KEX

Alexander Santi Straihamer

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0.1 Base Problem (Augmented-Node Formulation)

Sets and Parameters

- V: Set of all patients (indexed by i)
- K: Set of caregivers (indexed by k)
- σ_k: Start node of caregiver k
- τ_k: End node of caregiver k
- $V_k := V \cup \{\sigma_k, \tau_k\}$: Augmented set of nodes for caregiver k
- $c_{ij} \in \mathbb{R}_{\geq 0}$: Travel time from node i to node j
- $[e_i, l_i] \subset \mathbb{R}$: Time window for patient i (earliest e_i and latest l_i service start times)
- $s_i \in \mathbb{R}_{\geq 0}$: Service time at patient i
- $p_i^k \in \{0, 1\}$: Binary parameter indicating if caregiver k is qualified to treat patient i
- $M \in \mathbb{R}_{\geq 0}$: A sufficiently large constant

Decision Variables

- $x_{ij}^k \in \{0, 1\}$: Binary variable indicating if caregiver k travels directly from node i to node j. Defined for each $k \in K$ and $i, j \in V_k$ with $i \neq j$.
- $t_i^k \in \mathbb{R}_{\geq 0}$: Arrival time of caregiver k at node $i \in V_k$
- $d^k \in \mathbb{R}_{\geq 0}$: Down-time for caregiver k

Optimization Problem

$$
\begin{aligned}
\text{minimize} \quad & \sum_{k \in K} d^k && && \text{(V1)} \\
\text{subject to} \quad & \sum_{k \in K} \sum_{\substack{j \in V_k \\ j \neq i}} x_{ij}^k = 1, && \forall i \in V && \text{(V2)} \\
& \sum_{\substack{j \in V_k \\ j \neq i}} x_{ij}^k - \sum_{\substack{j \in V_k \\ j \neq i}} x_{ji}^k = 0, && \forall i \in V, \forall k \in K && \text{(V3)} \\
& \sum_{\substack{j \in V_k \\ j \neq \sigma_k}} x_{\sigma_k j}^k = \sum_{\substack{j \in V_k \\ j \neq \tau_k}} x_{j \tau_k}^k \leq 1, && \forall k \in K && \text{(V4)} \\
& \sum_{\substack{i \in V_k \\ i \neq \sigma_k}} x_{i \sigma_k}^k = \sum_{\substack{j \in V_k \\ j \neq \tau_k}} x_{\tau_k j}^k = 0, && \forall k \in K && \text{(V5)} \\
& x_{ij}^k \leq p_j^k, && \forall i \in V^k, j \in V, i \neq j, \forall k \in K && \text{(V6)} \\
& t_i^k \geq e_i, && \forall i \in V, \forall k \in K && \text{(V7)} \\
& t_i^k \leq l_i - s_i, && \forall i \in V, \forall k \in K && \text{(V8)} \\
& t_j^k \geq t_i^k + c_{ij} + s_i - M(1 - x_{ij}^k), && \forall i, j \in V_k, i \neq j, \forall k \in K && \text{(V9)} \\
& d^k \geq t_{\tau_k}^k - t_{\sigma_k}^k - \sum_{i \in V} s_i \Big(\sum_{\substack{j \in V_k \\ j \neq i}} x_{ji}^k \Big), && \forall k \in K && \text{(V10)}
\end{aligned}
$$

Possible extensions: lunch breaks, maximum shift length, and earliest/latest work times.

Constraint explanations:

- **Unique Visit Constraint** (V2): ensures each patient is visited exactly once by one caregiver.
- **Flow Conservation Constraint** (V3): ensures flow conservation for patients, ensuring continuous caregiver routes.
- **Route Completion Constraint** (V4): ensures caregivers either complete a full route or are not used.
- **Invalid Arc Prevention Constraint** (V5): prevents invalid arcs from entering start nodes or leaving end nodes.
- **Qualification Constraint** (V6): ensures caregivers only visit patients they are qualified for.
- **Earliest Arrival Constraint** (V7): ensures caregivers don't arrive at patients before their earliest service time.
- **Latest Arrival Constraint** (V8): ensures caregivers arrive early enough to complete services within time windows.
- **Temporal Feasibility Constraint** (V9): ensures temporal feasibility of visits, respecting travel and service times.
- **Downtime Definition Constraint** (V10): defines caregiver downtime as the total idle time between start and end nodes, excluding service durations.