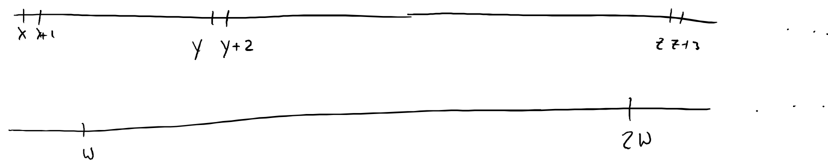
Lec 10/6

Friday, October 6, 2017 14:20

5) $\exists E \subset \mathbb{N}$, $d(E) = 0$, s.t. $\underbrace{E - E = \mathbb{Z}}_{\downarrow}$, $\overline{E/E} = [0, \infty)$



n^3, $2FS(3^n)$

$d(FS(a_n)) = 0$ where $\frac{a_{n+1}}{a_n} > \lambda > 3$ $\forall n$ (exercise)

maybe 2.

$\frac{1}{N} \sum_{n=1}^{N} \mu(n) \to 0$ μ is möbius function.

Problem 1:

$n > 11 \to n \longrightarrow$ remove & add 4 or 9

Pell's eqn take 4

$x^2 - Dy^2 = 1$

where D is nonsquare.

Problem 3:

$x^2 - 2y^2 = 1$

$3^2 - 2 \cdot 2^2 = 1$

$\begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}^2 = \begin{pmatrix} 17 & 24 \\ 12 & 17 \end{pmatrix}$

$\begin{vmatrix} x & 2y \\ y & x \end{vmatrix} = 1$

do powers of $\begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}$ give all solns to $x^2 - 2y^2 = 1$?

ex) find a problem in geometry leading to pell eqn

ex) connect contiued fraction relations btwn convergents to see if they look like pell eqn.

Maybe $a^2 + b^2$ on MT2.

Problem 3:

$x^2 + y^2 = z^2$ not solvable in primes $x^2 = (z-y)(z+y)$

Problem 4:

$x + 2y + 3z = 6w$ is solvable in P. (green-tao)

which eqns are always solvable ...

$x + 2y + 3z = 6w$ (translation invariance) (multiplying & shifting doesn't change solution). in sets of positive density or in AP-rich sets?

$x + y = z$ } 'partition regular' $\leftrightarrow$ $\forall$ finite partition $\mathbb{N} = \bigcup_{i=1}^{r} C_i$, one of C_i contains solutions

solvable in A if $d(A) > 0$ $\rightarrow$ $x + y = 2z$ }

$x + y = 3z$ $\leftarrow$ not partition regular (exercise)

$P \cap \{4n+1\}$ is AP-rich

so is $P \cap \{4n+3\}$.

Schur: $\forall r \in \mathbb{N}$, $\exists N = N(r)$ s.t. if $M \geq N$ and $\{1, 2, \ldots, M\} = \bigcup_{i=1}^{r} C_i$ then one of C_i contains $x, y, x+y$

(exercise) Cor: $\forall n \in \mathbb{N}$, if $p \in P$ is large enough, then $\exists \overbrace{x, y, z}^{\text{pairwise distinct}} \not\equiv 0 \mod p$ s.t. $x^n + y^n \equiv z^n \mod p$.

Hint: $\{1, \ldots, p-1\} \cong \mathbb{Z}_p^*$ (multiplicative group, coloring of $\{1, \ldots, p-1\}$ induces coloring of $\mathbb{Z}_p^*$).
$\Gamma = \{x^n ; x \in \mathbb{Z}_p^*\}$ (n fixed) is a subgroup of $\mathbb{Z}_p^*$
$\mathbb{Z}_p^* = \underbrace{\bigcup_{i=1}^{r} a_i \Gamma}_{\text{colorings}}$ so at least one has $x + y = z$

Van Der Warden Thm: Given any finite coloring $\mathbb{N} = \bigcup_{i=1}^{r} C_i$ at least one C_i is AP-rich.

Formulate finitistic version & prove equivalence (exercise)
$\downarrow$
see finite schur

"infinite schur" $\mathbb{N} = \bigcup_{i=1}^{r} C_i \Rightarrow$ in one C_i we have $x, y, x+y$.

Prove equivalence to regular schur (exercise)

Problem 7: $T \cap (T - x) \neq \emptyset$ $\forall x$ since cocountable sets intersect in $\mathbb{R}$.

$x \in T$, $2x - x = x$. $x \notin T \Rightarrow (x + \pi) - \pi = x$

Problem 8: $n!\,\alpha$ is not always u.d. mod 1. ex: $\alpha = e = \sum \frac{1}{n!}$

Reading: finish 11 for monday. (maybe skip transcendence of π for now)

$$\underbrace{\left|\frac{p}{q} - \xi\right|}_{\text{one } \frac{p_n}{q_n} \text{ convergent out of any 3 consecutive}} < \frac{1}{q^2\sqrt{5}}$$ for infinitely many $\frac{p}{q}$