

$$\bigcup_g gHg^{-1} = \{ghg^{-1} : h \in H, g \in G\} = \bigcup_h G_h.$$

$$|G| = \sum_{\substack{\text{orbits} \\ g}} |G_g|$$

$$|H \cap C| + [G:H] \cdot |H \setminus C| \quad ?$$

$$ghg^{-1} \in fHf^{-1} \cap C \Leftrightarrow h \in g^{-1}fHf^{-1}g \cap C$$

$h$  comm. w/  $g^{-1}f$ , not  $\Rightarrow$  all elts.

$$K = \bigcap_{g \in G} gHg^{-1}, \quad gHg^{-1}/K.$$

$$G/K = \bigcup_{g \in G} gHg^{-1}/K.$$

$$gHg^{-1}/K$$

$$\underline{[G:K] = [H:K] \cdot [G:H]}$$

$$\Rightarrow [G:H] = \# \{gHg^{-1} \mid g \in G\}$$

but  $|gHg^{-1}| = |H|$ , and  $gHg^{-1} \ni 1$ , so

$$G \leq [G:H] (|H|-1)$$

↓

$$X = \{gHg^{-1}\}, \quad G \curvearrowright X \text{ conj.}$$

$$|X| = \frac{|G|}{|\text{Stab}_G(H)|} \leq [G:H] \quad \text{also works.}$$