Lec 12/4

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Systems: $y'' = f(x, y, y')$

II $y'' = f(y, y')$, let $z = y'$, so $z\frac{\partial z}{\partial y} = f(y, z)$

I $y'' = f(x, y')$, let $z = y'$, so $z' = f(x, z)$

ex: $y'' = y y'$. $y(0) = 0$, $y'(0) = \frac{1}{2}$.

$y' = z$, $y'' = z\frac{\partial z}{\partial y}$

so $z\frac{\partial z}{\partial y} = yz$. $z = 0$ is a soln. but doesn't satisfy initial cond.

$\frac{\partial z}{\partial y} = y \Rightarrow z = \frac{1}{2}y^2 + c$

$\Rightarrow y' = \frac{1}{2}y^2 + \frac{1}{2}$ $y'(0) = z(0) = \frac{1}{2}$ since $y(0) = 0$

$\Rightarrow \frac{2\,\partial y}{1 + y^2} = \partial x$.

III $y'' = f(x, y)$

$y'' + 4y = 0$

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$$y = A\sin(2x)$$

$$y'' = -4y, \quad z' = y, \quad z\frac{\partial z}{\partial y} = -4y$$

$$z\,\partial z = -4y\,\partial y$$

$$\frac{z^2}{2} = -2y^2 + c$$

$$z^2 = -4y^2 + C$$

$$\frac{\partial y}{\partial z} = \pm\sqrt{c - 4y^2}$$

$$\frac{\partial y}{\sqrt{c - 4y^2}} = \partial z$$

So look at III $y'' = f(x,y)$ $\quad y(x_0) = y_0, \quad y'(x_0) = y_1$

$\varphi(x)$ solution $\varphi''(x) = f(x, \varphi(x)), \quad \varphi(x_0) = y_0, \quad \varphi'(x_0) = y_1$

$$\varphi'(x) = y_1 + \int_{x_0}^{x} f(t, \varphi(t))\,dt$$

$$\varphi(x) = y_0 + y_1(x - x_0) + \int_{x_0}^{x}\int_{x_0}^{s} f(t, \varphi(t))\,dt\,\partial s$$

$$y(x) = y_0 + y_1(x-x_0) + \int_{x_0}\int_{x_0} f(t,\varphi(t))\ \cdots$$

$$= y_0 + y_1(x-x_0) + \int_{x_0}^{x} dt \int_{t}^{x} \underbrace{f(t,\varphi(t))}_{\text{ind. of } s}\, ds$$

$$= y_0 + y_1(x-x_0) + \int_{x_0}^{x} f(t,\varphi(t))\,(x-t)\,dt$$

is the solution. this can probably be generalized

How about general case:

$$y^{(n)} = f(x, y, y', \ldots, y^{(n-1)})$$

$$y(x_0) = \alpha_0$$

$$y'(x_0) = \alpha_1$$

$$\vdots$$

$$y^{(n-1)}(x_0) = \alpha_{n-1}$$

Let $y_1 = y$, $y_2 = y'$, $\ldots$, $y_n = y^{(n-1)}$

equation becomes a system of order 1.

$$y_1' = y_2, \quad y_2' = y_3, \quad \ldots, \quad y_{n-1}' = y_n, \quad y_n' = y^{(n)}$$

$$= f(x, y_1, y_2, \ldots, y_n)$$

system of eqns

where y is defined

Let $Y = (y_1, y_2, \dots, y_n) : I \longrightarrow \mathbb{R}^n$

$$\downarrow F$$

$$Y' = (y_2, y_3, \dots, f(x, y_1, y_2, \dots, y_n))$$

So $Y' = F(x, Y)$, $Y(x_0) = \alpha = (\alpha_0, \dots, \alpha_{n-1})$

So $\Phi(x) = \alpha + \int_{x_0}^{x} F(t, \Phi(t))\, dt \in \mathbb{R}^n$

So can use approx. method for Φ.