Lec 8/28

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Counting orders of elements in S_5.

Cycle-types in S_5:	Their Orders	Ex
5	5	(15324)
4+1	4	(1325)
3+1+1	3	(154)
3+2	6	(154)(23)
2+1+1+1	2	(23)(14)
2+2+1	2	(12)
1+1+1+1+1	1	e

$\binom{5}{3} \times 2$ of type $3+2$.
$\quad \shortparallel$
$\quad 20$

$\frac{1}{2}\binom{5}{2}\binom{3}{2}$ of type $2+2+1$
$\quad \shortparallel$
$\quad 15$

$4!$ of type 5

Partition fn: $p(n)$ = # ways to write n as sum of positive integers.
= # of cycle types in S_n.

$H \leq G \rightsquigarrow G/H$ set of left cosets

$\uparrow$ subgroup $\quad \uparrow$ group

an element of this is a subset of G of the form $gH = \{gh : h \in H\}$ for some $g \in G$

not unique in general.

Defn $H \leq G$ is normal, denoted by $H \trianglelefteq G$, if $\forall x \in G$, $h \in H$, we have $xhx^{-1} \in H$. i.e. $xH = Hx$.

(Careful: <u>not</u> saying $xhx^{-1} = h$)

<u>Ex</u>: if G is abelian, every subgroup is normal.

<u>Ex</u>: $D_{2n} = \langle s, r \mid s^2 = e = r^n,\ srs^{-1} = r^{-1} \rangle$

$H = \{e, r, r^2, \dots, r^{n-1}\}$ is normal. $\underbrace{\text{(only need to check } s \,\&\, r)}_{\text{Since everything in } G \text{ can be represented as + powers.}}$

$sr^ks^{-1} = r^{-k} \in H$.

$rr^kr^{-1} = r^k \in H$.

<u>Ex</u>: $G = S_4 \geq H = \{e, (12)(34), (13)(24), (14)(23)\}$. $\overbrace{\text{Check that } H \text{ is a subgroup.}}^{\text{ex:}}$

H is normal since $\sigma(x_1\ x_2\ \cdots\ x_\ell)\sigma^{-1} = (\sigma(x_1)\ \sigma(x_2)\ \cdots\ \sigma(x_\ell))$.

and so conjugation doesn't change "permutation type."

Note: this is unique to S_4. $H \leq S_5$ but not normal.

Let G be a group and $N \trianglelefteq G$. Group structure on G/N:

"definition" of $*$: $(g_1 N) * (g_2 N) = (g_1 g_2) N$.

choices.

Only thing to check is: if $g_1 \sim g_1'$ and $g_2 \sim g_2'$ then $g_1g_2 \sim g_1'g_2'$

(it obviously satisfies other axioms)

This is not always true: $G = S_4 \geq S_3 = \{\sigma \in S_4 : \sigma(4) = 4\}$. come up w/ example.

$(14) \sim (14)$

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$(14) \sim (14)$
$(14) \sim (124)$ but $e \nsim (21)$.

We have $g_1^{-1} g_1', \; g_2^{-1} g_2' \in H$.

Thus $(g_1 g_2)^{-1}(g_1' g_2') = g_2^{-1} \underbrace{\boxed{g_1^{-1} g_1'}}_{\text{an elt } h_1 \in H} g_2' = \underbrace{\boxed{g_2^{-1} h_1 g_2}}_{\in H} \underbrace{\boxed{g_2^{-1} g_2'}}_{\in H}$.

So G/N is called a Quotient group

= the set of left (or right) cosets of N in G.

eg $G = S_3 \geq N = \{e, \underset{t}{(123)}, \underset{t^{-1} = t^2}{(132)}\}$. Easy $\sigma \pi \sigma^{-1}$ does not change cycle type of π.

$N \cong \mathbb{Z}/3\mathbb{Z}$

$$\left|G/N\right| = 2 = \frac{|G|}{|N|} = \frac{6}{3}. \quad \text{So } G/N \cong \mathbb{Z}/2\mathbb{Z}.$$

$$\left(\text{recall: } |H| = p \text{ prime} \iff H \cong \mathbb{Z}/p\mathbb{Z}\right).$$

eg $G = GL_2(\mathbb{R})$ = {2x2 matrices w/ non-zero det}. $\det TXT^{-1} = \det X$.
(general linear)

$\geq$

$N = SL_2(\mathbb{R})$ = { " " w/ det = 1 }
(special linear)

Preview: $GL_n(\mathbb{R}) / SL_n(\mathbb{R}) \cong \mathbb{R}^\times$