Lec 11/17

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<u>Theorem</u>: Any $x \in \mathbb{R}$ is a difference of two Liouville #s

If A is "conull" and the notion of largeness is s.t. $A-t$ is "conull" $\forall t$, then $A \cap A-t \neq \emptyset$

The set of Liouville #s (L) contains a shift of any countable set (this is "conull") (exercise)

If $A \subset \mathbb{N}$, $d^*(A) > 0$, $A-A$ is (better than) syndetic.

If $A \subset \mathbb{R}$, $\lambda(A) > 0$ (Lebesgue), then $A-A$ contains an interval $(-\varepsilon, \varepsilon)$ for some $\varepsilon > 0$.

(Steinhaus) (if $\lambda(A), \lambda(B) > 0$ then $A+B \supset (c,d)$)

Exercise: if $d(A) > 0$ then $A+A$ does not have to be syndetic

<u>Renling Jin</u>: If $d^*(A) > 0$, $d^*(B) > 0$, then $A+B$ is piecewise syndetic.

(Reconsideration of philosophy).

Convolution: $h(t) = \int f(x)\, g(t-x)\, dx$

μ (möbius function) inversion formula is some sort of convolution.

$\frac{n}{d} \approx t-x$, $d \approx x$, $\sum_{d \mid n} \approx \int$

$$\frac{n}{d} \approx t - x \,, \quad d \approx x \,, \quad \sum_{d \mid n} \approx \int$$

claim:

$\sqrt{p_n}$ are linearly independent over $\mathbb{Q}$ (exercise)

$G = \{g_1, \dots, g_n\}$

group algebra of G over (say) $\mathbb{R}$ is

$$\left\{ \sum_{i=1}^{n} \alpha_i g_i \; : \; \alpha_i \in \mathbb{R} \;\; \forall i \right\}$$

addition: $\sum \alpha_i g_i + \sum \beta_i g_i = \sum (\alpha_i + \beta_i) g_i$

multiplication: $\left(\sum \alpha_i g_i\right)\left(\sum \beta_i g_i\right) = \sum \alpha_i \beta_j g_i g_j = \sum \gamma_k g_k$

exercise: figure out formula for γ_k

<u>Green-Tao</u>: Any set of positive relative upper density in P is AP-rich.

$$\bar{d}_P(A) = \lim_{N \to \infty} \frac{|A \cap P \cap \{1, \dots, N\}|}{|P \cap \{1, \dots, N\}|}$$

Λ is used to prove this.

Assume $A \subset \mathbb{N}$ has same growth rate as P. Is A AP-rich?

?? (unknown)

$$\sum \frac{1}{n_i} = \infty \implies \{n_i\} \text{ AP-rich} \qquad \text{(Erdős - Turan)}$$

Reading for break:

Ch 18, 2 midterm packets, p-adics packets