Emissions Abatement with Permits - Exercise 13.9

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This document performs Exercise 13.9 from the Integrated Assessment Model chapter of Climate Economics. It runs the underlying climate and economic modeling tasks from sections 13.1 and 13.2. It also adds the new emissions reduction rates, $R_{i,t}$ to the model, calculates the relative and absolute cost of the emissions reductions, and explores the impact on carbon dioxide concentrations and warming.

Climate Module (Lab 1)

Emissions to CO2

Five Box model of carbon dioxide concentrations. Each box follows equation 13.1:

```
CO2_{i,t} = (1 - CO2decay) * CO2_{i,t-1} + CO2convert * CO2share_i * CO2emissions_{t-1}
```

Make this into a function for regular and repeated use.

```
C02difference <- function(C02previous, newC02emissions) {
    C02decay <- c(0, 1-exp(-1/363), 1-exp(-1/74), 1-exp(-1/17), 1-exp(-1/2))
    C02share <- c(0.13, 0.20, 0.32, 0.25, 0.10);
    C02convert <- 1/2.13/1000;

C02concnew = (1-C02decay)*C02previous + C02convert*C02share*newC02emissions # Equation 13.1
    C02concnew
}
```

Now we just need some emissions data and starter values for the model.

```
# emissions data
emissionsSince1750 <- read_sheet("https://docs.google.com/spreadsheets/d/15gnvwp5HWqPVb4_h0Dz6j1DlLiaQv.")</pre>
```

Load historic emissions data

```
## v Reading from "ECON269-Emissions-data".
## v Range '2:10000000'.
## New names:
## * `` -> `...1`
names(emissionsSince1750) <- c("year", "EmissionsMMTC")
# head(emissionsSince1750)
# Graph emissions over time
emissionsSince1750 %>%
    ggplot(aes(year, EmissionsMMTC)) +
    geom_line()
```

Prepare the data for the CO2 model calculations.

```
HistoricData$Box1 <- 0
HistoricData$Box2 <- 0
HistoricData$Box3 <- 0
HistoricData$Box4 <- 0
HistoricData$Box5 <- 0
HistoricData$Box1 == "Box1")</pre>
```

[1] 3

Now we implement it, year by year to use the previous year's box values and the emissions to calculate new box values.

```
for (i in 2:length(HistoricData$EmissionsMMTC)) {
HistoricData[i, Box1col:(Box1col+4)] <- CO2difference(HistoricData[i - 1, Box1col:(Box1col+4)],</pre>
                                                      HistoricData$EmissionsMMTC[i - 1])
}
HistoricData$CO2conc <- HistoricData$Box1 + HistoricData$Box2 + HistoricData$Box3 +
  HistoricData$Box4 + HistoricData$Box5
tail(HistoricData)
## # A tibble: 6 x 8
##
     year EmissionsMMTC Box1 Box2 Box3 Box4 Box5 CO2conc
##
                  <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <
     <dbl>
## 1 2015
                 10775. 300. 34.8 41.2 15.2 1.19
                                                         392.
                 11110. 300. 35.7 42.3 15.6 1.23
## 2 2016
                                                         395.
## 3 2017
                 11257. 301. 36.6 43.4 16.0 1.27
                 11467.
                         302. 37.6 44.5 16.4 1.30
## 4 2018
                                                         402.
## 5 2019
                 11477.
                         302. 38.6 45.6 16.8 1.33
                                                         405.
## 6 2020
                 11432. 303. 39.5 46.7 17.2 1.34
                                                         408.
We can plot the CO2 concentration over time.
HistoricData %>%
  ggplot(aes(x = year, y = CO2conc)) +
```

CO₂ concentrations to Temperatures

geom_line()

Radiative Forcing (Equation 13.3) and Temperature anomalies (Equations 13.4 and 13.5)

```
RadForc <-function(CO2) {
    5.35*log(CO2/275);
}

Temps <- function(atmtempold, oceantempold, radforc) {
    par1 <- 1.15
    par2 <- 0.0256
    par3 <- 0.00738
    par4 <- 0.00568</pre>
```

```
atmtempnew = atmtempold
      par2*(par1*radforc-atmtempold) +
      par3*(oceantempold-atmtempold)
  oceantempnew = oceantempold +
    par4*(atmtempold-oceantempold)
  temps <- c(atmtempnew, oceantempnew)</pre>
  names(temps) <- c("atm", "ocean")</pre>
  temps
HistoricData$RF <- RadForc(HistoricData$CO2conc)</pre>
HistoricData$TempAtm = 0
HistoricData$TempOcean = 0
StartYear <- 1850
EndYear <- 2020
for (i in (which(HistoricData$year == StartYear)+1):which(HistoricData$year == EndYear)) {
  temp <- Temps(HistoricData$TempAtm[i-1], HistoricData$TempOcean[i-1], HistoricData$RF[i])
  HistoricData$TempAtm[i] <- temp["atm"]</pre>
  HistoricData$TempOcean[i] <- temp["ocean"]</pre>
We can now plot temp over time
HistoricData %>%
  ggplot(aes(year, TempAtm)) +
 geom_line()
```

Future Scenario Example: Constant Emissions

Economic Module (Lab 2)

2100

Kaya Identity

$$Emissions = Pop \times \frac{GDP}{Pop} \times \frac{Energy}{GDP} \times \frac{Emissions}{Energy}$$

year

2200

2300

$$M^* = Pop \times \frac{GDP}{Pop} \times EnergyIntensity \times EmissionsIntensity$$

We will model Population, GDP, Energy Intensity, and Emissions Intensity over time and allow for variation across three economic regions of the world, the richest, middle income, and poorest regions.

Using Righ-Middle-Poor Region Data

Access the regional data from a Google Sheet

```
# regional kaya data
regional.df <- read_sheet("https://docs.google.com/spreadsheets/d/1fgR_jtz7zCLynudKfqpjCr2LbORW_tXqHrdG
## v Reading from "Lab 02 Regional Data".
## v Range 'Sheet1'.
## New names:
## * `` -> `...3`
## * `` -> `...4`
## * `` -> `...5`
## * `` -> `...7`
## * `` -> `...8`
## * `` -> `...9`
## * `` -> `...11`
## * `` -> `...12`
## * `` -> `...13`
## * `` -> `...15`
## * `` -> `...16`
# note the regional data has some issues with the names, etc.
# because the data starts on row 4, with regional labels, etc. in earlier rows
Rich.df \leftarrow regional.df[-(1:3), c(1, 2, 6, 10, 14)]
#head(Rich.df)
Rich.df <- Rich.df %>% mutate(across(Population:Emissions, as.numeric))
Middle.df \leftarrow regional.df[-(1:3), c(1, 3, 7, 11, 15)]
names(Middle.df) <- names(Rich.df)</pre>
Middle.df <- Middle.df %>% mutate(across(Population:Emissions, as.numeric))
Poor.df \leftarrow regional.df[-(1:3), c(1, 4, 8, 12, 16)]
names(Poor.df) <- names(Rich.df)</pre>
Poor.df <- Poor.df %>% mutate(across(Population:Emissions, as.numeric))
Rich.df$Region <- "R"
Middle.df$Region <- "M"
Poor.df$Region <- "P"
In order to model future population, we will calculate the population growth rate from 2019 to 2020.
long.df <- full_join(Rich.df, Middle.df) %>% full_join(Poor.df) %>%
 group_by(Region)
## Joining with `by = join_by(Year, Population, GDP, Energy, Emissions, Region)`
## Joining with `by = join_by(Year, Population, GDP, Energy, Emissions, Region)`
PopulationGrowth2020 <- long.df %>% mutate(g.Pop = Population/lag(Population) - 1) %>%
  filter(Year == 2020) %>% select(Region, g.Pop, Year)
Calculate the energy intensity of GDP as the emissions intensity of energy.
long.df <- long.df %>%
  mutate(EnInt = Energy/GDP,
         EmInt = Emissions/Energy)
head(long.df)
```

```
## # A tibble: 6 x 8
## # Groups:
               Region [1]
##
      Year Population
                        GDP Energy Emissions Region EnInt EmInt
##
     <dbl>
                <dbl> <dbl>
                             <dbl>
                                        <dbl> <chr>
                                                     <dbl> <dbl>
## 1
     1960
                 755. 7556.
                             1823.
                                        1635 R
                                                     0.241 0.897
## 2 1961
                 765. 7939.
                                        1684. R
                                                     0.236 0.901
                            1870.
## 3 1962
                 774. 8405.
                                        1776. R
                             1971.
                                                     0.235 0.901
## 4 1963
                 783. 8839.
                             2097.
                                        1875. R
                                                     0.237 0.894
## 5 1964
                 792. 9403.
                              2194.
                                        1976. R
                                                     0.233 0.900
                             2372.
## 6 1965
                 801. 9928.
                                        2044. R
                                                     0.239 0.862
```

And calculate the average growth rate of the intensities to use in modeling the future.

Before we model the future values of population, energy intensity and emissions intensity, we need to create the modeled GDP (Y^*) as well as capital values that we'll need for the future.

GDP

For GDP we will use the Cobb-Douglass Production function and the Solow Growth model.

$$GDP = A \times K^{\alpha} \times L^{(1-\alpha)}$$

where K is capital, L is labor, and A is the Total Factor Productivity.

```
CobbDouglassalpha <- .2
CobbDouglassGDP <- function(A, K, L, alpha = CobbDouglassalpha) {
    A*K^alpha*L^(1-alpha)
}</pre>
```

We are also told that capital also follows a difference equation, changing over time depending on depreciation, investment and savings.

$$K_t = K_{t-1} - \delta K_{t-1} + Investmentt$$

and because Investment = Savings, and we assume Savings is a constant share of output, Savings = s*GDP, then

$$K_t = K_{t-1} - \delta K_{t-1} + s * GDP_{t-1}$$

```
depreciation <- .1
savingsrate <- .2
Kapital <- function(Kold, GDPold, d = depreciation, s = savingsrate) {
   (1-d)*Kold + s*GDPold
}</pre>
```

Text says assume a starting value of capital with

$$K_{1960} = (\frac{s * A}{\delta})^{\frac{1}{1-\alpha}} * Population$$

We've already done the calibration of the GDP model and so we know some parameters.

```
GDPparameters <- data.frame(Region = c("R", "M", "P"),
                             A = c(5.5, 0.65, 0.283), # starter values
                             g.A = c(0.0183, 0.0282, 0.0205), # growth rates
                             Year = 1960) %>%
 left_join(long.df %>% select(Year, Population, GDP))
## Adding missing grouping variables: `Region`
## Joining with `by = join_by(Region, Year)`
# These values taken from previous calibration exercises
GDPparameters <- GDPparameters %>%
  mutate(K = (savingsrate*A/depreciation)^(1/(1-CobbDouglassalpha))*Population,
         Ystar = CobbDouglassGDP(A, K, Population))
long.df <- long.df %>% left_join(GDPparameters) %>% group_by(Region)
## Joining with `by = join_by(Year, Population, GDP, Region)`
for (year in 1961:2020) {
  long.df$K[which(long.df$Year == year)] <- Kapital(</pre>
    long.df$K[which(long.df$Year == year-1)],
    long.df$Ystar[which(long.df$Year == year-1)])
  long.df$g.A[which(long.df$Year == year)] <-</pre>
    long.df$g.A[which(long.df$Year == year-1)]
  long.df$A[which(long.df$Year == year)] <-</pre>
    long.df$A[which(long.df$Year == year-1)] *
    (1 + long.df$g.A[which(long.df$Year == year)])
  long.df$Ystar[which(long.df$Year == year)] = CobbDouglassGDP(
    long.df$A[which(long.df$Year == year)],
    long.df$K[which(long.df$Year == year)],
    long.df$Population[which(long.df$Year == year)])
}
Now let's grab the values for year 2020, and add the future values of years 2021-2300 to model future values.
future.df <- long.df %>% filter(Year == 2020) %>%
  left_join(PopulationGrowth2020) %>%
 left_join(KayaIntensities) %>%
 full_join(expand.grid(Year = 2020:2300, Region = c("R", "M", "P"))) %>%
 mutate(Mstar = NA)
## Joining with `by = join_by(Year, Region)`
## Joining with `by = join_by(Year, Region)`
## Joining with `by = join_by(Year, Region)`
Now we can model our future values
future.df$Mstar = future.df$Emissions
```

```
for (year in 2021:2300) {
  # Population growth rate and Population
  future.df$g.Pop[which(future.df$Year == year)] <-</pre>
    .95*future.df$g.Pop[which(future.df$Year == year-1)]
  future.df$Population[which(future.df$Year == year)] <-</pre>
    future.df$Population[which(future.df$Year == year-1)] *
    (1 + future.df$g.Pop[which(future.df$Year == year)])
  # Energy Intensity growth rate and Energy Intensity
  future.df$Ave.g.EnInt[which(future.df$Year == year)] <-</pre>
    future.df$Ave.g.EnInt[which(future.df$Year == year-1)]
  future.df$EnInt[which(future.df$Year == year)] <-</pre>
    future.df$EnInt[which(future.df$Year == year-1)] *
    (1 + future.df$Ave.g.EnInt[which(future.df$Year == year)])
  # Emissions Intensity growth rate and Emissions Intensity
  future.df$Ave.g.EmInt[which(future.df$Year == year)] <-</pre>
    future.df$Ave.g.EmInt[which(future.df$Year == year-1)]
  future.df$EmInt[which(future.df$Year == year)] <-</pre>
    future.df$EmInt[which(future.df$Year == year-1)] *
    (1 + future.df$Ave.g.EmInt[which(future.df$Year == year)])
  # Economic variables
  future.df$K[which(future.df$Year == year)] <- Kapital(</pre>
    future.df$K[which(future.df$Year == year-1)],
    future.df$Ystar[which(future.df$Year == year-1)])
  future.df$g.A[which(future.df$Year == year)] <-</pre>
    .99*future.df$g.A[which(future.df$Year == year-1)]
  future.df$A[which(future.df$Year == year)] <-</pre>
    future.df$A[which(future.df$Year == year-1)] *
    (1 + future.df$g.A[which(future.df$Year == year)])
  future.df$Ystar[which(future.df$Year == year)] = CobbDouglassGDP(
    future.df$A[which(future.df$Year == year)],
    future.df$K[which(future.df$Year == year)],
    future.df$Population[which(future.df$Year == year)])
  # Emissions (using adjusted Kaya Identity: GDP * Energy Intensity * Emissions Intensity)
  future.df$Mstar[which(future.df$Year == year)] =
    future.df$Ystar[which(future.df$Year == year)] *
    future.df$EnInt[which(future.df$Year == year)] *
    future.df$EmInt[which(future.df$Year == year)]
}
```

Bringing Together the Emissions Data across Regions

We have worked to create emissions predictions for three different regions. But we care about the global emissions.

```
future.df %>% group_by(Year) %>% summarise(Emissions = sum(Mstar))
## # A tibble: 281 x 2
      Year Emissions
##
##
      <dbl>
               <dbl>
   1 2020
              11432.
##
## 2 2021
              11775.
##
  3 2022
              12109.
##
  4 2023
              12445.
## 5 2024
              12784.
## 6 2025
              13125.
## 7 2026
              13468.
## 8 2027
              13812.
## 9 2028
              14157.
## 10 2029
              14504.
## # i 271 more rows
```

Calculate CO2 concentrations and temperature for these emissions.

```
Kaya.global <- left join(future.df %>%
                            group_by(Year) %>%
                            summarise(Emissions = sum(Mstar)),
                          HistoricData, by = c("Year" = "year") )
Box1col <- which(colnames(Kaya.global) == "Box1")</pre>
#Implement Five box model for CO2 concentrations
for (i in 2:length(Kaya.global$Year)) {
Kaya.global[i, Box1col:(Box1col+4)] <- CO2difference(Kaya.global[i - 1, Box1col:(Box1col+4)],</pre>
                                                         Kaya.global$Emissions[i - 1])
# Calculate total CO2 concentration in atmosphere
Kaya.global$CO2conc <- Kaya.global$Box1 + Kaya.global$Box2 + Kaya.global$Box3 +
 Kaya.global$Box4 + Kaya.global$Box5
# Calculate radiative forcing
Kaya.global$RF <- RadForc(Kaya.global$CO2conc)</pre>
# Calculate Temperatures
for (i in 2:length(Kaya.global$Year)) {
    temp <- Temps(Kaya.global$TempAtm[i-1], Kaya.global$TempOcean[i-1], Kaya.global$RF[i])
 Kaya.global$TempAtm[i] <- temp["atm"]</pre>
  Kaya.global$TempOcean[i] <- temp["ocean"]</pre>
}
Future.BAU <- Kaya.global
```

Abatement with Permits

Now we add a choice variable for each Region, G which is the amount of emissions reduced by the region in the next time period according to the allocation of emissions permits. We will also create variables for the equilibrium permit price, and now calculate the number of permits bought/sold, as well as the rate of

emissions reductions for each region as a function of the permit price.

Note that in the last lab $R_{r,t}$ was a choice variable, but now it gets calculated as a function of the permit price.

Set our overall reduction Rate,

```
GlobalR = 0.1
```

```
## Joining with `by = join_by(Year, Region)`
## Joining with `by = join_by(Year, Region)`
## Joining with `by = join_by(Year, Region)`
```

Now we can model our future values, but note that after estimating several of the economic variables, Y^* , M^* , etc., now we estimate the permit price first, then estimate the various other policy variables like R, etc.

$$\pi_{t} = \frac{\sum_{r} G_{r,t}}{\sum_{r} \frac{(M_{r,t}^{*})^{2}}{2\beta Y_{r,t}^{*}}}$$

```
GSUM <- future.df %>% filter(Year == 2020) %>% ungroup() %>% summarise(sum(G))

Denom <- future.df %>% filter(Year == 2020) %>% mutate(DenomReg = Mstar^2/2/.1/Ystar) %>%
    ungroup() %>% summarise(sum(DenomReg))

GSUM/Denom
```

```
## sum(G)
## 1 0.0598808
```

```
future.df$EnInt[which(future.df$Year == year)] <-</pre>
    future.df$EnInt[which(future.df$Year == year-1)] *
    (1 + future.df$Ave.g.EnInt[which(future.df$Year == year)])
  # Emissions Intensity growth rate and Emissions Intensity
  future.df$Ave.g.EmInt[which(future.df$Year == year)] <-</pre>
    future.df$Ave.g.EmInt[which(future.df$Year == year-1)]
  future.df$EmInt[which(future.df$Year == year)] <-</pre>
    future.df$EmInt[which(future.df$Year == year-1)] *
    (1 + future.df$Ave.g.EmInt[which(future.df$Year == year)])
  # Economic variables
  future.df$K[which(future.df$Year == year)] <- Kapital(</pre>
    future.df$K[which(future.df$Year == year-1)],
    future.df$Yadjusted[which(future.df$Year == year-1)])
  future.df$g.A[which(future.df$Year == year)] <-</pre>
    .99*future.df$g.A[which(future.df$Year == year-1)]
  future.df$A[which(future.df$Year == year)] <-</pre>
    future.df$A[which(future.df$Year == year-1)] *
    (1 + future.df$g.A[which(future.df$Year == year)])
  future.df$Ystar[which(future.df$Year == year)] <- CobbDouglassGDP(</pre>
    future.df$A[which(future.df$Year == year)],
    future.df$K[which(future.df$Year == year)],
    future.df$Population[which(future.df$Year == year)])
  future.df$Yadjusted[which(future.df$Year == year)] <-</pre>
    future.df$Ystar[which(future.df$Year == year)] *
    (1 - .1*future.df$R[which(future.df$Year == year-1)]^2)
  # Emissions (using adjusted Kaya Identity: GDP * Energy Intensity * Emissions Intensity)
  future.df$Mstar[which(future.df$Year == year)] =
    future.df$Yadjusted[which(future.df$Year == year)] *
    future.df$EnInt[which(future.df$Year == year)] *
    future.df$EmInt[which(future.df$Year == year)]
  # Set G at 10% of Mstar
  future.df$G[which(future.df$Year == year)] =
future.df$Mstar[which(future.df$Year == year)]*GlobalR
  # Calculate Pi, Permit Price which depends on G and YStar and Mstar from all regions
GSUM <- future.df %>% filter(Year == year) %>% ungroup() %>% summarise(sum(G))
Denom <- future.df %>% filter(Year == year) %>%
  mutate(DenomReg = Mstar^2/2/.1/Ystar) %>% ungroup() %>% summarise(sum(DenomReg))
future.df$Pi[which(future.df$Year == year)] = as.numeric(GSUM/Denom)
 # Now we can calculate R
future.df$R[which(future.df$Year == year)] =
  future.df$Pi[which(future.df$Year == year)] *
```

```
future.df$Mstar[which(future.df$Year == year)] /2/.1/
  future.df$Ystar[which(future.df$Year == year)]
# Now P, permit purchases
future.df$P[which(future.df$Year == year)] = future.df$G[which(future.df$Year == year)] -
  future.df$Pi[which(future.df$Year == year)]*
  future.df$Mstar[which(future.df$Year == year)]^2/2/.1/
  future.df$Ystar[which(future.df$Year == year)]
future.df$PermitCosts[which(future.df$Year == year)] =
  future.df$P[which(future.df$Year == year)] *
  future.df$Pi[which(future.df$Year == year)]
  future.df$TACosts[which(future.df$Year == year)] =
    .1*future.df$R[which(future.df$Year == year)]^2*
   future.df$Ystar[which(future.df$Year == year)]
  future.df$NetCosts[which(future.df$Year == year)] =
    future.df$TACosts[which(future.df$Year == year)] +
   future.df$PermitCosts[which(future.df$Year == year)]
  future.df$MACosts[which(future.df$Year == year)] =
      2*.1*future.df$R[which(future.df$Year == year)] *
      future.df$Ystar[which(future.df$Year == year)] /
      future.df$Mstar[which(future.df$Year == year)]
  future.df$Madjusted[which(future.df$Year == year)] =
    future.df$Mstar[which(future.df$Year == year)] *
    (1 - future.df$R[which(future.df$Year == year-1)])
}
```

Example values for year 2021.

```
future.df %% filter(Year==2021) %%
select(Region, Ystar, Mstar, G, Pi, R, P, TACosts, NetCosts, MACosts) %>%
head()
```

```
## # A tibble: 3 x 10
## # Groups:
              Region [3]
    Region Ystar Mstar
                            G
                                  Ρi
                                                 P TACosts NetCosts MACosts
     <chr>>
           <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
                                                      <dbl>
                                                               <dbl>
                                                                      <dbl>
            43368. 3505. 351. 0.0604 0.0244 265.
                                                                     0.0604
## 1 R
                                                      2.58
                                                               18.6
## 2 M
           15201. 7088. 709. 0.0604 0.141 -289.
                                                      30.1
                                                               12.7
                                                                      0.0604
## 3 P
            4459. 1170. 117. 0.0604 0.0792
                                              24.3
                                                      2.80
                                                               4.27 0.0604
```

In order to see the impacts on CO2 concentrations and warming, we need to add up the adjusted emissions across the three regions for each year and feed these into our Climate Model.

Calculate CO2 concentrations and temperature for these emissions.

```
Box1col <- which(colnames(Kaya.global) == "Box1")</pre>
#Implement Five box model for CO2 concentrations
for (i in 2:length(Kaya.global$Year)) {
Kaya.global[i, Box1col:(Box1col+4)] <- CO2difference(Kaya.global[i - 1, Box1col:(Box1col+4)],</pre>
                                                          Kaya.global$Emissions[i - 1])
}
# Calculate total CO2 concentration in atmosphere
Kaya.global$CO2conc <- Kaya.global$Box1 + Kaya.global$Box2 + Kaya.global$Box3 +
  Kaya.global$Box4 + Kaya.global$Box5
# Calculate radiative forcing
Kaya.global$RF <- RadForc(Kaya.global$CO2conc)</pre>
# Calculate Temperatures
for (i in 2:length(Kaya.global$Year)) {
    temp <- Temps(Kaya.global$TempAtm[i-1], Kaya.global$TempOcean[i-1], Kaya.global$RF[i])
  Kaya.global$TempAtm[i] <- temp["atm"]</pre>
  Kaya.global$TempOcean[i] <- temp["ocean"]</pre>
}
Future.RO.1 <- Kaya.global
ggplot(data = Future.RO.1, aes(x = Year)) +
  geom_line(aes(y = Emissions)) +
  geom_line(data = Future.BAU, aes(x = Year, y = Emissions), linetype = "dashed")
  30000 -
Emissions
  20000 -
  10000 -
                                2100
                                                           2200
                                                                                      2300
                                                Year
ggplot(data = Future.RO.1, aes(x = Year)) +
  geom_line(aes(y = TempAtm)) +
```

