Rec 9/5

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L.S. measure: is it complete?

<u>Thm</u>: Every measure can be completed.

<u>Construction</u>: $\mathcal{H}$, $\mathcal{M}(\mathcal{H}) = \mathcal{B}_{\mathbb{R}}$

<u>Example</u>: F step fn.

$$\rightsquigarrow \mu^* \; \& \; \mathcal{M}^* = \mathcal{P}(\mathbb{R})$$

general F : $\mathcal{M}^* \supsetneq \mathcal{P}(\mathbb{R})$

Any $F = \underset{\text{step}}{F_1} + \underset{\text{cts, behaves like } F(x) = x}{F_2}$

$(X, \mathcal{M}, \mu)$

<u>Example</u>

interesting case: $\mathcal{M} = \sigma$-alg of ctble & co-ctble sets $\quad (*)$

$\mu =$ counting measure (on $\mathcal{M}$).

defn $E \subset X$ is locally measurable if $E \cap A \in \mathcal{M}$ for any $A \in \mathcal{M}$ w/ $\mu(A) < \infty$.

$\tilde{\mathcal{M}}$ = collection of locally measurable sets

$\mathcal{M} \subset \tilde{\mathcal{M}}$, and if $\mathcal{M} = \tilde{\mathcal{M}}$ we say μ is saturated.

(a) μ σ-finite $\Rightarrow$ μ saturated.

(b) $\tilde{\mathcal{M}}$ is a σ-algebra.

(c) Def $\tilde{\mu}$ on $\tilde{\mathcal{M}}$ by $\tilde{\mu}(E) = \begin{cases} \mu(E) & E \in \mathcal{M} \\ \infty & \text{otherwise} \end{cases}$

(d) if μ is complete then $\tilde{\mu}$ is complete.

(e) Suppose μ is semifinite. define $\underline{\mu}$ on $\tilde{\mathcal{M}}$

$$\underline{\mu}(E) = \sup\{\mu(A) \mid A \in \mathcal{M},\ A \subset E\}$$

In the example $(*)$, $\tilde{\mu} = \underline{\mu}$.

(f) $\left.\begin{array}{l} X_1 = [0,1],\ X_2 = (1,2] \\ X = X_1 \cup X_2 = [0,2]. \end{array}\right\}$ $\mathcal{M}$ = ctble & co-ctble

$\mu_0 =$ counting measure on $P(X_1)$

$\mu(E) = \mu_0(E \cap X_1)$. find $\underline{\mu} \neq \hat{\mu}$