1 NR Band Model (Theory)

$$f(E_r,\lambda) = \lambda e^{-E_r\lambda} \tag{1}$$

$$\epsilon = E_r \frac{a}{2Z^2e^2} \tag{2}$$

$$\begin{aligned} g(\epsilon) &= 3\epsilon^{0.15} + 0.7\epsilon^{0.6} + \epsilon \\ \bar{\nu}(\epsilon) &= \frac{\epsilon}{1+kg(\epsilon)} \end{aligned} \tag{3}$$

$$Y = \frac{\epsilon - \bar{\nu}(\epsilon)}{\epsilon} = \frac{kg(\epsilon)}{1+kg(\epsilon)} \tag{4}$$

$$\begin{aligned} E_p &= E_r + \frac{YE_r}{\epsilon_\gamma}qV \\ E_r &= \frac{Ep}{1+\frac{YqV}{\epsilon_\gamma}} \end{aligned} \tag{5}$$

$$\begin{aligned} \sigma_p &= \sqrt{\alpha + \beta E_p + \gamma E_p^2} \\ \sigma_q &= \sqrt{\alpha + \beta E_q + \gamma E_q^2} \end{aligned} \tag{6}$$

$$N = \frac{YE_r}{\epsilon_\gamma} \tag{7}$$

$$\begin{aligned} \sigma_P &= \sqrt{\alpha + \beta E_p + \gamma E_p^2 + V^2NF} \\ \sigma_Q &= \sqrt{\alpha + \beta E_q + \gamma E_q^2 + \epsilon^2NF} \end{aligned} \tag{8}$$

Using an energy drawn from a normal distribution, with widths defined by σ_P and σ_Q, uncertainty is added to E_p and E_q by "smearing" or adding the drawn energy to E_p and E_q giving E_P and E_Q. $E_Q = E_q + "smear"$ and $E_Q = E_q + "smear"$ The reconstructed Yield is then:

$$Y = \frac{E_Q}{E_P - \frac{eV}{\epsilon_\gamma} E_Q} \tag{9}$$

2 Yield Variance and Fano factor. (Theory)

The yield variance is:

$$\sigma_y^2 = \frac{\partial^2 Y}{\partial E_q^2}(\sigma_q^2 + \epsilon^2 NF)^2 + \frac{\partial^2 Y}{\partial E_P^2}(\sigma_p^2 + V^2 NF)^2 \tag{10}$$

σ_p and σ_q are the resolutions without the added fano factor. Letting:

$$\begin{aligned} U_1 &= \frac{\partial Y}{\partial E_Q} = \frac{(E_P \epsilon^2)}{(V E_Q - E_P \epsilon)^2} \\ U_2 &= \frac{\partial Y}{\partial E_P} = \frac{-E_Q}{(E_P - \frac{E_q V}{\epsilon^2})^2} \end{aligned} \tag{11}$$

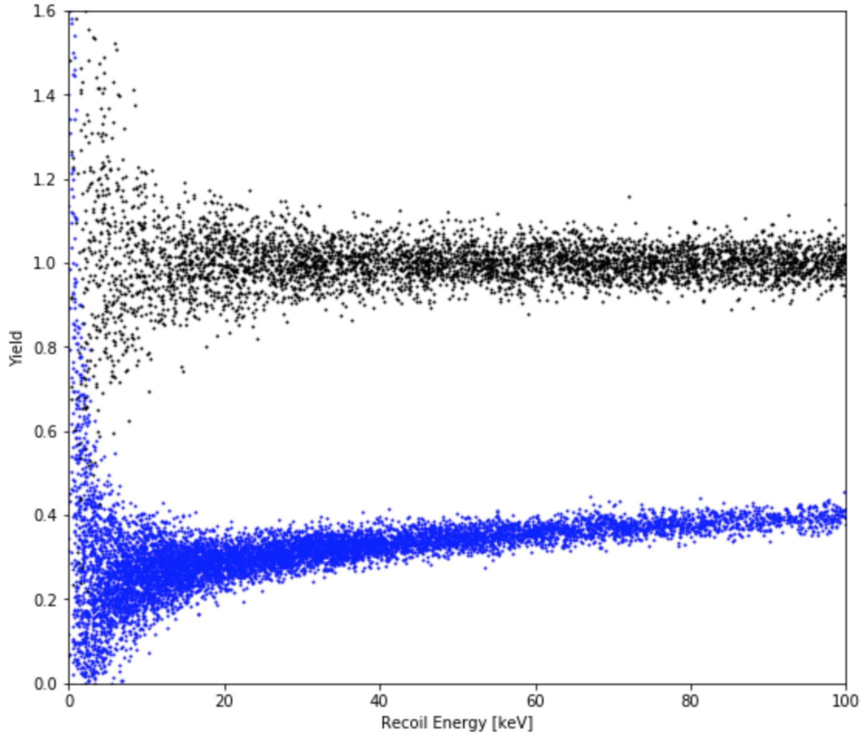
$$\sigma_y^2 = U_1^2(\sigma_q^2 + \epsilon^2 NF)^2 + U_2^2(\sigma_p^2 + V^2 NF)^2 \tag{12}$$

Solving for the fano factor, F:

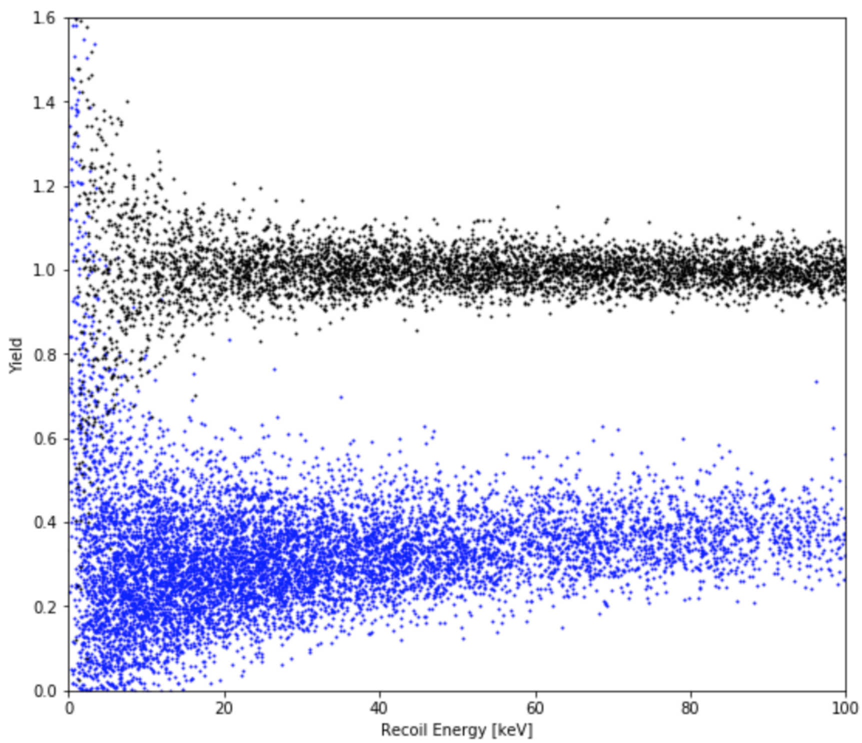
$$F = \frac{(-U_1^2 \sigma_q^2 - U_2^2 \sigma_p^2 + \sigma_y^2)}{(U_1^2 \epsilon^2 + U_2^2 V^2)} N \tag{13}$$

3 Fitting

With the exception of σ_Y, the quantities in F are all calculated using the randomly drawn energy from the exponential distribution. To find σ_Y, the data in Figure 1b is split into evenly spaced bins. A histogram for the Yield is made for each bin like shown in Figure 2a. Assuming a normal distribution, (may not be the case) each histogram is fit. The current fitting routine gives the mean, standard deviation, and amplitude. It also allows access to the covariance matrix so that we can see the error in each of the "fits." The standard deviation of the fit, (σ_Y), is then used to find the Fano factor for each bin. The reported fano factor is found via the same manner as σ_Y. Once calculated, the spectrum of fano factors for each bin are histrogramed, fit, and the mean extracted.

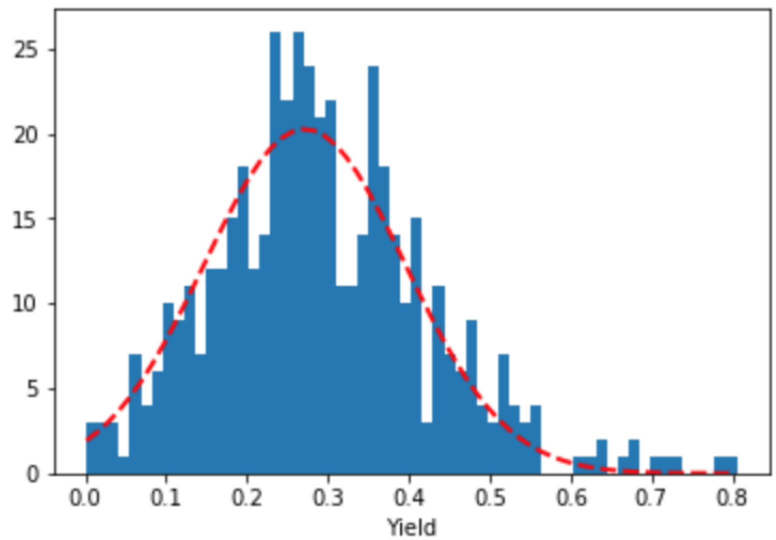


(a)

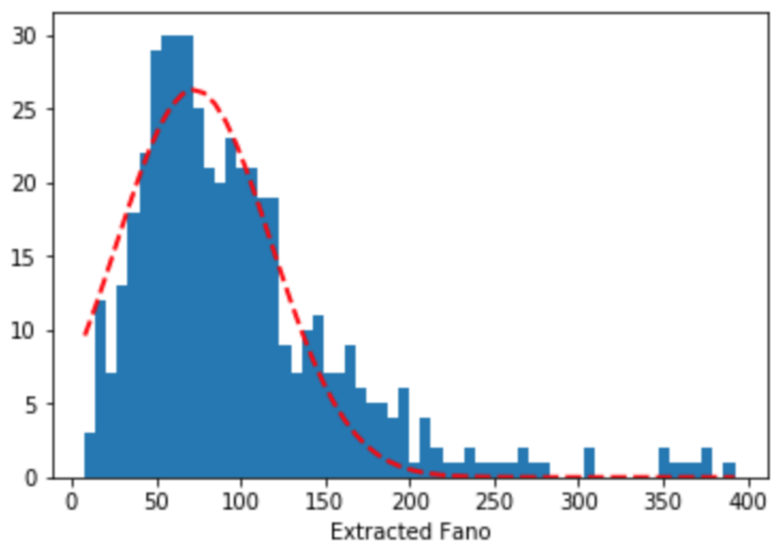


(b)

Figure 1: (a) Bands without fano factor, (b) bands with added fano factor, F = 100



(a)



(b)

Figure 2: (a) Yield fit. $\sigma_Y = 0.24 \pm 3 * 10^-5$, (b) Fano Fit, for F = 100, $F_{fit} = 76.81 \pm 7.8$