Introducing an AR Model

TIME SERIES ANALYSIS IN PYTHON

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Mathematical Description of AR(1) Model

$$R_t \quad = \quad \mu \quad + \quad \phi \quad R_{t-1} \quad + \quad \epsilon_t$$

- Since only one lagged value on right hand side, this is called:
  - AR model of order 1, or
  - AR(1) model
- AR parameter is ϕ
- For stationarity, $-1 < \phi < 1$

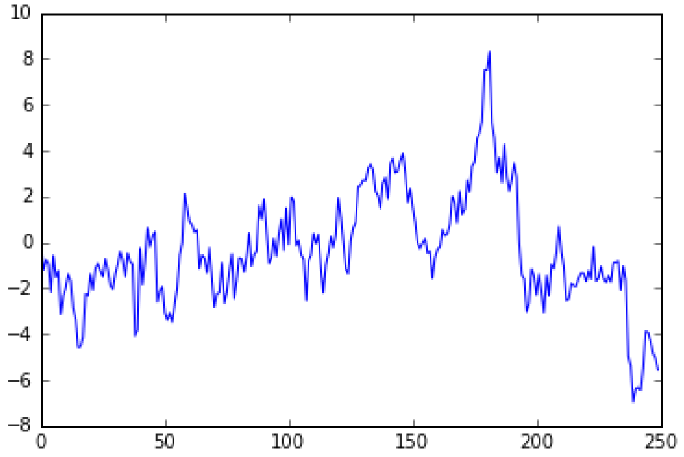
Interpretation of AR(1) Parameter

$$R_t \quad = \quad \mu \quad + \quad \phi \quad R_{t-1} \quad + \quad \epsilon_t$$

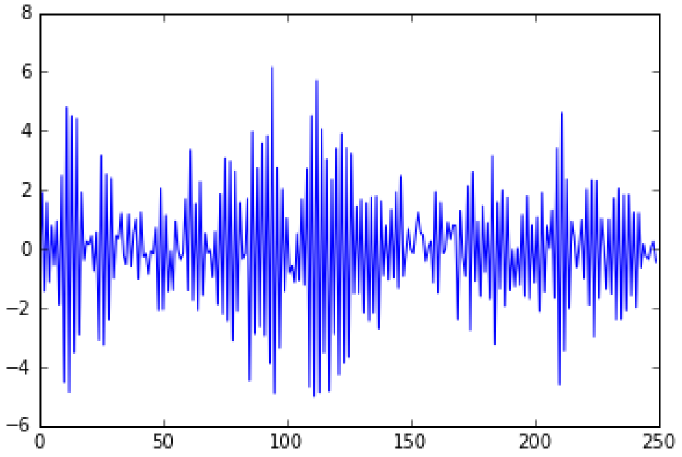
- Negative ϕ: Mean Reversion
- Positive ϕ: Momentum

Comparison of AR(1) Time Series

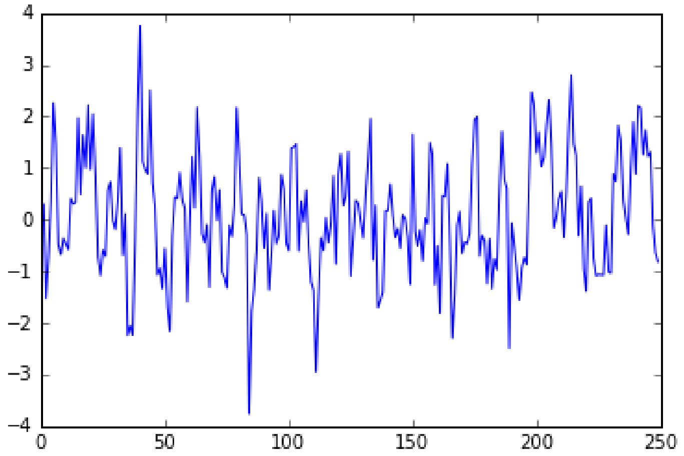
- $\phi = 0.9$



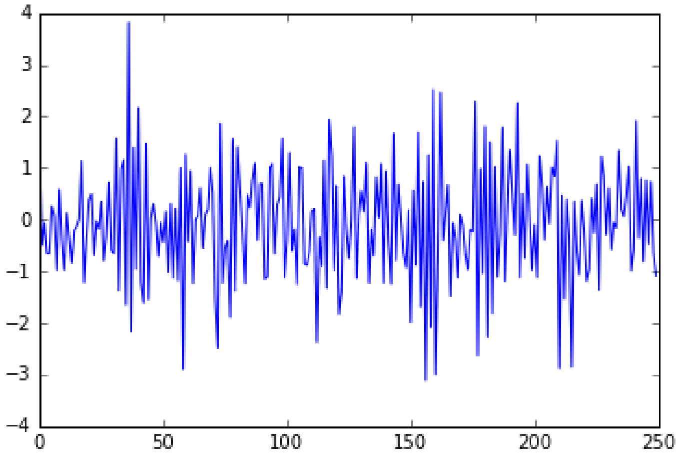
- $\phi = -0.9$



- $\phi = 0.5$

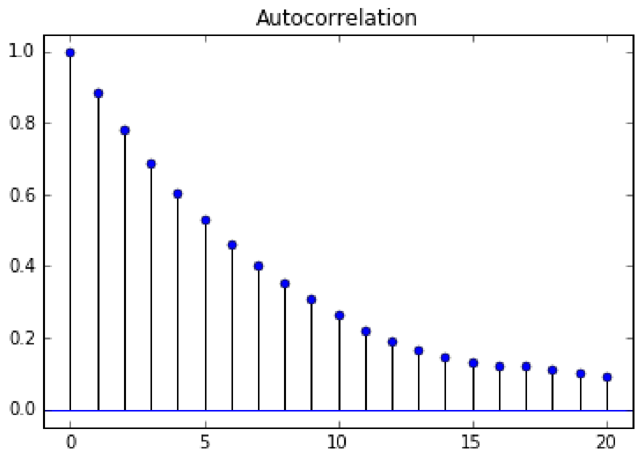


- $\phi = -0.5$

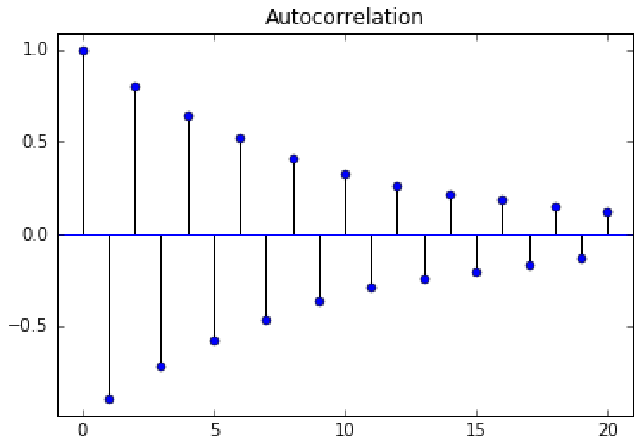


Comparison of AR(1) Autocorrelation Functions

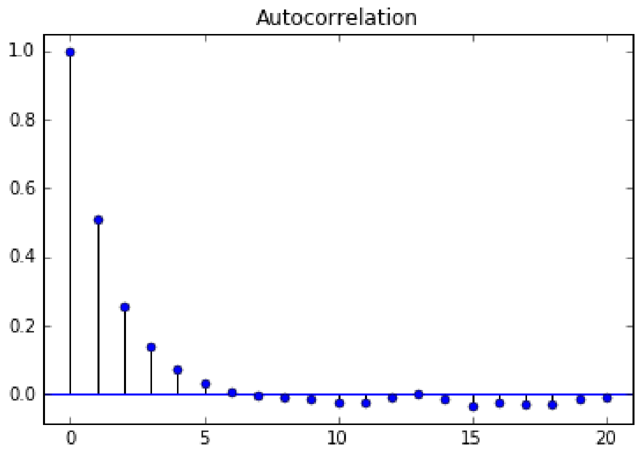
- $\phi = 0.9$



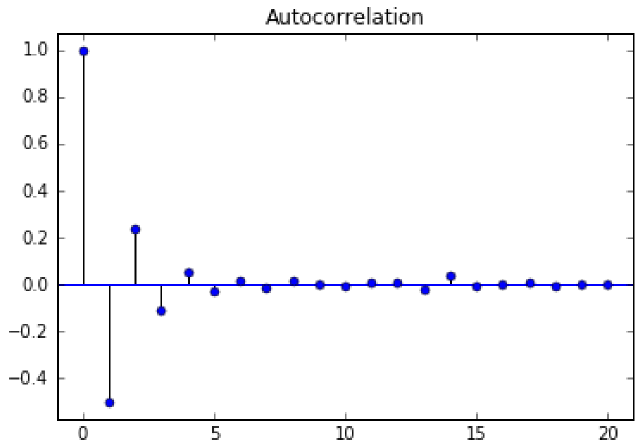
- $\phi = -0.9$



- $\phi = 0.5$



- $\phi = -0.5$



Higher Order AR Models

- AR(1)

$$R_t = \mu + \phi_1 R_{t-1} + \epsilon_t$$

- AR(2)

$$R_t = \mu + \phi_1 R_{t-1} + \phi_2 R_{t-2} + \epsilon_t$$

- AR(3)

$$R_t = \mu + \phi_1 R_{t-1} + \phi_2 R_{t-2} + \phi_3 R_{t-3} + \epsilon_t$$

- ...

Simulating an AR Process

```
from statsmodels.tsa.arima_process import ArmaProcess
ar = np.array([1, -0.9])
ma = np.array([1])
AR_object = ArmaProcess(ar, ma)
simulated_data = AR_object.generate_sample(nsample=1000)
plt.plot(simulated_data)
```

Let's practice!

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Estimating and Forecasting an AR Model

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Estimating an AR Model

- To estimate parameters from data (simulated)

```
from statsmodels.tsa.arima_model import ARMA
mod = ARMA(simulated_data, order=(1,0))
result = mod.fit()
```

Estimating an AR Model

- Full output (true $\mu = 0$ and $\phi = 0.9$)

```
print(result.summary())
```

```
                              ARMA Model Results
==============================================================================
Dep. Variable:                      y   No. Observations:                 5000
Model:                     ARMA(1, 0)   Log Likelihood               -7178.386
Method:                       css-mle   S.D. of innovations              1.017
Date:                Fri, 01 Dec 2017   AIC                          14362.772
Time:                        15:34:50   BIC                          14382.324
Sample:                             0   HQIC                         14369.625

==============================================================================
                 coef    std err          z      P>|z|      [95.0% Conf. Int.]
------------------------------------------------------------------------------
const         -0.0361      0.152     -0.238      0.812        -0.333     0.261
ar.L1.y        0.9054      0.006    151.020      0.000         0.894     0.917
                                    Roots
=============================================================================
                 Real           Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1           1.1045           +0.0000j            1.1045            0.0000
-----------------------------------------------------------------------------
```

Estimating an AR Model

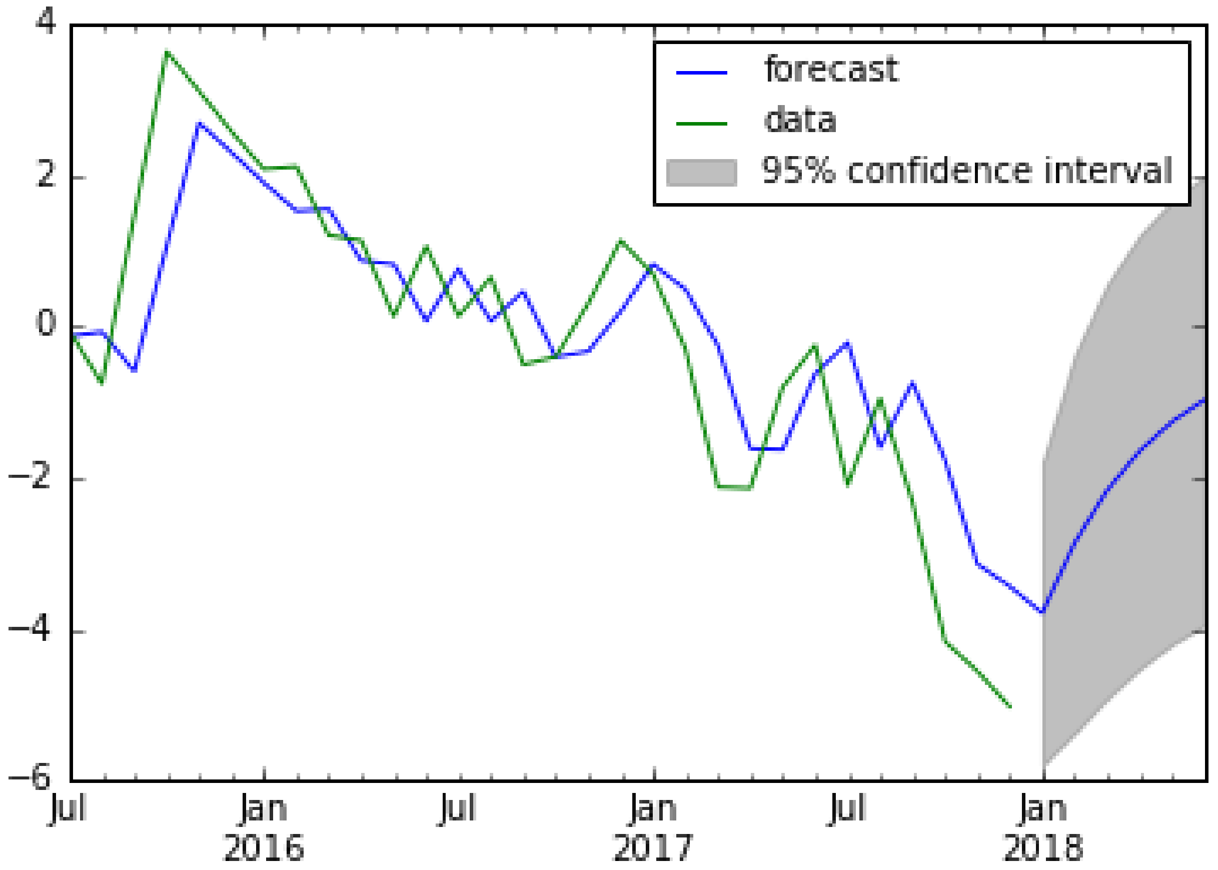
- Only the estimates of μ and ϕ (true $\mu = 0$ and $\phi = 0.9$)

```
print(result.params)
```

```
array([-0.03605989,  0.90535667])
```

Forecasting an AR Model

```
from statsmodels.tsa.arima_model import ARMA
mod = ARMA(simulated_data, order=(1,0))
res = mod.fit()
res.plot_predict(start='2016-07-01', end='2017-06-01')
plt.show()
```



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Choosing the Right Model

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Identifying the Order of an AR Model

- The order of an AR(p) model will usually be unknown
- Two techniques to determine order
  - Partial Autocorrelation Function
  - Information criteria

Partial Autocorrelation Function (PACF)

$$R_t = \phi_{0,1} + \boxed{\phi_{1,1}} R_{t-1} + \epsilon_{1t}$$

$$R_t = \phi_{0,2} + \phi_{1,2} R_{t-1} + \boxed{\phi_{2,2}} R_{t-2} + \epsilon_{2t}$$

$$R_t = \phi_{0,3} + \phi_{1,3} R_{t-1} + \phi_{2,3} R_{t-2} + \boxed{\phi_{3,3}} R_{t-3} + \epsilon_{3t}$$

$$R_t = \phi_{0,4} + \phi_{1,4} R_{t-1} + \phi_{2,4} R_{t-2} + \phi_{3,4} R_{t-3} + \boxed{\phi_{4,4}} R_{t-4} + \epsilon_{4t}$$

$$\vdots$$

Plot PACF in Python

- Same as ACF, but use `plot_pacf` instead of `plt_acf`
- Import module

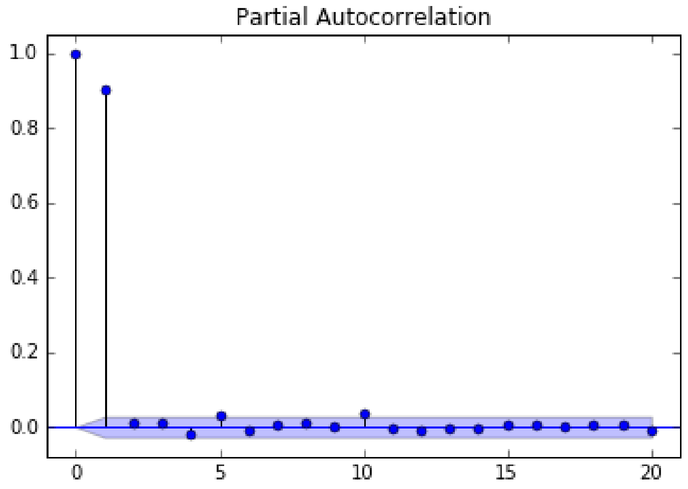
```
from statsmodels.graphics.tsaplots import plot_pacf
```

- Plot the PACF

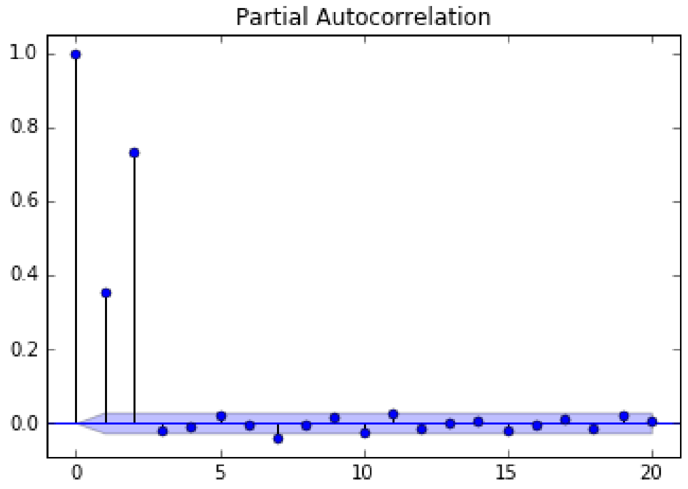
```
plot_pacf(x, lags= 20, alpha=0.05)
```

Comparison of PACF for Different AR Models

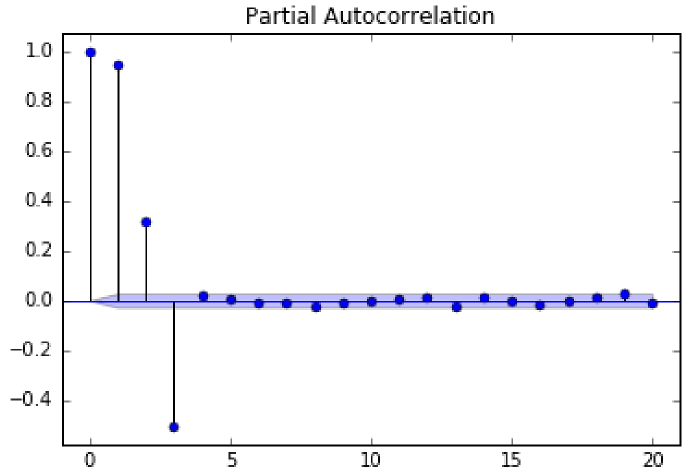
- AR(1)



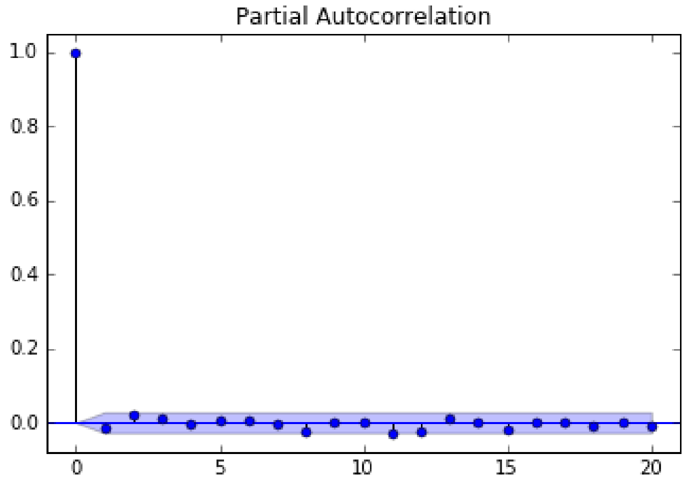
- AR(2)



- AR(3)



- White Noise



Information Criteria

- Information criteria: adjusts goodness-of-fit for number of parameters
- Two popular adjusted goodness-of-fit measures
  - AIC (Akaike Information Criterion)
  - BIC (Bayesian Information Criterion)

Information Criteria

- Estimation output

```
                              ARMA Model Results
==============================================================================
Dep. Variable:                      y   No. Observations:                 2500
Model:                     ARMA(2, 0)   Log Likelihood               -3536.481
Method:                       css-mle   S.D. of innovations              0.996
Date:                Fri, 29 Dec 2017   AIC                           7080.963
Time:                        22:53:24   BIC                           7104.259
Sample:                             0   HQIC                          7089.420

==============================================================================
                 coef    std err          z      P>|z|      [95.0% Conf. Int.]
------------------------------------------------------------------------------
const          0.0054      0.010      0.517      0.605        -0.015     0.026
ar.L1.y       -0.6130      0.019    -32.243      0.000        -0.650    -0.576
ar.L2.y       -0.3109      0.019    -16.351      0.000        -0.348    -0.274
                                    Roots
=============================================================================
                 Real           Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1          -0.9859           -1.4982j            1.7935           -0.3426
AR.2          -0.9859           +1.4982j            1.7935            0.3426
-----------------------------------------------------------------------------
```

Getting Information Criteria From `statsmodels`

- You learned earlier how to fit an AR model

```
from statsmodels.tsa.arima_model import ARMA
mod = ARMA(simulated_data, order=(1,0))
result = mod.fit()
```

- And to get full output

```
result.summary()
```

- Or just the parameters

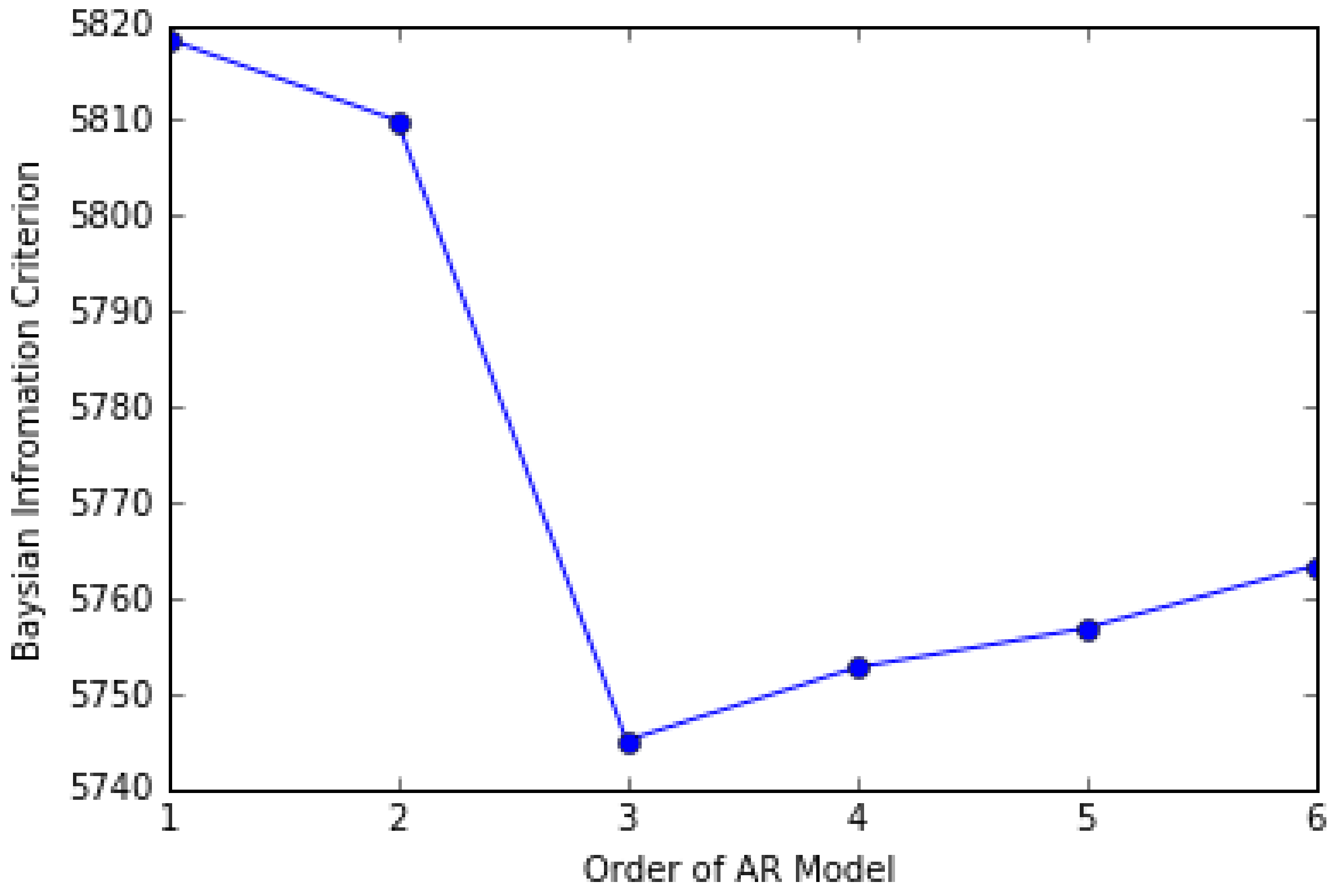
```
result.params
```

- To get the AIC and BIC

```
result.aic
result.bic
```

Information Criteria

- Fit a simulated AR(3) to different AR(p) models
- Choose p with the lowest BIC



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