Lec 11/9

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Diff Eqs w/ Analytic coefficients.

$a_i(x) = \sum_{k=0}^{\infty} \alpha_{ik} (x-x_0)^k$ abs conv on $|x-x_0| < r_0$.

$\varphi^{(i)}(x_0) = \alpha_i$

given IVP, $\exists!$ soln which converges on $|x-x_0| < r_0$.

Compare coefficients.

$y'' - xy' + y = 0$

$\varphi(x) = c_0 + c_1 x + c_2 x^2 + \cdots$

$\varphi(0) = \alpha_0, \quad \varphi'(0) = \alpha_1$

$$-x\varphi'(x) = \sum_{k=0}^{\infty} -kc_k x^k$$

$$\varphi''(x) = \sum_{k=2}^{\infty} k(k-1)c_k x^{k-2} = \sum_{k=0}^{\infty} (k+2)(k+1)c_{k+2} x^k$$

So $$\varphi''(x) - x\varphi'(x) + \varphi(x) = \sum_{k=0}^{\infty} \left[(k+2)(k+1)c_{k+2} - kc_k + c_k \right] x^k = 0$$

$$\Rightarrow \quad (k+2)(k+1)c_{k+2} = (k-1)c_k$$

$$\Rightarrow \quad c_{k+2} = \frac{k-1}{(k+1)(k+2)} c_k$$

$$c_0 = \alpha_0, \quad c_1 = \alpha_1, \quad c_2 = \frac{-1}{1\cdot 2} c_0 = -\frac{1}{2}\alpha_0$$

$$c_3 = 0 \qquad c_4 = \frac{1}{3\cdot 4} c_2 = -\frac{1}{4!}\alpha_0$$

$$c_5 = 0$$

$$c_4 = \frac{\quad}{3 \cdot 4} = \frac{\quad}{4!}\,\alpha_0$$

$$c_5 = 0$$

$$\vdots$$

$$c_6 = \frac{3}{5 \cdot 6} = -\frac{3}{6!}\,\alpha_0$$

$$c_{2k} = \frac{\prod_{n=0}^{k-1} (2n-1)}{(2k)!}\,\alpha_0$$

$$\varphi(x) = \alpha_0 + \alpha_1 x + \sum_{k=1}^{\infty} \frac{-\alpha_0}{2^k\, k!\, (2k-1)}\, x^{2k}$$