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Monday, September 23, 2019 14:59

Prime Decomposition

$$K = K_1 \# \cdots \# K_m$$

K_i = prime $\neq$ unknot

(Recall: $K = A \# B \Rightarrow g(K) = g(A) + g(B)$)

Theorem Suppose K is a (nontrivial, tame) knot in S^3. Then K admits a prime decomposition. Moreover, if it has two decompositions

$$K \approx K_1 \# \cdots \# K_m$$
$$\approx K_1' \# \cdots \# K_n'$$

Then $m = n$ and $\exists\, \sigma \in S_n$ s.t. $K_i = K'_{\sigma(i)}$.

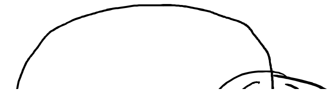
Theorem Suppose $K = P \# Q$ w/ P a prime knot, $Q \neq$ unknot. and $K = K_1 \# K_2$. Then P is a factor of K_1 or K_2.

so either $K_1 \approx P \# R_1$, $Q \approx R_1 \# K_2$

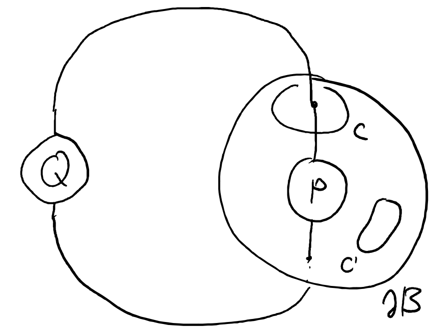
or $K_2 \approx P \# R_2$, $Q \approx R_2 \# K_1$

(This implies uniqueness by induction).

Proof Let B = ball containing P,

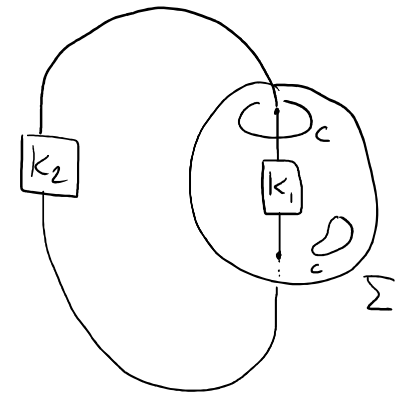


Proof Let B = ball containing P, and $\partial B \cap K = 2$ points.



Similarly, let Σ be separating sphere for $K_1 \# K_2$ - decomp.

Put ∂B and Σ in gen-pos $\partial B \pitchfork \Sigma$. That is,



$\partial B \cap \Sigma = \bigcup$ circles on either ∂B or Σ.

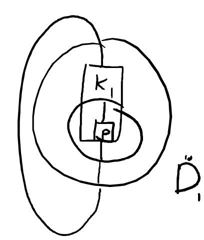
To each circle $C \subset \partial B \cap \Sigma$,

assign $\ell_c = |Lk(C, K)|$ (either 0 or 1, two types).

$\ell_c = 0 \rightsquigarrow C$ bounds disc in ∂B and in Σ

$\ell_c = 1 \rightsquigarrow C$ separates points in $\partial B \cap K$ and in $\Sigma \cap K$.

Suppose $\partial B \cap \Sigma = \emptyset$. $\partial B \subset S^3 - \Sigma$
$= \mathring{D}_1 \cup \mathring{D}_2$



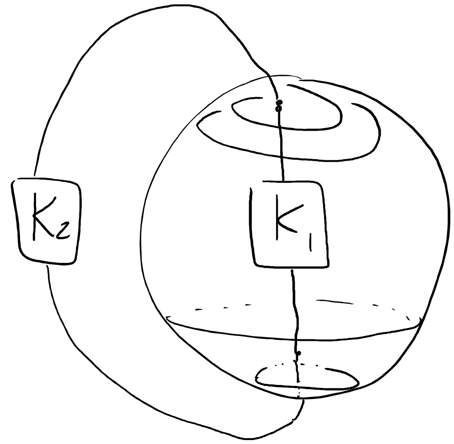
so B entirely in one of $\mathring{D}_1$ or $\mathring{D}_2$

if $B \subset \mathring{D}_i$, P is a factor of K_i. We are done.

Strategy: get rid of circles (via surgery).

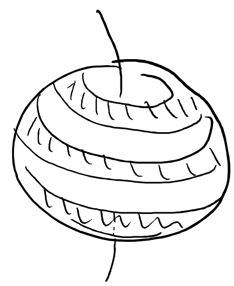
depending on where the knot is

if $\ell_c = 0$, just shrink or grow B to enclose or outclose Σ along c (start w/ innermost).



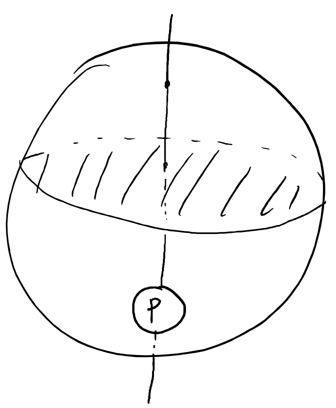
Keep going until $l_c = 1 \ \forall c$.

get "striped shirt":

$\Sigma \cap B =$ 

$\Sigma \cap B$ consists of at most 2 discs each intersecting K in 1pt, as well as several annuli.

Suppose $\Delta \subset \Sigma \cap B$ is a disc component

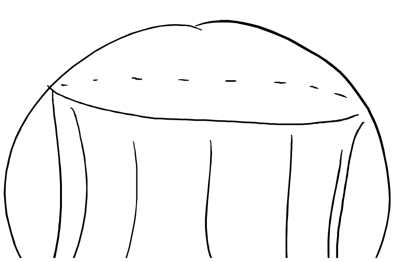


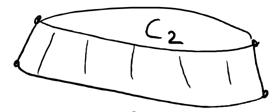
B

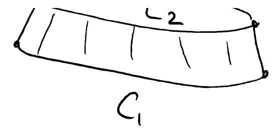
P is prime, so it's either above or below Δ.

So chop off the top or bottom to eliminate $\partial\Delta$.

If A is an annular component,







B

Annulus splits B into core $\cong$ Ball and full torus.

Chop out full torus to eliminate C_1 & C_2.

(Prime knot is entirely inside the core).

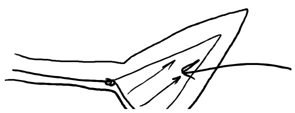
This removes all intersections so $\Sigma \cap B = \emptyset$. $\square$

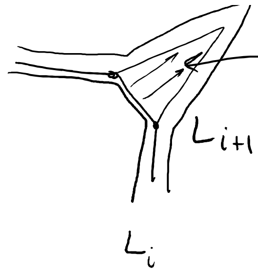
<u>Theorem</u> $K = K_1 \# K_2$, then K is fibered iff both K_1 & K_2 are fibered.

Presentations of Knots & Links

Generic proj for PL-knot

Combinatorial equivalence for PL-links





(triangle move. Can go either way).

$$L \sim L' \Leftrightarrow L = L_0 \underbrace{\longrightarrow L_1 \longrightarrow L_2 \longrightarrow \cdots \longrightarrow}_{\text{all } \Delta\text{-moves}} L_N = L'$$

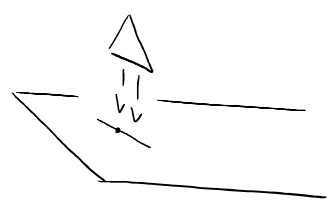
find projection for which all L_j are in general position at once.

$L_j \xrightarrow[T]{} L_{j+1}$ (PL-links)

$\hat{L}_j \xrightarrow[\hat{T}]{} \hat{L}_{j+1}$ (Diagrams)

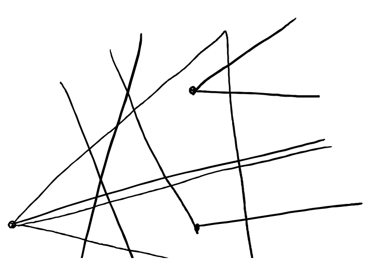
Degenerate case: $\hat{T}$ is a line segment

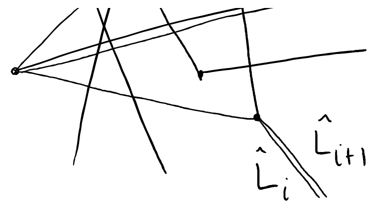
$\hat{L}_j = \hat{L}_{j+1}$ (modulo one subdivision pt).



Generic case:

$\hat{T}$

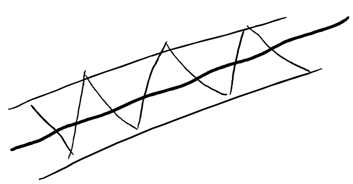


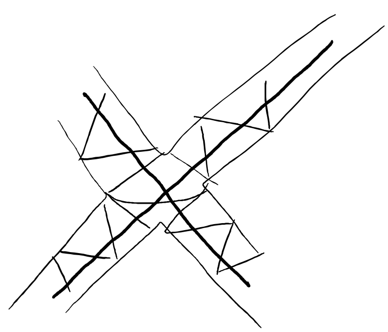


Subdivide $\hat{T}$ by triangulation
with nice local pictures.

then $\hat{T}$-move is the same as doing all the little moves.

Strategy for a good triangulation.





only 4 pictures:

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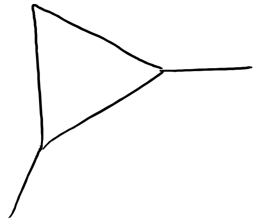

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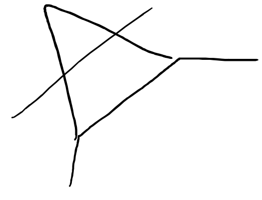
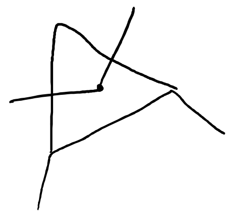
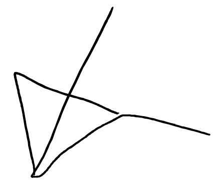


(2) 

(3) 

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