Complex and Social Networks

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Laboratory 1

1 Plot (a)

For this plot, we needed to show how the clustering coefficient and the average shortest-path change as a function of the parameter p of the Watts-Strogatz (WS) model. To do this, we first created the graph with p equal to 0, which is a one-dimensional ring with n = 1200 nodes and each node connected to `nei = 4` neighbors. The values for the clustering coefficient $C(0)$ and the average shortest path $L(0)$ were used to normalize the data. This allowed us to plot the ratios to keep all values within the range of [0,1]. We defined then a sequence of p values on a logarithmic scale (from 0.0001 to 1) to explore smaller probabilities in more detail, where changes in network structure are most significant. Specifically, for each p we generated the network 20 times to reduce the effect of randomness, calculated the clustering coefficient and the average shortest path for each realization and then took the mean values.
Finally, we normalized the mean values by $C(0)$ and $L(0)$ and stored them in two variables, which represent the relative change in network properties as p increases. This approach reduces the effect of randomness inherent in the network generation and provides a more robust estimate of the typical network properties at each p.
This is the resulting graph:

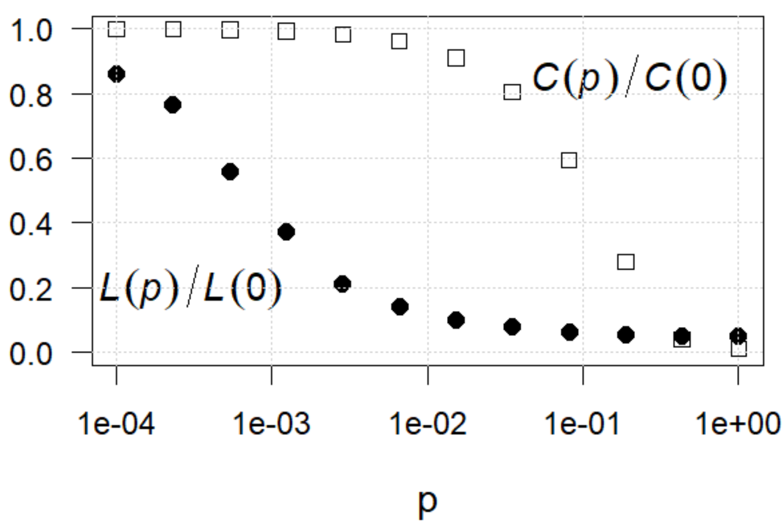

Figure 1: WS Model

As shown in the plot, increasing the parameter p from 0 to 1 causes a fast decrease in the average shortest-path, while the clustering coefficient decreases at a much slower rate. This is consistent with the small-world effect of the WS model.

2 Plot (b)

For the we second part of this laboratory session, we plotted the relationship between the average shortest-path length and the network size n in the ER model in order to prove that there was a logarithmic relationship. In addition, to demonstrate that the observed behavior of the curves is not due to a particular random configuration of the generated graph, multiple trials were performed to verify that the results are not coincidental but rather statistically significant. Therefore, after conducting these trials, we computed the average of the results.

To this end, we first created a vector of possible n values, ranging from 8 to 65536, where each value was double the previous one (i.e., 8, 16, 32, 64, 128...). For each value of n, we computed the parameter p using the formula $p = \frac{(1+\epsilon)\cdot\log(n)}{n}$, where $\epsilon = 0.1$. The choice of this parameter is crucial: if p is too small, the network is likely to contain isolated vertices. This selection of p therefore ensures that the graph is connected. As we stated in the previous paragraph, for every value of p we computed the average shortest-path 10 times.

This is the resulting graph:

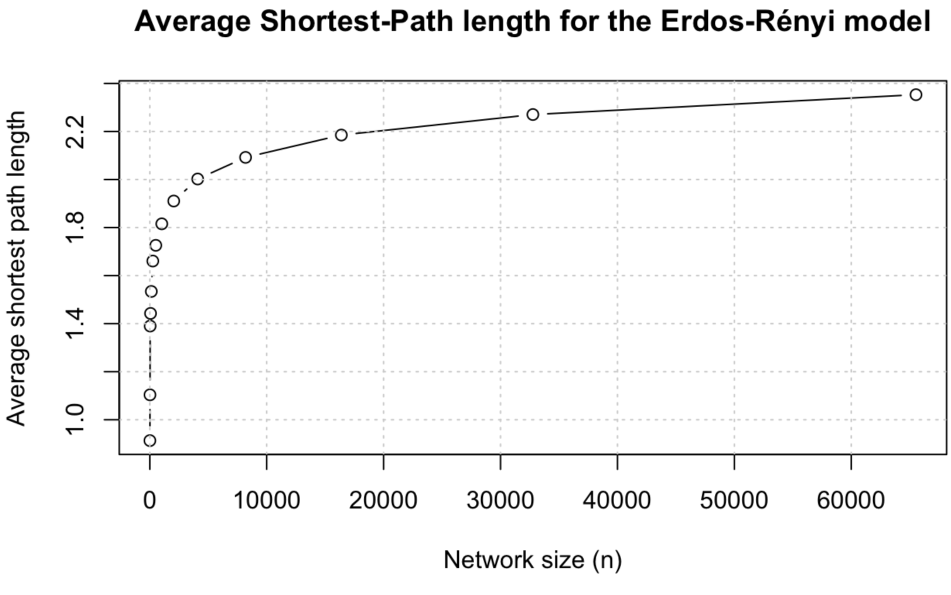

Figure (b) shows how the average shortest-path length in the Erdős-Rényi model grows logarithmically with network size. Starting from very small values (around 1.0) for tiny networks, the path length increases rapidly at first, then gradually levels off, reaching approximately 2.3 for networks of 65,000 nodes. This behavior exemplifies the 'small-world' property characteristic of random networks, where despite the network growing substantially in size, nodes remain relatively close to each other in terms of graph distance. The logarithmic growth pattern confirms the theoretical prediction that average path length scales as $\log(n)$ in Erdős-Rényi random graphs.