Lec 11/26

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Representation Theory

$GL(V) = \mathrm{Aut}_{gp}(V) \cong GL_n(\mathbb{C})$ if $\dim V = n$, $V =$ v.s. over $\mathbb{C}$.

Defn: Linear representation of G in V is a gp hom $\rho: G \longrightarrow GL(V)$.

eg: $V = \mathbb{C}$, $\rho_g = 1$ $\forall g$. (trivial rep).

eg: $G = S_3$, $V = \mathbb{C}^3$, permute coords.

Let $\rho: G \rightarrow GL(V)$ be a repn. $W \subseteq V$ (subspace) is invariant under G if $\rho_g(W) \subseteq W$ $\forall g \in G$.

$\rho^W: G \longrightarrow GL(W)$ is a repn of G in W

sub-representation

If $W, W' \subseteq V$ s.t. $V = W \oplus W'$,

$\rho: G \rightarrow GL(V)$, $\rho_g(x) = \rho_g(w) + \rho_g(w')$.

If W, W' invariant under G then $V = W \oplus W'$ as representations.

Lemma: $\rho: G \rightarrow GL(V)$ a rep'n, $W \subseteq V$ subspace invariant under G, then $\exists W^\circ \subseteq V$ s.t. $V = W \oplus W^\circ$

Thm: Every rep'n is a direct sum of irreducible rep'ns.

Pf use lemma & induct on $\dim V = n$.

Eg: $G = S_3$. 1) Trivial rep'n ($V = \mathbb{C}$) — irreducible

2) $V = \mathbb{C}$, alternating rep'n — irreducible

$\rho_\sigma(v) = \text{sign}(\sigma) \cdot v$ for $v \in \mathbb{C}$.

3) $S_3 \circlearrowright \mathbb{C}^3$ by permuting coords — <u>NOT</u> irreducible

$\text{Span}(1,1,1) \cong$ trivial rep'n $=: T$

Complementary: $U = \{(z_1, z_2, z_3) : z_1 + z_2 + z_3 = 0\}$ invariant

and $S_3 \circlearrowright U$ <u>is</u> invariant!

So $\mathbb{C}^3 = U \oplus T$

and U is the "standard" rep'n.

Gröbner Bases in the Weyl Algebra with Applications to ODE.