## Emissions Abatement - Exercise 13.6

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This document implements Exercise 13.6 - introducing emissions abatement as a choice variable to the model. We can then calculate the total and marginal costs of abatement. These are useful measures for future work examining the cost effectiveness and benefit/cost analysis of policy.

This document performs Exercise 13.9 from the Integrated Assessment Model chapter of Climate Economics. It runs the underlying climate and economic modeling tasks from sections 13.1 and 13.2. It also adds the new emissions reduction rates,  $R_{i,t}$  to the model, calculates the relative and absolute cost of the emissions reductions, and explores the impact on carbon dioxide concentrations and warming.

## Climate Module (Lab 1)

#### Emissions to CO2

Five Box model of carbon dioxide concentrations. Each box follows equation 13.1:

```
CO2_{i,t} = (1 - CO2decay) * CO2_{i,t-1} + CO2convert * CO2share_i * CO2emissions_{t-1}
```

Make this into a function for regular and repeated use.

```
C02difference <- function(C02previous, newC02emissions) {
   C02decay <- c(0, 1-exp(-1/363), 1-exp(-1/74), 1-exp(-1/17), 1-exp(-1/2))
   C02share <- c(0.13, 0.20, 0.32, 0.25, 0.10);
   C02convert <- 1/2.13/1000;

C02concnew = (1-C02decay)*C02previous + C02convert*C02share*newC02emissions # Equation 13.1
   C02concnew
}
```

Now we just need some emissions data and starter values for the model.

```
# emissions data
emissionsSince1750 <- read_sheet("https://docs.google.com/spreadsheets/d/15gnvwp5HWqPVb4_h0Dz6j1DlLiaQv.
Load historic emissions data</pre>
```

```
## v Reading from "ECON269-Emissions-data".
## v Range '2:10000000'.
## New names:
## * `` -> `...1`
names(emissionsSince1750) <- c("year", "EmissionsMMTC")
# head(emissionsSince1750)</pre>
```

```
# Graph emissions over time
emissionsSince1750 %>%
ggplot(aes(year, EmissionsMMTC)) +
geom_line()
```

Prepare the data for the CO2 model calculations.

```
HistoricData <- emissionsSince1750

HistoricData$Box1 <- 0
HistoricData$Box2 <- 0
HistoricData$Box3 <- 0
HistoricData$Box4 <- 0
HistoricData$Box5 <- 0
HistoricData$Box1[1] <- 275

(Box1col <- which(colnames(HistoricData) == "Box1"))</pre>
```

### ## [1] 3

Now we implement it, year by year to use the previous year's box values and the emissions to calculate new box values.

```
## # A tibble: 6 x 8
##
     year EmissionsMMTC Box1 Box2 Box3 Box4 Box5 CO2conc
##
    <dbl>
                <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <
                                                      <dbl>
## 1 2015
                10775. 300. 34.8 41.2 15.2 1.19
                                                       392.
                11110. 300. 35.7 42.3 15.6 1.23
## 2 2016
                                                       395.
## 3 2017
                 11257. 301. 36.6 43.4 16.0 1.27
                                                       398.
## 4 2018
                 11467. 302. 37.6 44.5 16.4 1.30
                                                       402.
                 11477.
                        302.
                              38.6 45.6 16.8 1.33
## 5 2019
                                                       405.
## 6 2020
                11432. 303. 39.5 46.7 17.2 1.34
                                                       408.
```

We can plot the CO2 concentration over time.

```
HistoricData %>%
   ggplot(aes(x = year, y = CO2conc)) +
   geom_line()
```

## ${ m CO2}$ concentrations to Temperatures

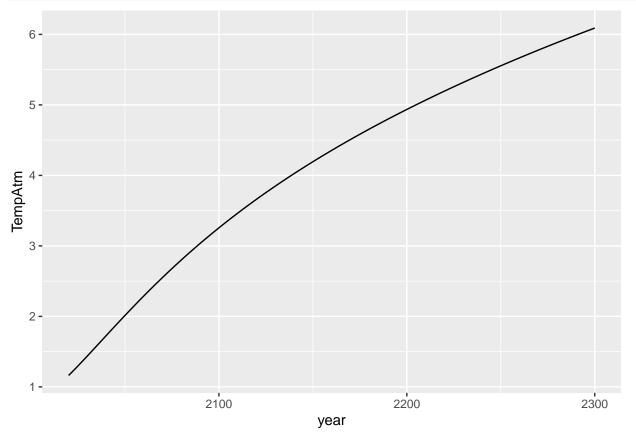
Radiative Forcing (Equation 13.3) and Temperature anomalies (Equations 13.4 and 13.5)

```
RadForc <-function(CO2) {
   5.35*log(CO2/275);
}</pre>
```

```
Temps <- function(atmtempold, oceantempold, radforc) {</pre>
  par1 <- 1.15
  par2 <- 0.0256
  par3 <- 0.00738
  par4 <- 0.00568
              = atmtempold
  atmtempnew
      par2*(par1*radforc-atmtempold) +
      par3*(oceantempold-atmtempold)
  oceantempnew = oceantempold +
    par4*(atmtempold-oceantempold)
  temps <- c(atmtempnew, oceantempnew)</pre>
  names(temps) <- c("atm", "ocean")</pre>
  temps
HistoricData$RF <- RadForc(HistoricData$CO2conc)</pre>
HistoricData$TempAtm = 0
HistoricData$TempOcean = 0
StartYear <- 1850
EndYear <- 2020
for (i in (which(HistoricData$year == StartYear)+1):which(HistoricData$year == EndYear)) {
  temp <- Temps(HistoricData$TempAtm[i-1], HistoricData$TempOcean[i-1], HistoricData$RF[i])
 HistoricData$TempAtm[i] <- temp["atm"]</pre>
 HistoricData$TempOcean[i] <- temp["ocean"]</pre>
}
We can now plot temp over time
HistoricData %>%
  ggplot(aes(year, TempAtm)) +
 geom_line()
```

## Future Scenario Example: Constant Emissions

```
# Calculate total CO2 concentration in atmosphere
Scenario1$C02conc <- Scenario1$Box1 + Scenario1$Box2 + Scenario1$Box3 +
    Scenario1$Box4 + Scenario1$Box5
# Calculate radiative forcing
Scenario1$RF <- RadForc(Scenario1$C02conc)
# Calculate Temperatures
for (i in 2:length(Scenario1$EmissionsMMTC)) {
    temp <- Temps(Scenario1$TempAtm[i-1], Scenario1$TempOcean[i-1], Scenario1$RF[i])
    Scenario1$TempAtm[i] <- temp["atm"]
    Scenario1$TempOcean[i] <- temp["ocean"]
}
Scenario1 %>% ggplot(aes(year, TempAtm)) +
    geom_line()
```



# Economic Module (Lab 2)

Kaya Identity

$$Emissions = Pop \times \frac{GDP}{Pop} \times \frac{Energy}{GDP} \times \frac{Emissions}{Energy}$$

$$M^* = Pop \times \frac{GDP}{Pop} \times EnergyIntensity \times EmissionsIntensity$$

We will model Population, GDP, Energy Intensity, and Emissions Intensity over time and allow for variation across three economic regions of the world, the richest, middle income, and poorest regions.

## Using Righ-Middle-Poor Region Data

Access the regional data from a Google Sheet

```
# regional kaya data
regional.df <- read_sheet("https://docs.google.com/spreadsheets/d/1fgR_jtz7zCLynudKfqpjCr2LbORW_tXqHrdG
## x Request failed [429]. Retry 1 happens in 2.1 seconds ...
## x Request failed [429]. Retry 2 happens in 3.8 seconds ...
## x Request failed [429]. Retry 3 happens in 20.7 seconds ...
## x Request failed [429]. Retry 4 happens in 28.1 seconds ...
## v Reading from "Lab 02 Regional Data".
## v Range 'Sheet1'.
## New names:
## * `` -> `...3`
## * `` -> `...4`
## * `` -> `...5`
## * `` -> `...7`
## * `` -> `...8`
## * `` -> `...9`
## * `` -> `...11`
## * `` -> `...12`
## * `` -> `...13`
## * `` -> `...15`
## * `` -> `...16`
# note the regional data has some issues with the names, etc.
# because the data starts on row 4, with regional labels, etc. in earlier rows
Rich.df \leftarrow regional.df[-(1:3), c(1, 2, 6, 10, 14)]
#head(Rich.df)
Rich.df <- Rich.df %>% mutate(across(Population:Emissions, as.numeric))
Middle.df \leftarrow regional.df[-(1:3), c(1, 3, 7, 11, 15)]
names(Middle.df) <- names(Rich.df)</pre>
Middle.df <- Middle.df %>% mutate(across(Population:Emissions, as.numeric))
Poor.df \leftarrow regional.df [-(1:3), c(1, 4, 8, 12, 16)]
names(Poor.df) <- names(Rich.df)</pre>
Poor.df <- Poor.df %>% mutate(across(Population:Emissions, as.numeric))
Rich.df$Region <- "R"</pre>
Middle.df$Region <- "M"
Poor.df$Region <- "P"
In order to model future population, we will calculate the population growth rate from 2019 to 2020.
long.df <- full_join(Rich.df, Middle.df) %>% full_join(Poor.df) %>%
 group_by(Region)
## Joining with `by = join_by(Year, Population, GDP, Energy, Emissions, Region)`
## Joining with `by = join_by(Year, Population, GDP, Energy, Emissions, Region)`
```

```
PopulationGrowth2020 <- long.df %>% mutate(g.Pop = Population/lag(Population) - 1) %>% filter(Year == 2020) %>% select(Region, g.Pop, Year)
```

Calculate the energy intensity of GDP as the emissions intensity of energy.

```
## # Groups:
               Region [1]
      Year Population
                        GDP Energy Emissions Region EnInt EmInt
##
                                       <dbl> <chr>
##
                <dbl> <dbl>
                             <dbl>
                                                    <dbl> <dbl>
                 755. 7556.
                                       1635 R
## 1 1960
                             1823.
                                                    0.241 0.897
                 765. 7939.
## 2 1961
                             1870.
                                       1684. R
                                                    0.236 0.901
## 3 1962
                 774. 8405.
                            1971.
                                       1776. R
                                                    0.235 0.901
## 4 1963
                 783. 8839.
                             2097.
                                       1875. R
                                                    0.237 0.894
                                       1976. R
## 5 1964
                 792. 9403.
                             2194.
                                                    0.233 0.900
## 6 1965
                 801. 9928.
                             2372.
                                       2044. R
                                                    0.239 0.862
```

And calculate the average growth rate of the intensities to use in modeling the future.

Before we model the future values of population, energy intensity and emissions intensity, we need to create the modeled GDP  $(Y^*)$  as well as capital values that we'll need for the future.

### **GDP**

For GDP we will use the Cobb-Douglass Production function and the Solow Growth model.

$$GDP = A \times K^{\alpha} \times L^{(1-\alpha)}$$

where K is capital, L is labor, and A is the Total Factor Productivity.

```
CobbDouglassalpha <- .2
CobbDouglassGDP <- function(A, K, L, alpha = CobbDouglassalpha) {
    A*K^alpha*L^(1-alpha)
}</pre>
```

We are also told that capital also follows a difference equation, changing over time depending on depreciation, investment and savings.

$$K_t = K_{t-1} - \delta K_{t-1} + Investmentt$$

and because Investment = Savings, and we assume Savings is a constant share of output, Savings = s\*GDP, then

```
K_t = K_{t-1} - \delta K_{t-1} + s * GDP_{t-1}
```

```
depreciation <- .1
savingsrate <- .2
Kapital <- function(Kold, GDPold, d = depreciation, s = savingsrate) {
   (1-d)*Kold + s*GDPold
}</pre>
```

Text says assume a starting value of capital with

$$K_{1960} = (\frac{s*A}{\delta})^{\frac{1}{1-\alpha}} * Population$$

We've already done the calibration of the GDP model and so we know some parameters.

```
GDPparameters <- data.frame(Region = c("R", "M", "P"),
                            A = c(5.5, 0.65, 0.283), # starter values
                            g.A = c(0.0183, 0.0282, 0.0205), # qrowth rates
                            Year = 1960) %>%
 left_join(long.df %>% select(Year, Population, GDP))
## Adding missing grouping variables: `Region`
## Joining with `by = join_by(Region, Year)`
# These values taken from previous calibration exercises
GDPparameters <- GDPparameters %>%
  mutate(K = (savingsrate*A/depreciation)^(1/(1-CobbDouglassalpha))*Population,
         Ystar = CobbDouglassGDP(A, K, Population))
long.df <- long.df %>% left join(GDPparameters) %>% group by(Region)
## Joining with `by = join_by(Year, Population, GDP, Region)`
for (year in 1961:2020) {
  long.df$K[which(long.df$Year == year)] <- Kapital(</pre>
    long.df$K[which(long.df$Year == year-1)],
    long.df$Ystar[which(long.df$Year == year-1)])
  long.df$g.A[which(long.df$Year == year)] <-</pre>
    long.df$g.A[which(long.df$Year == year-1)]
  long.df$A[which(long.df$Year == year)] <-</pre>
    long.df$A[which(long.df$Year == year-1)] *
    (1 + long.df$g.A[which(long.df$Year == year)])
  long.df$Ystar[which(long.df$Year == year)] = CobbDouglassGDP(
   long.df$A[which(long.df$Year == year)],
    long.df$K[which(long.df$Year == year)],
    long.df$Population[which(long.df$Year == year)])
}
```

Now let's grab the values for year 2020, and add the future values of years 2021-2300 to model future values.

```
future.df <- long.df %>% filter(Year == 2020) %>%
  left_join(PopulationGrowth2020) %>%
  left_join(KayaIntensities) %>%
```

```
full_join(expand.grid(Year = 2020:2300, Region = c("R", "M", "P"))) %>%
 mutate(Mstar = NA)
## Joining with `by = join_by(Year, Region)`
## Joining with `by = join_by(Year, Region)`
## Joining with `by = join_by(Year, Region)`
Now we can model our future values
future.df$Mstar = future.df$Emissions
for (year in 2021:2300) {
  # Population growth rate and Population
  future.df$g.Pop[which(future.df$Year == year)] <-</pre>
    .95*future.df$g.Pop[which(future.df$Year == year-1)]
  future.df$Population[which(future.df$Year == year)] <-</pre>
    future.df$Population[which(future.df$Year == year-1)] *
    (1 + future.df$g.Pop[which(future.df$Year == year)])
  # Energy Intensity growth rate and Energy Intensity
  future.df$Ave.g.EnInt[which(future.df$Year == year)] <-</pre>
    future.df$Ave.g.EnInt[which(future.df$Year == year-1)]
  future.df$EnInt[which(future.df$Year == year)] <-</pre>
    future.df$EnInt[which(future.df$Year == year-1)] *
    (1 + future.df$Ave.g.EnInt[which(future.df$Year == year)])
  # Emissions Intensity growth rate and Emissions Intensity
  future.df$Ave.g.EmInt[which(future.df$Year == year)] <-</pre>
    future.df$Ave.g.EmInt[which(future.df$Year == year-1)]
  future.df$EmInt[which(future.df$Year == year)] <-</pre>
    future.df$EmInt[which(future.df$Year == year-1)] *
    (1 + future.df\$Ave.g.EmInt[which(future.df\$Year == year)])
  # Economic variables
  future.df$K[which(future.df$Year == year)] <- Kapital(</pre>
    future.df$K[which(future.df$Year == year-1)],
    future.df$Ystar[which(future.df$Year == year-1)])
  future.df$g.A[which(future.df$Year == year)] <-</pre>
    .99*future.df$g.A[which(future.df$Year == year-1)]
  future.df$A[which(future.df$Year == year)] <-</pre>
    future.df$A[which(future.df$Year == year-1)] *
    (1 + future.df$g.A[which(future.df$Year == year)])
  future.df$Ystar[which(future.df$Year == year)] = CobbDouglassGDP(
    future.df$A[which(future.df$Year == year)],
    future.df$K[which(future.df$Year == year)],
    future.df$Population[which(future.df$Year == year)])
  # Emissions (using adjusted Kaya Identity: GDP * Energy Intensity * Emissions Intensity)
```

```
future.df$Mstar[which(future.df$Year == year)] =
  future.df$Ystar[which(future.df$Year == year)] *
  future.df$EnInt[which(future.df$Year == year)] *
  future.df$EmInt[which(future.df$Year == year)]
}
```

### Bringing Together the Emissions Data across Regions

We have worked to create emissions predictions for three different regions. But we care about the global emissions.

```
future.df %>% group_by(Year) %>% summarise(Emissions = sum(Mstar))
## # A tibble: 281 x 2
##
      Year Emissions
##
     <dbl>
             <dbl>
## 1 2020
           11432.
## 2 2021
             11775.
## 3 2022
           12109.
## 4 2023
           12445.
## 5 2024
           12784.
## 6 2025
            13125.
## 7 2026
           13468.
## 8 2027
           13812.
## 9 2028
             14157.
## 10 2029
             14504.
## # i 271 more rows
```

Calculate CO2 concentrations and temperature for these emissions.

```
Kaya.global <- left_join(future.df %>%
                            group_by(Year) %>%
                            summarise(Emissions = sum(Mstar)),
                          HistoricData, by = c("Year" = "year") )
Box1col <- which(colnames(Kaya.global) == "Box1")</pre>
#Implement Five box model for CO2 concentrations
for (i in 2:length(Kaya.global$Year)) {
Kaya.global[i, Box1col:(Box1col+4)] <- CO2difference(Kaya.global[i - 1, Box1col:(Box1col+4)],</pre>
                                                         Kaya.global$Emissions[i - 1])
# Calculate total CO2 concentration in atmosphere
Kaya.global$CO2conc <- Kaya.global$Box1 + Kaya.global$Box2 + Kaya.global$Box3 +
 Kaya.global$Box4 + Kaya.global$Box5
# Calculate radiative forcing
Kaya.global$RF <- RadForc(Kaya.global$CO2conc)</pre>
# Calculate Temperatures
for (i in 2:length(Kaya.global$Year)) {
    temp <- Temps(Kaya.global$TempAtm[i-1], Kaya.global$TempOcean[i-1], Kaya.global$RF[i])
 Kaya.global$TempAtm[i] <- temp["atm"]</pre>
 Kaya.global$TempOcean[i] <- temp["ocean"]</pre>
}
```

#### Abatement

Now we add a choice variable,  $R_{r,t}$  that represents the proportion of  $M_{r,t+1}^*$  that will get abated. (Note the text talks about the reductions in time t as a function of  $R_{r,t-1}$ ).

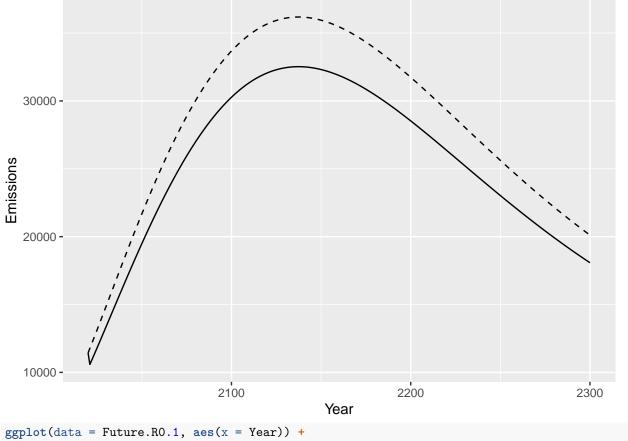
```
future.df <- long.df %>% filter(Year == 2020) %>%
  left_join(PopulationGrowth2020) %>%
  left_join(KayaIntensities) %>%
  full_join(expand.grid(Year = 2020:2300, Region = c("R", "M", "P"))) %>%
  mutate(Mstar = Emissions,
         Madjusted = Mstar,
         Yadjusted = Ystar,
         R = 0.1,
         TACosts = 0,
         MACosts = 0)
## Joining with `by = join_by(Year, Region)`
## Joining with `by = join_by(Year, Region)`
## Joining with `by = join_by(Year, Region)`
Now we can model our future values
for (year in 2021:2300) {
  # Population growth rate and Population
  future.df$g.Pop[which(future.df$Year == year)] <-</pre>
    .95*future.df$g.Pop[which(future.df$Year == year-1)]
  future.df$Population[which(future.df$Year == year)] <-</pre>
    future.df$Population[which(future.df$Year == year-1)] *
    (1 + future.df$g.Pop[which(future.df$Year == year)])
  # Energy Intensity growth rate and Energy Intensity
  future.df$Ave.g.EnInt[which(future.df$Year == year)] <-</pre>
    future.df$Ave.g.EnInt[which(future.df$Year == year-1)]
  future.df$EnInt[which(future.df$Year == year)] <-</pre>
    future.df$EnInt[which(future.df$Year == year-1)] *
    (1 + future.df$Ave.g.EnInt[which(future.df$Year == year)])
  # Emissions Intensity growth rate and Emissions Intensity
  future.df$Ave.g.EmInt[which(future.df$Year == year)] <-</pre>
    future.df$Ave.g.EmInt[which(future.df$Year == year-1)]
  future.df$EmInt[which(future.df$Year == year)] <-</pre>
    future.df$EmInt[which(future.df$Year == year-1)] *
    (1 + future.df$Ave.g.EmInt[which(future.df$Year == year)])
  # Economic variables
  future.df$K[which(future.df$Year == year)] <- Kapital(</pre>
    future.df$K[which(future.df$Year == year-1)],
    future.df$Yadjusted[which(future.df$Year == year-1)])
  future.df$g.A[which(future.df$Year == year)] <-</pre>
```

```
.99*future.df$g.A[which(future.df$Year == year-1)]
  future.df$A[which(future.df$Year == year)] <-</pre>
    future.df$A[which(future.df$Year == year-1)] *
    (1 + future.df$g.A[which(future.df$Year == year)])
  future.df$Ystar[which(future.df$Year == year)] <- CobbDouglassGDP(</pre>
   future.df$A[which(future.df$Year == year)],
    future.df$K[which(future.df$Year == year)],
    future.df$Population[which(future.df$Year == year)])
  future.df$Yadjusted[which(future.df$Year == year)] <-</pre>
    future.df$Ystar[which(future.df$Year == year)] *
    (1 - .1*future.df$R[which(future.df$Year == year-1)]^2)
  # Emissions (using adjusted Kaya Identity: GDP * Energy Intensity * Emissions Intensity)
  future.df$Mstar[which(future.df$Year == year)] =
    future.df$Yadjusted[which(future.df$Year == year)] *
    future.df$EnInt[which(future.df$Year == year)] *
   future.df$EmInt[which(future.df$Year == year)]
  future.df$Madjusted[which(future.df$Year == year)] =
    future.df$Mstar[which(future.df$Year == year)] *
    (1 - future.df$R[which(future.df$Year == year-1)])
  future.df$TACosts[which(future.df$Year == year)] =
    .1*future.df$R[which(future.df$Year == year)]^2*
    future.df$Ystar[which(future.df$Year == year)]
   future.df$MACosts[which(future.df$Year == year)] =
      2*.1*future.df$R[which(future.df$Year == year)] *
      future.df$Ystar[which(future.df$Year == year)] /
      future.df$Mstar[which(future.df$Year == year)]
}
Example values for year 2021.
future.df %>% filter(Year==2021) %>%
  select(Year, R, Region, Ystar, Yadjusted, Mstar, Madjusted, TACosts, MACosts) %>%
  head()
## # A tibble: 3 x 9
## # Groups:
               Region [3]
##
      Year
               R Region Ystar Yadjusted Mstar Madjusted TACosts MACosts
##
     <dbl> <dbl> <chr>
                         <dbl>
                                   <dbl> <dbl>
                                                    <dbl>
                                                            <dbl>
                                                                    <dbl>
## 1 2021
             0.1 R
                        43368.
                                  43325. 3505.
                                                    3155.
                                                            43.4
                                                                   0.247
## 2 2021
             0.1 M
                        15201.
                                  15186. 7088.
                                                    6379.
                                                                   0.0429
                                                            15.2
## 3 2021
             0.1 P
                         4459.
                                   4454. 1170.
                                                    1053.
                                                             4.46 0.0762
```

In order to see the impacts on CO2 concentrations and warming, we need to add up the adjusted emissions across the three regions for each year and feed these into our Climate Model.

Calculate CO2 concentrations and temperature for these emissions.

```
summarise(Emissions = sum(Madjusted)),
                          HistoricData, by = c("Year" = "year") )
Box1col <- which(colnames(Kaya.global) == "Box1")</pre>
#Implement Five box model for CO2 concentrations
for (i in 2:length(Kaya.global$Year)) {
Kaya.global[i, Box1col:(Box1col+4)] <- CO2difference(Kaya.global[i - 1, Box1col:(Box1col+4)],</pre>
                                                         Kaya.global$Emissions[i - 1])
}
# Calculate total CO2 concentration in atmosphere
Kaya.global$CO2conc <- Kaya.global$Box1 + Kaya.global$Box2 + Kaya.global$Box3 +
  Kaya.global$Box4 + Kaya.global$Box5
# Calculate radiative forcing
Kaya.global$RF <- RadForc(Kaya.global$CO2conc)</pre>
# Calculate Temperatures
for (i in 2:length(Kaya.global$Year)) {
    temp <- Temps(Kaya.global$TempAtm[i-1], Kaya.global$TempOcean[i-1], Kaya.global$RF[i])
  Kaya.global$TempAtm[i] <- temp["atm"]</pre>
 Kaya.global$TempOcean[i] <- temp["ocean"]</pre>
Future.RO.1 <- Kaya.global
ggplot(data = Future.RO.1, aes(x = Year)) +
  geom_line(aes(y = Emissions)) +
  geom_line(data = Future.BAU, aes(x = Year, y = Emissions), linetype = "dashed")
```



```
ggplot(data = Future.R0.1, aes(x = Year)) +
geom_line(aes(y = TempAtm)) +
geom_line(data = Future.BAU, aes(x = Year, y = TempAtm), linetype = "dashed")
```

