Lec 11/7

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$L(y) = y^{(n)} + a_1(x)\, y^{(n-1)} + \cdots + a_n(x) y$

$L(y) = 0$

assume φ_1 is a known solution.

$\varphi = u \varphi_1$

$(u\varphi_1)^{(k)} = \sum_{j=0}^{k} \binom{k}{j} u^{(k-j)} \varphi_1^{(j)}$

$\varphi' = u'\varphi_1 + u\varphi_1'$

$\vdots$

$\varphi^{(n)} = u^{(n)}\varphi_1 + n\, u^{(n-1)}\varphi_1' + \cdots + u\, \varphi_1^{(n)}$

$$L(\varphi) = \varphi_1 u^{(n)} + \cdots + \left(n\varphi_1^{(\varphi-1)} + a_1(n-1)\,\varphi_1^{(n-2)} + \cdots + a_n\varphi_1 \right) u' + \overbrace{\underbrace{\left(a_0\varphi_1^{(n)} + a_1\varphi_1^{(n-1)} + \cdots + a_n \varphi\right) u}_{\text{so this disappears}}}^{L(\varphi_1)=0}$$

If $L(\varphi) = 0$, u must satisfy this new eqn:

$\varphi_1 u^{(n)} + A_1 u^{(n-1)} + \cdots + A_{n-1} u' = 0$

$V = u'$

$\varphi_1 V^{(n-1)} + A_1 V^{(n-2)} + \cdots + A_{n-1} V = 0$ (reduction of order)

$J = \{ x \in I : \varphi_1(x) \neq 0 \}$

$V^{(n-1)} + B_1 V^{(n-2)} + \cdots + B_{n-1} V = 0$

suppose we can solve this & solutions are $\{v_2, \dots, v_n\}$ (linearly independent).

then we know solution to $L(y)=0$.

$$u_i = \int_{x_0}^{x} v_i(t)\, dt$$

and φ_1, $u_i \varphi_1$ $\quad {}_{2 \le i \le n}$ are n solutions.

$$(c_1 + c_2 u_2 + c_3 u_3 + \dots + c_n u_n)\varphi_1 = 0$$

$$\Rightarrow \quad c_2 u_2' + c_3 u_3' + \dots + c_n u_n' = 0$$

$$\Rightarrow \quad c_i = 0$$

They are linearly independent.

$$x^3 y''' - 3x^2 y'' + 6xy' - 6y = 0 \qquad x>0 \quad \text{or} \quad x<0$$

one solution is $\varphi_1(x) = x$

$\varphi = ux$	-6
$\varphi' = u'x + u$	$+6x$
$\varphi'' = u''x + 2u'$	$-3x^2$
$\varphi''' = u'''x + 3u''$	$+x^3$

$L(\varphi) = x^4 u'''$ solutions: x, x^2

thus x, x^2, x^3 are solutions of $L(y)=0$.

$$y'' + a_1(x)y' + a_2(x)y = 0$$

$$\varphi = u\varphi_1$$

$$\varphi' = u'\varphi_1 + u\varphi_1'$$

$$\varphi'' = u''\varphi_1 + 2u'\varphi_1' + u\varphi_1''$$

$$L(\varphi) = \varphi_1 u'' + (2\varphi_1' + a_1(x)\varphi_1)u' = 0$$

$$V = u'$$

$\Rightarrow \quad \varphi_1 v' + (2\varphi_1' + a_1(x)\varphi_1)V = 0 \qquad \varphi_1(x) \neq 0$ where we care.

$$\Rightarrow \quad V' + \left(\frac{2\varphi_1'}{\varphi_1} + a_1(x)\right)V = 0$$

$$\Rightarrow \quad \frac{V'}{V} = -\frac{2\varphi_1'}{\varphi_1} - a_1(x)$$

$$\Rightarrow \quad \log(v) = -2\log\varphi_1 - \overbrace{\int_{x_0}^{x} a_1(t)\,dt}^{A(x)}$$

$$\Rightarrow \quad V = e^{-2\log\varphi_1 - A(x)} = \varphi_1^{-2} e^{-A(x)}$$

So $\quad u = \displaystyle\int_{x_1}^{x} \varphi_1^{-2}(t)\, e^{-A(t)}\, dt$

So $\quad \varphi_2(x) = \varphi_1(x) \displaystyle\int_{x_1}^{x} \varphi_1^{-2}(t)\, e^{-A(t)}\, dt$