Rec 10/17

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Let $c = (a,b)$ in E. Then $c|a$ and $c|b$ and if $e|a,b$, $e|c$.

Then $d|c$. Show $c|d$ in E. $d \in (c)$.

$a \in (d)$, $b \in (d)$.

if $e|a$ & $e|b$ then $a,b \in (e)$.

$(a,b) = (d)$ in D.

$(a,b) \subset (d)$ so

$\underset{\text{PID}}{D} \subset \underset{\text{domain}}{E}$, $(a,b) = d$ in D

Then $(d) \supset (a,b) = (y)$ in D, and so $y|a$ and $y|b$ so $y|d$ meaning $(y) \supset (d)$,

So $(a,b) = (d)$.

Thus $d = ax + by$. So if $e \in E$ divides a & b, then $e \mid ax + by = d$ so $d = (a,b)$ in E too.

Every PID is factorial (UFD)

① Divisor Chain Condition $\quad (\nexists\ (a_1) \subsetneq (a_2) \subsetneq \cdots)$

② Irreducible $\Rightarrow$ prime.
(or GCD condition)

① idea : consider $I = \bigcup_{i=1}^{\infty} (a_i)$.

Then $I = (a)$ for some a.

So $a \in (a_i)$ for some i so $(a_i) \supset (a)$, but $(a) \supset (a_i)$ so $(a) = (a_i)$.

Thus $(a_j) = (a)\ \forall\ j \geq i$.

② $(a,b) = (d)$.

② If p is irreducible and $p \mid ab$, $\quad ab \in (p)$

so $ab = up$

$p \mid ab \Rightarrow p \mid a$ or $p \mid b$

$(p) \supset (ab) \supset$

$P \quad a \in P$ or $b \in P$

$(a) \supset (p)$ or $(b) \supset P \Leftarrow$

Euclidean Domain : $\exists\ \delta : D \longrightarrow \mathbb{Z}_{\geq 0}$ s.t. $\forall a,b \in D$, $b \neq 0$, $\exists\ q,r$
w/ $a = bq + r$ and $\delta(r) < \delta(b)$.

<u>Fact</u>: ED $\Rightarrow$ PID:

- Pick $I \subset D$, $a \in I$ with minimal δ.
- division thing shows $(a) = I$.