Lec 9/11

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$L(F^n, F^m) \longleftrightarrow M_{m,n}(F)$

matrix multiplication $\longleftrightarrow$ transformation composition.

So the set of matrices $M_{n,n}(F)$ is a ring.

Inv. $\exists T^{-1}$ s.t. $T^{-1} \circ T = I = T \circ T^{-1}$

for matrices: $I = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$

$A \in M_{n,n}(F)$ is invertible iff $Ax = 0 \Rightarrow x = 0$.

i.e. nullity $A = 0$

rank $A = n$

I —
- multiply ith row by λ $= \begin{pmatrix} 1 & & & & 0 \\ & \ddots & & & \\ & & \lambda & & \\ & & & \ddots & \\ 0 & & & & 1 \end{pmatrix} = D_i(\lambda)$
- add to ith row $\lambda \cdot j$th row $= \begin{pmatrix} 1 & & & & 0 \\ & \ddots & & & \\ & \lambda & 0 \cdot 1 & & \\ & & & \ddots & \\ 0 & & & & 1 \end{pmatrix} = B_{ij}(\lambda)$
- exchange ith & jth row $= \begin{pmatrix} 1 & & & \\ & \ddots & 0 & 1 \\ 0 & 1 & \ddots & 0 \\ & & & 1 \end{pmatrix} = P_{ij}(\lambda)$

$D_i(\lambda) A \sim A'$ (mult. a row by λ)

$B_{ij}(\lambda) A \sim A'$ (add a multiple of one to another)

$P_{ij}(\lambda) A \sim A'$ (interchange rows)