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$$C_k(X) = V_{F_2}(X_k)$$

$$\partial_k : C_k \longrightarrow C_{k-1}$$

$$\partial_k(\{v_0, v_1, \ldots, v_k\}) = \sum_j \{v_0, \ldots, \hat{v}_j, \ldots, v_k\}$$, extend linearly

<u>Thm</u>: $\forall k$,

$$\begin{tikzcd} C_{k+1} \arrow[r, "\partial_{k+1}"] \arrow[rr, bend right, "0"'] & C_k \arrow[r, "\partial_k"] & C_{k-1} \end{tikzcd}$$

. i.e. $\partial_k(\partial_{k+1}(v)) = 0 \ \forall v \in C_{k+1}$.

we write $\partial^2 = 0$.

<u>Pf</u> There are 2 ways to get each $(k-2)$-simplex from a k-simplex.

The subspace of k-cycles of X is $Z_k(X) = \ker(\partial_k^\times)$.

The subspace of k-boundaries of X is $B_k(X) = \mathrm{im}(\partial_{k+1}^\times)$.

<u>Prop</u>: $B_k \subseteq Z_k$

<u>Pf</u> we just did.

<u>Def</u> The k^{th} homology group of X is the vector space $H_k = Z_k / B_k$.