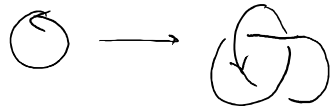
8/21

Wednesday, August 21, 2019 14:59

Knots & Links

Top. embedding $i : S^1 \hookrightarrow S^3$

often identify $K = i(S^1)$.

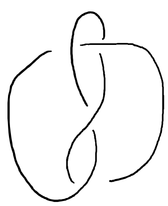


Left-handed Trefoil 3_1

link: $i : S^1 \sqcup \cdots \sqcup S^1 \hookrightarrow S^3$

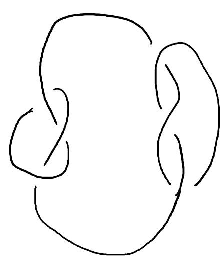
Hopf link 2^2_2

Figure-8



4_1

Granny-Knot:

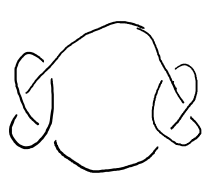


two trefoils "stuck together"

$3_1 \# 3_1$

Composite Knot

Square-knot



$3_1 \# \overline{3_1}$

Def K is a prime knot if it is not a composite like the ones above.

Tait (1867) crossing # ≤ 7

Kirkman, Little (1885) ≤ 10

TKL (1900) ≤ 11 alteranating. ← had duplicates

Alexander-Briggs (1927), Reidemeister (1932)
distinct tabulation ≤ 9

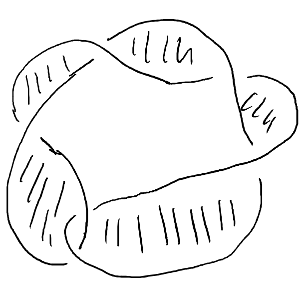
Rolfsen (1970s) ≤ 10, made only one mistake.
(one duplication)

$10_{161} \approx 10_{162}$ found by Perko

Caudron: correct ≤ 11

1998: Hoste/Thistlethwaite/Weeks ≤ 16

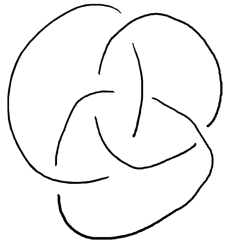
Crossing #	# of prime knots
3	1
4	1
5	2
6	3
7	7
8	21
9	49
10	165
11	552
12	2176



6_1 construction Generalizes

Steve dore knot

Borromean rings



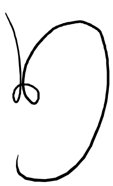
6^3_2

MOTIVATIONS

* Interesting by itself

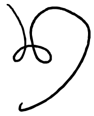
* Every closed, oriented 3-manifold can be represented by a (framed) link

* Every closed, oriented 4-manifold can be represented by a framed link w/ additional circles (Kirby dotted)

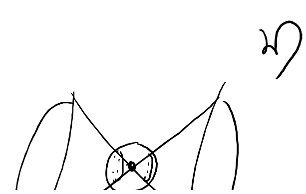
 $\subseteq \mathbb{CP}^2$ $\quad \mathrm{codim}_{\mathbb{C}} = 1$

$\cup$

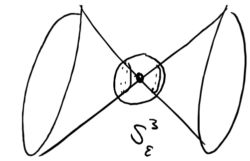
$\mathbb{C}^2$

 $\subseteq \mathbb{R}^4$ $\quad \mathrm{codim}_{\mathbb{R}} = 2$

↳ * singularity at 0.



$\mathcal{V}$* singularity at 0.



* consider a 3-sphere of ε-radius

$\mathcal{V} \cap S^3_\varepsilon$ is
generically dimension 1
and it's somehow
a link/knot

<u>Ex</u>

$$\mathbb{C}^2 \supset \mathcal{V}_{3,2} = \{(w,z) : w^3 + z^2 = 0\}$$

$$S^3 = \{(w,z) : |w|^2 + |z|^2 = 2\}$$

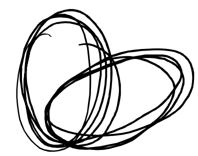
$$\mathcal{V} \cap S^3 = \text{trefoil} \qquad \text{(Exercise: check this!)}$$

$$\mathcal{V}_{p,q} = \{(w,z) : w^p + z^q = 0\}$$

$$\mathcal{V}_{p,q} \cap S^3 = \text{torus knots.}$$

* Dynamical Systems

Lorenz attractor ODE



$\vec{F}_\beta(\vec{x})$ in notes

Thm [Ghrist, Holmes].

Every knot/link type can be realized as a closed orbit of

$$\frac{d}{dt}\vec{X} = \vec{F}_\beta(\vec{X}) \quad \text{for some specific } \beta.$$

* Invariants

some kind of target space

$$f : \{\text{Links}\} \longrightarrow S$$

$$\underset{\text{equiv}}{L_0 \simeq L_1} \Rightarrow f(L_0) = f(L_1)$$

→ Operator Algebra

→ Hopf Algebras

→ Category Theory

* QFT

* DNA

* Celtic Knots

What about $S^1 \hookrightarrow S^2$? $S^1 \hookrightarrow S^4$?

Schönflies   too much space

$\underbrace{\hspace{6em}}_{\text{both trivial}}$

Can have knots $S^n \hookrightarrow S^{n+2}$ $\forall n$.

piecewise-linear $\Rightarrow$ codim-2 is only thing that works.

vs

smooth

Haefliger

$S^3 \xhookrightarrow{\text{knotted}} S^6$ because of differential structure.