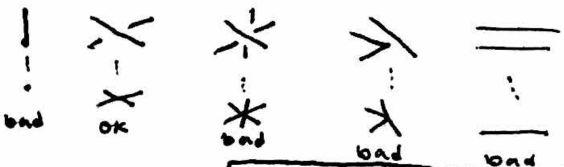
KNOT PROJECTIONS: $K \subset \mathbb{R}^3$ PL-KNOT, $p: \mathbb{R}^3 \to \mathbb{R}^2$ projection. $\bar{K} = p(K)$ = polygonal curve.

- Say K is in general position w.r.t. p if:
  - * Restriction of p to any line segment is embedding.
  - * $\bar{K}$ has at most double-points ($|p|_K^{-1}(c)| \le 2$ for $c \in \bar{K}$).
  - * If c is a double point of $\bar{K}$, anything in $p|_K^{-1}(c)$ is an interior of an edge.
  - * s', s'' line segments in $K \Rightarrow p(s')$ & $p(s'')$ overlap in at most one point.



$SO(3) = \{A \in M_3(\mathbb{R}),\ A^TA = 1,\ \det(A) = 1\}$
$= \mathbb{R}P^3$

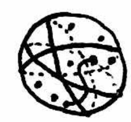
Proposition: Suppose $\{\mathcal{L}_i\}$ countable collection of PL-links in $\mathbb{R}^3$. Suppose $V \subset SO(3)$ is open. Then $\exists\, h \in V$ s.t. each $h(\mathcal{L}_i)$ is in gen. pos. wrt p. If $\{\mathcal{L}_i\}$ is finite, we can find a whole neighborhood of such h.

Proof: $h(\mathcal{L}_i)$ is in gen pos wrt $p \iff \mathcal{L}_i$ is in gen pos wrt $p_{\vec{n}}$ for $n = h^{-1}e_3$ ($p_{\vec{n}} = p \circ h$). For which $\vec{n} \in S^2$ is $\mathcal{L}_i$ in gen pos wrt $p_{\vec{n}}$? Strategy is to successively remove subsets in S^2 along which genericity condition can't hold:

* $s \to \bar{s}$ is 1-to-1, exclude 2 points $\pm d_s$ (direction of s).

* $\bar{v}\!\!\nearrow\!\bar{s} \in \bar{K}$.  all in a plane. exclude directions in that plane (a line in S^2) great circle

* $|| \cdots\!> |$ Same story: exclude the great circle of directions in that plane.



* exclude triple points: exclude directions of lines that intersect 3 segments of $\mathcal{L}_i$. (very technical, but this excludes some curves in S^2 w/ well-defined asymptotes (finite-length)).

"bad points" for $\mathcal{L}_i$ is $\bigcup_{\text{finite}}$ finite-length curves, so "good points" is an open & dense sets

$\bigcap_{\text{finite}}$ open & dense is open & dense. $\bigcap_{\text{countable}}$ open & dense is dense (Baire Category Theorem). ∎

Smooth embed. $\gamma: S^1 \hookrightarrow \mathbb{R}^3$ is in gen. pos w.r.t p if
* $p \circ \gamma$ is immersion
* no triple points
* transverse self-intersections

Thm The subspace $\mathcal{E}_g \subseteq \mathrm{Embed}^\infty(S^1, \mathbb{R}^3)$ of smooth embeddings which are in gen. pos wrt p is open & dense in $\mathrm{Embed}^\infty(S^1, \mathbb{R}^3)$.

(Use Multi-Jet-Transversality Thm)