Lec 10/31

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$$L = P(D) : C^{(n)}(\mathbb{R}) \longrightarrow C(\mathbb{R})$$

$P \in \mathbb{C}[x]$, $\deg(P) = n$, $D(f) = f'$

Solutions of $L(y) = 0$ are

$$e^{r_1 x}, \; x e^{r_1 x}, \; \dots, \; x^{m_1 - 1} e^{r_1 s}$$

$$\vdots$$

$$e^{r_s x}, \; \dots\dots, \; x^{m_s - 1} e^{r_s x}$$

Where $(r_1, \dots, r_s)$ are roots of P w/ multiplicity $(m_1, \dots, m_s)$.

These solns are linearly independent.

All solns of $L(y) = 0$ are linear combos of these:

Ex: $y''' - 3iy'' - 3y' + iy = 0$

$$r^3 - 3ir^2 - 3r + i = 0$$
$$\shortparallel$$
$$(r - i)^3$$

solns are e^{ix}, xe^{ix}, $x^2 e^{ix}$.

Initial Value problem:

$L(y) = 0$

$y(x_0) = \alpha_0$

$\vdots$

$y^{(n-1)}(x_0) = \alpha_{n-1}$

Thm 1: any IVP has a soln at least.

Proof: Let $\{\varphi_1, \dots, \varphi_n\}$ be the linearly independent solutions described above.

Look for some $\varphi = c_1\varphi_1 + \dots + c_n\varphi_n$

solve $c_1 \varphi_1(x_0) + \dots + c_n \varphi_n(x_0) = \alpha_0$

$\vdots$

$c_1 \varphi_1^{(n-1)}(x_0) + \dots + c_n \varphi_n^{(n-1)}(x_0) = \alpha_{n-1}$

Since $W(x) = e^{-a_1 x} W(x)$

Determinant of this system is $W(x_0) \neq 0$

Since φ_i are lin. indp. solns of $L(y)=0$.

So there is a soln.