Lec 9/21

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Homework: Submit the best 7 problems from end of chapter 6.

Ex: is it true that $A \cup A-1 = S \Rightarrow d(A) = \frac{d(S)}{2}$? (also consider $S = \mathbb{N}$).

d – additive largeness measure

d_x – multiplicative largeness measure.

"multiplicative version" of $3\mathbb{N}$: $\{n = 2^{c_1} 3^{c_2} 5^{c_3} \cdots p_k^{c_k} : \sum c_i \in 3\mathbb{N}\}$.

E_x: all n w/ $\sum c_i \in 2\mathbb{N}$.

O_x: all n w/ $\sum c_i \in 2\mathbb{N}-1$

Li Yi Theorem: $d_x(E_x) = d_x(O_x) = \frac{1}{2}$ (Exercise)

in fact, $d(E_x) = d(O_x) = \frac{1}{2}$!!!

Recall: $\bar{d}(S) = \bar{d}(S-t) = \bar{d}(S+t)$

Now: $\bar{d}_x(S) = \bar{d}_x(S/t) = \bar{d}_x(tS)$ (Exercise)

Cancellative Semigroup: Semigroup where $ax = ay \Rightarrow x = y$.

Non-cancellative semigroup: $(\mathbb{Z}/6\mathbb{Z}, \cdot)$, anything with a 0.

$\mathcal{F} =$ all finite subsets of $\mathbb{N}$.

① $A \cup B$

② $A \cap B$

③ $A \triangle B$

Ex: Let A be a finite n-element set.
then $\mathcal{P}(A)$ is a 2^n element group wrt operation $\triangle$.

$S = \{3^n : n \in \mathbb{N}\}$, $\quad S/_2 = \emptyset$

Any set which misses a prime has $d_x = 0$.

Ex. Let $S \subset \mathbb{N}$. define $M_S = \{2^{c_1} 3^{c_2} 5^{c_3} \cdots p_k^{c_k} : \sum c_i \in S\}$.
does $d_x(M_S) = d(S)$? What about for $\bar{d}_x$ and $\underline{d}$?

Ex. $\forall \alpha > 1$ let $S_\alpha = \{\lfloor n\alpha \rfloor : \alpha \in \mathbb{N}\}$. Show $d(S_\alpha) = \frac{1}{\alpha}$

$\alpha, \beta > 1$, $\alpha, \beta \notin \mathbb{Q}$

Ex: Claim: Let $\frac{1}{\alpha} + \frac{1}{\beta} = 1$. Prove $S_\alpha \cup S_\beta = \mathbb{N}$.

Ex: if $\alpha \notin \mathbb{Q}$, S_α does not contain an infinite progression.

Open-ended question: what is $d_x(S_\alpha)$?

Theorem: any finite field has p^n elements for some $n \in \mathbb{N}$, $p \in P$.

know this proof for midterm

Theorem: If F_1, F_2 are finite fields w/ $|F_1| = |F_2|$, $F_1 \cong F_2$.

3 & 2 are nonsquare in $\mathbb{F}_5$ so

$\{a + b\sqrt{3} : a, b \in \mathbb{F}_5\}$ and $\{a + b\sqrt{2} : a, b \in \mathbb{F}_5\}$

are fields. but they both have cardinality 25, so they are isomorphic.

Ex: check that these are fields & that they are isomorphic.

Ex: Create a field w/ p^2 elements $\forall p \in P$. (including $p=2$).

Little Fermat Theorem: $x^{p-1} \equiv 1 \mod p$.

Pf 1: $x^p = (\underbrace{1 + \cdots + 1}_{x \text{ times}})^p \equiv \underbrace{1 + \cdots + 1}_{x \text{ times}} = x \mod p$

So if $x \neq 0$, $x^{p-1} \equiv 1 \mod p$.

(multinomial coefficients are divisible by p). since p is prime. $\square$

Pf 2: for any x, the elements $x, 2x, 3x, \ldots, (p-1)x$ are distinct, and equal (in some order) to $1, 2, \ldots, p-1 \mod p$.

So $x \cdot 2x \cdot 3x \cdot \cdots \cdot (p-1)x \equiv 1 \cdot 2 \cdot 3 \cdot \cdots (p-1) \mod p$.

now everything cancels $(1 \cdot 2 \cdot 3 \cdot \cdots \cdot (p-1) \equiv 1 \mod p)$

So $x^{p-1} \equiv 1 \mod p$.