Lec 9/15

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Gram-Schmidt Orthonormalization Procedure:

- Start with a basis $\{v_1, \dots, v_n\}$ of V.
- $u_1 = \frac{v_1}{|v_1|}$,
- $w_2 = v_2 - \langle v_2, u_1 \rangle u_1$, so $w_2 \cdot u_1 = 0$ so $u_2 = \frac{w_2}{|w_2|}$

$S(u_1, u_2) = S(v_2, v_2)$

- $w_n = v_n - \sum_{i=1}^{n-1} \langle v_n, u_i \rangle u_i$, $\quad u_n = \frac{w_n}{|w_n|}$.

(by induction if $S(u_1, \dots, u_{n-1}) = S(v_1, \dots, v_{n-1})$ etc

then $\langle u_i, w_n \rangle = \langle u_i, v_n \rangle - \langle u_i, v_n \rangle + 0 = 0$, etc).

and $|w_n| \neq 0$ o.w. v_n would be a scalar multiple of $\{v_1, \dots, v_{n-1}\}$

and $u_n \in S(v_1, \dots, v_n)$ and $v_n \in S(u_1, \dots, u_n)$.

$V = C([-1,1])$, $\quad \langle f, g \rangle = \int_{-1}^{1} f(x) g(x)\, dx$

$\{\sin \pi x, \sin 2\pi x, \dots, \sin n\pi x, \dots\}$ is an orthonormal set.