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Technical Note

Stability analysis in nuclear reactor using Lyapunov exponent

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Abstract

A simple model for nuclear reactor is proposed. With increasing the fuel concentration, our minimal model shows two successive phases; subcritical and supercritical. In subcritical regime, the neutron population grows with increasing the fuel concentration. In the supercritical state, the Lyapunov exponent is positive implying that the neutron diffusion phenomena are spatiotemporal chaos. In the present paper, the infinite multiplication factor curve is qualitatively reproduced for fuel concentration. We have derived the critical fuel concentration based on the Lyapunov exponents. The basic objective of the work is to improve the prediction of the critical neutron population with respect to the fuel concentration.

1. Introduction

Over the past several years a number of models and computational codes have been developed and tests have been carried out to study and analyze instabilities in nuclear reactors (Lewins and Ngcobo, 1996). However, in the various models and codes that have been developed, either no attempt has been made to determine the stability boundary in an operating parameter plane, or the models or codes are mathematically and computationally so complex that only a few points approximating the stability boundary are computed (Koclas, 1998; Cavdar and Ozgener, 2004). It should be noted that nuclear reactors are typically of complex nonlinear and multivariable nature with high interactions between their state variables. Therefore, many of the identification methodologies and proposed intelligent control systems are not appropriate for real cases (Mosher, 2004). The introduction of nonlinear tools in stability analysis has demonstrated its great potential for the surveillance of nuclear reactors and their essential equipment, as well as the early detection of anomalies or failures (Otero, 2003; Liu, 2006). The purpose of the research reported here is to introduce the coupled map lattice (CML) (Kaneko, 1993) model for neutron diffusion. The introduced model is the starting point for a more complete, yet simple, model for the nonlinear dynamics of reactors. We consider the spectrum of Lyapunov exponents which has been proven to be the most useful dynamical diagnosis tool for instability of nuclear reactors (Suzudo, 2003).

2. Problem formulation

An experimental bare $100 \times 100 \times 100$ cm cubic thermal reactor, fueled with a homogeneous mixture of ^{235}U and ordinary water and operating at essentially room temperature and atmospheric pressure, is investigated in this work. The computational domain representing the aforementioned core is divided into $100 \times 100 \times 100$ nodes.

2.1. Formulation of dynamic processes

In the CML method, dynamic processes are usually formulated in mappings. In the present model, it is assumed that the neutron population is governed by the following physics and dynamics (Duderstat and Hamilton, 1967).

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$$\frac{1}{v}\frac{\partial \Phi(r,t)}{\partial t} - \nabla.D\nabla\Phi(r,t) + \sum_a \Phi(r,t) = v\Sigma_f\Phi(r,t) \quad (1)$$

This one speed diffusion equation is discretized using the mesh-cell-centered method, and is replaced by a coupled map lattice. The actual finite difference equations are obtained by using the seven-point scheme, leading to the following approximation for Eq. (1) (Vulkov et al., 1999):

$$\varphi^{n+1}_{i,j,k} = \frac{Dv\Delta t}{(\Delta x)^2}\left(\Phi^n_{i-1,j,k} + \Phi^n_{i+1,j,k}\right) + \frac{Dv\Delta t}{(\Delta y)^2}\left(\Phi^n_{i,j-1,k} + \Phi^n_{i,j+,k}\right) + \frac{Dv\Delta t}{(\Delta z)^2}\left(\Phi^n_{i,j,k-1} + \Phi^n_{i,j,k+1}\right) + (1 + Dv\Delta t\alpha)\Phi^n_{i,j,k} \quad (2)$$

where

$$\alpha = \left[\left(\frac{k_\infty - 1}{L^2}\right) - 2\left(\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2} + \frac{1}{(\Delta z)^2}\right)\right]$$

The boundary conditions for the problem are $\Phi(r)|_\Gamma = 0$, on Γ, where the latter parameter represents the reactor boundary. In this study the fuel concentration is considered as a control parameter that changes the neutron flux and reactor stability (Owen, 1963).

2.2. Lyapunov exponent

Sensitive dependence on initial conditions is the most prominent property of chaos and its characterization in terms of Lyapunov exponents. The signs of the Lyapunov exponent provide the qualitative picture of the system's dynamics. If a discrete nonlinear system is dissipative, a positive Lyapunov exponent quantifies a measure of chaos (Dorfman, 1999). The Lyapunov exponent can be simulated numerically with the finite difference method. In order to investigate the characteristics of Lyapunov exponents, we use the Jacobi matrix (Shibata, 1999). Jacobi matrix, J_n, gives the linear stability of the system and the disorderness of the field variables of the system. The eigenvalues of matrix J_n give the Lyapunov exponents. When we display the eigenvalues of J_n as $\{E^n_0, E^n_1, \ldots, E^n_N\}$, the mean Lyapunov exponents are given by

$$\lambda^n = \frac{1}{N}\sum_{i=0}^{N} \ln|E^n_i| \quad (i = 1, 2, \ldots, N)$$

3. Results and discussion

The results of an analysis for the stability of a nuclear reactor are shown in Fig. 1, where we show the variation of Lyapunov exponents with respect to the fuel concentration and thereby clarify the reactor super critical domain. The Lyapunov exponent takes negative values around the critical neutron population. As noted, the maximum value of the fuel concentration $\rho = 0.03$ results in positive values for Lyapunov exponents and thereby reactor enters into the critical domain (see Fig. 1a).

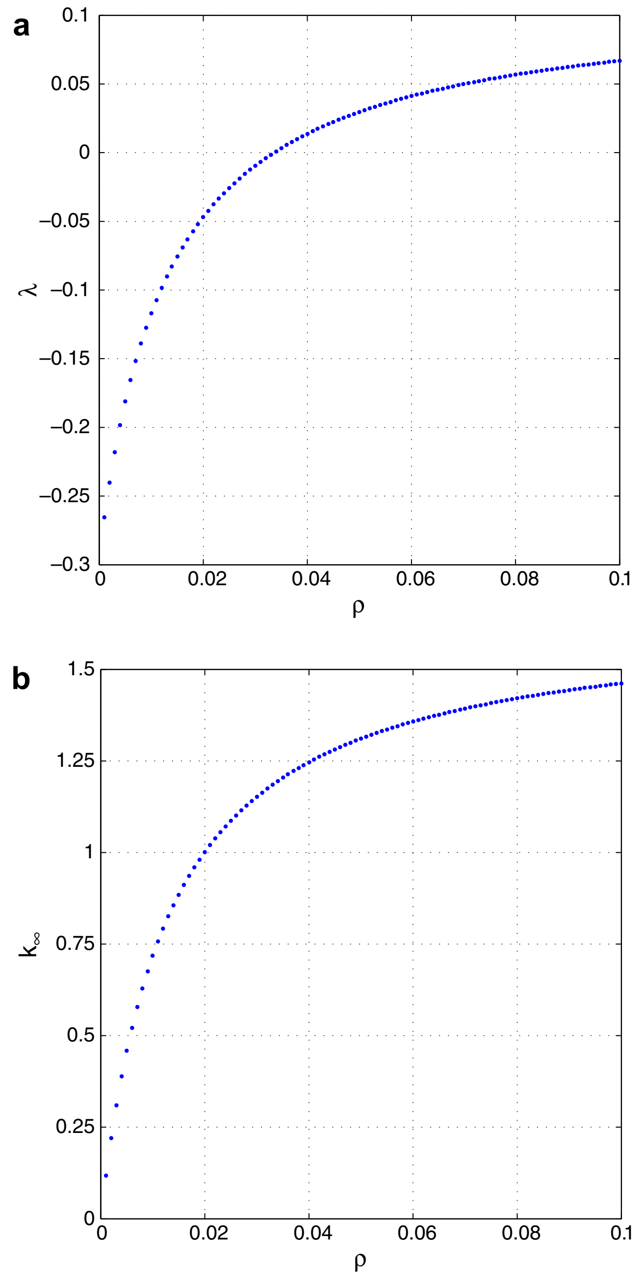

Fig. 1. (a) The variation of the Lyapunov exponent with respect to the fuel concentration and (b) distribution of the infinite multiplication factor k_∞ with respect to the fuel concentration.

Comparing the spectrum of the infinite multiplication factor (k_∞), with respect to the fuel concentration, with their Lyapunov exponent confirms the correctness of this prediction (see Fig. 1a and b). If $k_\infty < 1$, the number of neutrons decreases from generation to generation and the chain reaction dies out (Duderstat and Hamilton, 1967). We must thus have $k_\infty > 1$ in order to have any chance to achieve a critical chain reaction (see Fig. 1b). In the chaotic region, we have $k_\infty > 1$. In the nonchaotic region, the Lyapunov characteristic exponent is definitely negative, since in this region the fuel concentration result $k_\infty < 1$.

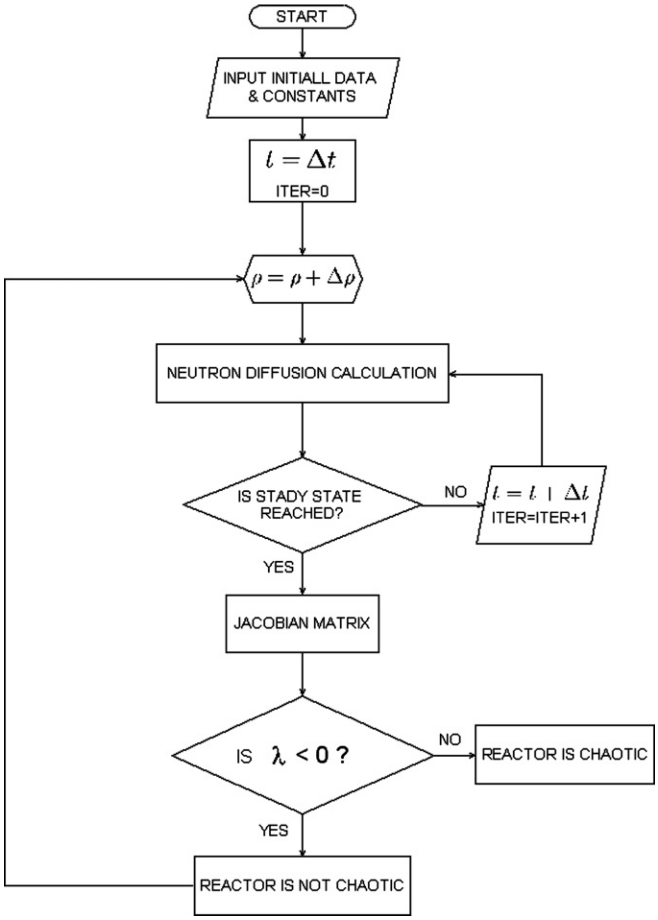

Fig. 2. Operation of a CML nuclear reactor stability simulation.

The flow chart of the overall solution algorithm is shown in Fig. 2.

4. Summery and conclusion

In a nuclear power plant, the neutron diffusion equations are simulated in order to obtain information about the unknown states of the nuclear reactor. The present 3D CML model has been able to capture the physics of neutron diffusion significantly better than the analytical and numerical models such as finite elements method. For the first time a coupled map lattice method which is essentially qualitative in nature has been able to predict the value of the critical fuel concentration for a sample nuclear reactor that was very close to the actual value (Dorfman, 1999). A sensitivity analysis shows that the model gives physically realistic and stable results.

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