$n(\pm 1) = 0$ but not homotopic to 0 in $\mathbb{C} \setminus \{\pm 1\}$.

<u>Theorem</u> Let X and Y be top. sp. Then on the set $C(X,Y)$ of all continuous maps $X \to Y$, the relation of being homotopic in Y is an equivalence relation.

<u>Notation</u> $\pi(X,Y)$ is the set of homotopy classes in $C(X,Y)$

<u>Thm</u> Compositions of homotopic maps are homotopic. Specifically, let X, Y, Z be top. sp. let $f_0, f_1 : X \to Y$ which are homotopic in Y, let $g_0, g_1 : Y \to Z$ which are homotopic in Z. Then $h_0 = g_0 \circ f_0$ and $h_1 = g_1 \circ f_1$ are homotopic in Z.

$$((f_0 \simeq f_1,\ g_0 \simeq g_1) \Longrightarrow h_0 \simeq h_1)$$

<u>Proof</u> Let $F : X \times [0,1] \to Y$ be a homotopy from f_0 to f_1 in Y, let $G : Y \times [0,1] \to Z$ be a homotopy from g_0 to g_1 in Z. Let $H : X \times [0,1] \to Z$ be defined by

$H(x,t) = G(F(x,t),t)$. Then H is a homotopy from h_0 to h_1 in Z. (check this) $\square$