

10) (CRT) $I_1, I_2 \subset R$ s.t. $I_1 + I_2 = R$.

Show if $a_1, a_2 \in R$ then $\exists x \in R$ s.t. $x \equiv a_1 \pmod{I_1}$
 $x \equiv a_2 \pmod{I_2}$

pf let $b_1 \in I_1$, $b_2 \in I_2$ s.t. $b_1 + b_2 = 1$.

$$\text{let } (b_1 + b_2)a_1$$

$$b_1 a_1 + b_2 a_1 =$$

$$r_1' + r_2' = a_1$$

$$r_1^2 + r_2^2 = a_2$$

$$\begin{matrix} \cap & \cap \\ I_1 & I_2 \end{matrix}$$

$$r_1' + r_2^2 \equiv a_2 \pmod{I_1}$$

$$r_2' = a_1 - r_1'$$

$$b_1 a_2 + b_2 a_1 = b_1 a_2 \pmod{I_2}$$

$$= b_1 a_2 + b_2 a_2 = a_2 \pmod{I_2},$$

Similar for I_1 .