<u>Recall</u> PL link in $\mathbb{R}^3$ can be rotated & projected to give local pictures X , / , ⌐ .
to reconstruct the knot, we keep track of "over" & "under" : X or X .
Then we can do :

since $\mathbb{R}^2 \subset \mathbb{R}^3$.

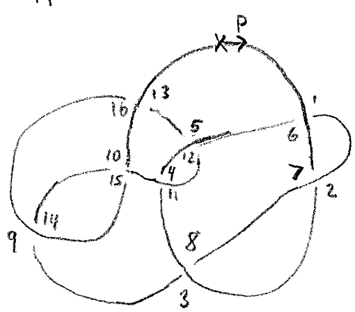
$$\overline{K} \longmapsto \overline{K}^{\#}$$

<u>Lemma</u> : $K \approx \overline{K}^{\#}$. <u>Pf</u> isotop up straight lines. use convex combination at crossings.

K = PL knot, $\overline{K}$ = diagram. equip K w/ orientation. get two distinct pictures @ crossings (not dependent on orientation for a knot)

<u>Dowker-Thistlethwaite Notation</u> :

$$8_{19} = T(3,4) = P(2,-3,-3)$$



* pick pt & orientation
* label crossings in order
* odd even : odd in order :

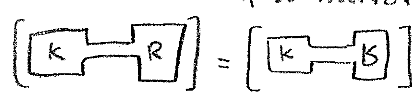
(1,6) (3,8) (5,12) (7,2) (9,14) (11,4) (13,16) (15,10)

$\rightsquigarrow$ 6 8 12 2 14 4 16 10

* keep track of over-under: $(\hat{1},\check{6})$ $(\hat{3},\check{8})$ ⋯ $(\check{9},\hat{14})$ ⋯ $(\check{13},\hat{16})$ $(\hat{15},\check{10})$

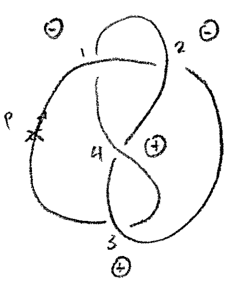
$\rightsquigarrow$ $[6\ \ 8\ \ 12\ \ 2\ \ -14\ \ 4\ \ -16\ \ 10]$ = DT-code.

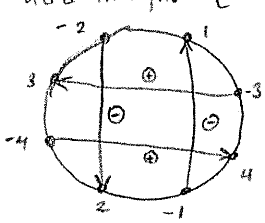
<u>Lemma</u> : if K is prime, DT-code determines K up to mirror (& orientation).
Not true if K isn't prime.



<u>Gauss-Code</u> :

* numbering, skipping already labeled, $1, \dots, n$ = crossing #
* second trip : $\begin{bmatrix} 1 & -2 & 3 & -4 & 2 & -1 & 4 & -3 \\ & - & - & + & + & & & \end{bmatrix}$ (negative for under crossings)
* add in signs : $\longrightarrow$ Determines knot diagram.

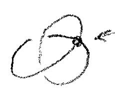




$\rightsquigarrow$ Finite type invariant theory.

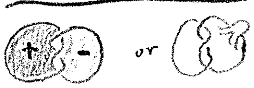
$\longrightarrow$ Can erase all the numbers (Gauss diagram)

<u>Do all codes give diagrams?</u>



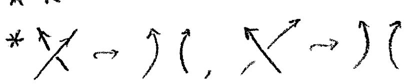
← problem.
Introduce virtual knots - some crossings are not crossings.

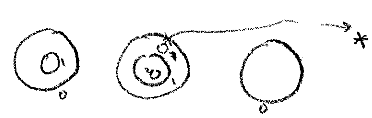
<u>SEIFERT SURFACES</u> : Suppose $K \subset S^3$ is PL-knot (smooth). A seifert surface is embedded (PL/smooth), oriented, compact surface $\Sigma \hookrightarrow S^3$ such that $\partial\Sigma = K \cong S^1$.



<u>Seifert Algorithm</u> :

* K ← orientation.





* yields oriented curve in $\mathbb{R}^2$, no crossings. (Seifert cycles)
* assign to each circle nesting #.
* to each seifert cycle $\mathcal{C}$ assign curve $\hat{\mathcal{C}} = \mathcal{C} \times \{n_{\mathcal{C}}\} \subset \mathbb{R}^3$ (nesting #)
* each $\mathcal{C} \times \{n_{\mathcal{C}}\}$ bounds a disk disjoint from other disks.

... to be continued.