Lec 11/27

Monday, November 27, 2017 14:13

Exercise: Create a proof of Completeness of $\hat{X}$ using X as a general metric space (similarly to Katok proof).

(Show that any metric space (X, d) is completable).

$d(A, B) := \mu(A \triangle B)$ where $A, B \subset \mathbb{R}$ are measureable.

if A, B finite, μ is counting, this makes sense.

d satisfies all but 1 axiom: $d(A, B)$ can be 0 when $A \neq B$

$$\mu(A \triangle B) = \int_X |1_A - 1_B| \, d\mu$$

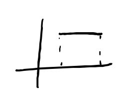
L^1-distance between 1_A and 1_B

So create equivalence classes: $A \sim B$ iff $\mu(A \triangle B) = 0$.

---

google: Hasse-Minkowski principle.

exercise: construct a function that approximates



and is C^∞.

Sums of Squares:

Prime $4n+1 = x^2 + y^2$ (first stated by Albert Girard in 1625, proved by Fermat in 1640).

$4n+3 = x^2+y^2+z^2+w^2$ (so every integer is sum of 4 squares (Lagrange)).

$X = 2^\alpha \prod p^\beta \prod q^\gamma = a^2 + b^2$ iff γ is always even.

$d^*(\text{Sums of Squares}) = 0$

$d^*(P) = 0$

Exercise: Let P_k = "span" of $p_1, \dots, p_k$ then $d^*(P_k) = 0$

exercise: give a proof that $f \in C[0,1]$ is unif. cts. w/o thinking

1. formulate negation
2. use compactness
3. get contradiction

$\forall z \in \mathbb{C}, \ \exists T_z : \mathbb{C} \to \mathbb{C}$ s.t. $T_z w = zw$.

matrix of T_z is: $(T_z(1,0),\ T_z(0,1)) = \begin{pmatrix} x & -y \\ y & x \end{pmatrix}$ where $z = x+yi$.

Can do same for $\mathbb{C} \to \mathbb{Q}[\sqrt{2}]$.

$|\mathbb{F}_q^{\cdot}| = q^n$

Book: Numbers of the form $x^2 + ny^2$ by Fox.

use Jstor

for wednesday: Find in AM Monthly the paper on Smith's proof of Sums of two squares theorem via continued fractions.

(containing Latin text & translation)

Know two proofs of a^2+b^2 for final.

maybe 3 including Smith.

Challenge: find interesting elementarily equivalent forms of PNT.

$$\frac{1}{N} \sum \mu(n) \to 0$$

$P((n,m)=1) = \frac{6}{\pi^2}$ (discuss next time)