Lec 10/21

Friday, October 21, 2016 9:08 AM

Midterm: ch 8, 9, 10, 11, 12, 22

<u>Problem</u> Graph (indicating important features) $f(x) = \frac{x}{1+x^2}$.

$$f'(x) = \frac{1+x^2 - 2x^2}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2}$$

$$f''(x) = \frac{-2x(1+x^2)^2 - (1-x^2)\,2(1+x^2)\,2x}{(1+x^2)^4}$$

$$= \frac{-2x\big((1+x^2) + 2(1-x^2)\big)}{(1+x^2)^3}$$

$$= \frac{-\cancel{2x}\ 2x^3 \cancel{\ldots}^2}{(1+x^2)^3} = (3-x^2)(-2x)$$

$f(1) = \frac{1}{2}$

$f(-1) = \frac{-1}{2}$

$f'(x) = 0 \Rightarrow 1-x^2 = 0 \Rightarrow x = \pm 1$

$f'(x)$ has same sign as $(1-x^2)$, so $f'(x) < 0$ if $x \in \underbrace{(-\infty, 1) \cup (1, \infty)}_{f \text{ decreasing}}$

$f'(x) > 0$ if $x \in (1, 1)$ ← f increasing.

$f''(x) = 0 \Rightarrow (3-x^2)(-2x) = 0 \Rightarrow x = \underbrace{0, \pm\sqrt{3}}$ inflection pts.

$f(\sqrt{3}) = \frac{\sqrt{3}}{4}$

$f(-\sqrt{3}) = -\frac{\sqrt{3}}{4}$

$f''(x) < 0$ if $x \in (-\infty, -\sqrt{3})$ → concave

> 0 $x \in (-\sqrt{3}, 0)$ → convex

< 0 $x \in (0, \sqrt{3})$ → concave

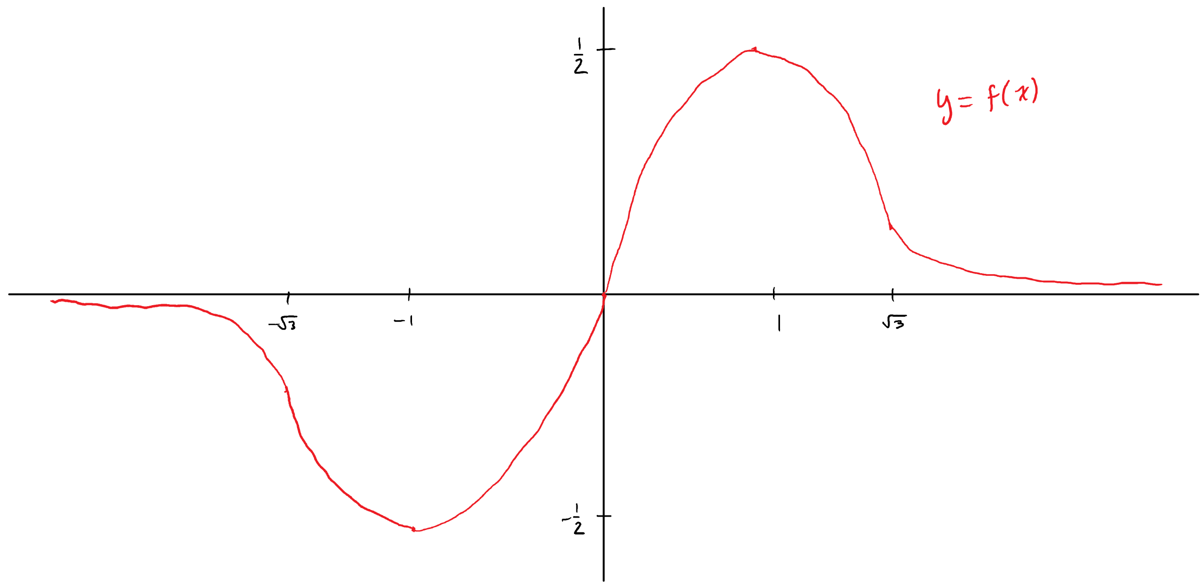
> 0 $x \in (\sqrt{3}, \infty)$ → convex

P.S. $f(0) = 0$

$f(-x) = -f(x)$

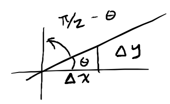
No vertical asymptotes: denominator > 0

$$\lim_{x \to \pm\infty} \frac{x}{1+x^2} = \lim_{x \to \pm\infty} \frac{x}{x^2(1+\frac{1}{x^2})} = \lim_{x \to \pm\infty} \frac{1}{x(1-\frac{1}{x^2})} = 0. \quad y = 0 \text{ horiz asymp}$$



What is the maximal angle through which the graph of $f(x)$ can be rotated counterclockwise and remain the graph of a function (same for clockwise).

$\tan(\theta) = \text{slope} = m$



ccw $\frac{\pi}{2} - \theta \to$ line is vert.

cw $\frac{\pi}{2} + \theta \to$ line is vert

$$f'(x) = \frac{1 - x^2}{(1 + x^2)^2}$$

$f''(x) = 0$ when $x \in \pm\sqrt{3}, 0$

Rotated graph is a function iff any line of slope m intersects the original graph in at most one point.

$f'(0) = 1$

$f'(\sqrt{3}) = \frac{1-3}{(1+3)^2} = \frac{-2}{16} = \frac{-1}{8}$

Data point: MVT $\Rightarrow$ if $y = mx + b$ intersects at 2 pts, then $f'(x) = m$ has a solution.

So the graph can be rotated CCW $\frac{\pi}{4}$ radians.

cw . . .

Take-home midterm problem.

$$r \sin\theta = \frac{r\cos\theta}{1 + r^2\cos^2\theta}$$

$$r\sin\theta + r^3 \sin\theta \cos^2\theta = r\cos\theta$$

$$r^2 = \cos\theta - \sin\theta$$

$$r\sin\theta + r^3\sin\theta\cos^2\theta = r\cos\theta$$

$$r^2 = \frac{\cos\theta - \sin\theta}{\sin\theta\cos^2\theta}$$