Lec 8/23

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Subgroups of $U(M(S))$ are called transformation groups.
$$\mathrm{Sym}(S) = U(M(S))$$

S_n is $U(M(\{1,\dots,n\}))$.

the dihedral group is a group of transformations of regular n-agon.

<u>Def</u> direct product of monoids: $M \times N = \{(m,n) : m \in M, n \in M\}$,

$$(m_1, n_1)(m_2, n_2) = (m_1 m_2, n_1 n_2).$$
$$1 = (1_1, 1_2).$$

Can be generalized to more than 2.

Also works for groups.

$M \times M = M^2$, $\underbrace{M \times \cdots \times M}_{n\text{-times}} = M^n$, etc.

<u>Ex</u>: $(\mathbb{R}^3, +, 0)$ is a group.

groups can be isomorphic.

for example: $(\mathbb{R}, +, 0) \cong (\mathbb{R}_{>0}, \cdot, 1)$

$$x \longleftrightarrow e^x$$

if G is a finite group of order n,

$G \simeq$ a subgroup of S_n. also $S_G \cong S_n$.

Cayley's Thm: further, G is isomorphic to a transformation group of G.

So a finite gp is $\cong \leq S_n$.

proof of cayley's thm:

$$g \longleftrightarrow \alpha_g : h \longmapsto gh.$$

A group is abelian sometimes.

If $A \subset G$, the centralizer of A is

$$C(A) = \{g \in G : ga = ag \ \forall a \in A\} = \bigcap_{a \in A} C(a)$$

$$C(a) = \{g \in G : ag = ga\}$$

$C(G)$ is called the center of G.

centralizers are groups

$$\begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix}^3 = \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2a & 2b+ae \\ 0 & 1 & 2c \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3a & 2b+ae+2ac+b \\ & 1 & 3c \\ & & 1 \end{pmatrix} = 1$$