Lec 10/11

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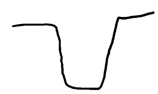
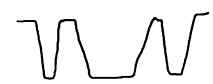
google what notion of largeness

Banach: "Typical" continuous function is nowhere differentiable.

Cesaro limits limit of average.

Assume $f_n(x) \to f(x)$ $\forall x$. (continuous) $f_n(x) = x^n \to$ 

Claim: f can have countably / uncountably many points of discontinuity.

$1_C = \lim f_n$ $f_1 =$  $f_2 =$  etc.

Points of discontinuity are C, since every point in C is a boundary point.

(exercise: flesh this out)

(exercise: $1_{\mathbb{Q} \cap [0,1]}$ cannot be a limit of continuous functions)

Baire category / class (google) (limits of limits etc).

Normal #s are of category 1.

Exercise: Van der Warden implies AP richness of P.W. syndetic sets

Maybe typical $d^*(A) > 0$ is p.w. syndetic.

$\{0,1\}^{\mathbb{N}} \cong$ set of subsets of $\mathbb{N}$ $\xleftrightarrow{\text{bij}}$ 

0 1

wn $\lim \frac{1}{N} \sum^N f(x_n) \to \int^1 f(x)\,dx$ $\forall f \in C[0,1]$

W.D. $\lim_{N-M\to\infty} \frac{1}{N-M} \sum_{n=M}^{N} f(x_n) \to \int_0^1 f(x)\,dx \quad \forall f \in C[0,1]$.

Since $n\alpha \bmod 1$ is w.d. in $[0,1]$ (weyl criterion works)

we have that $\forall\ 0 \le a < b \le 1$, $\{n : n\alpha \bmod 1 \in [a,b)\}$ is syndetic.

improvement on "denseness"

Can use any "quadratic irrationality" λ in $\mathbb{Z}[\lambda]$ or $\mathbb{Q}[\lambda]$

including i, ρ, etc. — ring — field

(Exercise: $\mathbb{Q} + \mathbb{Q}\sqrt[3]{2} + \mathbb{Q}\sqrt[3]{4}$ has inverses)

Also $\{a + a_2\sqrt[5]{2} + a_3\sqrt[5]{2^2} + a_4\sqrt[5]{2^3} + a_5\sqrt[5]{2^4},\ a_i \in \mathbb{Q}\}$ is a field

also $\{a + b\sqrt{2} + c\sqrt{3} + d\sqrt{6} : a,b,c,d \in \mathbb{Q}\}$ is a field

$\{n : n\alpha \bmod 1 \in (a,b)\}$ is syndetic since

$\lim_{N-M\to\infty} \frac{1}{N-M} \sum_{n=M}^{N} 1_{(a,b)}(n\alpha \bmod 1) = b-a$ ("any long interval has it")

$\{n : n^2\alpha \bmod 1 \in (a,b)\}$ is also syndetic lol.

Exercise: if $\bar{d}^*(A) > 0$ then $A-A$ is syndetic

exercise: Prove, ab ovo, that $\{n : n\alpha \bmod 1 \in (a,b)\}$ is syndetic

Same for $\{n : n^2\alpha \bmod 1 \in (a,b)\}$.

exercise: $\forall$ partition $\mathbb{N} = \bigcup_{i=1}^{r} C_i$, one C_i is piecewise syndetic.

bonus: if S is piecewise syndetic and $S = \bigcup_{i=1}^{r} C_i$ then one C_i is p.w. syndetic.

exercise: Sequence $2^n x \mod 1$ is never w.d.