# Rec 8/20

Tuesday, August 20, 2019 10:15

Keyfitz info: MW 454, keyfitz.2, MWF 2-3 pm office hours

<u>sets</u>:

$$\overline{\lim} E_n = \bigcap_{k=1}^{\infty} \bigcup_{n=k}^{\infty} E_n \quad : \quad x \in \overline{\lim} E_n \Leftrightarrow x \in E_k \text{ for } \infty\text{-many } k$$

$$\underline{\lim} E_n = \bigcup_{k=1}^{\infty} \bigcap_{n=k}^{\infty} E_n \quad x \in \underline{\lim} E_n \Leftrightarrow x \in E_k \text{ for large enough } k$$

<u>example</u>: $E_{2j} = \left(\frac{1}{j}, 1\right]$ $\quad \underline{\lim} E_n = \emptyset$.

$E_{2j+1}$ $\left[-1, -\frac{1}{j}\right)$ $\quad \overline{\lim} E_n = [-1, 1] \setminus \{0\}$.

<u>Sequences</u>:

$$\overline{\lim} a_n = \inf_k \sup_{n \geq k} a_n = \lim_{n\to\infty} \sup \{a_k : k \geq n\}$$

$$\underline{\lim} a_n = \sup_k \inf_{n \geq k} a_n = \lim_{n\to\infty} \inf \{a_k : k \geq n\}$$

} extended real #s: $\overline{\mathbb{R}} = \mathbb{R} \cup \{\pm\infty\} = [-\infty, \infty]$

$(\infty \cdot 0 = 0$ by defn$)$.

<u>De Morgan's Laws</u>

$$\left(\bigcup_{\alpha \in A} E_\alpha\right)^c = \bigcap_{\alpha \in A} E_\alpha^c$$

Is $(\overline{\lim}\, E_j)^c = \underline{\lim}\, E_j^c$ ? yes.

$X \times Y$ = collection of ordered pairs $(x, y)$

Relation from $X$ to $Y$ is a subset $R \subseteq X \times Y$

$x R y \Leftrightarrow (x, y) \in R$.

Equivalence Relation

3 transitive

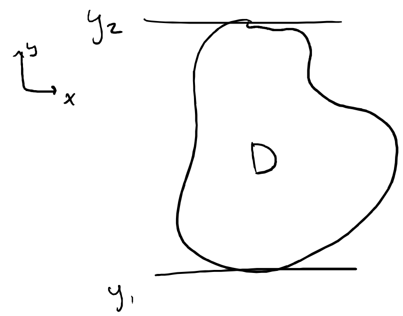
1 reflective / identity

2 symmetric / reciprocal

Mappings / Functions $X \to Y$ is a kind of relation

Cartesian products : $\prod_{\alpha \in A} X_\alpha$

~~eg~~




$x, y \in D$ $\quad X_y = \{x : (x,y) \in D\}$

$A = [y_1, y_2]$

$\prod_{y \in [y_1, y_2]} X_y = D$

order relation: $<, \leq$

partial order relation

"well-ordering" – Axiom of Choice / Zorn's Lemma

$a_n \in \mathbb{R}$, $\quad \sum_1^N a_n$ well defined

$\sum_1^\infty a_n$ : a limit

$a_\alpha \geq 0$ $\quad \sum_{\alpha \in A} a_\alpha = \sup \left\{ \sum_{\alpha \in F} a_\alpha : F \subseteq A \text{ finite} \right\}$

if $A = \mathbb{R}$, this is not the same as the integral.

Fact: if $f(\alpha) > 0$ for $\alpha \in A$ where $A$ is uncountable,

$$\sum_{\alpha \in A} f(\alpha) = \infty$$

open subset of $\mathbb{R}$:

Propn: Every open set in $\mathbb{R}$ is a disjoint union of open intervals.

0.6 Metric Spaces: Thursday

Quiz: material 0.1 – 0.5 excluding well-ordering thing.