Lec 11/17

Friday, November 17, 2017 10:31

$$J_\alpha(x) = \left(\frac{x}{2}\right)^\alpha \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,\Gamma(n+\alpha+1)} \left(\frac{x}{2}\right)^{2n} \quad \left(= \phi_1(x)\right)$$

$(\alpha = 0 \quad \text{case})$

$$x^2 \phi_2(x) = x^2 \ln|x| \; \phi_1(x) + x^3 \sigma_2(x)$$

$$x \phi_2'(x) = x\phi_1'(x) \ln x + \phi_1(x) + x\sigma_2(x) + x^2\sigma'(x)$$

$$x^2 \phi_2''(x) = x^2\phi_1''(x) \ln x + 2x\,\phi_1'(x) - \phi_1(x) + 2x\sigma_2'(x) + x^3\sigma_2''(x)$$

III $2\alpha \in \mathbb{N}$, $\quad \alpha = \frac{N}{2}$

I: $\quad \varphi_1(x) = x^{r_1}\sigma_1(x) \quad , \quad \varphi_2(x) = x^{r_2}\sigma_2(x)$

$$C_1\varphi_1 + C_2\varphi_2 = 0 \Rightarrow C_1 x^{r_1}\sigma_1(x) + C_2 x^{r_2}\sigma_2(x) = 0$$

so $\quad C_1 x^{\overset{r_1 - r_2}{\overset{\parallel}{\rho}}}\sigma_1(x) + C_2\sigma_2(x) = 0$ $\qquad$ Note: $|x^\rho| = e^{\gamma \ln x}$

$x \to 0 \Rightarrow \quad C_2\sigma_2(0) = 0$

so $\alpha_0 = 0$. similarly, $\beta_0 = 0$.

So simplify by x.