Lec 11/29

Wednesday, November 29, 2017 14:24

google: Hasse-Minkowski principle.

3 greek problems:

1. isecting tr an angle
2. Doubling a Cube
3. Squaring a Circle

look at wikipedia
Constructible numbers

[illegible]ature in why π is transcendental

Don Zagier sum of two squares

hantine Diop applications of p-adics

exercises

$$\ell^2_{\mathbb{R}} = \left\{ (x_1, x_2, \ldots) : x_i \in \mathbb{R}, \ \sum_{i=1}^{\infty} x_i^2 < \infty \right\}$$

$$d(x,y) = \sqrt{\sum_{i=1}^{\infty} (x_i - y_i)^2}$$

(taking for granted that $\mathbb{R}$ is complete)

exercise: Show that d is a metric space.

exercise: prove that $\ell^2_{\mathbb{R}}$ is a complete space w.r.t. d

$\sum a\, x^n$

$$\sum_{n \geq 0} a_n x^n$$

$$f(x) \sim \sum_{n=0}^{\infty} (a_n \sin(nx) + b_n \cos(nx)) \qquad x \in (0, 2\pi].$$

$$L^2[0, 2\pi] \approx \ell^2_{\mathbb{R}}$$

fns w/ integrable squares

countable basis $\cos(nx), \sin(nx)$

Scalar product: $\int_0^{2\pi} f\bar{g}\,dx$

fejer thm $\nearrow \searrow \nearrow$

$f \quad f' \quad xf'$

as long as $\underbrace{\int f \sin nx}_{a_n}$, $\underbrace{\int f \cos nx}_{b_n}$ are meaningful,

these are coeffs.

Want: recover function $\text{"}\lim_{N\to\infty}\text{"} \sum_1^N (\quad) \underset{\forall x}{=} f$ doesn't work.

Other Fejer thm: (for recovering f)

Call $S_N^{(f)}(x) = \sum^{N-1} (a_n \sin nx + b_n \cos nx)$

$n=0$

$$\sigma_n^{(f)} = \frac{S_0^{(f)} + \cdots + S_n^{(f)}}{n} \leftarrow \text{reweighting sines \& cosines}$$

Fejer: $f \overset{\forall x}{=} \lim_{n \to \infty} \sigma_n^{(f)}(x)$

Littlewood:

$$\lim_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N-1} |f - \sigma_n^{(f)}| \to 0$$

Exercise: in finite fields, what is e?

$\mathbb{F}_p = \mathbb{Z}/p\mathbb{Z}$

$$X = X_p = \prod \{0, \dots, p-1\} = \prod \mathbb{F}_p = \mathbb{Z}_p$$

$$a = \sum a_i p^i, \quad \sigma(a) = \sum ((p-1) - a_i) p^i$$

$$\underbrace{\sigma(\sigma(a)) = a.}_{\text{involution.}}$$

$$\sum_{i \geq 0} p^i = \frac{1}{1-p}$$

p-adic integers is an integral domain (final exam proof)

$p\mathbb{Z}_p$ is unique maximal ideal.

Topological Algebra:

group w/ continuous $\circ$ and $(\ )^{-1}$.

Division algebra:

a field w/o necessary commutativity.

(noncommutative field)

ex: $\mathbb{R}$.

only $_{\wedge}$ finite-dimensional div. algebras over $\mathbb{R}$: $\mathbb{R}, \mathbb{C}, \mathbb{H}$. (frobenius)

(formulate/Define)

exercise: what can you say about quadratic eqns. over $\mathbb{H}$?

$\mathbb{Z}_p$ is a topological group wrt $+$.

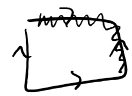
$\mathbb{Z}_p^\times$ is one wrt $\circ$

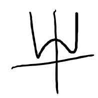
$\mathbb{R}/\mathbb{Z} = \mathbb{T}$

$\{ a + \mathbb{Z} \ \ a \in \mathbb{R} \}$

$\{a + \mathbb{Z},\ a \in \mathbb{R}\}$

$\{a+\mathbb{Z},\ a \in (0,1]\}$ set of cosets.

$\mathbb{R}^2/\mathbb{Z}^2 = \mathbb{T}^2 =$  $=$ 

$SL(2,\mathbb{R})/SL(2,\mathbb{Z})$ 

exercise: Show $R/\mathbb{Q}$ is not Hausdorff.

$\{\alpha n,\ n \in \mathbb{Z}\} \subset \mathbb{T}$ is a subgroup, dense if α irrational.

<u>for Next time</u>: Read first 10 pgs of Ch25 (elliptic curves)