Lec 3/10

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Tests regarding variances

$n_1 = 30$ $\quad n_2 = 20$

$= 0.21 s_1 \quad s_2 = 0.20 \quad \alpha = 0.02 \qquad H_0: \sigma_1 = \sigma_2 \qquad H_1: \sigma_1 \neq \sigma_2$

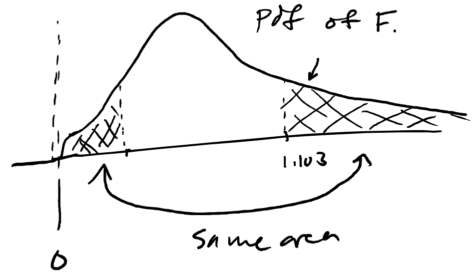
$$\frac{s_1^2}{s_2^2} = 1.103 \qquad \text{so} \quad s_1 > s_2$$

method 1: $f_{0.01, 29, 19} = 2.84 > 1.103$. Not suff. evidence $\sigma_1 \neq \sigma_2$.

method 2: p-value $=$

$$\mathbb{P}(F_{29,19} > 1.103) \cdot \underset{\text{both sides}}{2} > 0.02 = \alpha$$

fail to reject



§13.5 tests for proportions:

recall: LRT for $H_0: \theta = \frac{1}{2}$ vs $H_1: \theta \neq \frac{1}{2}$ was reject H_0 when $|X - \frac{n}{2}| \geq k$

LRT of $H_0: \theta = \theta_0$ and $H_1: \theta \overset{\neq}{\underset{<}{>}} \theta_0$ is $X \begin{array}{l} \notin [k_{\frac{\alpha}{2}}, k'_{\frac{\alpha}{2}}] \\ > k_\alpha \\ < k_\alpha \end{array}$

Ex: test $\theta < 20\%$. sample 26, 1 admits. is this evidence?

1: $H_0: \theta = 0.2 \qquad H_1: \theta < 0.2 \qquad \alpha = 0.05$

2: $X \sim Bin(26, 0.2)$, $\quad X = 1$

p-val $= \mathbb{P}(X \leq 1; \theta = 0.2) = 0.8^{26} + 26 \cdot 0.2 \cdot 0.8^{25} = 0.02267 < 0.05.$

3: yes this is evidence; reject H_0.

When n is large (>30), use normal approximation $X \sim N(n\theta, n\theta(1-\theta))$.

$$Z = \frac{\overline{X} - n\theta}{\sqrt{n\theta(1-\theta)}} \rightsquigarrow N(0,1) \quad \text{as } n \to \infty$$

Use this Z as test statistic to compute p-value.

Ex: $n = 200$, 110 positive. determine whether $\theta > 0.5$.

test: $H_0: \theta = 0.5$, $H_1: \theta > 0.5$.

Stat: $Z = \frac{x - n\theta}{\sqrt{n\theta(1-\theta)}} = \frac{110 - 100}{\sqrt{200 \cdot 0.5 \cdot 0.5}} = \frac{10}{\sqrt{50}} = \sqrt{2} = 1.41$

pval: $P(z > 1.41) = 0.07 > 0.05$

so fail to reject H_0.

§13.6 tests concerning differences of K proportions

let $X_1, \dots, X_K$ each be ind RVs w/ dist $Bin(n_i, \theta_i)$

When all n_i large, $Z_i = \frac{X_i - n_i\theta_i}{\sqrt{n_i\theta_i(1-\theta_i)}} \rightsquigarrow N(0,1) \quad \forall i \in \{1, \dots, k\}$

consider testing

$H_0: \theta_1 = \cdots = \theta_K = \underset{\text{hypothesized value}}{\theta_0}$ vs $H_1: \exists i$ s.t. $\theta_i \neq \theta_0$.

Consider $\chi^2 = \sum_{i=1}^{K} Z_i^2 \rightsquigarrow \chi^2_K$

$i=1$

Under H_0: $\theta_i = \theta_0$. so $\chi^2 = \sum_{i=1}^{k} \frac{(x_i - n_i\theta_0)^2}{n_i\theta_0(1-\theta_0)}$

CR: $\chi^2 \geq \chi^2_{\alpha,k}$