Lec 9/14

Wednesday, September 14, 2016 9:07 AM

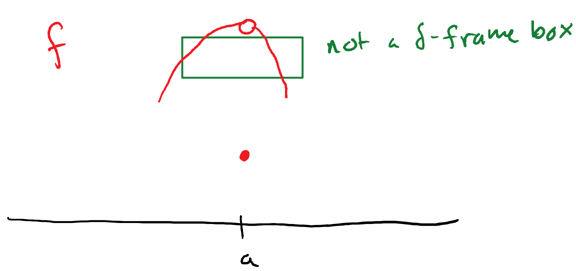
Finding a measure for discontinuity
- jump interval (or oscillation interval) at discontinuity.

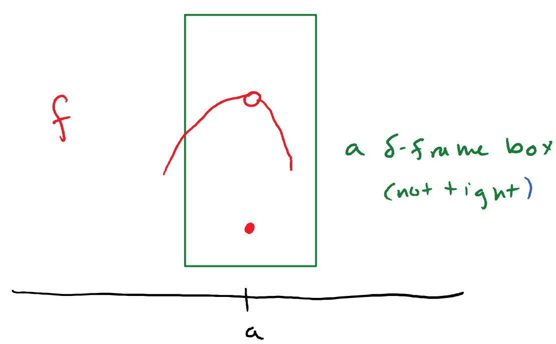
Given: f is discontinuous at a point $a \in \mathrm{dom}(f)$

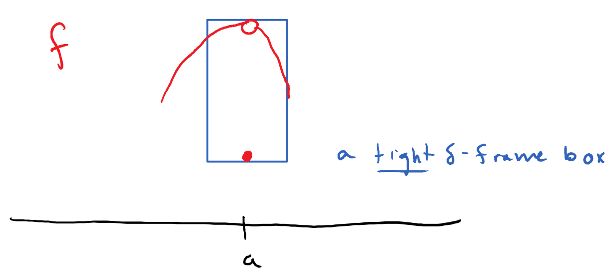
Definition: A (closed) box in $\mathbb{R}^2$ is a subset of the form $A \times B$ where A, B are closed intervals (and degenerate intervals and infinite closed intervals are allowed).

Definition: A δ-frame box at a point $a \in \mathrm{dom}(f)$ is a box $[a-\delta, a+\delta] \times B$ such that $\{f(x) : x \in [a-\delta, a+\delta] \cap \mathrm{dom}(f)\} \subseteq B$

We say that $[a-\delta, a+\delta] \times B$ is a <u>tight</u> δ-frame box if B is as small as possible.







$f(x) = c$ (constant)

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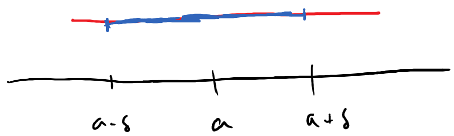
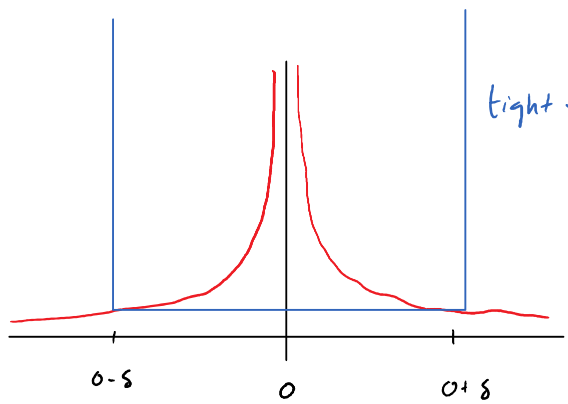
tight δ-frame box (B is degenerate)



$$f(x) = \begin{cases} \frac{1}{x^2} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

tight frame box (infinite B)



for each $\delta > 0$ there is a unique tight δ-frame box

$$[a-\delta, a+\delta] \times J_{f,a,\delta}$$

Proof:

$$[a-\delta, a+\delta] \times J_{f,a,\delta} = [a-\delta, a+\delta] \times \bigcap B$$

$\neq \emptyset$ since $f(a) \in$ all B so $f(a) \in \bigcap B$

over all B s.t. $[a-\delta, a+\delta] \times B$ is a δ-frame box

Theorem: Let $\{J_\alpha\}$ be any collection (non-empty)

$\bigcap_\alpha J_\alpha$ is one of the following:

(1) a (proper) closed interval

(2) a point

(3) the empty set $\emptyset$

Proof Indication:

Let $A =$ least upper bound of left endpoints of $\{J_\alpha\}$

$B =$ greatest lower bound of the right endpoints of $\{J_\alpha\}$

$A = \pm\infty$, $B = \pm\infty$ allowed

(1) occurs if $A < B$

(2) occurs if $A < B$ finite

(3) occurs if $B < A$ or $A = B = \pm\infty$

The jump interval at a point a in the domain of f

is $J_{f,a} = \bigcap_{\delta > 0} J_{f,a,\delta}$

f is continuous if $J_{f,a} = \{f(a)\}$

f is discontinuous if $J_{f,a}$ is a proper closed interval.