Lec 8/30

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1) V fin. gen vs $/F$, $W \leq V$ is a subspace

If $\dim(W) = \dim(V)$, $W = V$.

Pf If $\dim(W) = \dim(V) = n$, both W and V have bases $\{w_1, \dots, w_n\}$, $\{v_1, \dots, v_n\}$

Assume $W \neq V$. Then $\exists$ vector $u \in V \setminus W$ s.t. $u = \alpha_1 v_1 + \cdots + \alpha_n v_n$.

but then $V \supseteq \{w_1, \dots, w_n, u\}$ are lin. indp., contradicting $\dim(V) = n$.

2) $V = \mathbb{R}^3$. Subspace of dim 0 is $\{0\}$. of dim 1 are lines, of dim 2 are planes. thru origin.

3) $\mathcal{P}_4(\mathbb{R}) = \{p(x) = \alpha_4 x^4 + \alpha_3 x^3 + \alpha_2 x^2 + \alpha_1 x + \alpha_0 \ ; \ \alpha_i \in \mathbb{R}\}$

$W = \{p \in \mathcal{P}_4(\mathbb{R}) \ ; \ p(2) = 0\}$ find a basis of W.

$\{x-2, (x-2)^2, (x-2)^3, (x-2)^4\} \Rightarrow$ this is lin. indp. easy

$p(x) = (x-2)q(x)$ $\deg(q) \leq 3$

$= (x-2)(\lambda_3 x^3 + \lambda_2 x^2 + \lambda_1 x + \lambda_0)$

$= \lambda_3 (x-2)x^3 + \lambda_2 (x-2)x^2 + \lambda_1 (x-2)x + \lambda_0 (x-2)$

So lets use $\{(x-2), (x-2)x, (x-2)x^2, (x-2)x^3\}$

which is lin. indp.

if $0 = \lambda_3 (x-2)x^3 + \lambda_2 (x-2)x^2 + \lambda_1 (x-2)x + \lambda_0 (x-2)$,

$0 = \lambda_3 (x^3 + 3(x-2)x^2) + \lambda_2 (x^2 + 2(x-2)x) + \lambda_1 (x + (x-2)) + \lambda_0$

$\vdots$

$\lambda_i = 0$.

So $\dim(W) = 4$, $\dim(\mathcal{P}_4(\mathbb{R})) = 5$.

Now showing $\{(x-2)^i\}$ is a basis.

they are lin. indep. as in earlier by the same argument, and since there are 4 of them in a dim 4 subspace, they generate W.

$x = t+2 \Leftrightarrow t = x-2$, get a poly in t, easy to check.

4) $S = \{ p \in \mathcal{P}_4(\mathbb{R}) \,;\, \int_0^1 p = 0 \}$. find basis.

this is easily a subspace.

$\{ x - \frac{1}{2},\ x^2 - \frac{1}{3},\ x^3 - \frac{1}{4},\ x^4 - \frac{1}{5} \}$

if $\int_0^1 p = 0$, then $\frac{\alpha_4}{5} + \frac{\alpha_3}{4} + \frac{\alpha_2}{3} + \frac{\alpha_1}{2} + \alpha_0 = 0$

These vectors are linearly independent so, since $\dim(S) < 5$ (o.w. $S = \mathcal{P}_4(\mathbb{R})$ but $1 \notin S$), this set of 4 vectors must generate S.

5) $\{v_1, \ldots, v_n\} \subset V/_F$ linearly independent, $w \in V$

$\dim(S(v_1 + w, \ldots, v_n + w)) \geq n-1$

I $w \in S(v_1, \ldots, v_n) \Rightarrow S(v_1 + w, \ldots, v_n + w) \leq S(v_1, \ldots, v_n)$ so $n-1 \leq \dim \leq n$

II $w \notin S(v_1, \ldots, v_n) \Rightarrow$

all vectors are lin. indep.

$\lambda_1(v_1 + w) + \cdots + \lambda_n(v_n + w) = 0 \Rightarrow \lambda_1 v_1 + \cdots + \lambda_n v_n + (\lambda_1 + \cdots + \lambda_n) w = 0 \Rightarrow \lambda_i = 0$.

so $\dim(S(v_1 + w, \ldots, v_n + w)) = n$.