## EECE 598, Homework 02

Try to solve the problems by yourselves. Compare with your solutions after you are done. Exams will be similar formats as these.

- 1. **Problem 3-2.**
- 2. **Problem 3-3.**
- 3. **Problem 3-4.**
- 4. **Problem 3-13.**
- 5. **Problem 3-17.**
- Problem 3-18.

No submission is required. The solutions to select problems will be uploaded a week later.

## PROBLEMS -

- 3.1 Verify the expression given in Eq. (3.1c) that relates  $\alpha$ , which is in units of dB/km, to  $\alpha_p$ , which is in units of km<sup>-1</sup>.
- 3.2 A certain optical fiber has an attenuation of 0.6 dB/km at 1310 nm and 0.3 dB/km at 1550 nm. Suppose the following two optical signals are launched simultaneously into the fiber: an optical power of 150  $\mu$ W at 1310 nm and an optical power of 100  $\mu$ W at 1550 nm. What are the power levels in  $\mu$ W of these two signals at (a) 8 km and (b) 20 km?
- 3.3 An optical signal at a specific wavelength has lost 55 percent of its power after traversing 7.0 km of fiber. What is the attenuation in dB/km of this fiber?
- 3.4 A continuous 40-km-long optical fiber link has a loss of 0.4 dB/km.
  - (a) What is the minimum optical power level that must be launched into the fiber to maintain an optical power level of  $2.0 \mu W$  at the receiving end?
  - (b) What is the required input power if the fiber has a loss of 0.6 dB/km?
- 3.5 Consider a step-index fiber with a SiO<sub>2</sub>-GeO<sub>2</sub>

- radii less than 10 cm at  $\lambda = 1 \mu m$  for fibers having core radii of 4, 25, and 100  $\mu m$ .
- 3.9 Two common fiber jacket materials are Elvax® 265 ( $E_j = 21$  MPa) and Hytrel® 4056 ( $E_j = 58$  MPa), both made by DuPont. If the Young's modulus of a glass fiber is 64 GPa, plot the reduction in microbending loss as a function of the index difference  $\Delta$  when fibers are coated with these materials. Make these plots for  $\Delta$  values ranging from 0.1 to 1.0 percent and for a fiber cladding-to-core ratio of b/a = 2.
- **3.10** Assume that a step-index fiber has a *V* number of 6.0.
  - (a) Using Fig. 2.27, estimate the fractional power  $P_{\rm clad}/P$  traveling in the cladding for the six lowest-order LP modes.
  - (b) If the fiber in (a) is a glass-core glass-clad fiber having core and cladding attenuations of 3.0 and 4.0 dB/km, respectively, find the attenuations for each of the six lowest-order modes.
- 3.11 Assume a given mode in a graded-index fiber has a power density  $p(r) = P_0 \exp(-Kr^2)$ , where the factor K depends on the modal

where  $E = hc/\lambda$  is the photon energy and  $E_0$ and  $E_d$  are, respectively, material oscillator energy and dispersion energy parameters. In  $SiO_2$  glass,  $E_0 = 13.4$  eV and  $E_d = 14.7$  eV. Show that, for wavelengths between 0.20 and 1.0  $\mu$ m, the values of n found from the Sellmeier relation are in good agreement with those shown in Fig. 3.12. To make the comparison, select three representative points, for example, at 0.2, 0.6, and 1.0  $\mu$ m.

- 3.13 (a) An LED operating at 850 nm has a spectral width of 45 nm. What is the pulse spreading in ns/km due to material dispersion? What is the pulse spreading when a laser diode having a 2-nm spectral width is used?
  - (b) Find the material-dispersion-induced pulse spreading at 1550 nm for an LED with a 75-nm spectral width. Use Fig. 3.13 to estimate  $d\tau/d\lambda$ .
- **3.14** Verify the plots for b, d(Vb)/dV, and  $Vd^2(Vb)/dV$  $dV^2$  shown in Fig. 3.15. Use the expression for b given by Eq. (3.38).
- 3.15 Derive Eq. (3.13) by using a ray-tracing method.
- 3.16 Consider a step-index fiber with core and cladding diameters of 62.5 and 125  $\mu$ m, respectively. Let the core index  $n_1 = 1.48$ and let the index difference  $\Delta = 1.5$  percent. Compare the modal dispersion in units of ns/km at 1310 nm of this fiber as given by Eq. (3.13) with the more exact expression

$$\frac{\sigma_{\text{mod}}}{L} = \frac{n_1 - n_2}{c} \left( 1 - \frac{\pi}{V} \right)$$

where L is the length of the fiber and  $n_2$  is the cladding index.

Consider a standard G.652 non-dispersionshifted single-mode optical fiber that has a zero-dispersion wavelength at 1310 nm with a dispersion slope of  $S_0 = 0.0970 \text{ ps/(nm}^2 \cdot \text{km})$ . Plot the dispersion in the wavelength range 1270 nm ≤  $\lambda$  ≤ 1340 nm. Use Eq. (3.47).

3.18) A typical G.653 dispersion-shifted singlemode optical fiber has a zero-dispersion wavelength at 1550 nm with a dispersion slope of  $S_0 = 0.070 \text{ ps/(nm}^2 \cdot \text{km})$ .

- (a) Plot the dispersion in the wavelength range  $1500 \text{ nm} \le \lambda \le 1600 \text{ nm using Eq. 3.49}.$
- (b) Compare the dispersion at 1500 nm with the dispersion value for the non-dispersionshifted fiber described in Prob. 3.17.
- **3.19** Starting with Eq. (3.45), derive the dispersion expression given in Eq. (3.47).
- 3.20 Renner<sup>19</sup> derived a simplified approximation to describe the bend losses of single-mode optical fibers. This expression for the bending loss is

$$\alpha_{\text{simp}} = \alpha_{\text{conv}} \frac{2(Z_3 Z_2)^{1/2}}{(Z_3 + Z_2) - (Z_3 - Z_2)\cos(2\Theta)}$$

where the conventional bending loss is

$$\alpha_{\text{conv}} = \frac{1}{2} \left( \frac{\pi}{\gamma^3 R} \right)^{1/2} \frac{\kappa^2}{V^2 K_1^2 (\gamma a)} \exp \left( -\frac{2\gamma^3 R}{3\beta_0^3} \right)$$

where V is given by Eq. (2.57),  $\beta_0$  is the propagation constant in a straight fiber with an infinite cladding given by Eq. (2.46),  $K_1$  is the modified Bessel function (see App. C), and

$$Z_{q} \approx k^{2} n_{q}^{2} (1 + 2b/R) - \beta_{0}^{2}$$

$$\approx k^{2} n_{q}^{2} (1 + 2b/R) - k^{2} n_{2}^{2}$$
for  $q = 2, 3$ 

$$\Theta = \frac{\gamma^{3} R}{3k^{2} n_{2}^{2}} \left(\frac{R_{c}}{R} - 1\right)^{3/2}$$

$$\gamma = \left(\beta_{0}^{2} - k^{2} n_{2}^{2}\right)^{1/2} \approx k \left(n_{1}^{2} - n_{2}^{2}\right)^{1/2}$$

$$\kappa^{2} = k^{2} n_{1}^{2} - \beta_{0}^{2} \approx k^{2} \left(n_{1}^{2} - n_{2}^{2}\right)$$

$$R_{o} = 2k^{2} n_{2}^{2} b/\gamma^{2} = \text{the critical bend radius}$$

Using a computer, (a) verify the plot given in Fig. 3.27 at 1300 nm, and (b) calculate and plot the bend loss as a function of wavelength for 800 nm  $\leq \lambda \leq 1600$  m at several different bend radii (e.g., 15 and 20 mm). Let  $n_1 = 1.480$ ,  $n_2 = 1.475$ ,  $n_3$  $1.07n_2 = 1.578$ , and  $b = 60 \,\mu\text{m}$ .