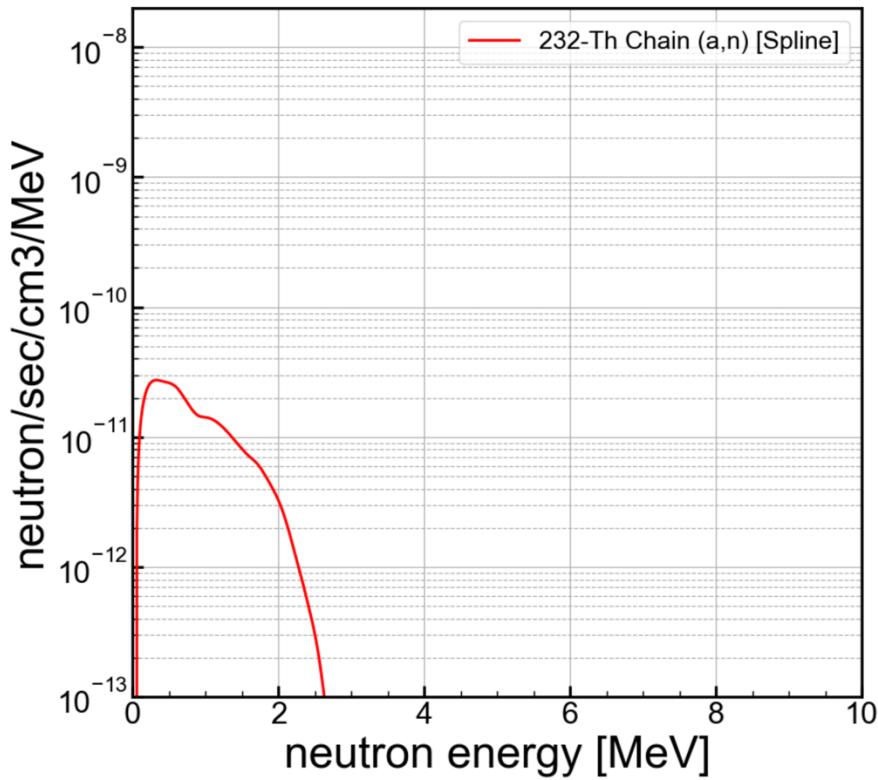


This is the neutron volumetric spectrum that SOURCES 4A (mod) predicts for Copper with 1 ppb Th contamination.

I want to use this to find out how many primary decays (from  $^{232}\text{Th}$ ) it takes to create 1 neutron.

Producing 1 neutron in the Cu volumes of SuperCDMS will obviously not amount to many registered interactions in the detectors. Many will not hit the detectors at all. However, we could test this by creating a volumetric neutron source in the Cu of SuperSim.



**NB:** In one year, 1 ppb contamination is equivalent to **0.00090 neutrons/cm<sup>3</sup>**. Equivalently, this corresponds to **1113 years** to get 1 neutron in each cm<sup>3</sup>.

Integrating this spectrum, I get a total volumetric source rate of  **$2.85 \times 10^{-11}$  neutrons/s/cm<sup>3</sup>/ppb**.

Remember, this is for 1 ppb of Th contamination. It is likely we have a lot more than this. However, in a simulation, to get meaningful background distributions we certainly need many counts. So, if I calculate how many decays it takes to get 1 neutron, that will be a benchmark because if we simulated the neutrons as a volumetric source, there would be a neutron for every primary simulated event.

I will calculate it like this (the bottom number is just the number above in bold):

Number of decays /s/cm<sup>3</sup>/1ppb

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Number of produced neutrons /s/cm<sup>3</sup>/1ppb

$^{232}\text{Pa}$ 1.31 d $\beta^-$ =100%	$^{233}\text{Pa}$ 26.975 d $\beta^-$ =100%	$^{234}\text{Pa}$ 6.671 h $\beta^-$ =100%
$^{231}\text{Th}$ 25.52 h $\beta^-$ =100%	<b><math>^{232}\text{Th}</math> 1.41e+10 y 99.98% <math>\alpha</math>=100% SF=1.1e-9%</b>	$^{233}\text{Th}$ 21.83 min $\beta^-$ =100%
$^{230}\text{Ac}$ 122 s $\beta^-$ =100% B-f=1.2e-6%	$^{231}\text{Ac}$ 7.5 min $\beta^-$ =100%	$^{232}\text{Ac}$ 119 s $\beta^-$ =100%

Probably, the way one would simulate the Th progeny in Geant4 is to place a  $^{232}\text{Th}$  atom in the contaminated material at a random location, and, let it decay. All the subsequent daughter decays will be captured in that event, no matter what the half life.

In practice, the  $\alpha$ -decays further down the chain will happen at the same rate as the Th decay, assuming secular equilibrium. That means for each Th decay there are a handful of  $\alpha$ -decays, each with the possibility of creating a neutron by the  $(\alpha, n)$  process.

The half life for  $^{232}\text{Th}$  is  **$1.41 \times 10^{10} \text{ y}$** , see left from [nndc.bnl.gov](http://nndc.bnl.gov)

## **8.96 g/cm<sup>3</sup> (Near room temperature)**

Near room temperature



First, we want to find out how many Cu atoms per cm<sup>3</sup> and then use 1 ppb to find the total number of Th atoms per cm<sup>3</sup> per ppb. Given that, we can use the decay equation to compute the denominator on the previous page.

1 amu is  **$1.674 \times 10^{-24}$  g** – I looked that up – it's based on carbon atoms being exactly 12 amu. So, the density of Cu is  **$5.35 \times 10^{24}$  amu/cm<sup>3</sup>**.

A copper atom is 63.5 amu. The number density of room temp. copper is therefore:  **$9.99 \times 10^{22}$  atoms/cm<sup>3</sup>**.

<b>232Pa</b> 1.31 d $\beta^-$ =100%	<b>233Pa</b> 26.975 d $\beta^-$ =100%	<b>234Pa</b> 6.671 h $\beta^-$ =100%
<b>231Th</b> 25.52 h $\beta^-$ =100%	<b>232Th</b> 1.41e+10 y 99.98% $\alpha$ =100% $SF=1.1e-9%$	<b>233Th</b> 21.83 min $\beta^-$ =100%
<b>230Ac</b> 122 s $\beta^-$ =100% $B-f=1.2e-6%$	<b>231Ac</b> 7.5 min $\beta^-$ =100%	<b>232Ac</b> 119 s $\beta^-$ =100%

Based on the density of Cu, and the level of 1 ppb, the number of Th atoms present in copper is **9.99x10<sup>13</sup> atoms/cm<sup>3</sup>/ppb**.

Then, we can calculate the number of decays in a certain time from the radioactivity equation.

$$N_d = N_0 - N$$

$N_d$  is the number of deays,  $N_0$  is the original number of Th, and  $N$  is the current number of Th. The radioactive law says:

$$N = N_0 \exp\left(-\frac{\ln(2)t}{\tau_{1/2}}\right)$$

Or, putting it together:

$$N_d = N_0 \left(1 - \exp\left(-\frac{\ln(2)t}{\tau_{1/2}}\right)\right)$$

$^{232}\text{Pa}$ 1.31 d $\beta^- = 100\%$	$^{233}\text{Pa}$ 26.975 d $\beta^- = 100\%$	$^{234}\text{Pa}$ 6.671 h $\beta^- = 100\%$
$^{231}\text{Th}$ 25.52 h $\beta^- = 100\%$	$^{232}\text{Th}$ 1.41e+10 y 99.98% $\alpha = 100\%$ $SF = 1.1e-9\%$	$^{233}\text{Th}$ 21.83 min $\beta^- = 100\%$
$^{230}\text{Ac}$ 122 s $\beta^- = 100\%$ $B-f = 1.2e-6\%$	$^{231}\text{Ac}$ 7.5 min $\beta^- = 100\%$	$^{232}\text{Ac}$ 119 s $\beta^- = 100\%$

For 1 yr the exponential factor in the parentheses is  $4.92 \times 10^{-11}$ . That gives **4,911 decays /year/cm<sup>3</sup>/ppb**.

Converting to per second gives:  **$1.56 \times 10^{-4}$  decays /s/cm<sup>3</sup>/ppb**

So, we can apply it to the equation:

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Number of decays /s/cm<sup>3</sup>/1ppb

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Number of produced neutrons /s/cm<sup>3</sup>/1ppb

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**$1.56 \times 10^{-4}$  decays /s/cm<sup>3</sup>/ppb**

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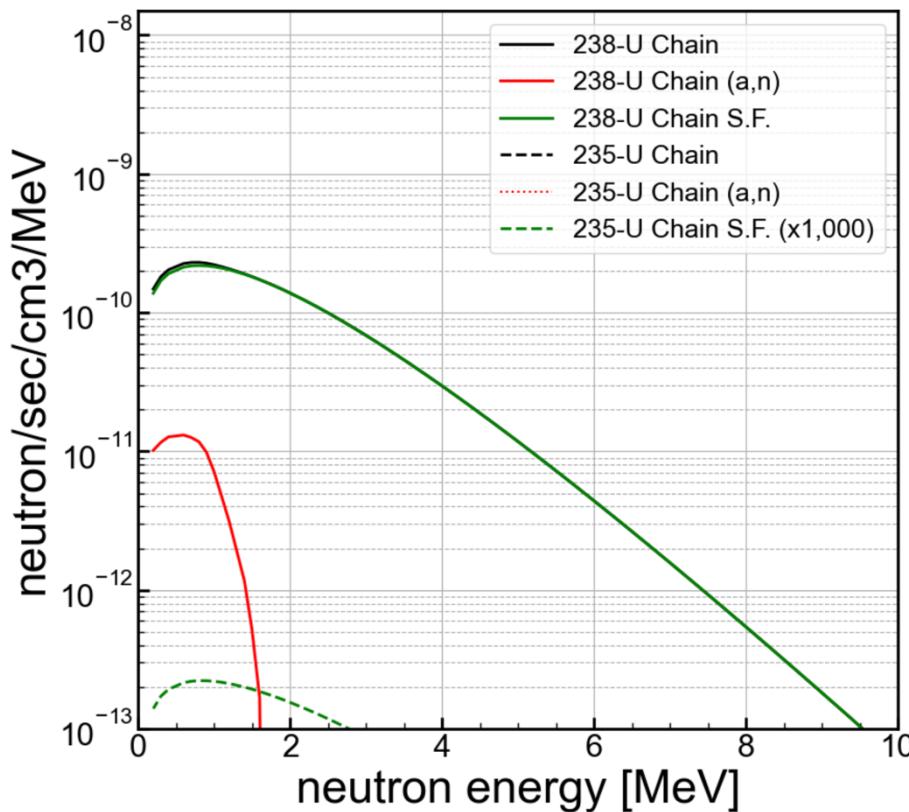
**$2.85 \times 10^{-11}$  neutrons/s/cm<sup>3</sup>/ppb**

And get: **5,403,509 decays/neutron**

$^{238}\text{Np}$ 2.10 d $\beta^-$ =100%	$^{239}\text{Np}$ 2.36 d $\beta^-$ =100%	$^{240}\text{Np}$ 61.9 min $\beta^-$ =100%
$^{237}\text{U}$ 6.752 d $\beta^-$ =100%	$^{238}\text{U}$ 4.46e+9 y 99.2742% $\alpha$ =100% SF=5.44e-5% $2\beta^-$ =2.2e-10%	$^{239}\text{U}$ 23.45 min $\beta^-$ =100%
$^{236}\text{Pa}$ 9.1 min $\beta^-$ =100%	$^{237}\text{Pa}$ 8.7 min $\beta^-$ =100%	$^{238}\text{Pa}$ 2.3 min $\beta^-$ =100%

Spontaneous fission is also a contributor to neutrons. Th has very little S.F. but  $^{238}\text{U}$  has a lot more (see left, branching of  $5.4 \times 10^{-5} \%$  compare with  $\sim 10^{-9} \%$  for Th).

In fact, our radiogenics are probably dominated by spontaneous fission of  $^{238}\text{U}$ . This could be simulated natively in SuperSim as well, if G4 has accurate S.F. simulation. I want to estimate how many decays we need for this one.



Integrating this spectrum for  $^{238}\text{U}$  S.F., I get a total volumetric source rate of  **$5.37 \times 10^{-10}$  neutrons/s/cm<sup>3</sup>/ppb**.

The same calculation as before: based on the density of Cu, and the level of 1 ppb, the number of  $^{238}\text{U}$  atoms present in copper is  **$9.99 \times 10^{13}$  atoms/cm<sup>3</sup>/ppb**.

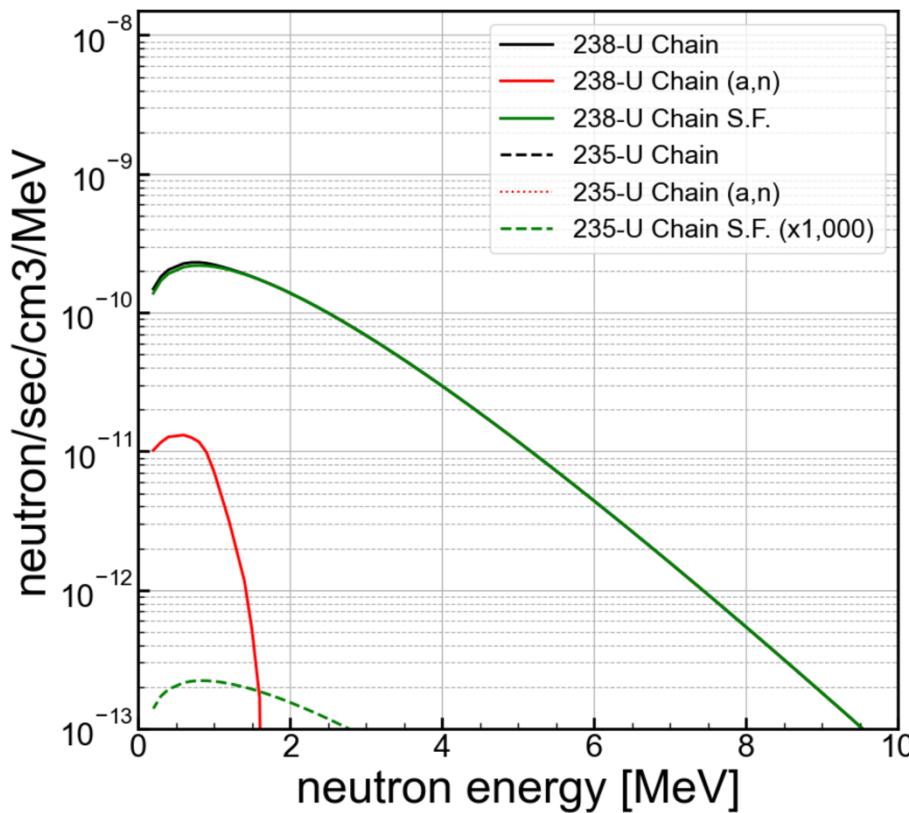
But now we got to use the halflife of  $^{238}\text{U}$ .

$$N_d = N_0 \left( 1 - \exp \left( -\frac{\ln(2)t}{\tau_{1/2}} \right) \right)$$

For 1 yr the exponential factor in the parentheses is  **$1.55 \times 10^{-10}$** .

That gives 15,526 decays /yr/cm<sup>3</sup>/ppb, but S.F. is only  $5.44 \times 10^{-5}$  % , so that is 8.4 S.F. /yr/cm<sup>3</sup>/ppb. Or:

$$\mathbf{2.68 \times 10^{-10} \text{ S.F. /s/cm}^3/\text{ppb}}$$



So, we can apply it to the equation:

$$\frac{\text{Number of S.F. /s/cm}^3/\text{1 ppb}}{\text{Number of produced neutrons /s/cm}^3/\text{1 ppb}}$$

$$2.68 \times 10^{-10} \text{ S.F. /s/cm}^3/\text{ppb}$$

$$5.37 \times 10^{-10} \text{ neutrons/s/cm}^3/\text{ppb}$$

And get: **0.499 S.F./neutron** or around **2.0 neutrons/S.F.**

This number is much more feasible than the (a,n), and it roughly aligns with how many neutrons expected from fission. We might in this case worry, however, that S.F. in G4 is modelled correctly—especially the correlations in neutrons emitted.