Lec 3/8

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Paired data. No independence assumption.

Data: (X_{1i}, X_{2i}) are dependent $((X_{1i}, X_{2i})$ indp from $(X_{1j}, X_{2j}))$.

For ex, measure "before" and "after" on same subject.

Look at difference $Y_i = \underset{\uparrow \text{before}}{X_{1i}} - \underset{\uparrow \text{after}}{X_{2i}}$. ← one Data pt for each subject.

$\Rightarrow$ $Y_1, \dots, Y_n$ new RS. assume $\dfrac{\bar{Y} - \mu_Y}{s_Y/\sqrt{n}} \sim t_{n-1}$ if $Y_i \sim$ Normal.

Perform a one-sample t-test of $H_0: \mu_Y = 0$ vs. $H_1: \mu_Y \overset{\neq}{\underset{<}{>}} 0$.

If n large, use $N(0,1)$ instead of t_{n-1}.

Ex: Data: # seizures before & after drug. $n=20$, $\alpha = 0.05$. $\bar{Y} = 12.5$, $s_Y = 14.12$.

$\dfrac{12.5 - 0}{14.12/\sqrt{20}} = 3.96$. p-value $= \mathbb{P}(|t| \geq 3.96) \cdots < 0.005$ (*).

CR: $|t| > t_{\frac{\alpha}{2}, n-1} = t_{0.025, 19} = 2.096$.

so Reject H_0.

So Reject H_0. There is a difference in # of seizures.

§ 13.4 tests concerning variance

One-sample test.

Ex: population SD is 0.17, but higher variation one day. test whether SD increased.

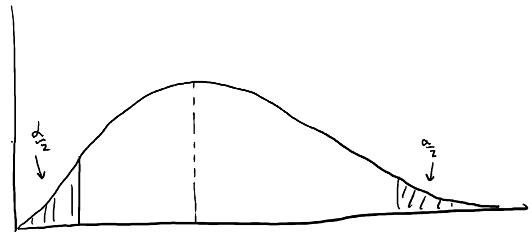
Test: $H_0: \sigma^2 = \sigma_0^2$ $(= 0.17^2)$ vs. $H_1: \sigma^2 \overset{\neq}{\underset{<}{>}} \sigma_0^2$

Write $\chi^2 = \frac{(n-1) S^2}{}$ ...

Note $X^2 = \frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$ (Thm 8.11) when H_0 is true.

To find a level α test, use χ^2_{n-1} dist to determine CR.

* Reject H_0 in favor of $H_1 : \sigma^2 \begin{smallmatrix}\neq\\<\\>\end{smallmatrix} \sigma_0^2$ if X^2 $\begin{smallmatrix}\notin [\chi^2_{1-\frac{\alpha}{2}, n-1}, \ \chi^2_{\frac{\alpha}{2}, n-1}] \\ < \chi^2_{1-\alpha, n-1} \\ > \chi^2_{\alpha, n-1}\end{smallmatrix}$.



Ex: $n=30$, $S = 0.21$, original pop $\sigma_0 = 0.17$. evidence that σ increased? $\alpha = 0.05$.

Sol: $H_0 : \sigma^2 = 0.17^2$ vs $H_1 : \sigma^2 > 0.17^2$

$$X^2 = \frac{(30-1)\, 0.21^2}{0.17^2} = 44.25.$$

Method 1: $\chi^2_{0.05, 29} = 42.557 < X^2 \Rightarrow$ reject H_0

Method 2: p-val $= \mathbb{P}(X^2 > 44.25) \in (0.025, 0.05) \Rightarrow$ reject H_0.

Two-sample test:

Compare SDs from independent samples.

Exercise 12.26. $\Rightarrow$ LRT based on $\frac{S_1^2}{S_2^2}$ when we assume the populations are normal.

Theorem 8.15: $F = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F_{n_1-1, n_2-1}$. n_1, n_2 sample size of the two samples.

Consider test $H_0 : \sigma_1^2 = \sigma_2^2$. under H_0, $F = \frac{S_1^2}{S_2^2} \sim F_{n_1-1, n_2-1}$ $= (f_{\frac{\alpha}{2}, n_2-1, n_1-1})^{-1}$

Test: reject H_0 in favor of $H_1 : \sigma_1^2 \begin{smallmatrix}\neq\\<\\>\end{smallmatrix} \sigma_2^2$ if $\begin{smallmatrix} S_1^2/S_2^2 \notin [f_{1-\frac{\alpha}{2}, n_1-1, n_2-1}, \ f_{\frac{\alpha}{2}, n_1-1, n_2-1}] \\ S_1^2/S_2^2 < f_{1-\alpha, n_1-1, n_2-1} \\ S_1^2/S_2^2 > f_{\alpha, n_1-1, n_2-1}\end{smallmatrix}$

Note: Can't use these formulas if not normal populations.