Lec 11/22

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<u>Adjoint:</u> V fd vs over F, $B : V \times V \longrightarrow F$ bilinear form

$T : V \longrightarrow V$. Adjoint T' of T wrt B satisfies

$$B(Tx, y) = B(x, T'y).$$

$\boxed{\text{char } F \neq 2}$

If $\eta \in O(V, Q)$ then $\eta^{-1} = \eta'$. $\left(B(\eta x, y) = B(\eta^{-1}\eta x, \eta^{-1} y) = B(x, \eta^{-1} y) \right)$.

In fact, $TT' = 1 \Rightarrow T \in O(V,Q)$ as well. $\left(B(Tx, Ty) = B(x, T'Ty) = B(x,y) \right)$.

<u>Prop</u> (i) if $T : V \longrightarrow V$, $\mathcal{U} \subset V$, $T(\mathcal{U}) \subset \mathcal{U}$,

Then $T'(\mathcal{U}^\perp) \subset \mathcal{U}^\perp$.

(ii) if in addition T is orthogonal, $T(\mathcal{U}^\perp) \subset \mathcal{U}^\perp$ as well.

<u>Pf</u> If $v \in \mathcal{U}^\perp$, $B(u, T'v) = B(Tu, v) = 0$, proving (i).

if $T \in O(V,Q)$, $T(\mathcal{U}) = \mathcal{U}$. so $T^{-1}(\mathcal{U}) = \mathcal{U}$.

<u>Def</u> $V = \mathcal{U}_1 \perp \cdots \perp \mathcal{U}_r$ (orthogonal direct sum) if $V = \mathcal{U}_1 \oplus \cdots \oplus \mathcal{U}_r$ & $\mathcal{U}_i \perp \mathcal{U}_j$.

In this case, $Q(\sum_1^r x_i) = \sum_1^r Q(x_i)$.

<u>Def</u> $\mathcal{U} \subset V$ is isotropic if it contains $\overset{\neq 0}{u}$ s.t. $Q(u) = 0$ (an isotropic vector).

$\mathcal{U} \subset V$ is totally isotropic if $Q|_{\mathcal{U}} = 0$.

<u>Def</u> a 2d vs V is called a <u>Hyperbolic plane</u> if it is non-degenerate & isotropic.

Def a pair (u,v) is hyperbolic if $B(u,v)=1=B(v,u)$ & $B(u,u)=0=B(v,v)$.

eg a hyperbolic base $(u,v) \rightsquigarrow Q(xu+yv)=xy$ (Hyperbolic)

($Q(xu+yv)=x^2+y^2$ would be cartesian).

Thm The following conditions on a 2d vs V w/ Q are equivalent

(i) V is a Hyperbolic Plane

(ii) V has a hyperbolic base

(iii) $\operatorname{discr} B = -1 F^{*2}$.

So, up to isometry, there is only one hyperbolic plane.

Thm The rotation gp $O^+(V,Q)$ is isomorphic to F^*.

* in fact, they look like $\begin{bmatrix} a & 0 \\ 0 & a^{-1} \end{bmatrix}$ in a hyperbolic base

* Also, every improper trans $\begin{bmatrix} 0 & a \\ a^{-1} & 0 \end{bmatrix}$ is a reflection S_{u-av}.