Lec 10/24

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$\underline{I} = (a,b) \subseteq \mathbb{R}$, $L : C^\infty(I) \to C^\infty(I)$, $L(y) = a_1(x)y^{(n)} + a_2(x)y^{(n-1)} + \dots + a_n(x)y$

$a_j \in C[I]$, $b \in C(I)$

Homogeneous: $L(y) = 0$ (N)

Nonhomogeneous: $L(y) = b(x)$ (H)

$C^\infty(I)/F$ where $F = \mathbb{R}$ or $\mathbb{C}$.

$$L(\lambda_1 y_1 + \lambda_2 y_2) = \lambda_1 L(y_1) + \lambda_2 L(y_2)$$

Strategy: 1. find general soln of H. 2. find particular soln of N.

$\Rightarrow$ the sum gives general soln of N.

find a basis of null(L) (which is finite Dim).

L: differential operator.

$$L = a_0 D^n + a_1 D^{n-1} + \dots + a_{n-1} D + a_n I$$

$c_j \in F$.

1: Soln to H: $\varphi_h = c_1\varphi_1 + \dots + c_n \varphi_n$ where $\{\varphi_1, \dots \varphi_n\}$ is a basis of $n(L)$.

2: soln to N: $\varphi_n = u_1\varphi_1 \dots + u_n\varphi_n$ where $u_i \in C^\infty(I)$.

$$\varphi = \varphi_h + \varphi_n.$$

Class of eqns:

Constant-coefficient: $a_i(x) = c_i$

take $n=1$, cc. $\quad L(y) = y' + \alpha y = \begin{cases} 0 \\ h(x) \end{cases}$

$$y' + \alpha y = 0 \Rightarrow y' = -\alpha y \Rightarrow \frac{y'}{y} = -\alpha \Rightarrow (\ln y)' = -\alpha \Rightarrow \ln y = -\alpha x + c$$

$$\Rightarrow y(x) = e^{-\alpha x + c}$$

$$= C e^{-\alpha x}$$

using integrating factor

$$\left(e^{\alpha x} y\right)' = 0$$

$$y' + a y = b(x) \iff (e^{ax} y)' = e^{ax} b(x) \iff e^{ax} y = \int_{x_0}^{x} e^{at} b(t)\, dt + C$$

$$\Rightarrow y = e^{-ax} \int_{x_0}^{x} b(t) e^{at}\, dt + C e^{-ax}$$

What if $y' + a(x) y = L(y)$?

$$\ln y = -\overbrace{\int_{x_0}^{x} a(t)\, dt}^{A(x)} + C \Rightarrow y = C e^{-A(x)}$$

$$\left(e^{-A(x)} y\right)' = 0 \checkmark$$