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Monday, October 28, 2019 15:03

General Properties of $\tilde{S}_K$.

$S_K = S^3 - N(k)$

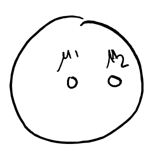
$\partial S_K = S^1_\lambda \times S^1_\mu$

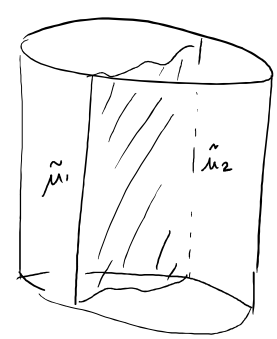
$$\begin{tikzcd}
S^1_\lambda \times \mathbb{R} = \partial \tilde{S}_k \arrow[r, hook] \arrow[d] & \tilde{S}_k \arrow[d] \\
S^1_\lambda \times S^1_\mu = \partial S_k \arrow[r, hook] & S_k
\end{tikzcd}$$

$$\pi_1(S^1_\lambda \times S^1_\mu) \longrightarrow \pi_1(S_K) \longrightarrow H_1(S_k)$$

$$\mathbb{Z}_\lambda \oplus \mathbb{Z}_\mu \longmapsto \mathbb{Z}$$

$K = K_1 \# K_2$

Annulus 

lifts to ribbon 

which splits $\tilde{S}_K = \tilde{S}_{K_1} \underset{\text{ribbon}}{\sqcup} \tilde{S}_{K_2}$

So $H_1(\tilde{S}_K) = H_1(\tilde{S}_{K_1}) \oplus H_1(\tilde{S}_{K_2})$

So

up to a unit

Lemma: $\Delta_K \doteq \Delta_{K_1} \Delta_{K_2}$

Note: $\Delta_{8_{18}} = \Delta_{3_1}^2 \Delta_{4_1}$

Also, the lemma doesn't help find prime knots since there are knots w/ $\Delta = 1$.

Can we obtain more invariants via higher Homology/Homotopy?

No: $*$ S_K is a homology circle

$*$ $\pi_2(S_K) = 1$ by Papakyriakopoulos Thm (Sphere thm)

$\vdots$

$\pi_j(S_K) = 1$ $(j>1)$ $(S_K = \text{aspherical})$

Seifert Matrices

$K \hookrightarrow S^3$ knot, and $S_K = S^3 \setminus K$.

$\Sigma \hookrightarrow S^3$ seifert surface.

Recall $\mathcal{C} \hookrightarrow S_k$ $\quad [\mathcal{C}] \in H_1(S_k)$

$\hookrightarrow$ intersection # $I(\Sigma, \mathcal{C})$

$M^\circ = S_k - \Sigma$, $\mathcal{C} \hookrightarrow M^\circ$ will not intersect Σ,

so $[\mathcal{C}] = 0$

So $\pi_1(M^\circ) \hookrightarrow P_*(\pi_1(\tilde{S}_k))$

cut up
by lifts of Σ

$$\begin{tikzcd}
\tilde{M}^\circ = \bigsqcup_{i \in \mathbb{Z}} M_i^\circ \arrow[r, hook] \arrow[d] & \tilde{S}_k \arrow[d] \\
M^\circ \arrow[r, hook] & S_k
\end{tikzcd}$$

similarly, $V = \Sigma \times (-1,1)$

lifts to $\tilde{V} = \bigsqcup_{i \in \mathbb{Z}} V_i$

So $\tilde{S}_k = \tilde{M}^\circ \sqcup \tilde{V}$. Let $V^+ = \Sigma \times (0,1)$, $V^- = \Sigma \times (-1,0)$

$V_i^\pm \hookrightarrow M_i^\circ$ so glue $\tilde{M}^\circ$ along Σ to form $\tilde{S}_k$.