Lec 9/2

Friday, September 2, 2016 9:08 AM

Archimedean Property $\mathbb{R}$ and any subfield

$\left(n \geq \frac{b}{a} \Leftrightarrow \frac{a}{b} \leq \frac{1}{n}\right)$

Given $a>0$, $b>0$, can find a positive integer n so that $na \geq b$

Geometrically: any length can be measured with a ruler

$\Leftrightarrow$ $\mathbb{R}$ contains no infinitesimals $\Leftrightarrow$ $\mathbb{R}$ contains no pseudo-infinite elements

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What is a function?

Informal: rule which assigns to any number in some subset of $\mathbb{R}$ another real number

Greek Geometers: $\underbrace{x^2}_{ft^2} + \underbrace{x}_{ft}$ is nonsensical don't add

Galileo: $h = 64 + 48t - 16t^2$ (an object falling in physics)

$h_0 + v_0 t + \frac{1}{2} a t^2$ so coefficients also come with units: $v_0 :: \frac{m}{s}$ which cancels s

$h = -16(t-4)(t+1)$

$0 = h \Rightarrow t = 4$ or $t = -1$ but $t = -1$ makes less sense

so this physical interpretation only makes sense for $t \in [0,4]$

So must distinguish between functions w/ different Domain

$h(t) = f(t) \quad \forall t \in \mathbb{R}$

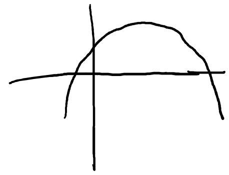
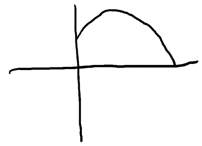
$h'(t) = f(t) \quad \forall t \in [0,4]$

Formal definition of function: (real-valued of one real variable)

A function f is a subset of $\mathbb{R} \times \mathbb{R}$ (a set of ordered pairs (x,y))

satisfying the property that $(x, y_1) \in f$ and $(x, y_2) \in f \Rightarrow y_1 = y_2$

Geometrically: vertical line test

h :  h' : 

notation: $f(x) = y := (x,y) \in f$

The set $\mathrm{dom}(f) = \{x : \exists y \text{ s.t. } (x,y) \in f\}$ is called the domain of f

$f : S \to T$ $:=$ S is the domain of f and $\forall x \in S$ $f(x) \in T$

<u>Not</u> saying the range/image of f is T.

T is the codomain.

<u>Ex</u>: $f : S \to [0, \infty)$ means $\sqrt{f(s)}$ is defined $\forall s \in S$

Convenient to specify functions by formulas. In that case, the domain is assumed to be $\{x \in \mathbb{R} : f(x) \text{ makes sense}\}$

Ex: $f(x) = \frac{x}{x^2-4}$ is shorthand for the function f defined $\forall x \neq \pm 2$ given by $f(x) = \frac{x}{x^2-4}$

<u>examples of functions:</u>

1) polynomial functions $f(x) = a_0 + a_1 x + a_2 x^2 + \dots a_n x^n$ where $a_0, a_1, \dots, a_n \in \mathbb{R}$

$\mathrm{dom}(f) = \mathbb{R}$

Two special cases: a) $f(x) = c$ constant function

b) $I(x) = x$ identity function

2) rational functions $f(x) = \frac{g(x)}{h(x)}$ where g and h are polynomial functions

$\mathrm{dom}(f) = \{x \in \mathbb{R} : h(x) \neq 0\}$

note: $f(x) = \frac{x-1}{x^2-1} \neq g(x) = \frac{1}{x+1}$ because $\underset{\mathbb{R} \setminus \{-1,1\}}{\underset{\parallel}{\mathrm{dom}(f)}} \neq \underset{\mathbb{R} \setminus \{-1\}}{\underset{\parallel}{\mathrm{dom}(g)}}$

3) $f(x) = |x|$ $f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$ $\mathrm{dom}(f) = \mathbb{R}$

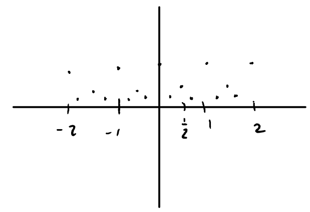
4) $g(x) = \lfloor x \rfloor$ floor of x - highest integer $n \leq x$ (well-ordering principle to show $dom(g) = \mathbb{R}$)

$h(x) = \lceil x \rceil$ Ceiling of x - lowest integer $n \geq x$ " "

5) $\emptyset$ is a function not defined for any $x \in \mathbb{R}$

6) $f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \notin \mathbb{Q} \end{cases}$ (discontinuous everywhere)

7) $f(x) = \begin{cases} \frac{1}{q} & \text{if } x = \frac{p}{q} \in \mathbb{Q} \text{ (in lowest terms, } q>0) \\ 0 & \text{if } x \notin \mathbb{Q} \end{cases}$



f in ex 7 is called "popcorn function"

it is continuous at all irrationals & discontinuous at all rationals