A category consists of rojects & morphisms.

for objects A, B in the category, we have a set of maphisms A-B.

If $\varphi: A \to B$, $\psi: B \to C$ are morphisms, then $\psi \circ \varphi: A \to C$ is defined,

 $\varsigma, \epsilon. \qquad \phi_{\iota} \circ (\phi_{2} \circ \phi_{3}) = (\phi_{\iota} \circ \phi_{2}) \circ \phi_{3}$

 \forall object A, \exists norphism $1_A:A\longrightarrow A$ sit. \forall $\psi:B\longrightarrow A$, $1_A\cdot \psi=\psi$.

A & B are isomorphic if \exists maphisms $\psi: B \to A$ s.t. $\psi: \psi: f = 1_B$, $\psi: \psi: f = 1_A$.

If and ψ are called isomorphisms.

An object A is called a universal repelling object if \forall object B, $\exists !$ morphism $A \rightarrow B^*$. "(attracting $\longrightarrow B \rightarrow A$).

These objects may not exist, but

Theorem if A, & A, are universal repelling (or affacting) objects, $A_1 \cong A_2$.

Proof: $\exists \ y_1 : A_1 \longrightarrow A_2 \ s \ y_2 : A_2 \longrightarrow A_1 \ and \ y_1 \circ y_2 : A_2 \longrightarrow A_2 \ but true is only one such morphism, <math>1_{A_2}$.

Ey: in Set, universal attractors on {x3, universal repellor is O.

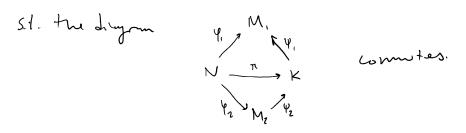
ey Category of groups with a fixed elements $(g_1, g_2, ..., g_n)$ and morphisms $(G_1, (g_1, ..., g_n)) \longrightarrow (H_1, (h_1, ..., h_n)) \text{ belong hom-smo} \quad f: G \rightarrow H_2(E_1, f) = h_1 \ \forall i.$ universal repellor = $(F_n = \langle a_1, ..., a_n \rangle)$, $(a_1, ..., a_n)$.

ey same as a bove but with abelian groups. Then (Z", (e,,...,en)) is universal regular

- g let R be a commutative ring. Consider the Category of R-Algebras with a marked element; morphisms $(A,a) \longrightarrow (B,b)$ are R-algebra home with a be.

 A universal repelling object is (R[x],x) with $R[x] \longrightarrow A$. $p(x) \longmapsto p(a)$
- eg R: unital ring. consider artigeny of R-modules ω/n marked elements $(M, (u_1,...,u_n))$.
 Universal repelling object: $(R^n, (e_1,...,e_n))$ with map $e_i \longmapsto u_i$.
- repelling a bject in the category of R-modules N with homomorphism $\{P_1,M_1\rightarrow N\leftarrow M_2, P_2\}$.

 (d) agrams $M_1 \stackrel{P_1}{\rightarrow} N \stackrel{P_2}{\leftarrow} M_2$) and marphism $(N_1,P_1,P_2) \longrightarrow (K_1,P_2)$ being home $T:N\rightarrow K$ s.t. the dron V_1, V_2, V_3 is committed.
 - $M, \oplus M_2$ is also the universal afteracting object in the category of (N, Ψ_1, Ψ_2) where $\Psi_1: N \to M_1$, $\Psi_2: N \to M_2$ with merphisms $(N, \Psi_1, \Psi_2) \to (K, \Psi_1, \Psi_2)$ being home π



Back to direct sums:

M, $M_2 \sim M$, $M \in M_2$.

Definition. Let M be an R-module of M_1 , M_2 be its Submodules. We say that M is a lived sum of M, and M_2 , $M = M_1 \oplus M_2$,
if $M \cong M_1 \oplus M_2$ so that the isomorphism is identical
on $M_1 \oplus M_2 \oplus M_3 \oplus M_4 \oplus M_4 \oplus M_4 \oplus M_4 \oplus M_4 \oplus M_5 \oplus M_4 \oplus$

Theoren: Let M, , M2 be submodules of M. Then M = M. & M2 iff my of the following:

- (i) Frem is uniquely representable as u=u, +uz where u,eM,, uzeMz.
- (ii) $M = M_1 + M_2$ and $M_1 \cap M_2 = 0$.
- (iii) The projection howers M M/M, is an isomorphism on Mz (when restricted).

 (iv) " M -> M/M2" " M,"
- Proof (i) define a mapping $\emptyset: M \longrightarrow M_1 \oplus M_2$ by if $u = u_1 + u_2 + u_1 = u_1 = u_2$.

 Prove this is an isomorphism a that it's identical on $M_1 \oplus M_2$.

 (ii) \Rightarrow (iii) . Let $\pi: M \longrightarrow M_M_1$. Then $\ker(\pi_1 \cup M_2) = M_1 \cap M_2 = 0$, and $\pi(M_2) = \pi(M_1 + M_2) = \pi(M_1) = \frac{M_1}{M_2}$.