Lec 9/22

Friday, September 22, 2017 10:40

$\dim V = n$, V/F. multilinear form $\underset{\Lambda^n V^*}{\Phi}: \underbrace{V \times \cdots \times V}_{n} \longrightarrow F$

(1-linear forms $\Phi: V \to F$, $\Phi \in V^*$

alternating: sign change on transposition.

$\Lambda^n V^*$ is a v.s. of dim 1 over F.

$A \in M_{n,n}(F)$, $A = (\alpha_{ij})$

$F^n \ni x \longmapsto Ax \in F^n$ linearly.

take $\Phi_{st} \in \Lambda^n (F^n)^*$ s.t. $\Phi_{st}(e_1, \ldots, e_n) = 1$.

then Define $\det(A) = \Phi_{st}(Ae_1, \ldots, Ae_n)$

$$= \sum_{\sigma \in S_n} \varepsilon(\sigma) \alpha_{\sigma(1)1} \alpha_{\sigma(2)2} \cdots \alpha_{\sigma(n)n}$$

$$= \sum_{\sigma \in S_n} \varepsilon(\sigma) \alpha_{1\sigma(1)} \alpha_{2\sigma(2)} \cdots \alpha_{n\sigma(n)}$$

then $\det(AB) = \det(A)\det(B)$

but $\det(A+B) \neq \det(A) + \det(B)$

$\det(\lambda A) = \lambda^n \det(A)$

$T \in L(V)$

fix a basis of V; $\{v_1, \ldots, v_n\}$, $\exists!$ $\Phi(v_1, \ldots, v_n) = 1$.

$\Phi(T(v_1), \ldots, T(v_n)) = \Phi_T(v_1, \ldots, v_n)$ (can be defined for any n vectors)

$\Phi_T \in \Lambda^n V^*$ so $\Phi_T = \lambda \Phi$

and $\lambda = \det(A)$ where A is the matrix assoc. w/ T wrt $\{v_1, \dots, v_n\}$

$=: \det(T)$ (is this well defined?)

take another basis $\{u_1, \dots, u_n\}$. Then $B = X^{-1} A X$ is the matrix of T wrt $\{u_1, \dots, u_n\}$ (X is the transition matrix)

so $\det(B) = \det(X^{-1}) \det(A) \det(X) = \det(A)$

So for any choice of basis, det (matrix of T in given basis) is the same, so $\det(T) := \det(A)$ is well defined.

Row/column expansion of determinant:

$$\underbrace{\det(A) = \sum_{i=1}^{n} (-1)^{i+1} \alpha_{ij} D_{ij}}_{\text{expansion along } j^{\text{th}} \text{ column.}}$$

where D_{ij} is det (A with i^{th} row & j^{th} col removed)

$$\underbrace{\det(A) = \sum_{j=1}^{n} (-1)^{j+1} \alpha_{ij} D_{ij}}_{\text{expansion along } i\text{th row.}}$$