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Def let $b < d$. The interval module $I[b,d)$ is the $\mathbb{R}$-indexed PVS w/

$$I[b,d)_r = \begin{cases} \mathbb{F} & : r \in [b,d) \\ 0 & : r \notin [b,d) \end{cases}$$

w/ linear maps $L_{r,s} = \begin{cases} \mathrm{Id}_{\mathbb{F}} & : r,s \in [b,d) \\ 0 & : \text{otherwise} \end{cases}$

Thm (Fund. Thm. of Persistent homology)

For a finitely presented PVS $V = \{V_r\}_{r \geq 0}$, $\exists\, b_i, d_i$, $i = 1, \dots, M$

iso of PVS (i.e. diagram commutes)

s.t. $V \cong I[b_1, d_1) \oplus \cdots \oplus I[b_M, d_M)$, and this repn is unique up to reordering.

i.e. every finite metric space has a unique barcode.

More Concepts From Linear Algebra:

Def A sequence

$$0 \xrightarrow{L_0} V_1 \xrightarrow{L_1} V_2 \xrightarrow{L_2} \cdots \xrightarrow{L_{n-1}} V_n \xrightarrow{L_n} 0$$

is called exact if $\mathrm{Ker}(L_{j+1}) = \mathrm{Im}(L_j)$ $\forall j$.

It's short if $n = 3$.

<u>Lemma</u> $\forall$ short exact sequence $0 \xrightarrow{L_0} V_1 \xrightarrow{L_1} V_2 \xrightarrow{L_2} V_3 \xrightarrow{L_3} 0$

L_1 is inj. & L_2 is surj.

<u>Thm</u> (The Snake Lemma): consider the commutative diagram

$$\begin{array}{ccccccccc}
0 & \longrightarrow & U_1 & \xrightarrow{p_1} & U_2 & \xrightarrow{p_2} & U_3 & \longrightarrow & 0 \\
 & & \downarrow f_1 & & \downarrow f_2 & & \downarrow f_3 & & \\
0 & \longrightarrow & V_1 & \xrightarrow[q_1]{} & V_2 & \xrightarrow[q_2]{} & V_3 & \longrightarrow & 0
\end{array}$$

If each row is exact, then there is a 6-term long exact sequence

$$0 \to \mathrm{Ker}(f_1) \xrightarrow[\bar{p}_1]{} \mathrm{Ker}(f_2) \xrightarrow[\bar{p}_2]{} \mathrm{Ker}(f_3) \xrightarrow[d]{} \mathrm{Coker}(f_1) \xrightarrow[\bar{q}_1]{} \mathrm{Coker}(f_2) \xrightarrow[\bar{q}_2]{} \mathrm{Coker}(f_3) \to 0$$

w/ $\mathrm{Coker}(f_j) = V_j / \mathrm{im}(f_j)$