Lec 3/3

Friday, March 3, 2017 14:54

Recall: p-value = $\mathbb{P}$(observing somthing as $\overset{\text{or more}}{\uparrow}$ extreme than what was observed if H_0 is true)

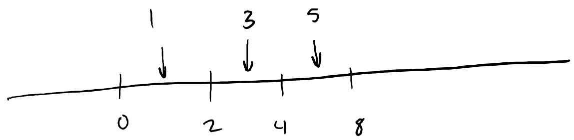
"extreme" defined in terms of alternative hypothesis. (in direction away from H_0).

e.g.

$H_0: \mu = \mu_0$

vs $H_1: \mu \neq \mu_0$

"extremeness" in this case = $|\bar{x} - \mu_0|$.



H_0 : μ is even

H_1 : μ not even

"extremeness" is binary if discrete, and dist to nearest even if cts.

Note: p-value does not give probability H_0 is true.

<u>Interpretation</u>: If p = p-value, when H_0 is true, there is a chance of prob. = p of observing the test statistic as $\overset{\text{or more}}{\nearrow}$ extreme as what was observed.

So if p-value $\leq \alpha$ then we reject H_0.

p-value small $\Rightarrow$ H_0 a poor explanation of data.

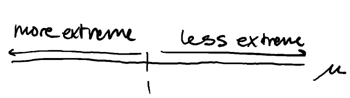
In general, will select a <u>cutoff for p-value</u> ahead of time (before collecting data)

↳ α-value or α-level of the test.

Q: what about testing $H_0: \mu = 1$ vs $H_1: \mu < 1$?

P-value : $\displaystyle\int_{-\infty}^{\bar{x}} \frac{1}{\sqrt{2\pi}\,0.17} \exp\left(-\frac{1}{2(0.17)^2}(t-1)^2\right) dt = \underbrace{0.004}_{}$

from example above.



P-value $= \mathbb{P}(\bar{X} \leq 0.959 \,;\, H_0) = \mathbb{P}(Z \leq -2.65) = 0.004.$

If $\alpha = 0.05$, reject H_0.

Example: the lifetime in hours of a 75-watt bulb is known to be approximately normally distributed w/ $\sigma = 45$ hours.

A RS of 20 bulbs has a mean lifetime $\bar{x} = 1014$ hours. Is this evidence to support the claim that $\mu > 1000$ hours?

$\mathbb{P}(\bar{X} \geq 1014 \,;\, \mu = 1000) = 0.082.$ So No, it is not significant evidence.

Note: always give interpretation of p-value on exam.

So far we've assumed normal w/ σ^2 known. What if σ^2 not known?

If n is large, substitute S^2 for σ^2 (even if dist is not normal).

If n is small & pop is normal, use t distribution for the test

→ if not normal, hopeless.

four steps for tests & Hypotheses.

1) Formulate H_0, H_1

2) (method 1) specify CR of size α

(method 2) specify test stat

3) evaluate test stat $\hat{\theta}$

4) (method 1) see if $\hat{\theta} \in CR$

(method 2) see if p-val for $\hat{\theta} < \alpha$.