Rec 10/1

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If $A \in M_2(\mathbb{F}_p) \setminus GL_2(\mathbb{F}_p)$, then $A^n = Tr(A)^{n-1} A$

Problem 3: $S \underset{\text{d.ring}}{\subsetneq} D$ division subring, $S^\times \trianglelefteq D^\times \Rightarrow S^\times \subseteq Z(D^\times)$

- ☐ S commutes w/ S
- ☐ S commutes w/ $D \setminus S$

goal show $[s,d] = 1$ for $d \in D^\times \setminus S^\times$.

↳ $s, d, d+1$ are all in $D^\times$ $(-1 \in S)$

↳ $[s,d]$ and $[s,d+1]$ are in S

$[s,d] = sds^{-1}d^{-1}$, $[s,d+1] = s(d+1)s^{-1}(d+1)^{-1}$

↳ $sds^{-1} = [s,d]d$, $s(d+1)s^{-1} = [s,d+1](d+1)$

Show that

☐ $s(d+1)s^{-1} = sds^{-1} + 1$

$\Downarrow$

☐ $[s,d+1](d+1) = [s,d]d + 1$

Then if $[s,d+1] = [s,d]$,

then $\quad [s,d](d+1) = [s,d]d+1$

$\Rightarrow \quad [s,d] = 1$

So show $[s,d+1] = [s,d]$

Then S commutes w $D \setminus S$.

pick $s,t \in S$. pick $d \in D^\times \setminus S^\times$.

Then $\quad st = s(d+t-d) = s(d+t) - sd = (d+t)s - ds = ts.$