

Smale's Horseshoe

Stochastic Matrices form a Semigroup

Group: Locally compact. Haar measure.

$T: X \rightarrow X$ gives \mathbb{Z} -action $x \mapsto T^n x$.

$T^n: f(x) \mapsto f(T^n x)$ so \mathbb{Z} action on functions on X .

$$\frac{1}{N} \sum f(T^n x) \xrightarrow{\text{a.e.}} \int f$$

$$\frac{1}{N} \sum T^n f \xrightarrow{\text{a.e.}} \int f$$

$$T(\alpha f + \beta g) = \alpha T f + \beta T g$$

group acting, counting, combinatorics.

Smale's Horseshoe - governed by dynamics on Cantor set.

$$A \in P+Q \iff \{n: (A-n) \in P\} \in \mathcal{I}.$$

by Ellis's Lemma, $\exists P = P+P$.

$$N = \bigcup_{i=1}^r C_i \Rightarrow \text{one } C_i = C \text{ is } P\text{-large (i.e. } C \in P).$$

$$C \in P = P+P \Rightarrow \{n: (C-n) \in P\} \in P.$$

So P -many shifts of C are P -large. So $\exists n_1 \in C$ s.t. $C - n_1 \in P$. So $C \cap C - n_1 \in P = P+P$

continue, letting

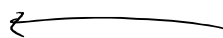
$$B_1 = C \cap C - n_1$$

and finding $n_2 \in B_1 = C \cap C - n_1$

$$B_2 = B_1 \cap B_1 - n_2 \in P = P+P.$$

etc.

$$FS(n_i) \subset C.$$



Ex: $\forall \varepsilon > 0, \exists A$ w/ $\bar{d}(A) > 1 - \varepsilon$ s.t. A contains no shift of an IP set.

→ find proof of Ellis Lemma online (VB survey).

Midterm Stuff from Handouts: (Read Derangement stuff, any interesting things can be asked)

- Automorphism groups/permutation groups

↳ 14.3 orbits & transitivity

Lagrange Thm

14.4.2 Schreier's Lemma

(Bell #s, Derangements, Catalan #s)

- Generating Functions

↳ Definitions of Generating functions (ordinary & exponential)

↳ Examples: eg 8.3 on p. 155. (problems like this)

↳ products of generating fns & interpretation (Thm 8.5 pg 156)

↳ examples: 8.7 & 8.8.

↳ Catalan $C(x) = \frac{1 - \sqrt{1-4x}}{2x}$ pg 161-162

↳ more examples w/ exponential GF.

• Matching, Covering & Packing.

↳ 3 or 4 theorems which are equivalent (max-min)

↳ find 3 applications of marriage lemma

König, Dilworth, Marriage theorem, Hall, Menger, König-Egervary -

find matrix version of all of these.

TO know → Hall thm. Defn of Stable matching. Something about path covers.

Know: König-Egervary & Bistochastic matrix theorem

Know: Caratheodory theorem. Browse Convexity Packet too