Rec 9/5

Thursday, September 5, 2019 11:28

Schrier's Lemma

$G = \langle S \rangle$, $G/_H$ "=" T

$h = g \cdots g 1 = g \cdots g t h = \cdots t h \cdots h$, so $t = 1$.

Other **Construction.**

- $H \leq G$
- S left $\underbrace{\text{transversal of } H \text{ in } G}_{\text{subset of } G \text{ containing 1 rep of each coset.}}$ s.t. representative of coset H is id.
- $\langle A \rangle = G$
- $B = \{ h \in H \mid h = (s')^{-1} a s, \ s, s' \in S, a \in A \}$.

Consider S_n.

- Show $H = \{ 1, (123), (132) \} \leq S_3$ is f.g.

- S_3 is f.g.,
  $$S_3 = \langle (1\,2), (1\,2\,3) \rangle$$

- can use Schrier's lemma to construct generating sets for gps.

<u>1.12 ex 5</u> Show H is normal if $[G:H] = p$ where p is smallest prime dividing order of G.

$$G = \bigsqcup_{1}^{p} x_i H$$

$$ghg^{-1}$$

$$G \circlearrowright G/H$$

$$G \xrightarrow[\varphi]{} S_p \circlearrowright G/H$$

$\operatorname{Ker}\varphi \leq H$,

$[G : \operatorname{Ker}\varphi] \mid p!$, $\quad p \mid [G : \operatorname{Ker}\varphi]$

$$\underbrace{|H : \operatorname{Ker}\varphi|}_{\substack{\text{divides } |G|, \\ \text{doesn't divide } p}} \cdot \underbrace{|G : H|}_{p} = \underbrace{|G : \operatorname{Ker}\varphi|}_{\text{divides } p!}$$

$|H : \operatorname{Ker}\varphi| = 1$, so $\underbrace{\operatorname{Ker}\varphi = H}_{\text{so } H \text{ is normal.}}$