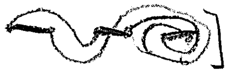
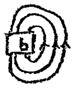
Bridge Numbers : Recovering bridge diagram from braid diagram.

⇝ movement of 2m points ⇝ ambient isotopy ϕ_t ⇝ $\phi_0 = [$  $] \to \phi_1 = [$  $]$.

So bridge $b(L)$ = smallest # of extrema in gen. pos diagram = smallest # of bridges

Thm [Schubert '54]: $b(K_1 \# K_2) = b(K_1) + b(K_2) - 1$. And $b(K) = 1 \Rightarrow K = O$, so $b(K) = 2$ → K prim

Alexander-Markov Thm / presentation : $cl(b) =$  . Connectivity depends only on image $\overset{S_n \leftarrow B_n}{\sigma \longleftrightarrow [b]}$.

Lemma : # of components in $cl(b)$ = # of disjoint cycles in $\sigma(b)$.

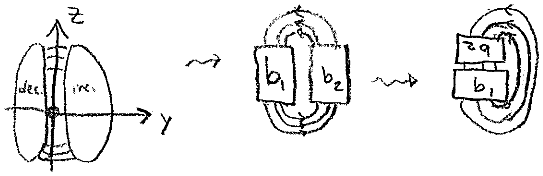
$B(L) \geq b(L)$

Def : $B(L)$ = smallest # of strands needed to close a braid b into $L = cl(b)$ = Braid index.

$cl(b)$ easily gives seifert surface : seifert cycles are right there. $B(L) \geq$ min # of seifert cycles.

Yamada '87 : $B(L)$ = min # of seifert cycles. $\boxed{\text{Alexander Thm: } L = cl(b) \text{ for some braid } b}$

Pf of Alexander Thm: choose a red-blue coloring as for bridge links (). Isotop the red intervals into standard position (vertical, next to each other, ↑) Extend to ambient isotopy. replace up-going blue segments by red segments. Now all blue goes down & all red goes up. Pull blue up, push red down. Make extrema all have different heights & look at it sideways:

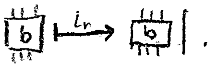
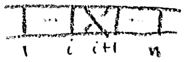


= braid representation of L. □

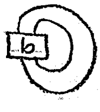
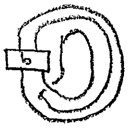
* If $b \sim b'$, then $cl(b) \sim cl(b')$. ⇝ $\underline{cl}([b]) = [cl(b)]$ is well-defined.

So $\underline{cl} : \bigcup_{n \geq 1} B_n \twoheadrightarrow$ {Classes of oriented links} by Alexander thm.

* $cl(b_2 \cdot b_1) \approx cl(b_1 \circ b_2)$ by sliding around, so $\underline{cl}(aba^{-1}) = \underline{cl}(b)$ $\forall a, b \in B_n$.

* $i_n : B_n \hookrightarrow B_{n+1}$,  . $\sigma_i =$ 

, $\sigma_1, \dots, \sigma_{n-1}$ generate B_n. $\sigma_n \in B_{n+1} \setminus B_n$.

$b \in B_n$ ⇝ $i_n(b)\sigma_n$ ⇝ $\underline{cl}(b) = \underline{cl}(i_n(b)\sigma_n)$ by (RI) :  =  .

Introduce eq. rel'n on $\bigcup_{n \geq 1} B_n$ gen. by $aba^{-1} \underset{M}{\sim} b$, $\underbrace{i_n(b)\sigma_n \underset{M}{\sim} b}_{\text{markov move}}$.

Thm (Markov '35) $\hat{\underline{cl}} : \bigcup_{n \geq 1} B_n / \underset{M}{\sim} \twoheadrightarrow$ {classes of oriented links} is a bijection.