

§ 16 Nonparametric tests

Relax assumptions about Dists under H_0 .

Q: what if we don't know exact distribution of population.
or if n is "not large"

A: Nonparametric tests.

Nonparametric \Rightarrow don't assume a particular dist.

- Some are distribution-free (no assumpt abt pop except Cts or Discrete).
- Assume some characteristic of pop like symmetry.

Parametric vs. nonparametric.

- NPM have wider applicability bc they make fewer assumptions.
- However, they generally have lower power than appropriate PM tests.
- If we don't know appropriate parametric test, or can't be sure assumptions of PM test are true, NPM tests are valuable.

Sec 16.2 The Sign test

- Nonparametric alternative to 1-sample t -test.
- Assume symmetric & Cts population.
- Testing $H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$ (or $\mu \geq \mu_0$)

Example $H_0: \mu = 10$

Data: 10.1, 10.2, 10.05, 10.4, 9.8, 10.5, 10.46, 10.19, 9.6, 10.2, 10.4
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if we use t-test we must assume pop is normal. $t = \frac{\bar{x} - 10}{s/\sqrt{n}} \sim t_{10}$
but we don't.

Since we assume symmetry, can consider count above & below μ_0 .

Let $V = \#$ of observations above μ_0 .

Under H_0 , $V \sim \text{Bin}(n, \frac{1}{2})$

So test $H_0: \mu = \mu_0$ vs $H_1: \mu \neq \mu_0$, have a simpler problem of deciding

whether $p = \frac{1}{2}$ or $p \neq \frac{1}{2}$ (if H_1 is 1-sided, use $p \geq \frac{1}{2}$ instead appropriately).
 H_0 H_1

obs. val of V is 9.

Compare this to a $\text{Bin}(11, \frac{1}{2})$ $P(V \geq 9 \text{ or } V \leq 2) = 0.0386$

So reject $H_0 \Rightarrow \mu \neq 10$.