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Wednesday, September 18, 2019 15:01

Gauss Integral formula:

$\gamma, \eta : S^1 \longrightarrow \mathbb{R}^3 \qquad K = \gamma(S^1), \quad R = \eta(S^1) \quad$ Knots,

then $$Lk(K, R) = -\frac{1}{4\pi} \int_0^1 \int_0^1 \left(\frac{d\vec{\gamma}(t)}{dt} \times \frac{d\vec{\eta}(s)}{ds} \right) \cdot \frac{(\vec{\gamma}(t) - \vec{\eta}(s))}{\| \vec{\gamma}(t) - \vec{\eta}(s) \|^2} \, dt\, ds.$$

Cohomology

M = closed orient mfd

$$H_k(M) \xrightarrow[\cong]{P} H^{m-k}(M)$$

Poincare duality

$a \in H_k(M)$, $b \in H_{m-k}(M)$

$$a \cdot b = P^{-1}(P(a) \cup P(b)) \in H_0(M) = \mathbb{Z}$$

$P(a) \in H^{m-k}$, $P(b) \in H^k$, $P(a) \cup P(b) \in H^m$

$$= \langle P(a) \cup P(b), [M] \rangle$$

intersection Product

= intersection #s.

$A, B \hookrightarrow M \qquad a + b = m$

j^A, j^B

$$J_*^A([A]) \cdot J_*^B([B]) = I(A,B)$$

<u>Diff'l forms</u> $\quad A \hookrightarrow N$

Θ - any $\overset{\dim A}{\overset{\|}{a}}$-form in $\Omega^a(N)$

$$\int_A \Theta = \int_N \Theta \wedge \xi_A \qquad \left(\xi_A \in \Omega^{n-a}(N)\right)$$

if $\quad A, B \hookrightarrow M, \quad a+b=m$

$$I(A,B) = \int \xi_A \wedge \xi_B$$

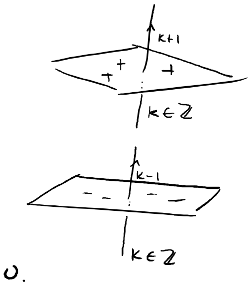
$\dim K + \dim L = \dim M - 1 \qquad$ (M = homology sphere)

$$\underset{\underset{\Omega^{m-k}}{\cap}}{\xi_K} = d\rho_K \qquad \underset{\underset{\Omega^{m-\ell}}{\cap}}{\xi_L} = d\rho_L$$

$$Lk(K,L) = \int \underbrace{\rho_K \wedge d\rho_L}_{m-k-\ell-1+m\ =\ m} = \int \rho_L \wedge d\rho_K .$$

<u>Lemma</u> If $K \sqcup R$ is a 2-comp link in S^3 and $Lk(K,R)=0$, then K has a seifert surface Σ_K disjoint from R.

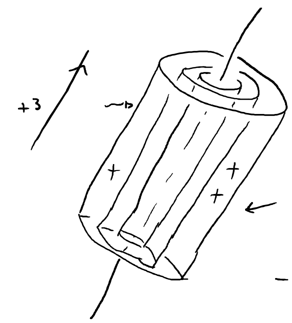
Pf Pick Seifert surface Σ° for K, $\Sigma^\circ \pitchfork R$.



Label intervals of R with integers s.t.:

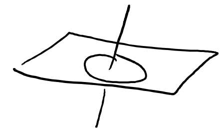
then the labeling is consistent since $Lk(K,R)=0$.

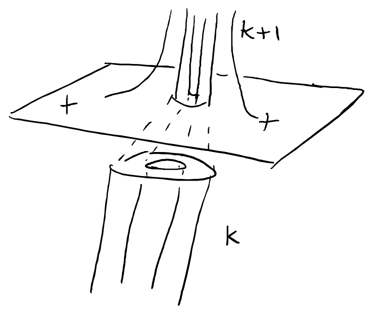
if an interval is labeled k, wrap it in k tubes:



$+$ if $k>0$

$-$ if $k<0$

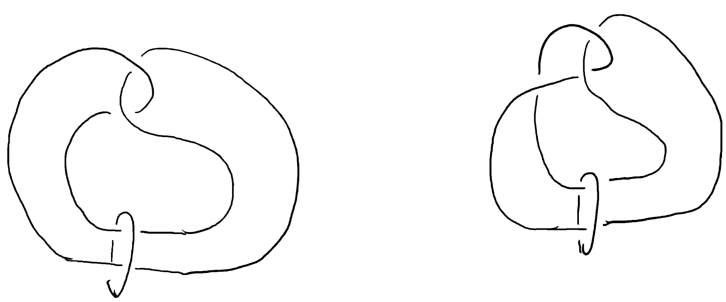
cut disks from  and glue in tubes:



then R doesn't intersect the modified Seifert surface. $\square$

[It is generally not possible to get Σ_K and Σ_R to be disjoint.]

eg



$L = K_1 \sqcup K_2 \qquad Lk(K_1, K_2) = 0$

$\{K_1\} \subseteq \pi_1(S^3 - K_2, *)$

$\uparrow$

conjugacy class, lives in commutator $[\pi_1, \pi_1]$

Since it is 0 in homology ($0 = \mathrm{Lk}$).

$L = K_1 \cup K_2 \cup K_3$

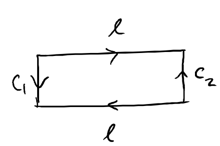
$\Sigma_1 \quad \Sigma_2 \quad \Sigma_3$ not disjoint.

<u>Composition & factorization</u> $\#$

K_1, K_2

$S^3 = B_1^3 \cup_{S^2} B_2^3$

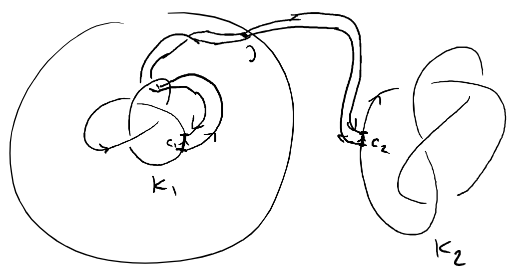
$*$ Place $K_i \hookrightarrow B_i$

$*$ Strip $R = [0,1]^2 =$ 

Embed $R \longrightarrow S^3$ s.t.

$*$ $R \cap K_i = c_i$ (so $\operatorname{int} R \subset S^3 - K_1 \cup K_2$) (orientation too

$\ast$ $R \cap S_2$ in exactly an interval $J \subset S^2$



remove c_1° & c_2° from K_1 & K_2, add ℓ°-segments.

<u>Lemma</u> $\underbrace{[K_1 \#_R K_2]}_{\substack{\text{ambient} \\ \text{isotopy equiv.} \\ \text{class}}}$ does not depend on $R \hookrightarrow S^3$.

(only depends on $[K_1]$ & $[K_2]$).

<u>Corollary</u> $[K_1 \#_R K_2] = [K_1] \# [K_2]$

$\uparrow$ well-defined composition

$\rightsquigarrow$ Monoid (identity is unknot).