

$$x^4 - 2 \in \mathbb{Z}_3[x]$$

//

$$x^4 + 1 = (x^2 - x - 1)(x^2 + x - 1)$$

$$G \cong \mathbb{Z}_2.$$

\mathbb{F}_{3^2} - the only quadratic
extn of \mathbb{F}_3 .

So this is spl. field of f .

Note: all galois groups of finite
fields are cyclic.

5) have a comp series which leads
to a tower:

K

// p

F_{r-1}

// p

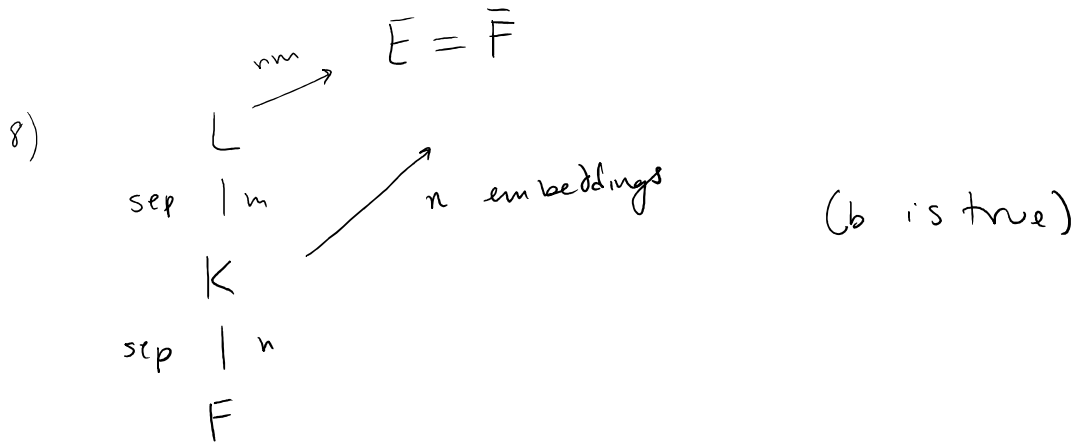
\vdots

"

$$\forall i, F_i = F_{i-1}(\sqrt[p]{a_i})$$

f is irreducible over \mathbb{F}_2 of deg 4, so $\text{Gal} = \mathbb{Z}_4$.

2) powers of 2.



$$\begin{array}{ccc}
 m_{\mathbb{Q}(\sqrt[4]{2}, \mathbb{Q}(\sqrt{2})}(\alpha) & & \mathbb{Q}(\sqrt[4]{2}) \\
 = (X^2 - \sqrt{2}) & \longrightarrow & \mid \\
 & & \mathbb{Q}(\sqrt{2}) \\
 & & \parallel \\
 & & \mathbb{Q}
 \end{array}$$

but

$\mathbb{Q}(\sqrt[4]{2})/\mathbb{Q}$ is not normal
(doesn't contain $\pm i\sqrt[4]{2}$)