Lec 9/14

Wednesday, September 14, 2016 7:53 AM

These ideas extend to n RVs (D or C)

Marginal Distributions §3.6

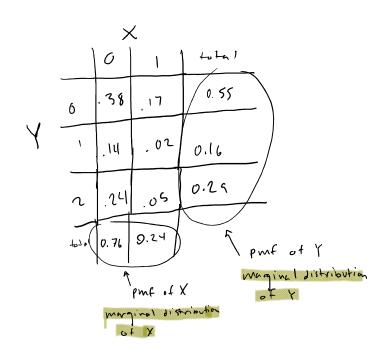
Get probabish for Y;

If X, Y are DRVs w/ joint pmf p(x,y), run the marginal dists of Y and X are as follows:

$$P_{\chi}(y) = \sum_{\chi} p(\chi, y)$$

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If X, Y are CRVs w/ 10mt pop f(x,y) then the marginal dists of x and Y are +5 collows:



$$f_{\chi}(x) = \int_{-\infty}^{\infty} f(x,y) dy \qquad f_{\chi}(y) = \int_{-\infty}^{\infty} f(x,y) dx$$

$$\underbrace{\xi x}$$
: $f(x,y) = \begin{cases} 3x & \text{if } 0 \leq y \leq x \leq 1 \\ 0 & \text{6.w.} \end{cases}$

find marginal distributions of X and Y

$$\int_{X} (x) = \int_{0}^{\infty} f(x,y) dy = \int_{0}^{\infty} 3x dy = 3xy \Big|_{0}^{x} = 3x^{2} \quad \text{so} \quad \int_{X} (x) = \begin{cases} 3x^{2} & \text{for } 0 \leq x \leq 1 \\ 0 & \text{o.w.} \end{cases}$$

$$f_{\gamma}(y) = \int_{-\infty}^{\infty} f(x,y) dx = \int_{3x}^{3x} dx = \frac{3}{2}x^{2} \Big|_{y}^{y} = \frac{3}{2} - \frac{3}{2}y^{2} + co + f_{\gamma}(y) = \begin{cases} \frac{3}{2} - \frac{3}{2}y^{2} & \text{for each } y \in \mathbb{N} \\ 0 & \text{e.w.} \end{cases}$$

Start w/ Joint pmf or paf for n RVs. you can find the mighal distribution of any subset of n RVs by summing/integrating out the rest of the n RVs.

f(x, x2,..., x,)

$$f(x_1, x_3, x_4) = \int_{-\infty}^{\infty} \int_{-\infty}^{x_1} f(x_1, \dots, x_n) dx_1 \dots dx_n$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{x_2} f(x_1, \dots, x_n) dx_1 \dots dx_n$$

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Conditional Distributions \$3.7

Recall:
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Now, look at DRVs X and Y. let A := (X=2) and B:= (Y=y)

$$P(X=x|Y=y) = P(X=x, Y=y) = \frac{P(x,y)}{P(Y=y)}$$

$$P(X=x|Y=y) = P(X=x, Y=y) = P(x,y)$$

$$P(Y=y) = P$$

Formally, if x, y are DRVs with sornt put P(x, y) and norginal puts $P_{x}(x)$ and $P_{y}(y)$ then conditions. We given by:

$$p(x|y) = \frac{p(x,y)}{p_{y}(x)} \qquad p(y|x) = \frac{p(x,y)}{p_{y}(x)}$$

valid as long as governot dividing by b.

(eno. dist of Y given X = 0
$$P(010) = P(Y=0 \mid X=0) = \frac{P(0,0)}{P_{y}(0)} = \frac{0.38}{0.76} = 6.5$$

$$P(110) = P(Y=1 \mid X=0) = \frac{P(0,1)}{P_{y}(0)} = \frac{6.10}{0.76} = 0.1842$$

$$P(210) = P(Y=2 \mid X=0) = \frac{P(0,1)}{P_{y}(0)} = \frac{6.24}{0.76} = 6.3158$$

If X, Y CRVs ω / suint radf f(x,y) and marginal distributions $f_{\chi}(x), f_{\chi}(y), \text{ the cond distributions}$ $f(\chi|y) = \frac{f(\chi,y)}{f_{\chi}(y)} \qquad f(y|\chi) = \frac{f(\chi,y)}{f_{\chi}(\chi)}$

$$\mathcal{L}_{x}: f(x,y) = \begin{cases} 3x & \text{if } o(y \leq x \leq 1) \\ 0 & \text{o.} \omega \end{cases}$$

$$f_{x}(x) = \begin{cases} 3x^{2} & \text{if } o(x \leq 1) \\ 0 & \text{o.} \omega \end{cases}$$

$$f_{y}(y) = \begin{cases} \frac{3}{2}, \frac{3}{2}y^{2} & \text{if } 0 \leq y \leq 1 \\ 0 & \text{o.} \omega \end{cases}$$

Combine reatrictions

of two functions you're using

3x

2x

if Oly (x = 1)

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$$f(x|y) = \frac{3x}{\frac{3}{2}(1-y^2)} = \frac{2x}{1-y^2}$$
 if $0 \le y < x \le 1$

$$f(y|x) = \frac{3x}{3x^2} = \frac{1}{x} \quad \text{if} \quad o(y \leq x \leq 1)$$