Lec 2/15

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Hypothesis testing Ch 12

Test something about parameter based on sample.

Ex: 1) Is mean weight above allowable cutoff?
2) Is there a difference in mean weight on 2 different days?
3) is the percent of boxes w/ wicking 10%?

first translate each to mathematical, more precise, statements.

1) μ = mean weight. is $\mu \geq 1.0$ lbs?

2) μ_1, μ_2 = mean weight on days 1 and 2. is $\mu_1 = \mu_2$?

3) p = proportion of boxes w/ wicking. is $p = 0.1$?

These statements will be used to form a "statistical hypothesis" aka guess, conjecture about distribution of 1 or more RVs. (Def 12.1).

Simple Hypothesis: hypothesis that along w/ some other assumptions completely specifies Distribution.

Ex: 3) $p = 0.1$, along w/ knowledge of sample size n and assumption of independency (and possibly asymp. normality)
the dist is $Bin(n, 0.1)$ or $N(n(0.1), n(0.1)(1-0.1))$.

Composite Hypothesis hypothesis specifies a family of distributions, not just 1 distribution.

Ex: 1) $\mu \geq 1$ implies dist. with mean ≥ 1

Ex: 2) $\mu_1 = \mu_2$ etc.

To formulate test of hypothesis, we must specify alternative hypothesis:

Ex. 1) $\mu < 1.0$ works. (also $0 < \mu < 1$).

Ex: 2) $\mu_1 \neq \mu_2$

Ex: 3) $p \neq 0.1$ (or $p = 0.4$ or $P < 0.1$).

alternative hypothesis can also be classified as simple or composite

Remark: It is common to formulate hypotheses of no difference, even if we're trying to show there is one.

i.e. Ex:2) might suspect $\mu_1 > \mu_2$ but we test the hypothesis $\mu_1 = \mu_2$ against $\mu_1 > \mu_2$.

why? for the hypothesis $\mu_1 = \mu_2$, we know what to expect. (i.e. we know some distribution). This wouldn't be the case for $\mu_1 \neq \mu_2$ or $\mu_1 > \mu_2$.

This leads to H_0 the null hypothesis and H_A (or H_1) the alternative hypothesis.

Sec 12.2 Testing a Hypothesis

idea: choose between H_0 and H_A based on data.

- Base our decision on test statistic.
- test partitions the value of statistic into 2 parts.
  1) acceptance region for H_0
  2) rejection region for H_0 → critical region

Steps:
1. collect data
2. compute test statistic
3. make decision

Possible outcomes:

	Accept H_0	Reject H_0
H_0 true	right decision	type I error
H_0 false	type II error	right decision

α = prob of type I error = P(reject H_0 when H_0 is true)

↳ the level of significancy of a test.

β = prob of type II error = P(accept false H_0)

Ex: wicking: suppose all boxes result in wicking 40% of the time, and a new box supplier promises only 10% wicking

Hypotheses: $H_0: p = 0.1$ vs. $H_A: p = 0.4$

Data: 20 boxes $X_1, \dots, X_{20} \in \{0,1\}$ 0: no wicking 1: wicking.

X = number of boxes w/ wicking $= \sum_{i=1}^{20} X_i \sim \text{Bin}(20, 0.1)$ if H_0 is true.

↑ test statistic.

Test: Accept H_0 if $X \le 5$, otherwise reject H_0.

Acceptance Region $= \{0, \dots, 5\}$

Critical region $= \{6, \dots, 20\}$

$\alpha = P(X \ge 6 \,;\, p = 0.1) = 0.0112$ by table 1.

$\beta = P(X \le 5 \,;\, p = 0.4) = 0.1255$

So we have a 1.12% chance of type 1 error and a 12.55% chance of type 2 error.

but if we reduce acceptance region to $\{0, \dots, 4\}$

then $\alpha = 0.0431$ ← increased

$\beta = 0.0509$ ← decreased

Can't reduce probability of both w/o collecting more data.