Rec 11/12

Tuesday, November 12, 2019 11:30

3 B, C bilinear forms on V

B ← non-degenerate

$\boxed{\text{I}}$ Show $\exists!\ L_c : V \longrightarrow V$ s.t. $C(x,y) = B(L_c x, y)$

$\boxed{\text{II}}$ C non-degen iff L_c bijective

$\boxed{\text{III}}$ $\exists!\ P : V \to V$ s.t. $B(y,x) = B(Px, y)$.

P ← bijective linear

$$(c_{ij}) = \underbrace{(c_{ij})(b_{ij})^{-1}}(b_{ij})$$

$$B(Lx, y) = B(L_c x, y) \rightsquigarrow B(Lx - L_c x, y) = 0 \Rightarrow L - L_c = 0.$$

8 $u \otimes v : x \longmapsto B(x,u)v$

$$x \longmapsto \langle x)(b_{ij})(u) \cdot v$$

$$e_i \longmapsto (e_i\ (b_{ij})(u))\, v$$

$$e_i(b_{ij}) = i^{th} \text{ row } (b_{ij})$$

$$(b_{ij})_j \cdot (u_j) \cdot v$$

i^{th} entry: $\left(\sum_j b_{ij} u_j\right) v_i =$

$$\operatorname{tr} u \otimes v = \sum_{ij} v_i b_{ij} u_j$$

$T : V \longrightarrow V$, $(t_{ij}) = \left((b_{ij}) \cdot (u)\right) \cdot v$