algebraic Closure of finite fields:

$$\mathbb{F}_{p} \subseteq \mathbb{F}_{p^{2}!} \subseteq \mathbb{F}_{p^{3}!} \subseteq \cdots$$

$$\overline{\mathbb{F}} = \bigcup_{n=1}^{\infty} \mathbb{F}_{p^n}$$

An embedding of an extension K/F to an extension E/F

a hom.sm Y:K-)E 5.6. P/F = 1dF

K Which is always injective (Since K is a field)

An isomorphism K/F 4 K/F is called an automorphism of K/K.

Automorphisms of K/F form a group, Aut (K/E). if [K:F] is finite, any embedding K/F > K/F is an aut-sm.

If $K \subseteq E$, let $\alpha \in K$. Let $\varphi: K \longrightarrow E$ be an embedding of extensions (over F).

Let $f = m_{\alpha,F} \in F(x)$. Then $\varphi(f) = f$, so $\varphi = \varphi(f(\alpha)) = \varphi(\varphi(\alpha))$. So $\varphi(\alpha)$ is a conjugate of α over φ .

 $Q(\sqrt{2}) \longrightarrow E$ $\sqrt{2} \longrightarrow \pm \sqrt{2}$ $i \longmapsto \pm i$ $\sqrt{2} \longmapsto \omega^{k} \sqrt{2} \quad \text{where } (k,n) = 1.$

Let $K = F(\alpha)$, $n = \deg_F \alpha$.

then in any extr E/F with KSE, d has at most n conjugates in E.

Any embedding K/F F/F is defined by P(a) which is a conjugate of a.

So I at most n embeddings K/F - E/F.

I exactly a embedding K/F -> E/F iff

Mais separable & splits completely in E.

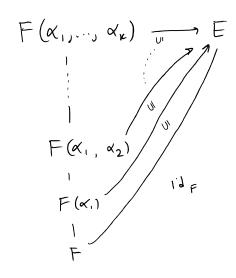
Let
$$F_{i} \subseteq E$$
. Let $\alpha \in E_{j}$
and let $\varphi: F_{i} \to E_{j}$, $\varphi(F_{i}) =: F_{2}$.
Let $f_{i} = m_{\alpha_{i}, F_{i}}$. Let $f_{2} = \varphi(f_{i})$.
Let $\widetilde{\varphi}: F_{i}(\alpha) \to E$ be a hom-sm set. $\widetilde{\varphi}|_{F_{i}} = \varphi$
Then $\widetilde{\varphi}(f_{i}(\alpha)) = \varphi(f_{i})(\widetilde{\varphi}(\alpha)) = f_{2}(\widetilde{\varphi}(\alpha))$
 $F_{i}(\alpha) \xrightarrow{\widetilde{\varphi}} E_{i}$ So $\widetilde{\varphi}(A)$ is a root of f_{2} .
 f_{i} is sire. if f_{2} is ire.

So
$$\# \{ \tilde{\varphi} : F_{r}(\alpha) \rightarrow E : \tilde{\varphi} |_{F_{r}} = \varphi \} = \# \text{ so to of } f_{2} \text{ in } E$$

$$\leq \deg f_{2} = \deg f_{1} = \deg f_{2} = \deg f_{2} = \deg f_{3} = \deg f_{4} = \deg f_{5} = \deg f_{5$$

let (K:F) be finite. Then K=F(x1,...,xr)
Tower of Simple extensions:

Page 3



$$K \subseteq E$$
.

embeddings

 $K/_F \longrightarrow E/_F$?

Let $N = [K:F] = (\deg_F \alpha_i)(\deg_{F(\alpha_i)} \alpha_2) \dots (\deg_{F(\alpha_{i,m,d_{k-1})}} \alpha_k)$.

We have = deg x, embeddings F(x,)/F - E/F.

of embedding P: F(x,) / => E/f, we

nove at most deg f(x,1) x2 embeddings F(x,1,x2)/F -> E/F

extending 4.

So we have at most n embeddings K/F = E/F.

If it, is separable over F& mxi, F splits completely in E, then there are exactly

this is nur

Sufficient nembeddings K/F -> E/F.

Not recessary)

to see the last part is true, add a bottom floor $F(\alpha)$ to the tower.

there is a wrong number of extensions at this step. So total # will be wrong.

Corollary: if K is generated by separable elements,

then every elment of K is separable.

(i.e. K is separable).

If additionally, min poll-lo of generating elements split in E, then tack, major splits in E.

this is the because the splittability of the minual polynomials is controllable: we can just pick the right E.