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Bayesian joint modelling of life expectancy and healthy life expectancy and valuation of retirement village contract

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ABSTRACT

Life expectancy and healthy life expectancy are both important indicators of population wellbeing, reflecting the expected length of life and duraiton of healthy living respectively. While extensive research has focused on projecting life expectancy, very few have investigated the projection and simulation of healthy life expectancy and examined its potential applications. In this paper, we establish a Bayesian approach to jointly model life expectancy and healthy life expectancy. We apply this approach to the population data of various countries, demonstrating its ability to provide accurate forecasts, generate probability intervals to capture future uncertainty, and estimate missing data values. We then illustrate an application of the Bayesian model in valuing a retirement village contract. Many Australian seniors choose retirement villages for the community support they offer, transitioning to aged care facilities as their health declines. The value of the accommodation service depends on the resident's length of stay, which can be approximated by healthy life expectancy. The proposed Bayesian model can produce a sampled distribution of the contract's net present value, providing a useful risk management tool for retirement village operators. Our findings emphasise the importance of considering healthy life expectancy in policies on public health and retirement.

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Healthy life expectancy; life expectancy; retirement village; Bayesian analysis; retirement age

1. Introduction

It is a worldwide phenomenon that human life expectancy continues to rise. For instance, female (period) life expectancies at birth in the UK and Australia have increased from 76.6 and 78.2 in 1980 to 83.3 and 85.4 in 2019 respectively (Figure 1). For many decades, this measure has widely been used as an estimate of the expected length of life and so an indicator of the overall wellbeing of a population. Recently, there has been a rising concern over the health-related quality of life, rather than just the quantity of life. A natural question is: does higher life expectancy come with more years in good health? The answer to this question requires looking beyond the usual mortality patterns into the healthy and unhealthy states (e.g. Majer et al. 2013).

A useful alternative measure is the so-called healthy life expectancy. Broadly speaking, it refers to a range of population health indicators that are based on both mortality and morbidity information. They have been designed to estimate how long an individual is expected to live and also remain healthy. Some examples include health expectancy (Nusselder & Looman 2004), health-adjusted life expectancy (Australian Institute of Health and Welfare 2017), disability-free life expectancy (Jagger

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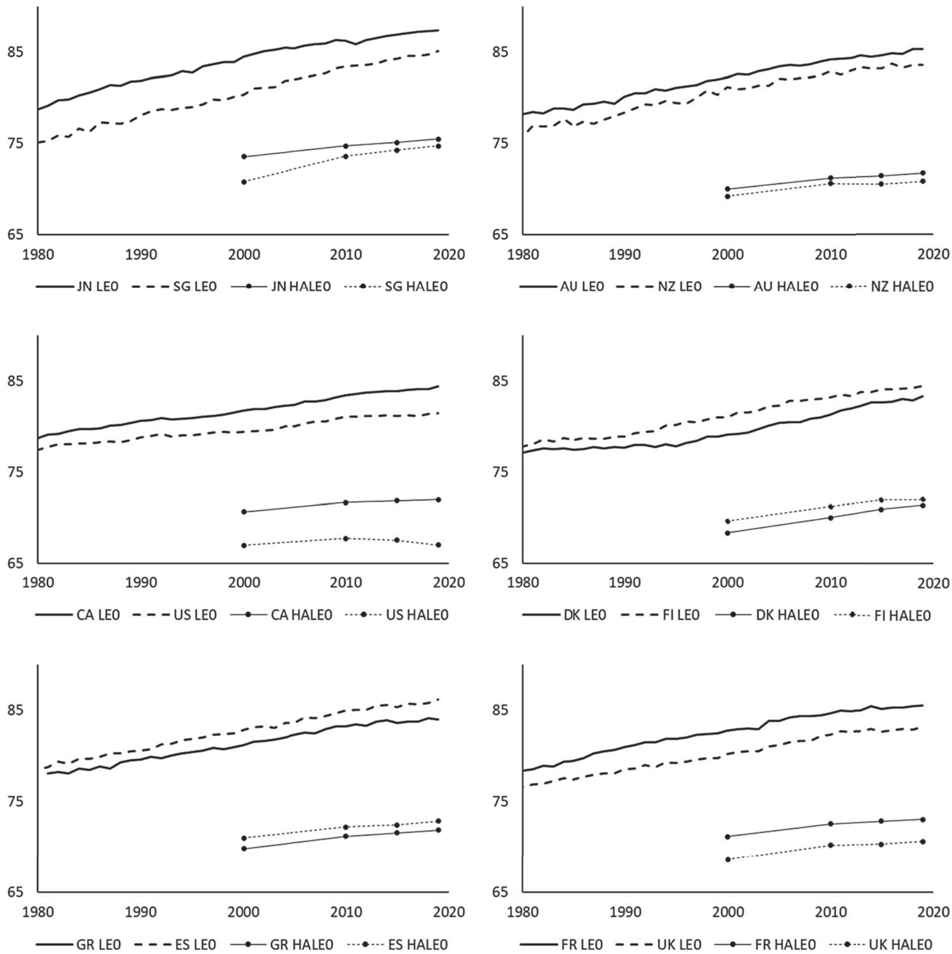

Figure 1. Female life expectancies at birth (LE0) and healthy life expectancies at birth (HALE0) of 12 countries from 1980 to 2019.

et al. 2016), and healthy working life expectancy (Lynch et al. 2022). It is useful to complement the analysis of life expectancy with that of healthy life expectancy. In general, there are three scenarios of their relationship (Jivraj et al. 2020). The first case is an 'expansion of morbidity', where life expectancy increases faster than healthy life expectancy does if there is no postponement of morbidity or no recovery from poor health. The second is a 'compression of morbidity', in which the pace of the starting age of morbidity being shifted to older ages is faster than the improvement rate in life expectancy, resulting in shorter duration in morbidity. Finally, a 'dynamic equilibrium' means that there is a mix of the two cases above, where the prevalence rate increases with more years in mild or moderate disability but fewer years in severe disability. There is also a subtle difference between 'absolute' and 'relative' compression / expansion of morbidity. The former refers to the absolute number of years while the latter is expressed as a proportion of life expectancy. The future gap between life expectancy and healthy life expectancy has significant implications for government policies on public health and retirement system.

As the population ageing problem aggravates, governments and industry practitioners are actively seeking feasible solutions to address the funding issues arising from increasing longevity. A senior person faces not only the longevity risk of exhausting his or her financial resources, but also other personal risks such as feeling lonely and suffering from social isolation. The latest ideas range from the reverse mortgages in the US and UK (Institute and Faculty of Actuaries 2005), the social integration of seniors in Singapore (Civil Service College 2019a), to the creation of 'care-bots' in Japan (Smith 2021). Retirement village is an attractive retirement solution with increasing demand in countries like Australia, Singapore, and the UK (Bernard et al. 2012, Crisp et al. 2013, Civil Service College 2019b). More and more seniors seek this kind of 'age-friendly' accommodation, while still in relatively good health. Retirement villages are purpose-built residential complexes where reasonably healthy retirees of a similar age can live as neighbours in a community (Hu et al. 2017), and they are very different to nursing homes or health care facilities. The common reasons for entering into such a contract include the social trend of living separately from the extended family, more support from the other residents, being 'asset rich, cash poor', and inadequate financial resources for retirement. It is designed for those who are capable of living independently. The contract will terminate and the resident will move out into an aged care facility when the person's health declines beyond a certain level. For instance, the Australian retirement village market size growth is about 3% per annum over the period of 2017–2022 (IBISWorld 2022). In the current market, there are different types of accommodation structures with a variety of services and facilities. Retirement villages are typically managed by financial institutions for profit, though a few are organised by religious and charity groups.

The value of a retirement village contract depends on the resident's length of stay, which is an unknown variable. We propose using healthy life expectancy as a proxy for the expected length of stay, as the traditional life expectancy measure does not contain morbidity information. Accordingly, we estimate the expected net present value and the corresponding distribution for a retirement village contract, based on the simulated values of healthy life expectancy from our Bayesian process. While many research papers have worked on the topic of forecasting life expectancy, very few indeed have investigated the projection and simulation of healthy life expectancy and explored its potential applications.[1] This problem is understandable, as health data are generally much less available than mortality data and the modelling required for dealing with multiple states of health is more complex. In this paper, we construct a pragmatic Bayesian approach for co-modelling life expectancy and healthy life expectancy. We recognise the historical relationships between the two measures and exploit them to project and simulate future outcomes, that is, taking an extrapolation approach. We apply the proposed method to the population data of a range of countries and find that it can generate decent forecasting performance, produce suitable probability intervals to describe the uncertainty of future outcomes, and also estimate the missing values in the data. The Bayesian model and results on healthy life expectancy would be useful for governments in planning health care policies and setting retirement age. Moreover, a viable retirement village operation requires appropriate modelling to project the cash flows and to manage the underlying risks. The proposed Bayesian model allows for systematic longevity risk and morbidity risk and can be used to produce a sampled distribution of the contract's net present value. It can be a useful addition to the risk management toolkit for retirement village operators.

The remainder of the paper is organised as follows. Section 2 reviews the life expectancy and healthy life expectancy trends of various countries and identifies their historical patterns. Section 3 sets forth the proposed Bayesian approach for modelling life expectancy and healthy life expectancy jointly, and analyses the results produced and the implications for policymakers. Section 4 demonstrates an application of the Bayesian model to the valuation of a benchmark retirement village contract and discusses the underlying risks. Section 5 gives the concluding remarks. The Appendix provides some more simulation details.

[1] Cao et al. (2020) applied multiple linear regression to project healthy life expectancy. Lynch et al. (2022) used a three-state model and the Lee-Carter model to project healthy working life expectancy. In this paper, we take a different approach and devise a Bayesian joint model for both life expectancy and healthy life expectancy, allowing for the underlying relationships between the two measures.

2. Life expectancy and healthy life expectancy

We obtain healthy life expectancy data from the WHO (World Health Organization)[2] for years 2000, 2010, 2015, and 2019, and life expectancy and mortality data from the HMD (Human Mortality Database 2022)[3] for the period of 1950–2019. The WHO data are not collected and disseminated every single year and so there are a lot of missing values regarding our modelling process. Following Tuljapurkar et al. (2000) and Lee & Miller (2001), the starting year of the HMD data period is chosen as 1950 in order to exclude past structural changes in mortality. We cover a number of geographic regions including Asia (Japan, Singapore), Australasia (Australia, New Zealand), North America (Canada, United States), Northern Europe (Denmark, Finland), Southern Europe (Greece, Spain), and Western Europe (France, UK).

The WHO adopts Sullivan's method, which

> uses the equivalent lost healthy year fraction (adjusted for comorbidity) at each age in the current population (for a given year) to divide the hypothetical years of life lived by a period life table cohort at different ages into years of equivalent full health and equivalent lost healthy years.

The equivalent lost healthy year fractions are computed as the 'all-cause years lost due to disability rate per capita, adjusted for independent comorbidity, by age, sex, and country'. It is the most popular method for estimating healthy life expectancy as it is prevalence-based and requires only cross-sectional data (Sullivan 1971). However, it does not allow for health state changes over time explicitly. A more precise method is to build a multiple-state model, but it needs longitudinal data, which usually have a limited sample size and are not widely available. It has been suggested that both methods would deliver similar results if transition rates are smooth and regular across time without abrupt changes (Mathers & Robine 1997).

Figure 1 shows the trends of female (period) life expectancies at birth and healthy life expectancies at birth of 12 countries. Generally speaking, all these expectancies have been rising steadily over the recent decades. It means that not only the number of years that a life is expected to live but also the duration that a life is expected to stay healthy increases across time. It is interesting to observe different patterns of convergence / divergence in different regions. The differences in life expectancy at birth reduce over time between Japan and Singapore, between Australia and New Zealand, and between Denmark and Finland, while the situation is reverse for Canada and the US, for Greece and Spain, and for France and the UK. In particular, Singapore female life expectancy at birth has been catching up at a fast pace, whereas US female life expectancy at birth is lagging increasingly more. On the other hand, the differences in healthy life expectancy at birth reduce over time for Japan and Singapore, for Denmark and Finland, for Greece and Spain, and for France and the UK, and the other regions demonstrate an opposite trend. Similarly, Singapore female healthy life expectancy at birth is improving relatively quickly, but US female healthy life expectancy at birth is gradually falling short of the others.

Table 1 provides the expected duration in morbidity (i.e. life expectancy minus healthy life expectancy) and also the healthy life expectancy value as a percentage of the corresponding life expectancy value for each sex and country. On average over the populations considered and the four years of data, the expected duration in morbidity of a newborn is 12.1 years for females and 9.2 years for males, and that of a life aged 60 is 6.5 years for females and 5.0 years for males. While females are expected to live longer than males do, they also spend more time in morbidity. There is an absolute expansion of morbidity, in which the average annual rate of increase in the expected duration in morbidity at birth (age 60) is 0.5% (0.8%) for females and 0.6% (1.0%) for males over the recent

[2] https://www.who.int/data/gho/data/indicators/indicator-details/GHO/gho-ghe-hale-healthy-life-expectancy-at-birth. https://www.who.int/data/gho/data/indicators/indicator-details/GHO/gho-ghe-hale-healthy-life-expectancy-at-age-60.

[3] In the current literature, the calculation of healthy life expectancy is typically based on data from multiple sources, as there is generally a lack of health data. Moreover, in order to obtain some annual data of different countries for our modelling process, we resort to using both the WHO and HMD datasets.

Table 1. Expected durations in morbidity and healthy life expectancy percentages at birth and at age 60 of females and males in 12 countries in 2000, 2010, 2015, and 2019.

Females	Expected Duration in Morbidity (Years)				Healthy Life Expectancy at Birth (%)			
Country	2000	2010	2015	2019	2000	2010	2015	2019
JN	11.0	11.5	11.9	12.0	87.0	86.6	86.4	86.3
SG	9.6	9.9	10.0	10.4	88.1	88.2	88.1	87.8
AU	12.3	13.0	13.3	13.6	85.0	84.5	84.3	84.0
NZ	12.0	12.4	12.7	12.8	85.2	85.1	84.7	84.7
CA	11.1	11.8	12.1	12.5	86.4	85.9	85.6	85.3
US	12.5	13.3	13.7	14.5	84.3	83.5	83.2	82.2
DK	10.8	11.3	11.8	12.0	86.4	86.1	85.8	85.6
FI	11.4	12.0	12.2	12.5	86.0	85.5	85.5	85.2
GR	11.4	12.1	12.1	12.1	86.0	85.5	85.6	85.6
ES	11.9	12.8	12.9	13.3	85.7	85.0	84.9	84.5
FR	11.7	12.1	12.3	12.6	85.9	85.7	85.5	85.3
UK	11.6	12.2	12.4	12.6	85.5	85.2	85.0	84.8
Males	**Expected Duration in Morbidity (Years)**				**Healthy Life Expectancy at Birth (%)**			
Country	2000	2010	2015	2019	2000	2010	2015	2019
JN	8.2	8.4	8.7	8.7	89.4	89.5	89.2	89.3
SG	7.9	8.1	8.2	9.0	89.6	89.7	89.7	88.9
AU	9.8	10.7	10.9	11.1	87.3	86.6	86.5	86.3
NZ	9.6	10.1	10.7	10.4	87.3	87.2	86.7	87.1
CA	8.5	9.2	9.5	9.8	88.9	88.4	88.2	87.8
US	9.5	10.6	10.8	11.3	87.1	86.2	85.9	85.3
DK	7.7	8.3	8.7	8.8	89.6	89.2	89.0	89.0
FI	8.4	8.9	9.1	9.3	88.7	88.4	88.4	88.3
GR	8.1	9.0	9.0	9.1	89.3	88.4	88.5	88.5
ES	8.8	9.2	9.2	9.6	88.4	88.4	88.4	88.2
FR	7.8	8.3	8.5	8.7	89.6	89.4	89.2	89.1
UK	8.9	9.7	10.0	9.9	88.2	87.7	87.3	87.5
Females	**Expected Duration in Morbidity (Years)**				**Healthy Life Expectancy at Age 60 (%)**			
Country	2000	2010	2015	2019	2000	2010	2015	2019
JN	6.3	6.9	7.2	7.3	76.5	75.6	74.9	74.9
SG	4.9	5.3	5.3	5.6	78.3	79.1	79.6	79.1
AU	6.5	7.1	7.4	7.7	74.0	73.0	72.4	72.0
NZ	6.2	6.5	6.8	6.9	74.3	74.4	73.7	73.4
CA	6.0	6.5	6.8	7.1	75.5	74.8	74.3	73.6
US	6.5	7.3	7.5	8.0	71.9	70.3	69.8	68.1
DK	5.4	5.8	6.2	6.4	75.7	75.6	74.9	74.7
FI	5.8	6.4	6.6	7.0	75.5	74.9	74.8	73.6
GR	5.8	6.4	6.4	6.5	75.4	75.0	75.2	75.1
ES	6.5	7.1	7.1	7.5	74.3	73.5	73.6	73.0
FR	6.1	6.5	6.8	7.0	76.0	75.9	75.2	74.8
UK	5.7	6.3	6.5	6.8	75.1	74.7	74.0	73.5
Males	**Expected Duration in Morbidity (Years)**				**Healthy Life Expectancy at Age 60 (%)**			
Country	2000	2010	2015	2019	2000	2010	2015	2019
JN	4.7	4.9	5.1	5.1	78.0	78.5	78.2	78.6
SG	4.2	4.4	4.5	4.8	78.7	80.0	80.5	79.7
AU	5.0	5.7	6.0	6.2	76.4	75.3	74.9	74.5
NZ	4.8	5.3	5.7	5.7	76.5	76.7	75.6	75.9
CA	4.5	5.1	5.3	5.6	78.0	77.6	77.3	76.5
US	5.0	5.9	6.1	6.4	74.8	72.4	72.2	70.8
DK	3.8	4.3	4.6	4.7	79.7	79.1	78.8	79.0
FI	4.3	4.8	5.0	5.3	77.4	77.3	77.4	76.8
GR	4.3	4.9	4.9	5.2	78.5	77.5	77.6	76.9
ES	4.7	5.2	5.2	5.5	76.9	76.9	77.2	76.5
FR	4.1	4.5	4.8	5.0	80.0	79.9	79.1	78.8
UK	4.3	5.0	5.3	5.4	77.6	77.2	76.4	76.6

Table 2. Selected autoregressive orders p for females and males in 12 countries.

Selected p	Females		Males	
Country	Age 0	Age 60	Age 0	Age 60
Japan	3	2	3	2
Singapore	1	1	1	2
Australia	1	1	1	4
New Zealand	1	1	1	3
Canada	1	1	1	2
United States	1	1	1	3
Denmark	1	1	1	2
Finland	1	1	1	3
Greece	1	1	1	1
Spain	1	2	1	2
France	1	1	1	1
United Kingdom	1	2	1	5

two decades. These observations have significant implications for a country's welfare and health care policies, as increasing time in morbidity would lead to aggravating financial burden on the society.

In line with the differences between both sexes as discussed above, the average healthy life expectancy percentage of a newborn is 85.5% for females and 88.2% for males, and that of a life aged 60 is 74.5% for females and 77.2% for males. Females are expected to spend a higher proportion of lifetime in morbidity. There is also a relative expansion of morbidity generally (except for Singapore females and males at age 60 and Japan males at age 60), where the average healthy life expectancy percentage at birth (age 60) decreases by 0.9% (1.4%) for females and by 0.7% (1.0%) for males from 2000 to 2019.

It can be observed in Figure 1 that the increasing life expectancy trends are persistent and quite steady for different countries. Torri & Vaupel (2012) forecasted the so-called best-practice life expectancy using the ARIMA model.[4] Pascariu et al. (2018) used the ARIMA model to forecast the gap between the best-practice life expectancy and the country-specific trend. In a similar vein, we adopt the ARIMA (p, 1, 0) structure under the Bayesian framework to model the life expectancy trends (see Section 3). Rosenberg & Young (1999) suggested that it is more convenient to incorporate the AR models, rather than the MA models, into a Bayesian setting. Moreover, when an adequate autoregressive order is chosen, the resulting effects are comparable to those from a given ARIMA model. Accordingly, based on the usual Box–Jenkins approach and the corresponding sample autocorrelations and partial autocorrelations, Table 2 lists the selected autoregressive order p for each sex and country. It appears that Japan life expectancies and generally male life expectances at age 60 tend to involve more autoregressive effects over the data period of 1950–2019.

As noted earlier, the healthy life expectancy percentages tend to decline over time. As a reference, Pascariu et al. (2018) assumed a linear model for the decreasing best-practice life expectancy trend. Considering these declining trends of different countries from 2000 to 2019 and the limited number of data points, we also adopt a linear model structure with time as the main covariate to describe the healthy life expectancy percentages, but under the Bayesian framework (see Section 3). Given the data limitations, the resulting average R squared values of 0.85 (females) and 0.72 (males) are actually quite high and suggest that the linear model structure captures the declining trends reasonably well, in which the model fitting appears to be better for females overall.

[4] Torri and Vaupel (2012) chose ARIMA(2, 1, 1) and ARIMA(1, 1, 1) for the female and male best-practice life expectancies at birth. These two models are more extended than those selected in Table 1 (except for Japan), probably because their data period of 1900–2006 is much longer and the best-practice life expectancy is composed of life expectancy values of different best performers over the period of more than a hundred years. Their data patterns are likely to be more complex and require higher ARIMA orders.

3. Bayesian joint modelling

We propose the following joint model structure for both measures of life expectancy and healthy life expectancy. First, the ARIMA (p, 1, 0) structure for life expectancy is given below:

$$\Delta le_t = \varphi_0 + \sum_{i=1}^{p} \varphi_i \Delta le_{t-i} + \varepsilon_t,$$

where Δ is the first difference operator, le_t is the life expectancy value in year t, φ_i's are the autoregressive parameters, and $\varepsilon_t \sim \text{Normal}(0, \sigma_\varepsilon^2)$ is the error term in year t. Then, the linear model structure for healthy life expectancy is specified as follows:

$$r_t = \frac{hale_t}{le_t} = \alpha_0 + \alpha_1 t + \omega_t,$$

in which $hale_t$ is the healthy life expectancy value in year t, r_t is the ratio of $hale_t$ to le_t, α_0 and α_1 are the linear model parameters, and $\omega_t \sim \text{Normal}(0, \sigma_\omega^2)$ is the error term in year t. The prior distribution for the unknown autoregressive and linear parameters is assumed as multivariate normal, in which (as for the hyperprior distributions) the mean vector is treated as following multivariate normal and the inverse covariance matrix is treated as Wishart. The inverse covariance matrix of the two error terms is also assumed as having a Wishart prior.

As pointed out in Li (2014), there are several advantages in working under the Bayesian framework.[5] First, prior information can readily be fed into the modelling process. In this work, the estimated results of all the countries are used to set the parameters of the hyperprior distributions for a particular country.[6] In this way, the information content can be enhanced via 'borrowing' extra information from the other countries, and the problem of limited data size can be alleviated. Second, the two model structures specified above can be integrated coherently within the same framework. The parameters of both structures are computed in the same Bayesian simulation process, rather than via two separate steps of estimation, to avoid any estimation bias. Third, the missing values in the healthy life expectancy data are imputed automatically during the simulation procedure. This output can provide an approximation of the healthy life expectancy trend over a longer time period. Moreover, in terms of the application in valuations (see Section 4), this Bayesian approach allows for both process error and parameter error and generates probability intervals for future outcomes. Both types of uncertainties should be properly assessed in order to reflect the underlying level of risks adequately. We use the software WinBUGS (Spiegelhalter et al. 2003) to perform the Bayesian MCMC (Markov chain Monte Carlo) simulations for the proposed joint model structure.

Figure 2 demonstrates the observed / imputed (posterior means) and projected (predictive means) trends of life expectancies and healthy life expectancies at birth for both sexes in six of the countries under study, with the corresponding 95% prediction intervals. It can be seen that all the life expectancy and healthy life expectancy values are projected to continue to rise for the period till 2050. Based on these simulated results, a life is expected to live and stay healthy for a longer time in the foreseeable future. For all the populations considered, the average female (male) healthy life expectancy values at birth observed are 70.0, 71.4, 71.7, and 72.0 (67.1, 69.1, 69.8, and 70.2) in 2000, 2010, 2015, and 2019, respectively. The average projected values in 2030 and 2050 are 73.6 and 76.5 (72.1 and 75.3). Back in 1960 and 1980 (where healthy life expectancy data are not available), the

[5] The posterior distribution of the unknown parameters is deduced from $f(\theta|D) \propto f(D|\theta) f(\theta)$. The predictive distribution of the future variables is deduced from $f(x|D) = \int f(x|\theta) f(\theta|D) \, d\theta$.

[6] For the multivariate normal hyperprior, the mean and variance are taken as the sample mean and sample variance (divided by the number of countries involved) of the model parameters estimated across different countries. For the Wishart hyperprior, the matrix parameter is taken as the sample covariance (times the degree of freedom) between the estimated model parameters, and the degree of freedom is set equal to the number of model parameters plus two such that the Wishart variance is large while the Wishart mean still exists. Similarly, for the Wishart prior for the two error terms, the degree of freedom is set as four so that the mean exists and the variance is large.

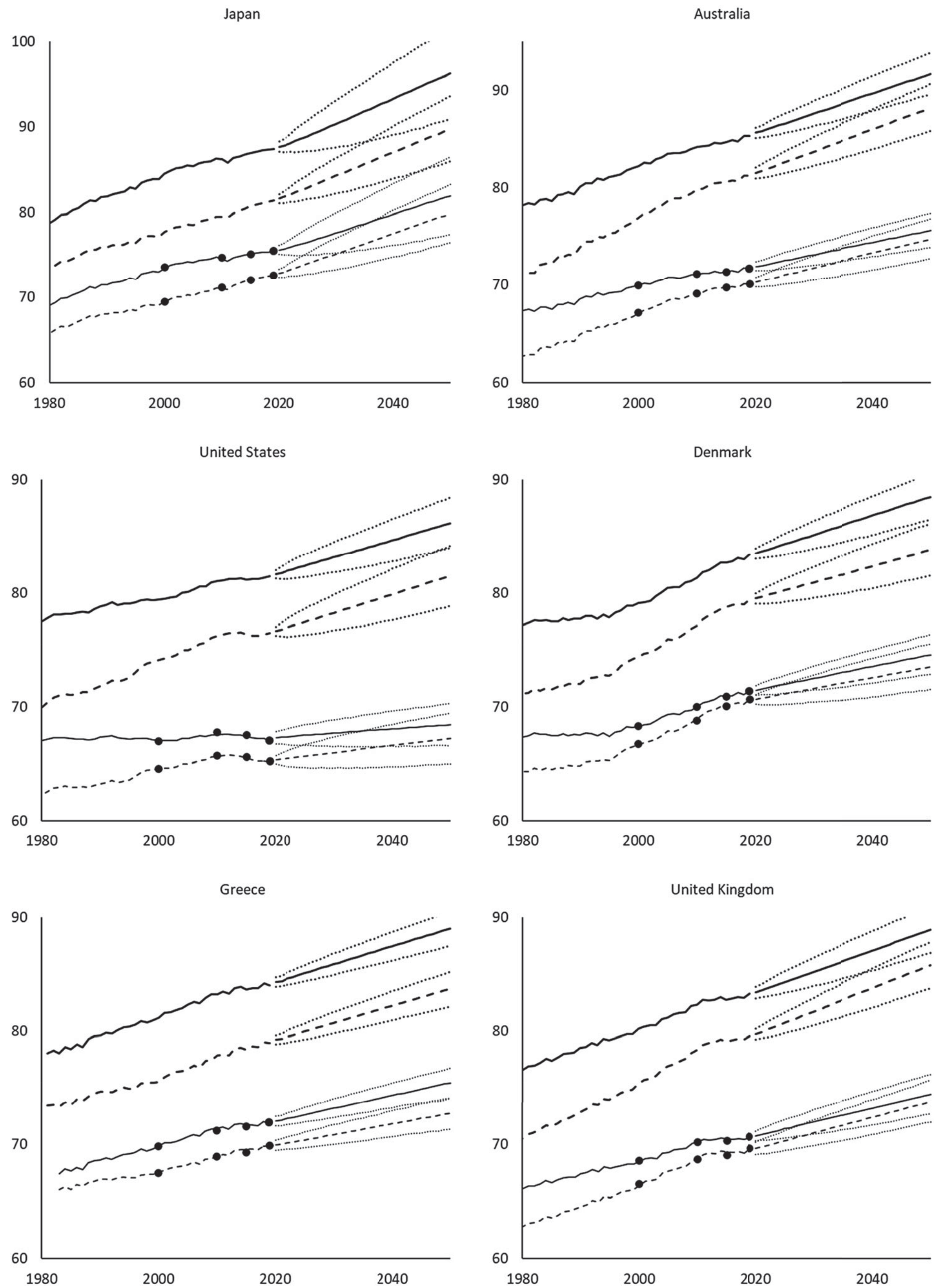

Figure 2. Life expectancies at birth (top two, thicker lines) and healthy life expectancies at birth (bottom two, thinner lines) for females (solid lines) and males (dashed lines) in six countries. The data period is from 1980 to 2019 and the projected values cover 2020–2050, accompanied with the corresponding 95% prediction intervals (dotted lines).

Note: The four *observed* healthy life expectancy values of each sex and country in 2000, 2010, 2015, and 2019 are shown as separate round dots, while the *imputed* healthy life expectancy values from 1980 to 2019 are plotted along the four observed values as solid / dashed lines (bottom two, thinner lines).

average imputed values are 64.1 and 67.4 (60.6 and 63.3). It is also noted that the average gap between female and male healthy life expectancies at birth decreases from around 3–4 years before 2000 to 2–3 years in the recent two decades, and that this gap is projected to reduce further to about 1–2 years in

Table 3. Results of out-of-sample analysis on healthy life expectancies at birth and at age 60 in 2019 for both sexes.

Out-of-Sample	Females		Males	
Results	Age 0	Age 60	Age 0	Age 60
	Bayesian Joint Model			
MAPE	0.0019	0.0044	0.0026	0.0052
MAE	0.1317	0.0858	0.1858	0.0933
MSE	0.0413	0.0138	0.0645	0.0175
	Random Walk with Drift			
MAPE	0.0033	0.0084	0.0049	0.0107
MAE	0.2381	0.1614	0.3451	0.1878
MSE	0.0986	0.0425	0.1654	0.0515

Table 4. Bayesian 95% prediction intervals (in brackets) and observed values of healthy life expectancies (in years) at birth and at age 60 of females and males in 12 countries in 2019.

	Females		Males	
Country	Age 0	Age 60	Age 0	Age 60
JN	75.48 (74.99, 76.02)	21.85 (21.49, 22.20)	72.64 (72.27, 73.25)	18.82 (18.49, 19.17)
SG	74.72 (74.53, 75.49)	21.05 (20.86, 21.61)	72.39 (72.38, 73.64)	18.81 (18.68, 19.53)
AU	71.73 (71.36, 72.28)	19.71 (19.36, 19.97)	70.15 (69.69, 70.68)	18.25 (17.88, 18.48)
NZ	70.82 (70.28, 71.59)	19.17 (18.73, 19.64)	69.64 (68.94, 70.27)	17.91 (17.51, 18.32)
CA	71.98 (71.69, 72.33)	19.72 (19.57, 19.96)	70.51 (70.19, 70.82)	18.21 (18.08, 18.42)
US	67.02 (67.25, 67.91)	17.06 (17.11, 17.51)	65.21 (65.05, 65.78)	15.59 (15.46, 15.88)
DK	71.38 (70.86, 71.72)	18.86 (18.50, 19.16)	70.67 (69.88, 70.74)	17.56 (17.04, 17.64)
FI	72.03 (71.68, 72.69)	19.50 (19.38, 19.98)	69.93 (69.32, 70.47)	17.34 (17.03, 17.59)
GR	71.87 (71.50, 72.24)	19.50 (19.25, 19.82)	69.86 (69.26, 70.01)	17.32 (17.13, 17.64)
ES	72.85 (71.96, 73.70)	20.33 (19.91, 20.68)	71.26 (70.41, 72.41)	17.98 (17.53, 18.39)
FR	73.06 (72.74, 73.78)	20.82 (20.58, 21.36)	71.06 (70.62, 71.67)	18.48 (18.13, 18.91)
UK	70.62 (70.17, 71.07)	18.85 (18.52, 19.10)	69.64 (68.84, 69.67)	17.62 (17.14, 17.66)

the next few decades. The proposed Bayesian approach exploits the annual life expectancy data available and the historical relationships between life expectancy and healthy life expectancy, producing a reasonable estimation of the healthy life expectancy trend over a long period of time, despite the data limitations.

In 1960 and 1980, the average imputed female (male) healthy life expectancy values at age 60 are 15.0 and 16.7 (12.7 and 13.5). In 2000, 2010, 2015, and 2019, the average observed values are, respectively, 18.1, 19.3, 19.5, and 19.7 (15.6, 17.1, 17.5, and 17.8). In 2030 and 2050, the average projected values are 20.6 and 22.1 (18.7 and 20.4). There is some small decrease in the average gap between female and male healthy life expectancies at age 60 over time.

As noted in Hammond et al. (2016), many of the OECD countries are increasing the retirement age for their populations to 67 or above. Some countries have further linked future retirement age increases to life expectancy changes. We deem that, however, considering life expectancy alone is inadequate. One should instead put more focus on estimating how long a life is expected to be able to continue working with the same, full capacity. In this regard, healthy life expectancy can be a useful indicator for the possible time of staying in the labour force. For instance, the current UK retirement age is 66, which will rise to 67 by 2028, but based on our Bayesian estimation, the projected increase in UK healthy life expectancy from 2022 to 2028 is only 0.8 years. This comparison suggests that after the revision by 2028, more people may not be able to work until the (new) retirement age when compared to the past. There are also other complications, such as the ability to work in the same job at older ages differing between white-collar, blue-collar, and first responder (police officers, paramedics, firefighters, etc) jobs. Those workgroups who are unable to meet the physical demands of their jobs at older ages would find it hard to stay in the labour force until the new retirement age and so they would face a higher risk of having low pensions and insufficient funds. While rising longevity has long been taken as the main consideration for increasing the retirement age, the ability to work at older ages should actually be set as the key factor and be assessed thoroughly.

We now leave the year 2019 healthy life expectancy data out and perform an out-of-sample analysis. That is, we apply the proposed Bayesian model to the data before 2019 to produce forecasts for 2019 and then compare the forecasted and observed values. Table 3 presents the resulting MAPE (Mean Absolute Percentage Error), MAE (Mean Absolute Error), and MSE (Mean Squared Error) values from the Bayesian joint model. As a comparison, we also obtain similar forecasts from applying the random walk with drift to only the healthy life expectancy data. The Bayesian joint model clearly outperforms the random walk with drift for both sexes and both ages under all the three error measures, highlighting again the advantage of making use of the life expectancy data availability and the historical relationships between life expectancy and healthy life expectancy. The general forecasting performance is better for females than for males, probably because mortality / morbidity rates have varied more for males in recent years. Moreover, the relative forecasting errors (i.e. MAPE) are larger for age 60 than for age 0, as old-age morbidity / mortality patterns are often more volatile than those at younger ages. Table 4 gives the corresponding 95% prediction intervals for the Bayesian forecasts. For 46 of the 48 cases, the prediction intervals cover the actual values, that is, the overall percentage of observed values falling within the prediction intervals is 95.8%, which is in line with the prescribed level of probability of the intervals. The Bayesian simulation process provides a useful and reasonable estimation of the distribution of the variable of interest here.

4. Retirement village: contract description and valuation

Retirement villages have gradually become a feasible retirement solution in a number of countries. In Australia, they are intended for those seniors who are capable of living independently while enjoying the benefits of community living (Kyng et al. 2021). The minimum entry age is usually 55, and most residents are aged 70 and older. Many Australian seniors are 'asset rich, cash poor' and would find retirement villages a good solution to address their financial concerns. There are various kinds of accommodation structures with different types of services and facilities. They are specifically designed as being age-friendly, in which site maintenance and landscaping are managed by the operators. Younger retirees would be more attracted to the retirement communities that provide opportunities for social interactions and activities and fitness facilities. Older retirees would be attracted to the support from peers, health services, and safety arrangements. However, retirement villages are not suitable for those who have a serious need for a high level of care – they are not aged care centres or nursing homes. Depending on the contract, a resident should leave the retirement village and move out into an aged care facility when the person can no longer perform certain daily activities. While in some cases there may be an associated aged care facility, one is required to end the existing retirement village contract and enter into a new, separate contract with the aged care facility to obtain its access. Retirement villages generally do not offer full-time health care support, though the residents can purchase and bring in their own health care services. In 2022, the Australian retirement village market size is estimated to be (AUD) $4.7 billion (IBISWorld 2022). There are currently almost 200,000 Australian seniors staying in retirement villages.

In Singapore, Kampung Admiralty is the first integrated vertical kampung with elderly housing and facilities under one roof (Civil Service College 2019b). It takes a whole-of-government approach to manage the development. It incorporates health care facilities such that medical staff and volunteers can work actively with the residents, especially on the management of chronic illness. It is designed to be barrier-free and wheelchair-friendly, with good connectivity and ample rest-stops. There are plenty of communal spaces for social interactions and community activities and events.

Retirement villages are fairly new to the UK unlike Australia and are often confused with aged care homes. Being specifically built for those who are aged 55 and over, they are designed to support independent living and good health and wellbeing for as long as possible (Davey 2021, Green 2022).

Some choose to move in retirement villages because they want to downsize their homes and to avoid the trouble of home maintenance. Others may feel safer living in a community than living alone. The entry price can range from around £200,000 to more than £1 million. The UK retirement villages provide a wide range of services, facilities, and activities. The accommodation structures include houses, flats, and apartments. There tend to be more than 100 homes in each retirement village. Traditionally, retirement villages have provided limited health care to the residents. Recently, retirement village operators are moving towards more flexible structures and arrangements, making aged care accessible within or close to the retirement village setting. These newer setups provide the option to pay and upgrade the level of care when needs arise. The residents are required to pay monthly or annual management fees for maintaining the communal areas and facilities. The management fee can range from about £500 to more than £1,000 per month, depending on the operator, location, and facilities. There are also exit fees to be paid when the resident moves into full-time health care or dies and the property is sold. Some charge low rates at 1% to 3% of the resale price, while others have fees at 10% to 15%. For some cases, the fee depends on how long the resident have stayed in the accommodation and can be as high as 30% of the property value. Despite retirement village being a relatively new concept in the UK, the market is growing rapidly, as more and more seniors realise this option and choose to live their retirement years independently in custom-built properties with community life and age-friendly facilities, free of the duty of site maintenance.

Retirement village contracts are many and varied and have a complex financial structure. While there are some previous papers in the literature examining the qualitative aspects of retirement villages, very few have indeed investigated the quantitative, financial side of retirement village contracts. The authors of Kyng et al. (2021) have collected and analysed a large sample of such contracts offered in the Australian market, under the funded project of designing and developing the Retirement Village Cost Calculator.[7] Kyng et al. (2021) estimated the expected term of residence using mortality and health transition data and calculated a rent metric for consumers to use, based on deterministic assumptions on the interest rate and inflation rate. In this section, we study this kind of financial contract from an Australian operator's perspective and adopt a stochastic (Bayesian) approach instead to estimate the distribution of its net present value. We use healthy life expectancy as a proxy for the length of stay, allowing for systematic longevity risk and morbidity risk.

Kyng et al. (2021) summarised the key financial features of the various types of retirement village contracts. First, the customer pays an *entry fee* in the form of a lump sum before using the accommodation. This payment gives the person the right to live in the retirement village, in which the right terminates when the resident passes away, becomes unable to live independently, or moves to another accommodation for other reasons. Many residents use part of the sale proceeds of their original family homes to make this payment. It is usually lower than the cost of a similar non-retirement property in the same suburb. In 2019, the entry fee for a two-bedroom unit is (AUD) \$459,000 on average, and the national median house price across the capital cities in Australia is \$638,000.

Second, during the stay, the resident pays an ongoing *maintenance fee*, which covers the costs of general services, maintenance of accommodation and facilities, and management and staff. This fee is not intended for making profit. In 2019, the maintenance fee is \$536 per month on average. Moreover, this fee may be subject to further increases based on the inflation rate.

On exiting the retirement village, the departing resident receives a refund of the entry fee, less any exit fees, from the operator. One type of exit fee is called the *deferred management fee*. It is usually calculated as a percentage of the entry fee multiplied by the length of stay, the total amount of which is often subject to an upper limit. For many retirement villages, this upper limit is about 30%, and it would take around 5–10 years to reach the cap. The deferred management fee is one main source of profit. Moreover, under some contracts, the departing resident may receive / pay a prespecified percentage of the capital gain / loss, which is the difference between the entry fee paid by the next

[7] https://rvcalculator.mq.edu.au/#/.

incoming resident and the entry fee paid initially by the departing resident. There may also be other renovation and refurbishment costs.

In effect, a retirement village contract combines the concepts of life annuity, life insurance, and financial option on real estate. The ongoing maintenance fees are like a life annuity structure with increasing payment size; the final refund, less the exit fees, is like a life / health insurance with varying sum insured; and the sharing of capital gain / loss is like an option on housing property. The payments are triggered by the underlying stochastic processes governing the morbidity and mortality levels and the house prices. The proposed Bayesian framework is very suitable for modelling the stochastic nature of these payments and providing an estimation of the contract cash flows.

Based on the industry findings and observations as noted above, we construct and valuate a hypothetical retirement village contract in the following as a benchmark case for demonstration. Suppose the entry fee is \$500,000 for a new retirement village contract for a person aged 60 in 2023. The ongoing maintenance fee is \$2,000 (subject to quarterly adjustments based on the consumer price index) payable at the end of each quarter during the residence. The deferred management fee is 6% per annum, with an upper limit of 30% on the total fee. The resident is not involved in any capital gain / loss sharing. The net refund (i.e. entry fee minus deferred management fee plus capital gain shared) is paid at the end of the residence. It is assumed that the exit occurs at the quarter end and there are no other renovation and refurbishment costs. Suppose the market value of a similar non-retirement dwelling is \$750,000.[8]

There are no publicly available demographic studies on Australian retirement village residents, so we rely on the WHO and HMD data used in Section 2. Incorporating both process error and parameter error, the Bayesian MCMC process simulates the predictive distribution of healthy life expectancy at age 60, which serves as a proxy for the resident's length of stay. We allow for only involuntary exits due to disability or death, but not voluntary exits due to financial or other reasons. We assume that the retirement village portfolio of the operator is homogeneous and is sufficiently large such that sampling error due to individual uncertainty (i.e. non-systematic longevity and morbidity risks) is negligible on the aggregate level.

We obtain the quarterly residential property price index and consumer price index data from the Australian Bureau of Statistics (ABS),[9] and the 3-month bank accepted bills yields from the Reserve Bank of Australia (RBA),[10] for the period of 2003–2022. The property price index is used merely as a proxy, whereas in practice each retirement property is rather unique and does not have too much liquidity. As illustrated in Figure 3, while the Australian property market has enjoyed significant growth in the past two decades, the property value moves upward with some cyclical patterns. Its quarterly growth rate fluctuates between −3.0% and 6.7% during the period. There are also cyclical movements in the inflation rate and interest rate. The quarterly inflation rate ranges from −1.9% to 2.1%, and the quarterly interest rate ranges from almost 0% to 2.0%. Figure 4 shows clearly that many of the sample autocorrelations and cross-correlations in between the property growth rate, inflation rate, and interest rate are statistically significant (exceeding the t-test critical value of 0.23 in magnitude). As such, the widely used geometric Brownian motion is not suitable for these financial variables. After inspecting thoroughly the sample partial autocorrelation matrices and the resulting model residuals, we select the VAR(3) process (i.e. a maximum lag of three quarters) to model the three financial time series. We also find that the sample autocorrelations of the squared residuals are insignificant, so there is no need to add a GARCH-type structure. We assume that the retirement village portfolio has homogenous price movements across the retirement properties.

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[8] According to PwC (2020), the ratio of the average entry fee of a retirement village unit to the median house price in the same suburb is 67% in Australia.

[9] https://www.abs.gov.au/statistics/economy/price-indexes-and-inflation/residential-property-price-indexes-eight-capital-cities/latest-release#data-downloads. https://www.abs.gov.au/statistics/economy/price-indexes-and-inflation/consumer-price-index-australia/latest-release#data-downloads.

[10] https://www.rba.gov.au/statistics/tables/#interest-rates.

Figure 3. Australian quarterly residential property price index values (left) and quarterly property growth rates, inflation rates, and interest rates (right) from 2003 to 2022.

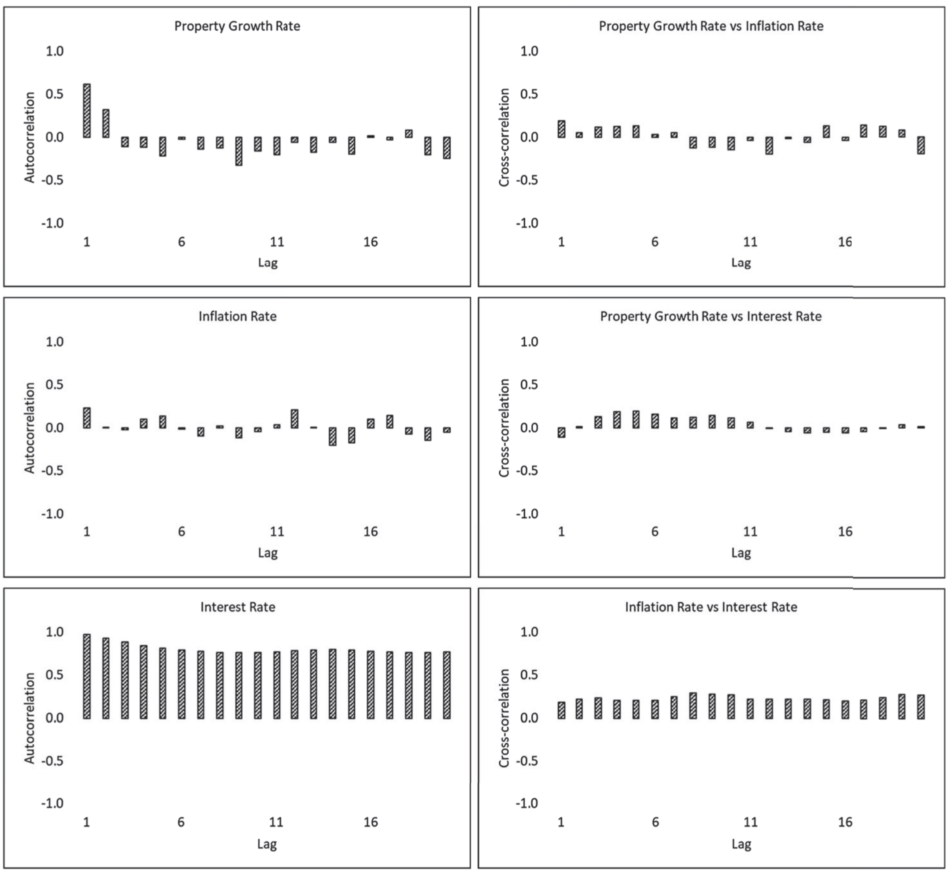

Figure 4. Sample autocorrelations (left) and cross-correlations (right) of Australian quarterly property growth rates, inflation rates, and interest rates from 2003 to 2022.

The VAR(3) process is specified as follows:

$$\begin{pmatrix} r_{t,1} \\ r_{t,2} \\ r_{t,3} \end{pmatrix} = \begin{pmatrix} \theta_1^{(0)} \\ \theta_2^{(0)} \\ \theta_3^{(0)} \end{pmatrix} + \begin{pmatrix} \theta_{1,1}^{(1)} & \theta_{1,2}^{(1)} & \theta_{1,3}^{(1)} \\ \theta_{2,1}^{(1)} & \theta_{2,2}^{(1)} & \theta_{2,3}^{(1)} \\ \theta_{3,1}^{(1)} & \theta_{3,2}^{(1)} & \theta_{3,3}^{(1)} \end{pmatrix} \begin{pmatrix} r_{t-1,1} \\ r_{t-1,2} \\ r_{t-1,3} \end{pmatrix} + \begin{pmatrix} \theta_{1,1}^{(2)} & \theta_{1,2}^{(2)} & \theta_{1,3}^{(2)} \\ \theta_{2,1}^{(2)} & \theta_{2,2}^{(2)} & \theta_{2,3}^{(2)} \\ \theta_{3,1}^{(2)} & \theta_{3,2}^{(2)} & \theta_{3,3}^{(2)} \end{pmatrix} \begin{pmatrix} r_{t-2,1} \\ r_{t-2,2} \\ r_{t-2,3} \end{pmatrix}$$

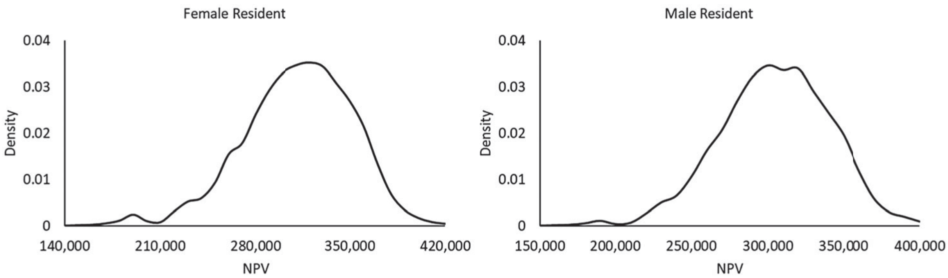

Figure 5. Densities of net present values of retirement village contract for female and male residents.

$$+\begin{pmatrix}\theta_{1,1}^{(3)} & \theta_{1,2}^{(3)} & \theta_{1,3}^{(3)}\\ \theta_{2,1}^{(3)} & \theta_{2,2}^{(3)} & \theta_{2,3}^{(3)}\\ \theta_{3,1}^{(3)} & \theta_{3,2}^{(3)} & \theta_{3,3}^{(3)}\end{pmatrix}\begin{pmatrix}r_{t-3,1}\\ r_{t-3,2}\\ r_{t-3,3}\end{pmatrix}+\begin{pmatrix}\epsilon_{t,1}\\ \epsilon_{t,2}\\ \epsilon_{t,3}\end{pmatrix},$$

where r's represent the property growth rate, inflation rate, and interest rate in quarter t, θ's are the vector autoregressive coefficients, and ϵ's are multivariate normal error terms with zero means and a Wishart prior for their inverse covariance matrix. The prior distribution for the unknown autoregressive coefficients is assumed as multivariate normal, in which the mean vector is zero and the hyperprior for the inverse covariance matrix is Wishart. Moreover, it is assumed that longevity and morbidity (demographic) risks and market (financial / economic) risks are independent.

Based on the predictive means estimated from the MCMC simulations, for a female (male) resident, the expected present value of the fees charged for general services, maintenance, and management is equal to about \$159,000 (\$148,000), and the expected net present value (i.e. entry fee at onset minus expected present value of net refund on exit) of the contract is \$280,000 (\$273,000), assuming those fees will cover the actual maintenance costs adequately. Using the market value of a similar non-retirement dwelling, the effective annual rental yield of the contract (i.e. converting the expected net present value into a series of annuitised values)[11] is then expected to be 2.4% (2.5%). Figure 5 displays the kernel density of the simulated net present values of the contract for both female and male residents. The 95% prediction interval is from \$156,000 (\$156,000) to \$377,000 (\$368,000). There are fairly comparable potential upside and downside outcomes (though skewed to the downside), and the decision to run a retirement village depends on the risk appetite of the institution and the desired risk-return trade-off.

In the Australian market, the terms of a retirement village contract usually do not depend on the age and sex of the resident. Under the same deferred management fee, the effective rent for a male resident would be higher than that for a female resident, as the length of stay for the male would be shorter than that for the female. It then becomes a more attractive product for the female, as she faces the same fee structure but would stay in the village for a longer time. The same argument applies to an older resident and a younger resident, in which the effective rent for the older resident is higher.

Table 5 lists the effective annual rental yields of the retirement village contract for both sexes under various fee structures, including the case when the departing resident can share some of the capital gain, if any (but not capital loss). First, the effective rent for a male resident is consistently higher than that for a female resident. As noted above, the male would stay in the village for a shorter period, and so his rent is effectively higher given the same fee structure. Second, as the entry fee increases, the operator's yields rise moderately, and vice versa. The entry fee setting in practice, however, is subject to the demand and supply forces and the marketing strategy of the operator. Third, as the capital gain sharing percentage increases, the yields for the operator reduce sharply. It appears that

[11] The annuity formula is $\mathrm{PV}=\mathrm{CF}\frac{(1-(1+i)^{-n})}{i}$. We suggest that the resulting effective yield is a very convenient measure here for comparison between the yields of different kinds of complex contracts and also the market yields.

Table 5. Effective annual rental yields (in %) of retirement village contract for female and male residents under different fee structures.

Female Resident (Deferred Management Fee Upper Limit = 18%)						
	Capital Gain Sharing Percentage					
Entry Fee	0%	10%	20%	30%	40%	50%
$400,000	1.7	1.3	0.9	0.6	0.2	−0.2
$450,000	1.9	1.5	1.1	0.6	0.2	−0.2
$500,000	2.1	1.6	1.2	0.7	0.2	−0.2
$550,000	2.3	1.8	1.3	0.8	0.3	−0.3
$600,000	2.5	2.0	1.4	0.8	0.3	−0.3
Female Resident (Deferred Management Fee Upper Limit = 30%)						
	Capital Gain Sharing Percentage					
Entry Fee	0%	10%	20%	30%	40%	50%
$400,000	2.0	1.6	1.2	0.8	0.4	0.1
$450,000	2.2	1.8	1.4	0.9	0.5	0.1
$500,000	2.4	2.0	1.5	1.0	0.6	0.1
$550,000	2.7	2.2	1.7	1.1	0.6	0.1
$600,000	2.9	2.4	1.8	1.2	0.7	0.1
Female Resident (Deferred Management Fee Upper Limit = 42%)						
	Capital Gain Sharing Percentage					
Entry Fee	0%	10%	20%	30%	40%	50%
$400,000	2.2	1.8	1.5	1.1	0.7	0.3
$450,000	2.5	2.1	1.6	1.2	0.8	0.4
$500,000	2.8	2.3	1.8	1.4	0.9	0.4
$550,000	3.1	2.5	2.0	1.5	1.0	0.5
$600,000	3.3	2.8	2.2	1.6	1.1	0.5
Male Resident (Deferred Management Fee Upper Limit = 18%)						
	Capital Gain Sharing Percentage					
Entry Fee	0%	10%	20%	30%	40%	50%
$400,000	1.7	1.4	1.0	0.6	0.3	−0.1
$450,000	1.9	1.5	1.1	0.7	0.3	−0.1
$500,000	2.2	1.7	1.2	0.8	0.3	−0.1
$550,000	2.4	1.9	1.4	0.9	0.4	−0.1
$600,000	2.6	2.0	1.5	0.9	0.4	−0.1
Male Resident (Deferred Management Fee Upper Limit = 30%)						
	Capital Gain Sharing Percentage					
Entry Fee	0%	10%	20%	30%	40%	50%
$400,000	2.0	1.6	1.3	0.9	0.6	0.2
$450,000	2.3	1.9	1.4	1.0	0.6	0.2
$500,000	2.5	2.1	1.6	1.1	0.7	0.2
$550,000	2.8	2.3	1.8	1.3	0.8	0.3
$600,000	3.0	2.5	1.9	1.4	0.8	0.3
Male Resident (Deferred Management Fee Upper Limit = 42%)						
	Capital Gain Sharing Percentage					
Entry Fee	0%	10%	20%	30%	40%	50%
$400,000	2.3	1.9	1.6	1.2	0.8	0.5
$450,000	2.6	2.2	1.8	1.4	0.9	0.5
$500,000	2.9	2.4	2.0	1.5	1.0	0.6
$550,000	3.2	2.7	2.2	1.7	1.2	0.7
$600,000	3.4	2.9	2.4	1.8	1.3	0.7

the capital gain is a significant element for the operator and the sharing structure has to be designed with caution. In the current market, about half of the retirement villages have capital gain sharing arrangements (PwC 2020). Moreover, the operator's yields increase steadily with the upper limit on the total deferred management fee. Some retirement villages set an upper limit of even up to 36%, and some may also take an extra sinking fund charge of up to 5%. The results show that the deferred management fee is a major source of profit for the operator. The interplay between the entry fee, capital gain sharing, and deferred management fee gives the operator much flexibility to generate its desired yield level and cater for the needs of the customers.

Overall, the effective annual rental yields in Table 5 are lower than the current non-retirement property rental yields of around 3.5% to 4% per annum.[12] In fact, the average entry age of moving into a retirement village in Australia is about 75 years old (PwC 2020). While the WHO data in Section 2 cover age 60 only, we make an approximate adjustment by deducting (say) 15 years from the simulated healthy life expectancy values and recalculate the yields. The resulting effective annual rental yields of the benchmark contract become 5.4% for a female resident and 6.9% for a male resident, which are much higher than those in Table 5 and also the non-retirement property rental yields. If this yield level represents the overall return required on the retirement village portfolio, its exceedance over the comparable market yield level can be seen as a compensation for the risks in operating the business. Note that a typical retirement village portfolio in practice would be composed of residents of both sexes at different ages with varying contract terms. It is likely that there is some subsidy across different subgroups in the portfolio, as some cases (low entry fee, high capital gain sharing percentage, and / or low total deferred management fee upper limit, especially for male residents) are not financially viable for the operator (particularly, like those negative yields in Table 5), and that there are economies of scale in managing many similar properties in the same area.

Notwithstanding the data limitations, the proposed Bayesian method provides a reasonable approach to assess the profitability of a retirement village contract. Proper management of a retirement village portfolio requires an appropriate model to project the future cash flows and to evaluate the underlying risks. Our Bayesian model can serve as a useful pricing and risk management tool in this aspect. The approach can further be modified in cases where more detailed health data are accessible. For instance, the prevalence rate is the proportion of lives at a certain age who have a particular health condition at a specific point of time. If such data are available, one can estimate healthy life expectancy by multiplying the survival rates with the corresponding prevalence rates (into a disability state) and summing the multiples over different ages. In this way, healthy life expectancy at different old ages can be projected, provided that there is also a suitable model to project the prevalence rates into the future.

5. Concluding remarks

In this paper, we develop a pragmatic Bayesian approach for modelling healthy life expectancy by exploiting its historical relationships with life expectancy, where the data for the latter measure are generally much more available than those for the former. We show that the proposed approach can yield decent forecasting performance and generate probability intervals to describe the uncertainty of future outcomes adequately. It can also automatically provide estimates for the missing values in the healthy life expectancy data. Our study suggests that the life expectancy and healthy life expectancy values of various countries would continue to increase steadily for the next few decades. However, the results raise the concern that the pace of increase in how long a life can continue working may not be as fast as the growth in life expectancy. When many countries are now linking future retirement age increases to life expectancy changes, there is a risk that more people would actually become unable to work until the new retirement age(s) in the future. These people may then suffer from having insufficient pensions and funds for their retirement. Although life expectancy has been the key argument

[12] https://www.smh.com.au/property/news/where-are-the-highest-rental-yields-in-australia-20220211-p59vqr.html.

for revising the retirement / pension age, we deem that the focus should be on healthy life expectancy instead, as the accuracy in assessing the working capacity of a population has a significant impact on the planning of retirement funding and health and aged care policies and so the wellbeing of retirees.

We also apply the proposed Bayesian approach to perform a valuation of a benchmark retirement village contract. Based on the numerical results, we find that the effective annual rental yield can be very sensitive to the specific terms of the fee structure, including the entry fee, the capital gain sharing percentage, and the total deferred management fee upper limit. Moreover, we notice that the effective rent is higher for a male resident or an older resident, due to their shorter length of stay. The proposed Bayesian framework can also simulate the distribution of the contract cash flows and help evaluate the downside risks. It can readily be modified to incorporate additional health data (e.g. prevalence rates, transition rates), which may further improve the model performance.

There are a number of interesting aspects for future research. As noted in Huang et al. (2021), most of the healthy life expectancy studies have focused on physical health but not cognitive health. As the deterioration of cognition function is a significant factor in the quality of old-age life, it should be incorporated more properly in the calculation of healthy life expectancy. Moreover, while the average number of residents per accommodation in Australian retirement villages is only 1.21 currently, it would still be useful to extend the modelling to cater for residents who are couples. Besides the usual (probably overly) simplifying assumption of mortality independence, one may use a copula function to model the last survivor status of a couple. Finally, the proposed approach here is extrapolative by nature, which means that the projection and simulation results would be limited by what has happened within the data period. An (rather arbitrary) allowance for possible future medical breakthroughs (e.g. cure for cancer, gene therapy, artificial organs) by embedding (say) a compound Poisson process into the modelling framework can be considered.

Disclosure statement

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Appendix

We simulated 6,000 MCMC samples (with thinning of 20, i.e. samples being collected from every 20th iteration) and discarded the first 1,000 samples in order to eliminate the effects of the starting values. Figure A1 exhibits the autocorrelations over successive simulated samples and also the sampling history of the model parameters for Australian females aged 60. Note that p[1] to p[4] represent φ_0, φ_1, α_0, and α_1. We notice that the autocorrelations are negligible and there are no particular trends in the sampling history, strongly suggesting that the level of convergence is adequate in the MCMC simulations. The Monte Carlo errors are all smaller than 4% of the sample standard deviations, which provide further evidence of convergence (Spiegelhalter et al. 2003). The simulated results remain stable when different sets of starting values are adopted.

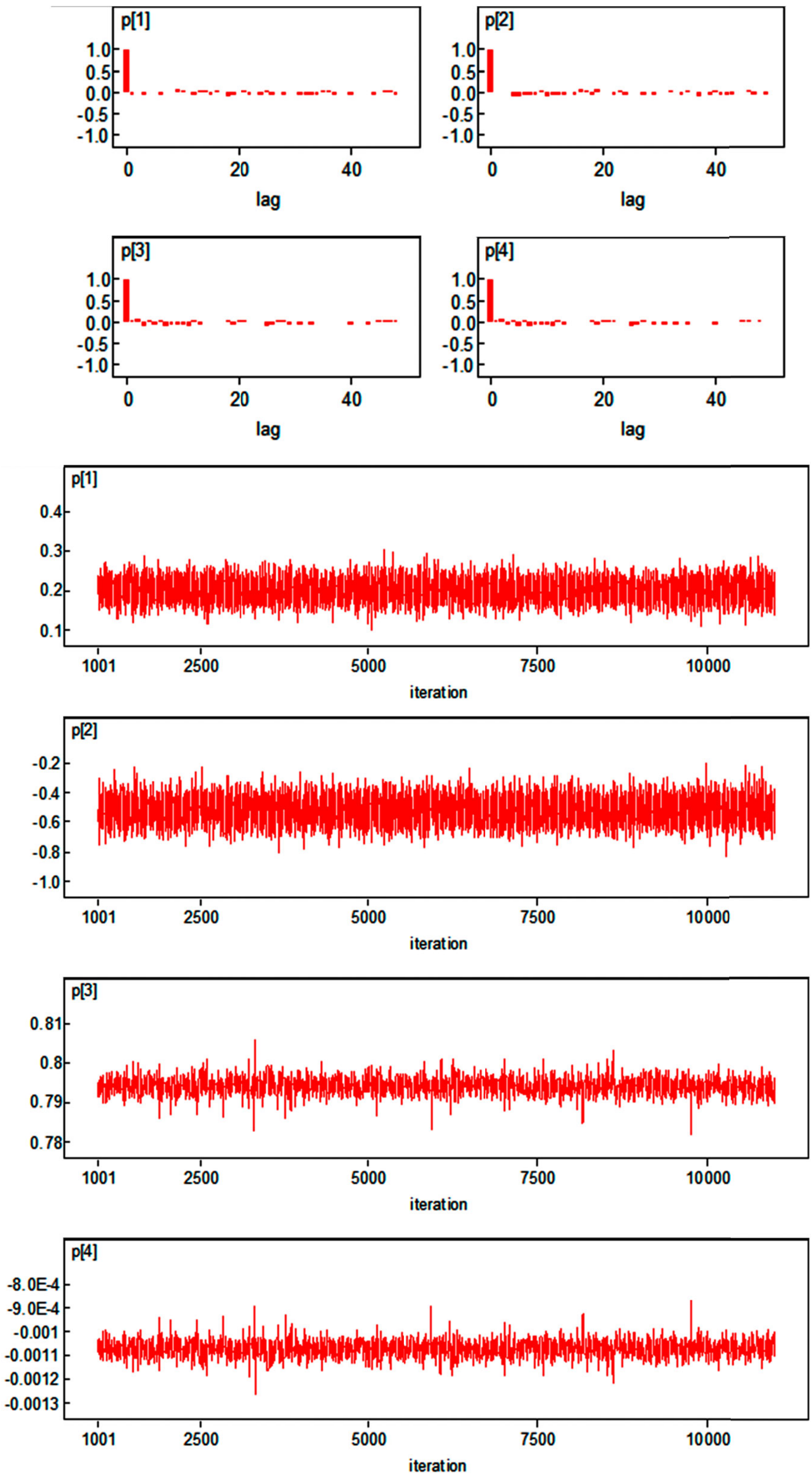

Figure A1. Sample autocorrelations over successive MCMC samples and sampling history of model parameters for Australian females aged 60.