$$\arg\min J(w)$$

$$\cancel{\propto}\ \arg\max_{w} \prod_{i=1}^{N} \exp\left(-\frac{1}{2}(t_i - y_i)^2\right) \cdot \prod_{j=1}^{M} \exp\left(-\lambda \cdot w_j^2\right)$$

$$\propto \arg\max_{w} N(t \mid y, 1) \cdot N(w \mid 0, 1/\lambda)$$

also observe that $e_i = t_i - y_i$; so:

$$\propto \arg\max_{w} N(E \mid 0, 1) \cdot N(w \mid 0, 1/\lambda)$$

① Least Squares assumes Normal ERROR distribution

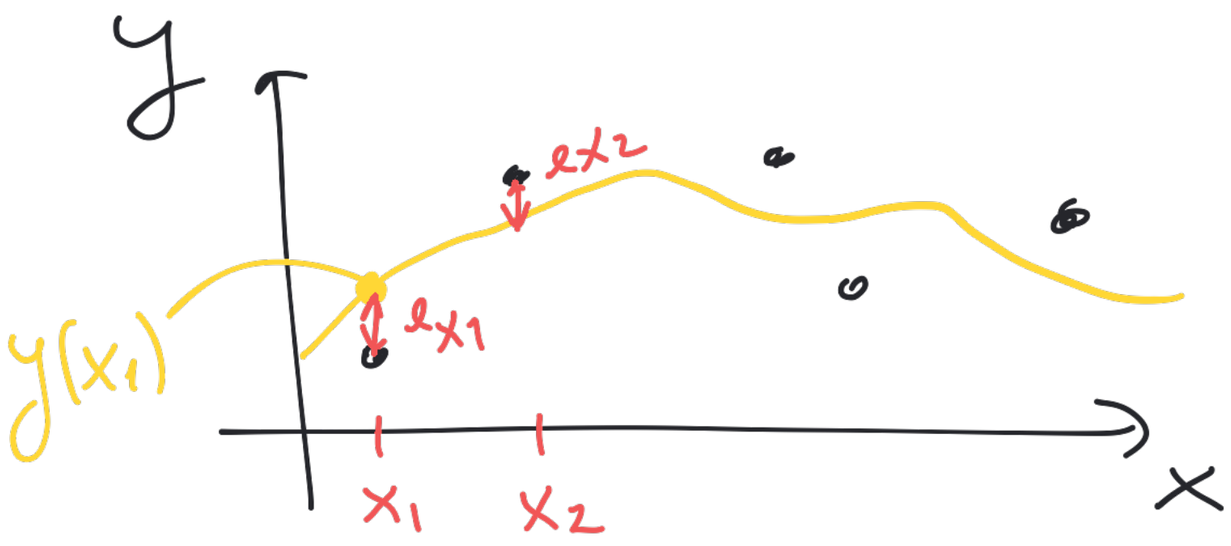
$$\arg\max_{w} \underline{\underline{P}}(E \mid w) \cdot \underline{\underline{P}}(w)$$

Conjugate Prior

L_D is such that its shape is the same as the posterior prob.

$$\arg\max_w P(E|w) \cdot P(w) \propto \arg\max_w P(w|E)$$

- $P(E|w)$: data likelihood
- $P(w)$: PRIOR
- $P(w|E)$: posterior
- $\propto$: Bayes' Rule



$$\begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{bmatrix} = \begin{matrix} t_1 - y_1 = t_1 - y(x_1) \\ t_2 - y_2 \\ \\ t_N - y_N \end{matrix}$$

Examples of Conjugate Prior:

DATA Likelihood Prior Posterior

Gaussian × Gaussian = Gaussian

$$\arg\max_w P(E|w) \cdot P(w)$$

Data

$$\{x_i, t_i\}_{i=1}^{N}$$

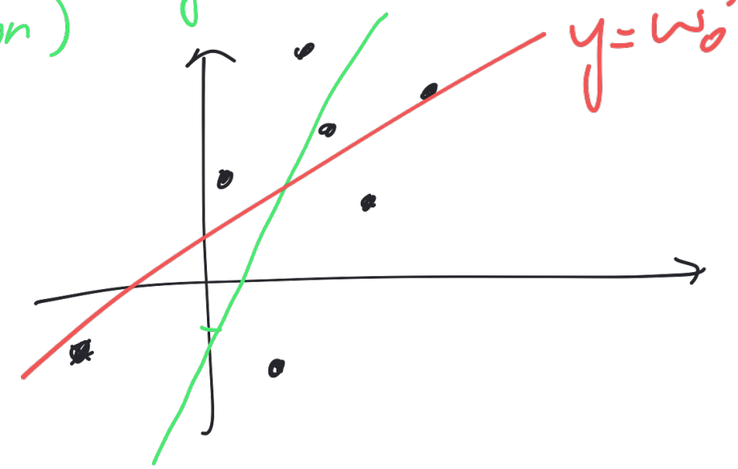
Model

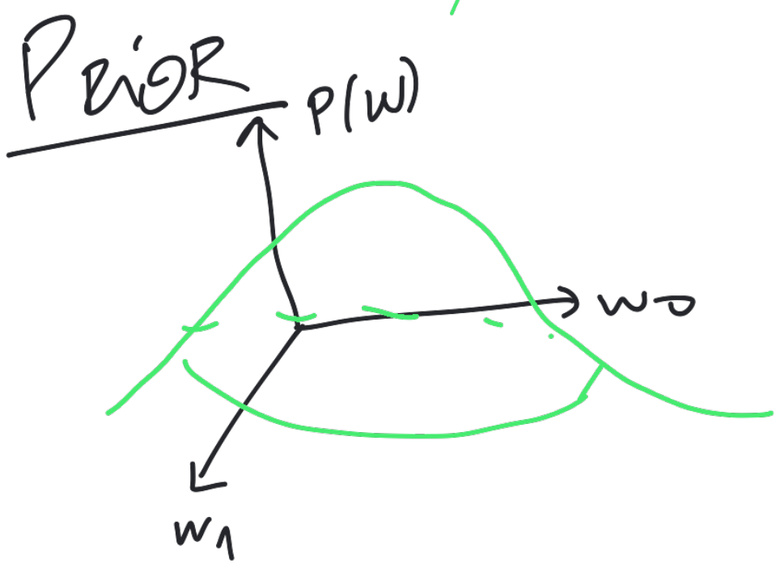
$$y(x, w) = w_0 + w_1 \cdot x$$

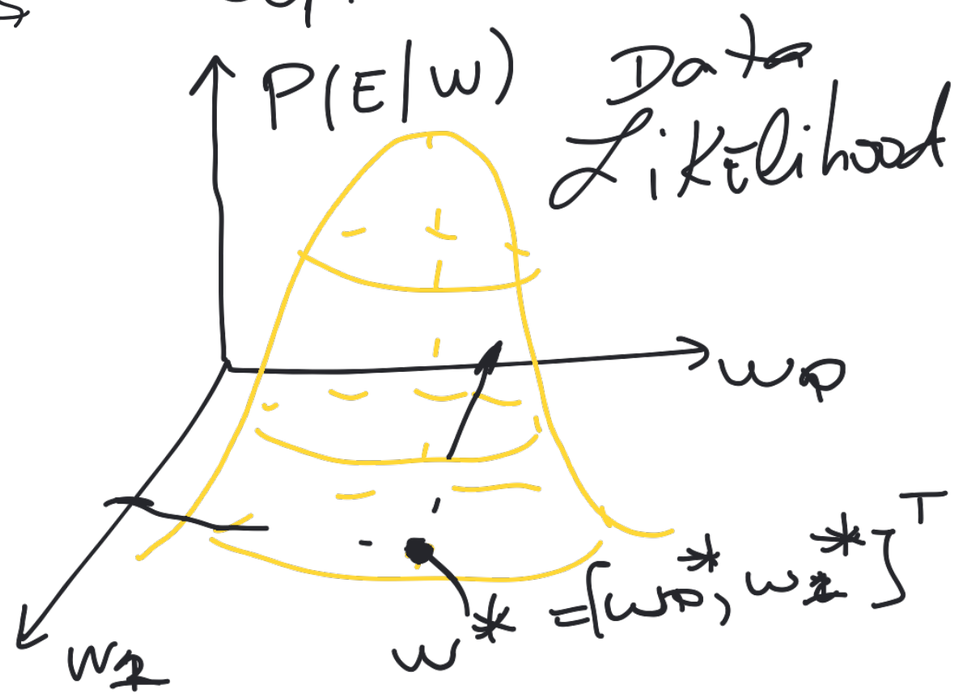
TRUE function (unknown): $y = -0.3 + 0.5x$

$y = w_0^* + w_1^* \cdot x$

I want to find w_0 and w_1.







Multivariate Gaussian distribution:

$$N(x \mid \underline{\mu}, \underline{\underline{\Sigma}}) = \frac{1}{(2\pi)^{1/2} \cdot |\Sigma|^{d/2}} \cdot \exp\left(-\frac{1}{2}(x-\mu)^T \cdot \Sigma^{-1}(x-\mu)\right)$$

$d \equiv$ dimensionality (2D, $d = 2$)

$|\Sigma| \equiv$ determinant of covariance

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_1 \cdot \sigma_2 \\ \sigma_2 \cdot \sigma_1 & \sigma_2^2 \end{bmatrix}$$

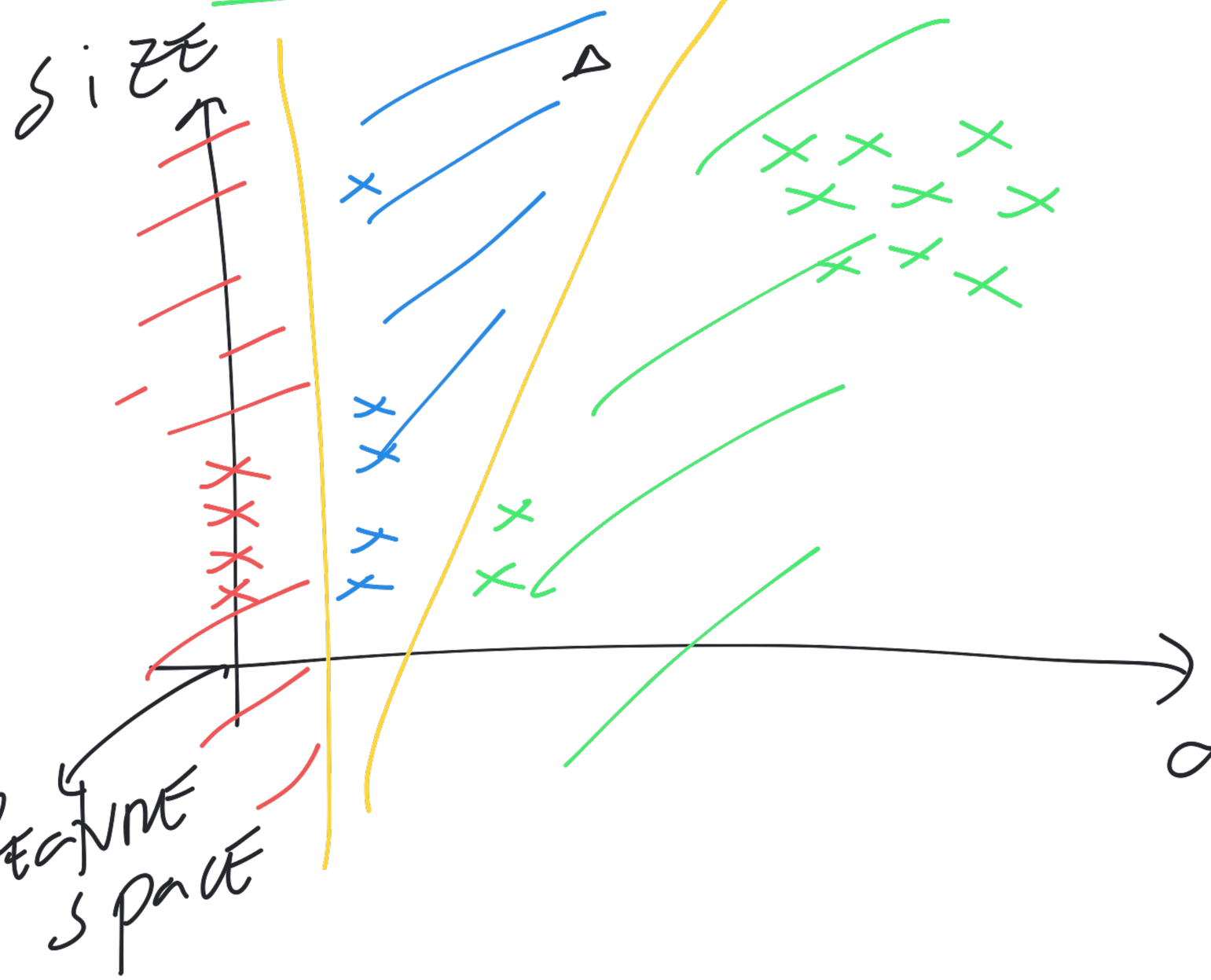
ASSUME that

$$\Sigma = \underline{\underline{\sigma^2 \cdot I}}$$

isotropic

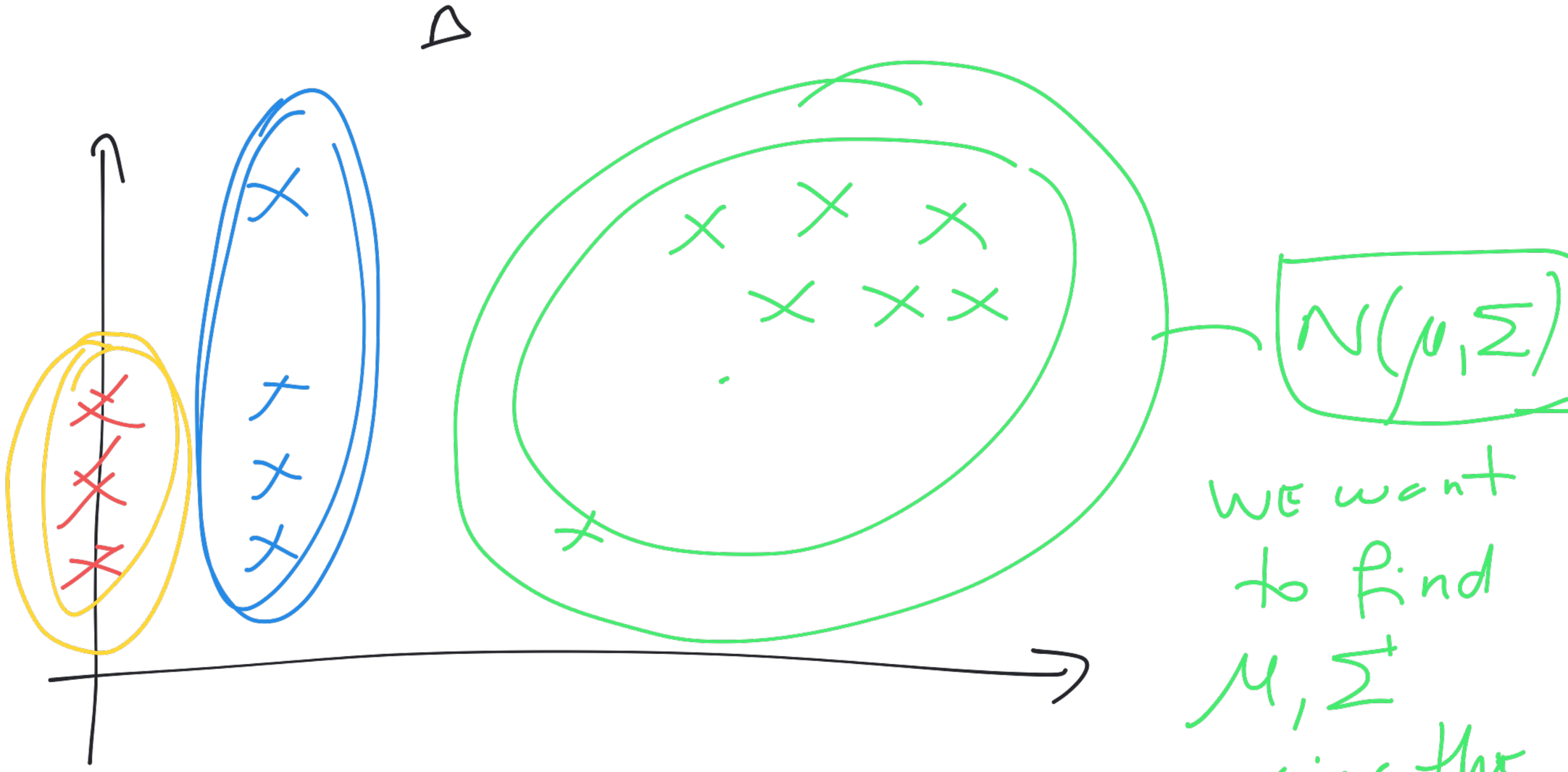
$$\Sigma = \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix}$$

{dog, cats, birds} ← classification labels



Discriminative classification

△ ≡ new animal



we want to find μ, Σ using the dog samples

GENERETIVE CLASSIFIER

$p(\Delta|dogs) > p(\Delta|cats) \Rightarrow \Delta$ is a dog