Lec 10/9

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Midterm: 5 probs + bonus

Solvable $\iff$ N & G/N solvable $\iff$ $\exists\ \Sigma$ s.t. grps are abelian.

Examples: Abelian $\subsetneq$ Nilpotent $\subsetneq$ Solvable (upper triangular 2×2)

Know stuff about D_{2n}, $A_n \trianglelefteq S_n$, matrices. eg #9 of PPP.pdf.

(A_n: simple for $n \geq 5$)

3 problems of type Examples / Statements of Thms / Defns

(Statements: Classification of finite ab gps. J-H thm.)

+1 Aut(G) for some familiar groups

$$W = \mathbb{Z}/p\mathbb{Z} \Rightarrow \mathrm{Aut}(W) = \mathbb{Z}/(p-1)\mathbb{Z}$$

$$W = \mathbb{Z}/p^n\mathbb{Z} \Rightarrow \mathrm{Aut}(W) = \mathbb{Z}/p^{n-1}(p-1)\mathbb{Z} \text{ if } p \text{ odd}$$

$$= \mathbb{Z}/2 \times \mathbb{Z}/2^{n-2} \text{ if } p = 2.$$

$$W = \mathbb{Z}/m\mathbb{Z} \times \mathbb{Z}/n\mathbb{Z} \Rightarrow \mathrm{Aut}(W) = \underset{\underset{(\mathbb{Z}/m\mathbb{Z})^\times}{\cong}}{\mathrm{Aut}(\mathbb{Z}/m\mathbb{Z})} \times \mathrm{Aut}(\mathbb{Z}/n\mathbb{Z})$$

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if $(m,n) = 1$.

$$\mathrm{Aut}(D_{2n}) \cong \mathbb{Z}/n\mathbb{Z} \rtimes \mathrm{Aut}_{gp}(\mathbb{Z}/n\mathbb{Z}).$$

(+1) Semidirect product: must know defn. $N \rtimes_\alpha H$.

$$\underline{h n h^{-1} = \alpha(h)(n)}$$

everything starts & ends here.

$$\left\langle x_1, x_2, y \;\middle|\; \begin{array}{ll} x_1^2 = e,\ x_2^2 = e, & y^7 = e \\ x_1 y x_1^{-1} = y & x_1 x_2 = x_2 x_1 \\ x_2 y x_2^{-1} = y^{-1} & \end{array} \right\rangle \cong D_{28}$$

$$r \longmapsto x_1 y \qquad s \longmapsto x_2$$

Comes from $(\mathbb{Z}/7) \rtimes_\alpha (\mathbb{Z}/2 \times \mathbb{Z}/2)$

$$\begin{array}{ccc} \mathbb{Z}/2 \times \mathbb{Z}/2 & \longrightarrow & \mathrm{Aut}_{gp}(\mathbb{Z}/7) \\ (1, 0) & \longmapsto & \mathrm{id}_{\mathbb{Z}/7} \\ (0, 1) & \longmapsto & \{y \mapsto y^{-1}\} \end{array}$$

T/F: $S = G/N$ simple $\Longrightarrow$ $\exists\, H \leq G$, $H \cong G/N$

$$G = \begin{bmatrix} d_1 & x & z \\ 0 & d_2 & y \\ 0 & 0 & d_3 \end{bmatrix} \trianglerighteq N = \begin{bmatrix} 1 & a & 0 \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}$$

$$G/N \cong \underline{\mathbb{F}_p^* \times \mathbb{F}_p^* \times \mathbb{F}_p^*}$$

$\hookrightarrow$ pf: every in G can be written as diag $\cdot\, N$.

Commutator Subgroups

$$[A;B] = \langle \{[a;b] : a \in A, b \in B\} \rangle$$

(1) $[A;B] \trianglelefteq G$ assuming $A, B \trianglelefteq G$.

(2) $[G;G] \trianglelefteq G$ is smallest normal subgp N s.t. G/N is abelian

$$C : G \longrightarrow \mathrm{Aut}_{gp}(G)$$

$$g \longmapsto \{x \mapsto gxg^{-1}\}$$

$C(g) \in \mathrm{Aut}_{gp}(G)$,

$C(gh) = C(g)C(h)$

$\varphi([a,b]) = [\varphi(a);\varphi(b)]$ $\forall$ gp hom φ.

(3) $[G;A] = \{e\} \iff A \subset Z(G)$

$G \cong N \rtimes H \Rightarrow G/N \cong H.$