Lec 10/9

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Transcendentality

Q is Champernowne transcendental? yes

Q Is π normal (in base 10 say)? idK.

Van der Warden Numbers:

VDW Finitistic: $\forall \ell, r$, $\exists N$ s.t. if $\{1, \dots, N\} = \bigvee_{i=1}^{r} C_i$ then one C_i contains a length ℓ AP.

VDW #: $W(\ell, r) = N$ in above statement.

Absolutely Normal – Normal in any base.

Theorem Almost every $x \in (0,1)$ is absolutely normal (normal in all bases $b \geq 2 \in \mathbb{N}$).

Proof Cover x non normal in base b by intervals of total length $\frac{\varepsilon}{2^b}$.

Nice Problems (all exercises)

1: Champernowne # is transcendental.

2: The banach upper density of sums of squares is 0.

3: $\delta^*(P) = 0$.

4: The set of square-free numbers is not syndetic

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4.5: actually it's not piecewise syndetic. A is piecewise syndetic if $A \supset \overset{\text{syndetic}}{S} \cap \text{Thick}$

5: $\forall$ finite partition $\mathbb{N} = \bigcup_{i=1}^{r} C_i$ at least one C_i is piecewise syndetic.

being p.w.s. is a partition regular property

5.5: actually if S is piecewise syndetic and $S = \bigcup_{i=1}^{r} C_i$ then one C_i is piecewise syndetic.

Midterm:

8: T/F: $\forall x \notin \mathbb{Q}, \overset{1}{x}$ $\{x^n\}$ is dense in $[0,1]$ False.

$\ast$ Let $x = (1+\sqrt{2})$, then $x^n + \underbrace{(1-\sqrt{2})^n}_{\to 0} \in \mathbb{Z}$ so $x^n \bmod 1 \to 0$. not dense.

Theorem (Koksma) for a.e. $x > 1$, x^n is u.d. mod 1.

Also Let $n_i \nearrow \infty$, $n_i \in \mathbb{N}$. Then for a.e. $x \in \mathbb{R}$, $(n_i x)_{i \in \mathbb{N}}$ is u.d. mod 1

Claim: uncountably many x that don't work.

(exercise) give countably many counterexamples to problem 8 using idea $\ast$ above.

google: Pisot-Vijayaraghavan numbers

9: $n^2\alpha + \log n$ is u.d. mod 1.

Ingredients: 1. vdC for u.d. 2. If $y_n \to 0$ and x_n u.d. then $x_n + y_n$ u.d.

<u>Harald Bohr</u>: around 1925 introduced notion of almost periodic fn.

Special Case: $f: \mathbb{Z} \to \mathbb{R}$ is almost periodic if $\forall \varepsilon > 0$,

set of ε periods $\to$ $\{\tau : \sup_{n \in \mathbb{Z}} |f(n+\tau) - f(n)| < \varepsilon\}$ is syndetic.

(exercise) if f_1, f_2 are almost periodic, then $f_1 \cdot f_2$ and $f_1 + f_2$ are too.

("same" def for $f: \mathbb{R} \to \mathbb{R}$).

$\sin(n)$ is Bohr almost periodic $\leftarrow$ (exercise)

1_A where $A = \{\lfloor n\alpha \rfloor, n \in \mathbb{Z}\}$ is B.A.P.

$\sin(x) + \cos(\sqrt{2}\, x)$ is B.A.P. on $\mathbb{R}$