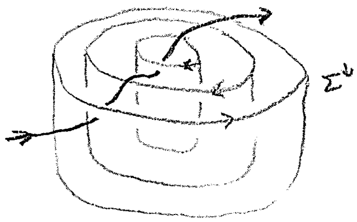
Recall: if $P, Q \subseteq N$ mfds and $p+q=n$, and $P \pitchfork Q$, we can define $I(P,Q)$ as sum of signs of intersections.

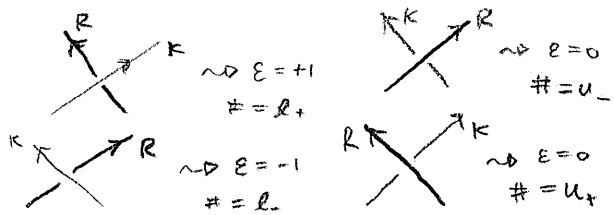
- If ℓ_1, ℓ_2 oriented curves in $\mathbb{R}^2$, $I(\ell_1, \ell_2) = 0$: "if you go in, you have to go out."
- $I(P,Q) = (-1)^{pq} I(Q,P)$. • η curve in $M-K \xrightarrow{\varphi} S^1 \ni 1$, $\Sigma = \varphi^{-1}(1)$ regular seifert surface, and $\eta \pitchfork \Sigma$, then $H_1(M-K) \xrightarrow{\cong} H_1(S^1)$ by $I(\eta, \Sigma) = \underbrace{\text{winding \# of } \varphi\circ\eta = I(\varphi\circ\eta, \{1\})}_{\text{linking number of } \eta \,\&\, K}$.

$[\eta] = I(\Sigma, \eta)[\mu]$  $\rightsquigarrow \varepsilon = +1$  $\rightsquigarrow \varepsilon = -1$.

Is there a better way to calculate linking #? probably:

Make seifert surface into "cans" (disks) and look from above:





$I(\Sigma^\downarrow, \eta) = \ell_+ - \ell_-$

if we pushed cans up instead:

$I(\Sigma^\uparrow, \eta) = u_- - u_+$.

These should be equal.

Notice that if $K, R \subseteq \mathbb{R}^3$, $I(K,R) = 0$, but this is $\ell_+ + u_+ - \ell_- - u_-$. So $\overbrace{u_+ + \ell_+ = u_- + \ell_-}^{\text{so they are}}$.

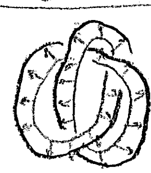
$I(\Sigma, \eta) = \frac{1}{2}(\ell_+ - u_+ + u_- - \ell_-) = Lk(K,R)$, the linking number. (nice symmetric formulation).

Note: $Lk(K^{-1}, R) = -Lk(K,R)$, and $Lk(K,R) = Lk(R,K)$

Self-linking of Framed Knot:

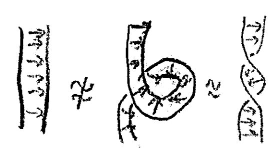
"push K off along the normal direction" to get $K^\#$. The self-linking # is $Lk(K, K^\#)$.

Blackboard Framing for a diagram



"always keep ↑ within the blackboard"

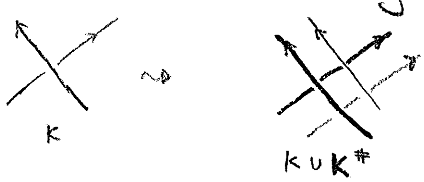
This depends on diagram

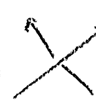


but any framing can be realized as some blackboard framing.

(note: it's the same if you go the other way)

How to read off self-linking of blackboard-framed diagram:



$\rightsquigarrow -1$ and similarly  $\rightsquigarrow +1$

so self-linking # is $\underbrace{\text{\# positive crossings} - \text{\# negative crossings}}_{\text{AKA writhe of diagram.}}$

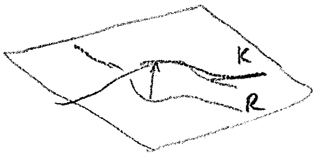
For framed link $L = K_1 \cup \cdots \cup K_m$, let $A(L) = (Lk(K_i, K_j^\#))$ - linking matrix.

$L = \underset{\gamma}{K} \cup \underset{\eta}{R} \subseteq \mathbb{R}^3$, $(s,t) \longmapsto (\gamma(s), \eta(t))$

$S^1 \times S^1 \longrightarrow \mathbb{R}^3 \times \mathbb{R}^3 - \Delta \xrightarrow[\text{eq.}]{\text{homotopy}} S^2$. $\Delta = \{(x,x) \mid x \in \mathbb{R}^3\}$

$(x,y) \longmapsto \dfrac{x-y}{|x-y|}$

So $\Gamma(s,t) = \dfrac{\gamma(s) - \eta(t)}{|\gamma(s) - \eta(t)|}$, $\underbrace{\Gamma : T^2 \longrightarrow S^2}_{\text{Gauss map}}$.



$\underset{p \in S^2 \text{ regular point}}{\deg_p \Gamma} = \sum_{x \in \Gamma^{-1}(p)} \mathrm{sgn}(\deg(d\Gamma_x))$

p = north pole: $\deg_p \Gamma = -$ linking # of K & R.

indep. of p? probably...