# Lec 8/31

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$V/F$ with basis $\{v_1, \ldots, v_n\}$

Let $a_1, \ldots, a_m \in V$. Find a basis for $S(a_1, \ldots, a_m)$.

ex. $V = P_4(\mathbb{R})$ basis $\{x^4, x^3, x^2, x, 1\}$

$a_1 = x^4 + 2x^3 + x^2$ $\quad a_2 = 2x^4 + 3x^3 - x$ $\quad a_3 = x^4 + 2x^3 + x^2 - x$ $\quad a_4 = x$

$v_1 + 2v_2 + v_3$ $\quad 2v_1 + 3v_2 - v_4$ $\quad v_1 + 2v_2 + v_3 - v_4$ $\quad v_4$

$1\ 2\ 1\ 0\ 0$ $\quad 2\ 3\ 0\ {-1}\ 0$ $\quad 1\ 2\ 1\ {-1}\ 0$ $\quad 0\ 0\ 0\ 1\ 0$

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 2 & 3 & 0 & -1 & 0 \\ 1 & 2 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$ coefficient matrix.

Operations that don't affect subspace:

$R_i - \alpha R_j$ $\qquad S(a_1, \ldots, a_i, \ldots, a_j, \ldots, a_n) = S(a_1, \ldots, a_i, \ldots, a_j - \alpha a_i)$

$\beta R_j$ for $\beta \neq 0$. $\qquad$ etc.

$R_j \leftrightarrow R_i$

<u>Row-Echelon form</u>

$A'$ = row echelon form of $A$.

Nonzero rows of $\mathrm{ref}\left[\begin{pmatrix} a_1 \\ \vdots \\ a_m \end{pmatrix}\right]$ give a basis for $S(a_1, \ldots, a_n)$

elementary operations give row-equivalent matrices

Thm Let $A$ be the coeff matrix of $\{a_1, \ldots, a_m\}$ wrt basis

(1) $\exists A' \underset{row}{\sim} A$ in Row-echelon form.

(2) $A'$ can be $0$ or $\exists k \in [m]$ s.t. the first $k$ rows of $A$ are in Row-echelon form and the last $m-k$ rows are all $0$.

(3) The vectors $a_i$ corresponding to the first $k$ rows of $A'$ are linearly independent.

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 \\ -1 & 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 1 & 1 \\ 0 & -1 & -1 & -1 & 0 \end{pmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 & 1 \\ 0 & -2 & 0 & 2 & -2 \\ 0 & 2 & 2 & 2 & 0 \end{bmatrix} \sim \begin{pmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & 2 & -1 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$V = S(1, \cos x, \sin x, \cos 2x, \sin 2x) \subseteq C^\infty(\mathbb{R})$

Periodic

$f(x + k2\pi) = f(x)$

$0 = \lambda_1 + \lambda_2 \cos x + \lambda_3 \sin x + \lambda_4 \cos 2x + \lambda_5 \sin 2x$

$\Downarrow$

$\lambda_i = 0 \ \forall i$. (take derivatives, plug in values).