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A novel lyapunov exponent approach for stability analysis of the simple nuclear reactor

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Abstract

Stability monitoring of nuclear reactors has been extensively investigated in the last few decades. However, the identification methods and proposed intelligent control systems are not appropriate for real cases [1]. In this study stability analyses of nuclear reactors have been done by using a dynamical system approach. We elaborated the relation between nuclear fission and spatial chaos. The relation among neutron effective multiplication coefficient **k**, concentration rate of uranium and chemical shim, indicates that the nuclear self-sustaining chain fission and nuclear fusion have literally spatial nonlinear dynamical behavior. Stability boundaries are obtained in several three and two-dimensional parameter spaces. We have found the absolutely stable area of nuclear reactor, which is markedly different from the formerly mentioned areas.

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Key words: *Neutron diffusion, Fuel concentration, Chemical Shim, Infinite multiplication factor, Coupled map lattice, Lyapunov exponent*

1. Introduction

Monitoring stability of nuclear reactors is one of the attractive subjects in the nuclear energy field. A nuclear reactor is always required to be under control and amenable to adjustments within accurate limits in order to have a safe, controlled nuclear (fission or fusion) energy release. Since it is relevant to evaluating the safety of a nuclear system, efforts both from theoretical and practical viewpoint have been made to correctly detect the early instability of reactors during last decades [2]. Nuclear reactor is a complex system that requires an elaborate computer model to reproduce the dynamics of the system with enough details to perform accurate steady state calculation or transient analysis [3].

Over the past several years a wide range of models and computational codes have been developed and many tests have been carried out to study and analyze instabilities in nuclear reactors [4, 5]. However, in the various models and codes developed, either no attempt has been made to determine the stability boundaries in an operating parameter plane, or the models or codes are mathematically and computationally so complex that only a few points approximating the stability boundaries are computed [6-8]. It should be noted that nuclear reactors are typically of complex nonlinear and multivariable nature with high interactions between their state variables. Therefore, many of the identification methodologies and proposed intelligent control systems are not appropriate for real cases [9]. The introduction of nonlinear tools in stability analysis has demonstrated its great potential in the surveillance of nuclear reactors and their fundamental equipment, as well as the early detection of anomalies or failures [10-13]. The coupled map lattices (CMLs) method is based on a dynamic system with continuous field variables with discrete space and time. The purpose of this study is to introduce a CML model of Neutron diffusion [14]. The introduced model is a starting point for a more complete, yet simple, model for the nonlinear dynamics of reactors. We consider the spectrum of Lyapunov exponents which has been proven to be the most useful dynamical diagnosis for instability of nuclear reactors [15, 16].

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2. Formulation of dynamical systems

In the CML method, mostly dynamic processes are formulated in mappings. In the present model, it is assumed that Neutron population is governed by the following physics and dynamics.

2.1. Neutron Diffusion

The neutron transport equation is the most fundamental and exact description of the distribution of neutrons in space, energy, and direction (of motion) and is the starting point for approximative methods. It can be derived from a particle balance on an infinitesimal volume using only a few assumptions which remove unimportant phenomena, such as neutron neutron interactions, in most applications [17].

The diffusion equation can be derived by considering an additional assumption that the angular flux has a linearly anisotropic directional dependence on problems with isotropic sources and scattering [18]. This allows the removal of the directional variables from the neutron density and simplifies the governing equation and associated numerical methods. The neutron diffusion equation is a partial differential equation which describes the neutron population behavior inside a nuclear reactor. In order to carry out better and faster safety analysis, the equation needs to be solved as fast as possible while keeping the accuracy requirements. By considering a one-speed diffusion theory

$$\frac{1}{v}\frac{\partial \Phi(r,t)}{\partial t} - \nabla . D\nabla\Phi(r,t) + \Sigma_a\Phi(r,t) = v\Sigma_f\Phi(r,t), \tag{1}$$

where D is the Diffusion coefficient, Φ neutron velocity, and Σ_f , Σ_a are absorption and fission cross-sections. One speed diffusion equation introduced as Eq. (1) is discretized and replaced by coupled map lattice. The actual finite difference equations are obtained via using the seven-point scheme (Fig. 1 shows a mesh point). Let its subscripts be **i, j, k** according to the coordinates **x, y, z,** respectively. This figure shows the planes drawn perpendicularly to the coordinate axes at half way between the neighboring mesh points. In this way, we reach finally the following approximation for the Eq. (1) [18].

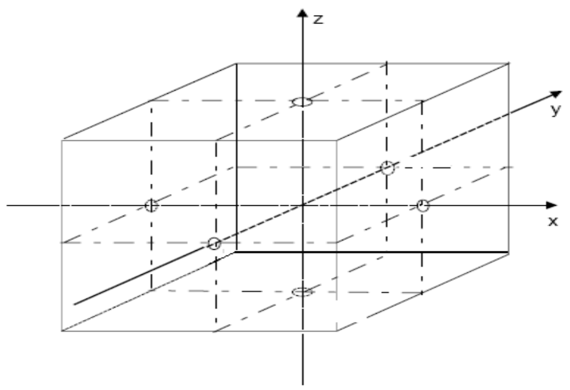

Fig. 1. Computational domain and lattices.

$$\varphi_{i,j,k}^{n+1} = \frac{Dv\Delta t}{(\Delta x)^2}\left(\Phi_{i-1,j,k}^n + \Phi_{i+1,j,k}^n\right) + \frac{Dv\Delta t}{(\Delta y)^2}\left(\Phi_{i,j-1,k}^n + \Phi_{i,j+1,k}^n\right) + \frac{Dv\Delta t}{(\Delta z)^2}\left(\Phi_{i,j,k-1}^n + \Phi_{i,j,k+1}^n\right) + (1 + Dv\Delta t\alpha)\Phi_{i,j,k}^n, \tag{2}$$

where:

$$\alpha = \frac{v\Sigma_f - \Sigma_a}{D} - 2\left(\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2} + \frac{1}{(\Delta z)^2}\right).$$

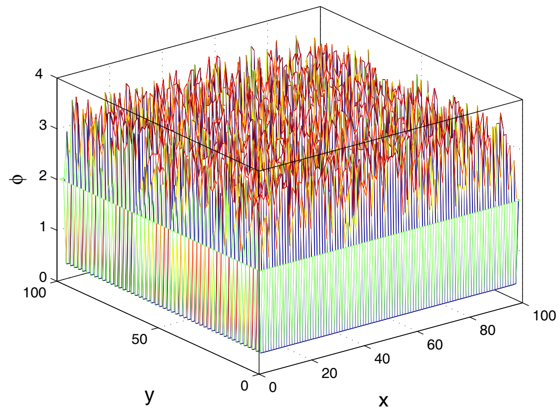

Fig. 2. Variation of the neutron density in chaotic regime.

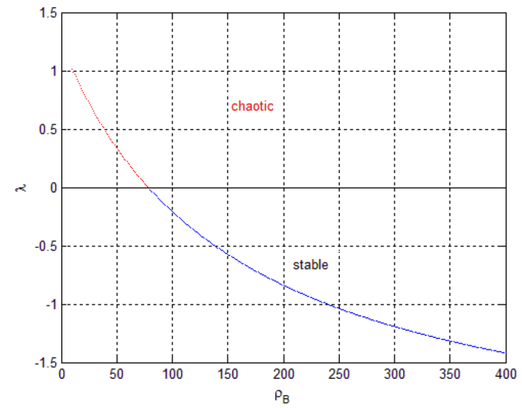

Fig. 3. (a) The variation of the Lyapunov exponent with respect to the fuel concentration, ($\boldsymbol{\rho_{235_U} = 0.0145}$) .

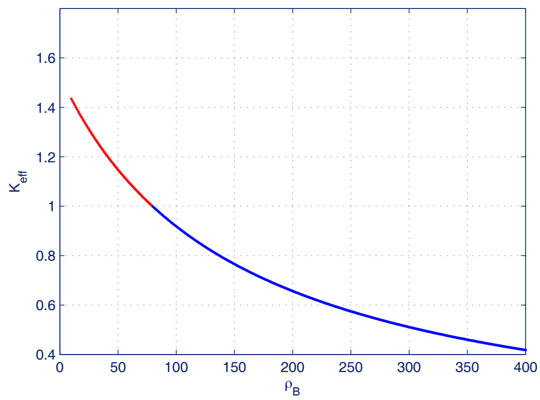

Fig. 3. (b) Distribution of the Infinite multiplication $\boldsymbol{k_\infty}$ with respecting to the Boron concentration. ($\boldsymbol{\rho_{235_U} = 0.0145}$)

The boundary conditions for the problem are $\Phi(r)|_\Gamma = 0$, where Γ is the reactor boundary [19].

A nuclear reactor must be provided with a system to control the reactor output. The control and safety of nuclear fission reactors are linked with

- The rate of neutron generation,
- The rate of neutron loss by leakage,
- The rate of neutron loss by parasitic absorption in the core.

The rates of neutron production and losses are directly related to the influences of reactivity and control of the reactor. It is clear that Boron is one of the best elements to absorb the excess thermal or near thermal neutrons in reactor configurations. This is due to its relatively large neutron capture in the thermal range. The processes for concentrating the natural boric acid solutions used as chemical shims in reactor coolant systems have already been defined [17]. In this study fuel concentration and Boron concentration are considered as control parameters which change the neutron flux and reactor stability.

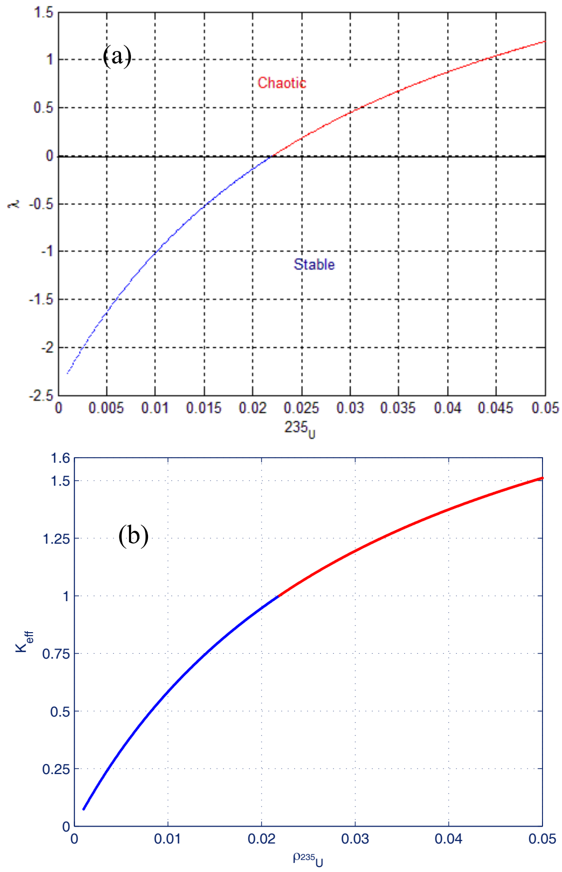

Fig. 4. (a) The variation of the Lyapunov exponent with respect to the fuel concentration, ($\rho_B = 150$), (b) Distribution of the Infinite multiplication k_∞ with respecting to the fuel concentration ($\rho_B = 150$).

2.2. Computational domain and lattices

An experimental bare cubic thermal reactor, 100 × 100 × 100, fueled with a homogeneous mixture of ^{235}U and ordinary water, operating at essentially room temperature and atmospheric pressure is investigated; the reactor can be controlled in part through varying the concentration of boric acid H_3BO_3 in the water. The concentration of H_3BO_3 is usually specified in parts per million ppm of water. The computational domain is divided into a 100 × 100 × 100 lattice. There are 100 grid points in the horizontal direction, 100 in the vertical direction, and 100 grid points in the direction perpendicular to the plane of paper. The interaction of solid lines in Fig. 1 represents location of grid points. The broken lines in the same figure indicate the faces of lattices.

Each lattice contains one grid point at its center. A grid point is designated by **i, j, k** with **i** increasing in the **x** direction, **j** in the **y** direction, and **k** in the **z** direction. In this study uranium concentration and boron concentration are considered as control parameters which change the neutron flux and reactor stability. In the present case, δx = 0.1 cm, δy = 0.1 cm, δz = 0. 1 cm and δt = 0.001 are used to ensure numerical stability.

2.2. Lyapunov exponent

One of the most striking features of chaotic systems is their unpredictability beyond a particular length of time. This property is a consequence of the inherent sensitive dependence on initial conditions. Thus, the trajectories of two points that are spatially close, diverge exponentially in chaotic systems. A good measure of this divergence is given by the "Lyapunov exponents" which are properly averaged exponents of the increase in the separation between two spatially close points as they evolve in time. If a discrete nonlinear system is dissipative, a positive Lyapunov exponent quantifies a measure of chaos [20]. The Lyapunov exponent can be simulated numerically with the finite difference method. In this section we introduce Lyapunov exponent and its derivation for neutron diffusion equation. Suppose that there is a small change δx(0) in the initial state x(0). At time t, this has changed to δx(t) given by

$$\delta x(t) \cong \delta x(0)\left|\frac{d\Phi\big(x(0)\big)}{dx}\right| = \delta x(0)\left|\Phi\big(x(t-1)\big)\Phi\big(x(t-2)\big)\dots\Phi\big(x(0)\big)\right|$$

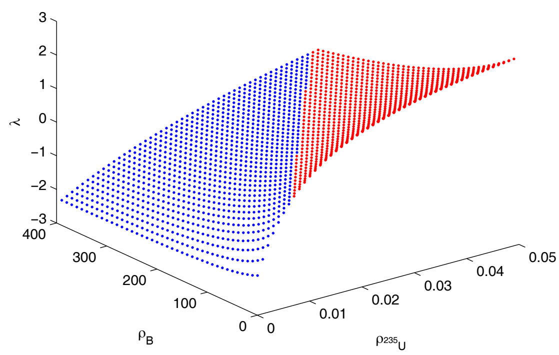

Fig. 5. The variation of the Lyapunov exponent with respect to the fuel concentration and Chemical Shim.

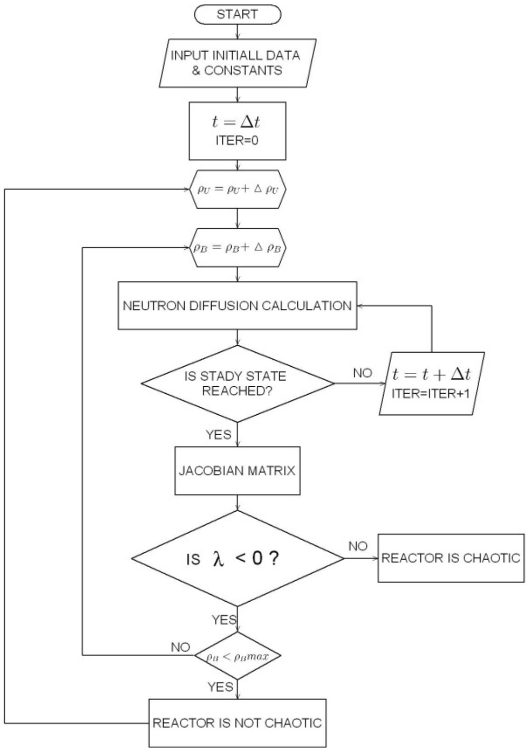

Fig. 6. Operation of a CML nuclear reactor stability simulation.

where we have used the chain rule to expand the derivative of Φ. In the limit of infinitesimal perturbations δx(0) and infinite time, we get an average exponential amplification, the Lyapunov exponent λ [21].

$$\lambda = \lim_{t\to\infty} \frac{1}{t} \ln \left|\frac{\delta x(t)}{\delta x(0)}\right|$$
$$= \lim_{t\to\infty} \sum_{k=0}^{t-1} \ln\left|\Phi\big(x(k)\big)\right|.$$

Now we take up a CML model described by

$$\Phi^{n+1}(i) = f\big(\Phi^n(i-1), \Phi^n(i), \Phi^n(i+1)\big),$$

where $\Phi^n(i)$ is the observable dynamical parameter with n as its time step and i as spatial site. In order to investigate the characteristics of Lyapunov exponents, we introduce the Jacobi matrix [22]

$$J_n = \begin{pmatrix} \frac{\partial\Phi^{n=1}(1)}{\partial\Phi^n(1)} & \frac{\partial\Phi^{n=1}(1)}{\partial\Phi^n(2)} & \cdots & \frac{\partial\Phi^{n=1}(1)}{\partial\Phi^n(N)} \\ \frac{\partial\Phi^{n=1}(2)}{\partial\Phi^n(1)} & \frac{\partial\Phi^{n=1}(2)}{\partial\Phi^n(2)} & \cdots & \frac{\partial\Phi^{n=1}(2)}{\partial\Phi^n(N)} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial\Phi^{n=1}(N)}{\partial\Phi^n(1)} & \frac{\partial\Phi^{n=1}(N)}{\partial\Phi^n(2)} & \cdots & \frac{\partial\Phi^{n=1}(N)}{\partial\Phi^n(N)} \end{pmatrix},$$

where N is the number of system size. This Jacobi matrix gives the linear stability of the system and the disorderliness of the field variables of the system at time n. The eigenvalues of matrix J_n give the Lyapunov exponents. When we write down the eigenvalues of J_n as $\{E_0^n, E_1^n, \ldots, E_n^n\}$ the Lyapunov exponents are given by

$$\lambda^n = \frac{1}{N}\sum_{i=0}^{N} \ln|E_i^n| \quad (i = 1,2,\ldots,N),$$

where $|E_i^n|$ means the absolute value of E_i^n. The mean Lyapunov exponent is defined as

$$\Lambda_n = \frac{1}{N}\sum_{j=0}^{N-1} \lambda_j.$$

3. Conclusion & Result

The aim of this paper was to predict the core dynamics behavior and safety of nuclear reactors, establishing the full power output and transient and natural convection circulation behavior of the reactor. Influencing the flux distribution and the dynamic reactivity coefficients in the reactor core predicting the safety limits for reactivity insertion in a reactor because it's importance from safety analysis as well as the reactor operation point of view. For the design and the safety reasons, nuclear power plants need fast and accurate plant simulators. From its spatial nonlinear system analysis, we investigated the relationship between chain fission and Lyapunov exponent; concentration rate of uranium, chemical shim, effective reproduction factor **k** of neutron; neutron multiplication theory and other aspects of spatial nonlinear system.

The results of the analysis for the stability of nuclear reactor are shown in Figs. 2-5. We have shown that the variation of Lyapunov exponents with respect to the fuel concentration and chemical shim clarifies the reactor super critical domain (See Fig. 3.a and Fig. 4.a). The Lyapunov exponents take negative values around the critical neutron population. Comparing effective multiplication factor spectrum with respect to the fuel concentration (see Figs. 3.a and 4.a) with their Lyapunov exponent, confirms this prediction (see Figs. 3.b and 4.b). In chaotic region, we have k_{eff} then the neutron flux increases each generation (see Fig. 2). In the non-chaotic region, the Lyapunov characteristic exponent is definitely negative, therefore in this region, the neutron production is less than the absorption and leakage, the reactor is then said to be sub critical.

As it was shown in Fig. 5 this analysis could be extended in a more general condition to consider the interaction of control parameters in the stability of reactor, too. The flow chart of the overall solution algorithm is shown in Fig. 6.

Most of previous studied models that simulate the behavior of the reactor, are strongly dependent on the special features of the reactors. However, we have used only methodical ideas to develop our model. The results showed indicate that the Lyapunov exponent can be an alternative stability indicator for the stable behavior of reactor core.

Results obtained here using a simple model suggest that further studies along these lines, with more detailed models, are needed to identify operating conditions in current and next generation of BWRs.

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