Lec 8/28

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<u>Thm</u> a subgp of a cyclic gp is cyclic.

<u>Pf</u> Let $H \leq \langle a \rangle$. if $H = \{1\}$ then $H = \langle 1 \rangle$.

if not, let s be the smallest positive integer s.t. $a^s \in H$. Then $\langle a^s \rangle \leq H$.

Also, if $a^m \in H$ then $m = qs + r$ where $0 \leq r < s$.

But $a^{qs} \in H$ so $a^{-qs} a^m = a^r \in H$, so $r = 0$. $\square$

<u>Thm</u> if $\langle a \rangle$ is infinite, then subgps $\neq 1$ are infinite.

and $s \mapsto \langle a^s \rangle$ is a bijection between $\mathbb{Z}_{\geq 0}$ and the set of subgps of $\langle a \rangle$.

<u>Thm</u> if $\langle a \rangle$ is finite of order r, then the order of every subgp divides r. also, $s \mapsto \langle r^s \rangle$ is a bijection between positive divisors of r and subgps of $\langle a \rangle$.

<u>Pf</u> $H \leq \langle A \rangle$, $H = \langle a^s \rangle = \{1, a^s, a^{2s}, \ldots, a^{(q-1)s}\}$ where $r = qs$.

(write $r = qs + t$, $0 \leq t < s$. $a^t = a^r (a^s)^{-q} = (a^s)^{-q} \in H$, so $t = 0$)

<u>Def</u> Let G be a finite gp. The exponent of G, $\exp(G)$, is the smallest integer e s.t. $a^e = 1$ $\forall a \in G$.

Thm Let G be a finite ab. gp. Then G is cyclic iff $\exp G = |G|$.

cycle decomposition:

Def a cycle (or r-cycle) γ of the symmetric gp S_n is an element that permutes distinct numbers $i_1, \dots, i_r$ cyclically: $i_1 \to i_2 \to \cdots \to i_r$.

$r > 1$

Also, it fixes $\{1, \dots, n\} \setminus \{i_1, \dots, i_r\}$.

γ is denoted $(i_1\ i_2\ \cdots\ i_r)$.

two cycles are disjoint if they don't act on any common number.

Prop (i) $\gamma = (i_1\ i_2\ \cdots\ i_r) \implies |\gamma| = r$.

$[\ \gamma^k(i_j) = i_{\underbrace{j+k}_{\text{mod } r}} \neq i_1$ unless $r | k\ ]$.

(ii) disjoint cycles commute.

(iii) $\alpha = (i_1 \cdots i_r)(j_1 \cdots j_s) \cdots (l_1 \cdots l_u)$ is a product of disjoint cycles.

Then $|\alpha| = \mathrm{lcm}(r, s, \dots, u)$.

Prop any permutation is a product of disjoint cycles (algorithm). This is **essentially** unique

<u>Prop</u> any permutation is a product of 2-cycles.
this is not unique.