11/4

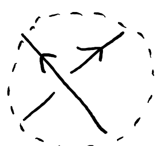
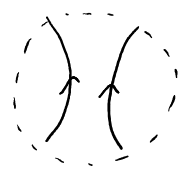
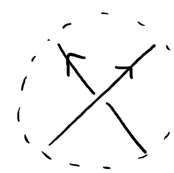
Monday, November 4, 2019 14:59

Alexander Modules for links & Skein Relations

Def $\nabla_L(z)$ conway polynomial $\in \mathbb{Z}[z]$

$$\nabla_L(t^{1/2} - t^{-1/2}) = \Delta_L \in \mathbb{Z}[t^{1/2}, t^{-1/2}].$$

Consider link diagrams

$L_- =$  $L_0 =$  $L_+ =$ 

Pick seifert surface Σ_0 for L_0

So Seifert surfaces $\Sigma_\pm$ for $L_\pm$ are obtained from Σ_0 by gluing in a strip.

$$H_1(\Sigma_\pm) = \underset{f_1, \dots, f_d}{H_1(\Sigma_0)} \oplus \underset{f}{\mathbb{Z}_{[f]}}$$

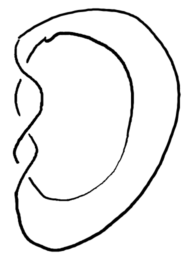
Thm $\Delta \;-\; \Delta \quad (t^{1/2} \;\; t^{-1/2}) \; \Delta$

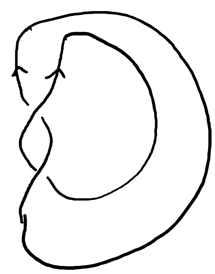
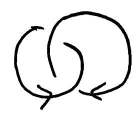
$$\Delta_{L_+} - \Delta_{L_-} = (t^{\frac{1}{2}} - t^{-\frac{1}{2}})\,\Delta_{L_0}$$

$$\nabla_{L_+} - \nabla_{L_-} = z\,\nabla_{L_0}$$

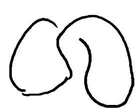
<u>example</u>:

$L_+ =$  $= T$

$L_- =$  $= O$

$L_0 =$  $=$  $= H_+$

$H_- =$  $= O\ O$

$H_0 =$  $= O$

$\nabla_O = 1$, so

$$\nabla_T = 1 + z^2 + z\,\nabla_{OO}$$

Lemma if L is a splittable link

$(L = L_1 \sqcup L_2,\ \ L_1 \,\&\, L_2$ separated by sphere$)$
$L_1, L_2 \neq \emptyset$

Then $\nabla_L = \Delta_L = 0$

So $\nabla_{oo} = 0 \rightsquigarrow \nabla_T = 1 + z^2$

$$\Delta_T = 1 + (t^{1/2} - t^{-1/2})^2$$
$$= t + t^{-1} - 1$$

Thm: There is a unique fn

$$\nabla : \text{Link Classes} \longrightarrow \mathbb{Z}[z]$$

determined by

- $\nabla_O = 1$
- $\nabla_{L_+} - \nabla_{L_-} = z\nabla_{L_0}$
- $\nabla_L = 0$ if L is splittable.