Rec 9/12

Thursday, September 12, 2019 10:23

μ^* on $P(X)$ from μ_0 on $\mathcal{A}$, $\mathcal{M} = \mathcal{M}(\mathcal{A})$

$\mathcal{M}^* \subset P(X)$ measurable sets. $\mathcal{M}^* \supset \mathcal{M}$

L-S: $\mathcal{A} = \mathcal{A}(\{(a,b]\})$ $\quad \mathcal{M}(\mathcal{A}) = \mathcal{B}_{\mathbb{R}}$

$\mathcal{M}^* = \mathcal{L} \supsetneq \mathcal{B}_{\mathbb{R}}$

or even $\mathcal{M}$

<u>Interesting fact</u> for any $E \in P(X)$, $\exists F \in \mathcal{M}^*$ s.t. $\mu^*(E) = \mu^*(F)$.

(used on HW2)

What is λ^* of Vitali counterexample?

Recall: N = one element from every eq. class:

Can choose $N_\varepsilon \subset [0, \varepsilon)$, so $\lambda^*(N_\varepsilon) \leq \varepsilon$

<u>But</u> for each ε we get a <u>different</u> set N_ε

<u>And</u> if $\lambda^*(N) = 0$ then N is mble.

<u>Moral</u>: this is mysterious...

<u>prob 3/8</u>: if $E \notin M^*$ then $\mu^*(F \setminus E) > 0$

<u>Prob 4/a</u> $\{0,1\}$ μ^* not from a premeasure...

$\nexists F \in M^*$ w/ $\mu^*(E) = \mu^*(F)$...

Cantor Set

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$C_0 = [0, 1]$

length of each interval

C_n , 2^n a_n

$\bigcap C_n = C$

$$\eta_p(C) = 0 \quad \Leftarrow \quad 2^n a_n^p \longrightarrow 0 \quad \forall p > 0.$$