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Wednesday, November 6, 2019 14:58

$K \hookrightarrow S^3$ knot

$S_K = S^3 \setminus K$ $\qquad k \in \mathbb{N}$

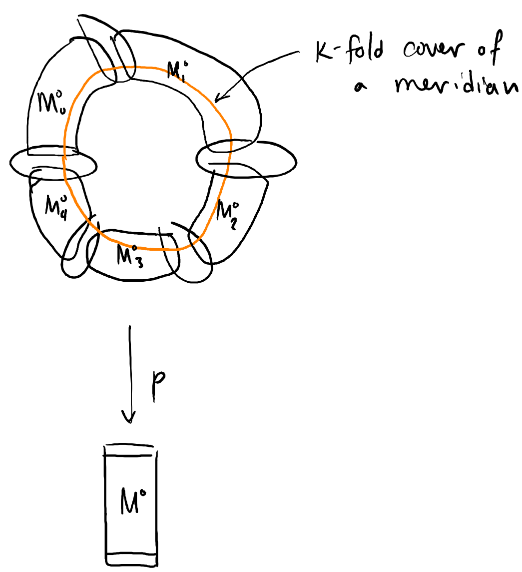
$\eta : \pi_1(S_K) \longrightarrow H_1(S_K) = \mathbb{Z} \longrightarrow \mathbb{Z}/k$

associate cyclic k-fold cover $\tilde{S}_K^{(k)}$ → compact

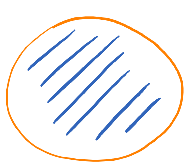
$1 \longrightarrow \pi_1(\tilde{S}_K^{(k)}) \longrightarrow \pi_1(S_K) \longrightarrow \mathbb{Z}/k \longrightarrow 1$ $\quad = \mathcal{D}$, deck transformation grp.

$H_1(\tilde{S}_K^{(k)})$ is a $\Lambda^{(k)} = \mathbb{Z}[\mathcal{D}] = \mathbb{Z}[\mathbb{Z}/k] = \mathbb{Z}[h]/(h^k - 1) = \mathbb{Z}[h]$ -module

Extra Homology Cycle:



So glue in a disc:



or glue in a full torus so that Mfd is closed.
(w disc as cross section).

This is a way to create 3-mfds.

<u>Thm</u> If no roots of Δ_K are k^{th} roots of unity, then

$$\left|H_1(\hat{S}_K^{(k)})\right| = \left|\prod_{j=0}^{k-1} \Delta_K(\zeta_k^j)\right| .$$

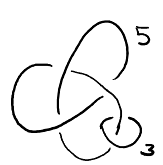
$\qquad\qquad\qquad\qquad\|$

And $\qquad\infty\qquad$ otherwise.

Create 3-manifolds as regular branch covers over links:

* $L \subset S^3$, assign k_j to j^{th} component $L_j \hookrightarrow L$.
* $\pi_1(S_L) \longrightarrow H_1(S_L) \longrightarrow \bigoplus \mathbb{Z}[\mu_j] = \mathbb{Z}/_{k_1} \oplus \cdots \oplus \mathbb{Z}/_{k_m}$
* find resp (abelian) cover $S_L^{(\vec{k})}$
* Glue in $D^2_{\mu_j} \times S^1_\lambda$ as w/ knot.

<u>eg</u>:



represents a 3-manifold.

$z \longmapsto z^k$

perturb to $z \longmapsto z^k - k\varepsilon z \qquad \varepsilon > 0$