Lec 9/27

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$P \in F[X] \overset{\sim}{\longmapsto} \tilde{P} : F \to F$ (evaluation map)

$P(x) = \alpha_0 + \alpha_1 x + \cdots + \alpha_n x^n$ $\quad \tilde{P}(\xi) = \alpha_0 + \alpha_1 \xi + \cdots + \alpha_n \xi^n$ for $\xi \in F$.

$\sim$ is a homomorphism (preserves ring structure)

$\widetilde{(P+Q)} = \tilde{P} + \tilde{Q}$, $\quad \widetilde{PQ} = \tilde{P}\tilde{Q}$

of division w/ remainder

<u>Cor 1</u> Let $P \in F[X]$, $\xi \in F$, $G(X) = X - \xi$.

Then $P = (X - \xi)Q + R$ where $R \in F$. so $R = \lambda \in F$.

<u>Cor 2</u> Let $P \in F[X]$, $\xi \in F$ s.t. $P(\xi) = 0$. Then $P = (X - \xi)Q$ (and vice versa)

not x but 'times'.

Units: invertible polynomials $\cong F \setminus \{0\} = F^{\times}$

↳ universal divisors.
trivial

$P \in F[X]$ is prime if whenever $P = gh$, $g, h \in F[X]$ then either g or h is a unit.

Prop if $p \mid fg$ then $p \mid f$ or $p \mid g$.

unique up to units

<u>Thm</u> $\exists !$ gcd of $f_1, \dots, f_n \in F[X]$. (d is $\gcd(f_1, \dots, f_n)$ if $d \mid f_i$ $\forall i$ and any $d' \mid f_i$ $\forall i \Rightarrow d' \mid d$.)

<u>pf</u> tomorrow.