Rec 9/17

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for <u>exam</u>: Know Burnsides Lemma

find a group of order 12.

$12 = 2^2 \times 3$.

$n_3 = 1 \bmod 3,\ n_3 \mid 4 \Rightarrow n_3 = 1$ or 4.

$n_2 = 1 \bmod 2,\ n_2 \mid 3 \Rightarrow n_2 = 1$ or 3.

$n_3 = 1,\ n_2 = 1 \Rightarrow G = P_3 \times P_2 = \underbrace{\mathbb{Z}_3 \times \mathbb{Z}_4}_{= \mathbb{Z}_{12}}$ or $\underbrace{\mathbb{Z}_3 \times \mathbb{Z}_2 \times \mathbb{Z}_2}_{= \mathbb{Z}_6 \times \mathbb{Z}_2}$.

$n_3 = 4,\ n_2 = 3 \Rightarrow$ 4 3-gps intersect only at 1, get $2 \times 4 = 8$ elements of order 3.
must have 3 elts of order 2,

<u>Burnsides Thm</u> Let $G \circlearrowright S$ with r orbits. Then

$$r = \frac{1}{|G|} \sum_{g \in G} |\underbrace{\mathrm{fix}_g(S)}_{= \{x \in S \mid gx = x\}}|.$$

$G \circlearrowright S$ ($|S|>1$) transitively $\Longrightarrow \exists g \in G$ with no fixed points.

pf : $|G| = \sum_{g \in G} |\mathrm{fix}_g(S)|$, but $|\mathrm{fix}_1(S) = S| = |S| > 1$.

do this!

Application: $G \circlearrowright \{xHx^{-1}\}$, can show $\bigcup xHx^{-1} = G \Rightarrow$ every g has a fixed pt.

So action would not be transitive $\lightning$

normal series

$$G = G_1 \rhd G_2 \rhd \cdots \rhd G_s \rhd G_{s+1}$$

is a normal series in G.

Eg $S_n \rhd A_n \rhd 1$.

If G_i / G_{i+1} is abelian and $G_{s+1} = 1$, then G is solvable.

Eg $\underbrace{S_3 \rhd \overset{\mathbb{Z}_3}{\overset{\parallel}{A_3}}}_{S_3/A_3 = \mathbb{Z}_2} \rhd 1$ is solvable

Sylow Theorems

- Sylow p-subgroups exist. $\rightsquigarrow$ there are groups of p^k $\forall\ p^k \mid |G|$.
- All sylow p-subgroups are conjugate.
- Every p-subgroup is contained in some Sylow p-subgroup.

Let n_p = # of Sylow p-subgroups in G.

- $n_p \equiv 1 \mod p$
- $n_p \mid [G : P]$ where P is a Sylow p-subgroup.

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