Rec 8/29

Thursday, August 29, 2019 10:20

$\mu_0 : \mathcal{A} \longrightarrow [0, \infty]$

μ_0 ↑ premeasure

$\mathcal{A}$ ↑ algebra

satisfies:

⓪ $\mu_0(\emptyset) = 0$.

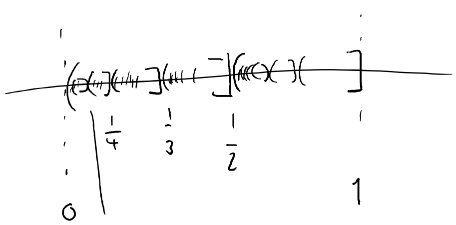
① if $(A_n) \subset \mathcal{A}$ disjoint & $\bigcup A_n \in \mathcal{A}$,

then $\mu_0(\bigcup A_n) = \sum \mu_0(A_n)$.

$$X = \mathbb{R}$$

$\mathcal{A}$: h-intervals

$(a, b]$



$\frac{1}{n} = a_n$

$a_n \longrightarrow 0$.

Can get <u>many</u> accumulation points.

but <u>can't</u> get a dense set of accumulation points.

<u>Folland #5</u>

$X, \quad \mathcal{E} \subset \mathcal{P}(X).$

<u>Claim</u>: $\mathcal{M}(\mathcal{E}) = \bigcup_{\substack{\mathcal{F} \subset \mathcal{E} \\ \mathcal{F} \text{ ctble}}} \mathcal{M}(\mathcal{F}) \equiv \mathcal{N}.$

Clearly $\mathcal{N} \subset \mathcal{M}(\mathcal{E})$.

otoh, $\mathcal{N}$ is a σ-algebra, so $\mathcal{N} \supset \mathcal{M}(\mathcal{E})$

<u>Proof</u>: $\mathcal{N}$ is nonempty & closed under complements.

let $(E_i) \subset \mathcal{N}$.

$E_i \in \mathcal{M}(\mathcal{F}_i)$.

Then $\bigcup E_i \in \mathcal{M}(\overbrace{\bigcup \mathcal{F}_i}^{\text{still countable!}}) \subset \mathcal{N}$. $\square$