Rec 11/14

Thursday, November 14, 2019 10:27

$$\underbrace{\ell^1 \subset C_0 \subset C}_{\text{separable}} \subset \ell^\infty$$

ℓ^∞: not

$|f|$ = seminorm

Let $Tx = \lim_{n\to\infty} T_0 x_n$. Well-defined since (x_n) Cauchy $\Rightarrow (T_0 x_n)$ Cauchy

$\|T_0(x_n - x_m)\| \leq \|T_0\| \|x_n - x_m\|$

Then if $x_n, y_n \longrightarrow x$

$\exists N$ s.t. $\|x_n - x\|, \|y_n - x\| \leq \frac{\delta}{2} \Rightarrow \|x_n - y_n\| \leq \delta \Rightarrow \|T_0 x_n - T_0 y_n\| < \varepsilon$

so $T_0 x_n - T_0 y_n \longrightarrow 0$, so $T_0 x_n, T_0 y_n \longrightarrow$ same limit.

T linear obv. $T|_{X_0} = T_0$ obv.

T unique obv. $x_n \to x$, $\|T_0 x_n\| \leq \|T_0\| \|x_n\| \implies \|Tx\| \leq \|T_0\| \|x\|$

Quiz

$g \in C^*$ given by $g(x) = \lim_{n\to\infty} x_n$

can be extended to ℓ^∞

Variant: $C([a,b]) \subset L^\infty([a,b])$

$\|f\| = \sup_x |f(x)|$

ptwise eval: $\varphi_x(f) = f(x)$

can be extended to

$L^\infty([a,b])$ is not separable

X^* separable $\Longrightarrow$ X separable.