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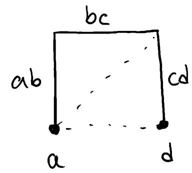
Monday, February 25, 2019 10:38

$$C_1(X) \xrightarrow{\partial_1} C_1(X)$$

edges — vertices

Vs over $\mathbb{F}_2$



$$\partial_1(xy) = x + y$$

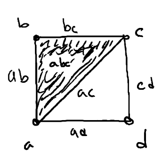
$\mathrm{Ker}(\partial_1)$ = closed loops.

row reduce matrix for ∂_1 to find a matrix w/ same kernel.

In the example,

$$\mathrm{Ker}(\partial_1) = \mathrm{Span}\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$$= \mathrm{Span}\{ab + bc + ac,\ ac + cd + ad\}.$$

Ex: $Y =$ 

$C_0(Y) = \mathrm{Span}(a, b, c, d)$.

$C_1(Y) = \mathrm{Span}(ab, ac, ad, bc, cd)$.

$C_2(Y) = \mathrm{Span}(abc)$.

$$C_2(Y) \xrightarrow{\partial_2} C_1(Y) \xrightarrow{\partial_1} C_0(Y)$$

$\uparrow$

$\mathrm{Ker}(\partial_1) = \mathrm{Span}(ab+bc+ac,\ ac+cd+ad)$

doesn't pick up on the fact that has been filled.

$\partial_2(abc) = ab + bc + ac$

So 1^{st} homology vector space of Y is

$$H_1(Y) = \mathrm{Ker}(\partial_1) / \mathrm{Im}(\partial_2)$$

<u>Goal</u>: define $H_k(Y)$ for general sp cpx Y and $k = 0, 1, 2, \ldots$

<u>Defn</u>: Let S be a set & F a field. The free v.s. over F generated by F is the set of functions $f : S \longrightarrow F$. it's $V_F(S)$.

Let Y be an abstract sp cpx.

<u>Def</u> the k^{th} chain group of Y is $C_k(Y) = V_{\mathbb{F}_2}(Y_k)$.

<u>Want</u>: define $\partial_k : C_k(Y) \longrightarrow C_{k-1}(Y)$

Show $\text{Im}(\partial_{k+1}) \subseteq \text{Ker}(\partial_k)$

Define $H_k = \text{Ker}(\partial_k) / \text{Im}(\partial_{k+1})$