# Lec 8/22

Tuesday, August 22, 2017 09:31

We talked about "regular" induction (I)

and complete M.I. (II)

clearly $\mathrm{II} \Rightarrow \mathrm{I}$

but $\mathrm{I} \Rightarrow \mathrm{II}$ too.

Let $Q(n) := P(1) \wedge P(2) \wedge \ldots \wedge P(n)$

So $\mathrm{I}$ on $Q$ $\Leftrightarrow$ $\mathrm{II}$ on $P$.

Let $S \subseteq \mathbb{N}$. If $S \neq \emptyset$ then $S$ has a least element

(that is, $\exists\, n_0 \in S$ s.t. $\forall n \in S,\ n_0 \leq n$).

This statement is equivalent to induction (well-ordering principle). W

<u>Proof that $\mathrm{II} \Rightarrow W$:</u> by contrapositive (prove $S$ has no least elt $\Rightarrow S = \emptyset$)

(A) $1 \notin S$ since $1 \leq n\ \forall n \in \mathbb{N}$.

(B) if $1, 2, \ldots, n \notin S$ then $n+1 \notin S$ since o.w. $n+1$ would be the least elt.

So $S = \emptyset$.

<u>Proof that $W \Rightarrow \mathrm{I}$:</u> (by contrapositive) Let $S = \{ n \in \mathbb{N} \mid P(n) \text{ false} \}$

If $S \neq \emptyset$ then $\exists$ a least element $k$. if $k = 1$ then $P(1)$ false, else $P(k-1) \not\Rightarrow P(k)$

since $P(k-1)$ true but $P(k)$ false.