Persistence Vector Spaces

<u>Defn</u> A PVS is $\mathbb{V} = \{V_r\}_{r\geq 0}$ w/ linear maps $L_{r,s} : V_r \to V_s$ $\forall\, r \leq s$,

s.t. $\forall$ $r \leq s \leq t$,

$$\begin{tikzcd} V_r \arrow[r] \arrow[rr, bend right] & V_s \arrow[r] & V_t \end{tikzcd}$$

commutes.

<u>Ex</u>: Let $\{K_r\}_{r\geq 0}$ be a filtered sp cpx. Then

$\{H_k(K_r)\}_{r\geq 0}$ is a PVS $\forall$ $k = 0, 1, 2, \ldots$

<u>Defn</u> a finite PVS is a PVS which has finitely many V_r, and each is finite-dimensional.

Thus is repnted by

$$V_{r_0} \xrightarrow[L_{r_0 r_1}]{} V_{r_1} \xrightarrow[L_{r_1 r_2}]{} \cdots \xrightarrow[L_{r_{N-1} r_N}]{} V_{r_N}.$$

Sometimes we don't care about the particular index set, so we write

$$V_1 \xrightarrow{L_1} V_2 \xrightarrow{L_2} \cdots \xrightarrow{L_{N-1}} V_N$$

For any finite PVS, we obtain an $\mathbb{R}$-indexed PVS as follows:

$\mathbb{V} = \{V_i\}^N$ ~~~~~~ $\overline{\mathbb{V}} = \{\overline{V}\}$

$$V = \{V_{r_j}\}_{j=0}^{N} \rightsquigarrow \overline{V} = \{\overline{V}_r\}_{r \geq r_0}$$

by
$$\overline{V}_r = \begin{cases} V_{r_0} & r_0 \leq r < r_1 \\ V_{r_1} & r_1 \leq r < r_2 \\ \vdots & \vdots \\ V_{r_N} & r_N \leq r \end{cases}.$$

and linear maps do the obvious thing.

$\mathbb{R}$-indexed PVS is finitely presented if it arises from this construction.