Lec 12/6

Wednesday, December 6, 2017 14:21

Sin n is not u.d. but it is dense and

$$\frac{1}{N}\sum f(x_n) \longrightarrow \int_0^1 f\,d\nu = \int_0^1 f(x)\,g(x)\,dx$$

Log n is u.d. in this sense:

$$\frac{1}{\log N}\sum_{n=2}^{N}\frac{f(x_n)}{n} \longrightarrow \int_0^1 f\,dx$$

$$A \cap A-f(n) \cap A-2f(n) \cap \cdots \cap A-kf(n)$$

$\lfloor n^\alpha \rfloor$ is "good" for Sz.

$\forall k \in \mathbb{N}$, $d^*(A) > 0 \Rightarrow \exists$ (many) n:

$$A \cap A - \lfloor f(n) \rfloor \cap \cdots \cap A - k\lfloor f(n) \rfloor \neq \emptyset$$

if f is "good" for Sz.

Log n does not introduce anything since $\lfloor \log \mathbb{N} \rfloor = \mathbb{N}$

Kuipers & Niederreiter for more u.d. mod 1.

Almost Periodic Functions:

for "reasonable" f which satisfies this;

$f(x) = f(x+\tau)$ (periodic)

there exists a minimal τ.

unreasonable f: $f(x)=C$, $f(x) = \begin{cases} 0 & x \in \mathbb{Q} \\ 1 & x \notin \mathbb{Q} \end{cases}$

Def: $f: \mathbb{Z} \to \mathbb{R}$ is $\overset{\text{Bohr}}{\vee}$ Almost Periodic if $\forall \varepsilon > 0$, $\{ \tau : \sup_{n \in \mathbb{Z}} |f(n+\tau) - f(n)| < \varepsilon \}$ is syndetic.
(H. Bohr)

A continuous $f: \mathbb{R} \to \mathbb{R}$ if $\forall \varepsilon > 0$ $\{ \tau : \sup_{x \in \mathbb{R}} |f(x+\tau) - f(x)| \}$'s gaps are bounded.

$AP(\mathbb{Z})$, $AP(\mathbb{R})$ are algebras (closed under $+$ and $\times$).

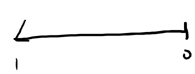
Besicovitch

Kakeya's Needle problem (1910 ± 3)

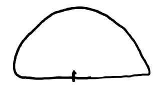
needle:

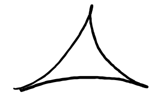
rigid motions are allowed.

move it through to



minimize area it sweeps through





Area can be arbitrarily small:

inf of swept area is 0.

Δ^* intersects any $A-A$ where A infinite.

$\sin x + \sin\sqrt{2}x$ ① is A.P.

② is NOT Periodic.

Bohr Compactification (see after final)

$$\sum a_n \sin nx + b_n \cos nx$$

$$= \sum c_i e^{2\pi i n x}$$

List of Topics:

1. Primes (equivalent forms of PNT via arithmetic functions

(Dirichlet thm proof in LeVeque intermediate book).

· In other fields

2. Extensions of $\mathbb{Q}$ such as $\mathbb{Q}[\sqrt{2}, \sqrt{3}]$ or $\mathbb{Q}[i]$

3. u.d. $\xleftarrow{\text{normal \#}}$ ergodic theory

4. Additive Combinatorics $(S_Z, S_{\acute{a}}, d, \bar{d}, d^*, etc)$
Vow

5. Arithmetic functions

6. Continued fractions (pell equation)

7. Diophantine equations

8. P-Adic numbers

9. Diophantine Approximations (Liouville theorem)

10. Minkowski Business

11. Modular stuff

Farey fractions
& Ford circles

Babylonian & Egyptian math

Indian pell eqn via
Chakravala

Sierpiński