Lec 10/8

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$\mathbb{Z}/4 \triangleright \mathbb{Z}/2 \triangleright \{e\}$

$(\mathbb{Z}/2)^2 \triangleright \mathbb{Z}/2 \triangleright \{e\}$

#3(3) $H \underset{\beta}{\overset{\alpha}{\rightrightarrows}} \mathrm{Aut}(N)$ $j : H \longrightarrow N$ s.t. $\alpha(h)(n) = j(h) \cdot \left(\beta(h)(n)\right) \cdot \left(\beta(h)(j(h)^{-1})\right)$

$$\Rightarrow \quad N \rtimes_\alpha H \xrightarrow{\sim} N \rtimes_\beta H$$
$$(n, h) \longmapsto (n \cdot j(h), h)$$

#4

$$\begin{array}{ccc} G \rtimes_c G & & (g_1, g_2) \\ \wr\downarrow & & \downarrow \\ G \times G & & (g_1 g_2, g_2) \end{array}$$

#14 $\mathrm{Aut}(\mathbb{Z}/2 \times \mathbb{Z}/4)$

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all options ok — None ok — all ok.

$$\left\{ \begin{array}{l} (1,0) \longmapsto (1,0) \text{ or } (0,2) \text{ or } (1,2) \\ (0,1) \longmapsto (0,1) \text{ or } (1,1) \text{ or } (0,3) \text{ or } (1,3) \end{array} \right\}$$

So there are 8 elements.

$\mathrm{Aut}(S_n) \cong S_n$?

$$S_n \xrightarrow{c} \mathrm{Aut}_{gp}(S_n)$$
$$\sigma \longmapsto \{x \mapsto \sigma x \sigma^{-1}\}$$

$\cup$

$\sigma \longmapsto \{x \mapsto \sigma x \sigma^{-1}\}$

$\mathrm{Ker}(C) = Z(S_n) \overset{\text{check.}}{=} \{e\}$

$\varphi : S_n \longrightarrow S_n$ gp iso

T/F : Let $f : G \longrightarrow G$ be gp iso.

Does f preserve conjugacy classes?

(orbits of $G \circlearrowright G$ by conjugation).

i.e. is it true that $f(x) \underset{\text{conjugacy.}}{\sim} x$

No: Abelian groups: each element is its own conj. class.

In general, if $x \sim y$, $f(x) \sim f(y)$.

Consider $S_5 \longrightarrow S_5$

$(12) \overset{①}{\longmapsto} (x_1\ x_2)$

$(12) \overset{②}{\longmapsto} (x_1\ x_2)(x_3\ x_4)$

2^{nd} case $\Rightarrow$ # 2 cycles = # 2-2 cycles

which is false

$$\mathrm{Aut}(S_n) \cong S_n \,?$$

$$\varphi \begin{cases} (1\ 2) \longmapsto (x_1\ \ x_2) \\ (2\ 3) \longmapsto (x_2\ \ x_3) \\ \quad \vdots \qquad\qquad \vdots \\ (n\text{-}1\ \ n) \longmapsto (x_{n-1}\ \ x_n) \end{cases} \qquad \varphi = \text{conj. by } \{ j \longmapsto x_j \}$$

#1: $S_5 = A_5 \rtimes \mathbb{Z}/2\mathbb{Z}$?

To check: $A_5 \trianglelefteq S_5$, $\mathbb{Z}/2\mathbb{Z} \leq S_5$, $A_5 \cap \mathbb{Z}/2\mathbb{Z} = \{e\}$

$A_5 \cdot \mathbb{Z}/2\mathbb{Z} = G$.

Now $A_5 \trianglelefteq S_5$.

$$\mathbb{Z}/2\mathbb{Z} < S_5$$
$$\parallel$$
$\langle (12) \rangle$ (not $\langle (12)(34) \rangle$ since $(12)(34) \in A_5$ lmao)

$$A_5 \cap \langle (1\,2) \rangle = \{e\}$$

$$\begin{array}{c} \{e, (12)\} \\ \downarrow \quad \downarrow \\ 1 \quad -1 \end{array} = S_5/A_5 \cong \{\pm 1\}$$

$$\Rightarrow A_5 \cdot \mathbb{Z}/2\mathbb{Z} = S_5$$

So $S_n = A_n \rtimes \mathbb{Z}/2\mathbb{Z}$