Lec 10/19

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Adjoint transformation: T^* so that $\langle Tv, w\rangle = \langle v, T^* w\rangle$ for any $v, w \in V$.

$T^* \rightsquigarrow \overline{A^T}$ if $T \rightsquigarrow A$.

U unitary $\Leftrightarrow$ $A^* = A^{-1}$ $\Leftrightarrow$ $U^* = U^{-1}$ $\langle v, w\rangle = \langle Uv, Uw\rangle = \langle U^* U v, w\rangle$

for any v, w, so $U^* U = I$.

$T^{**} = T$ since $\langle T^{**} v, w\rangle = \langle v, T^* w\rangle = \langle T v, w\rangle$.

$(\lambda_1 T_1 + \lambda_2 T^2)^* = \bar{\lambda}_1 T_1^* + \bar{\lambda}_2 T_2^*$ since $A^* = \overline{A^T}$.

$(T_1 T_2)^* = T_2^* T_1^*$

since

$\langle T_1 T_2 v, w\rangle = \langle T_2 v, T_1^* w\rangle = \langle v, T_2^* T_1^* w\rangle$

Analogue of symmetric $\langle Tv, w\rangle = \langle v, Tw\rangle$ over $\mathbb{R}$

$T^* = T$ self-adjoint or hermitian symmetric.

Def T is Normal if $T^* T = T T^*$.

Spectral Theorem: T normal $\Leftrightarrow$ $\exists$ $OB(V)$ of eigenvectors of T.

Proof Idea: $h(x)$ = char. poly. of T with distinct roots $\{\lambda_1, \dots, \lambda_r\}$.

$V_i = n(T - \lambda_i I)$, want to prove $V = V_1 \oplus \cdots \oplus V_r$ and $\langle v_i, v_j\rangle = 0$ if $i \neq j$.

$T^*(V_i) \subseteq V_i$ since $T T^* v_i = T^* T v_i = T^* \lambda_i v_i = \lambda_i T^* v_i$