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$(\prod M_\alpha) \otimes N \cong \prod (M_\alpha \otimes N)$? No.

$0 \longrightarrow I \xrightarrow{i} R$ exact

$0 \longrightarrow I \otimes M \longrightarrow M$ exact

16) R: ID. $Q = (R \setminus 0)^{-1} R$ field of fractions, M: R-mod

$\Rightarrow$ $Q \otimes_R M$ is a Q-vector space, $M \xrightarrow{\varphi} Q \otimes M$, $u \longrightarrow 1 \otimes u$

Theorem: $\operatorname{Ker}\varphi = \operatorname{Tor}(M)$.

12) a : $\mathbb{Z}_2 \otimes_{\mathbb{Z}} G = \mathbb{Z}_2 \oplus 0 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2^2 = \mathbb{Z}_2^4$

c : $\mathbb{Q}$ kills torsion: so $\mathbb{Q} \otimes_{\mathbb{Z}} G = \mathbb{Q}^2$.

22): $R = F[x, y]$, $M = I = (x, y)$ is not flat

23): Proj $\Rightarrow$ flat : Proj $\Leftrightarrow$ direct summand of free module

b) field of fractions works: $\mathbb{Q}$-flat but not projective.

$$\begin{array}{ccccc} M & \xrightarrow{\pi} & N & \longrightarrow & 0 \\ & {}_{\psi}\nwarrow & & \nearrow_{\varphi} & \\ & & P & & \end{array}$$

P proj iff $\forall \pi$, $\forall \varphi$, $\exists \psi$.

Consider

$$\begin{tikzcd} M \arrow[r] & \mathbb{Q} \arrow[r] & 0 \\ & \mathbb{Q} \arrow[u, "2"'] \arrow[ul] & \end{tikzcd}$$

free containing $\mathbb{Q}$