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1. a) There are two conditions for c so that f(x) is a valid probability density function:

- $\int_0^3 cx^2\,dx = 1$ Since the probability of any possible outcome is 1.

- f(x) must be non-negative for $0 \leq x \leq 3$.

b) Since it is a continous function the odds of X being exactly 1 is $\underline{\underline{0}}$. You would need an intervall to have a probability.

c) $E(X) = \int_0^3 x \cdot f(x)\,dx = \int_0^3 \frac{x^3}{9}\,dx$

$= \left[\frac{x^4}{36}\right]_0^3 = \frac{3^4}{36} - 0 = \frac{81}{36} = \frac{9}{4} = \underline{\underline{2.25}}$

$V(X) = E(X^2) - E(X)^2$

$E(X^2) = \frac{1}{9}\int_0^3 x^4\,dx = \frac{1}{9}\left[\frac{x^5}{5}\right]_0^3 = \frac{1}{9}\left(\frac{3^5}{5} - 0\right) =$

$= \frac{1}{9} \cdot \frac{243}{5} = \frac{27}{5}$

$V(X) = \frac{27}{5} - \frac{9^2}{4^2} = \frac{27}{5} - \frac{81}{16} = \underline{\underline{0.3375}}$

Answer: $E(X) = 2.25$ & $V(X) = 0.3375$

2\. a) There are 2 events:

Anna takes bus 1 or bus 2.

The mean of the time Anna spends walking to the bus (x) is the sum of both events divided by 2: $E(X) = \frac{5+10}{2} = 7{,}5$

$$V(x) = E(x^2) - E(x)^2$$

$$E(x^2) = \frac{5^2+10^2}{2} = 62{,}5$$

$V(x) = 62{,}5 - 7{,}5^2 = 6{,}25$ Answer: 6,25.

b) Let Y be the event Anna is on time.

Let A be the event she took bus 1.

Let B be the event she took bus 2.

$$P(Y) = P(Y|A)\cdot P(A) + P(Y|B)\cdot P(B)$$

$P(Y|A) = 1 - 0{,}15 = 0{,}85$ $\quad P(A) = 0.5$

$P(Y|B) = 1 - 0{,}05 = 0{,}95$ $\quad P(B) = 0.5$

$$P(Y) = 0{,}85\cdot 0{,}5 + 0{,}95\cdot 0{,}5 = 0.9$$

Answer: The probability that Anna is on time is 0.9.

c) Bayes' rule:

$$P(A|Y) = \frac{P(Y|A)\cdot P(A)}{P(Y)} = \frac{0{,}85\cdot 0{,}5}{0{,}9} \approx 0{,}47$$

Answer: If Anna is on time the probability that she took bus 1 is 0,47

3. a) We can use the Central limit theorem since the sample size is 55 we get normal distribution.

$\mu = 5 \qquad \sigma = 2 \qquad n = 55$

Standardize $\quad z = \dfrac{Y - 55\cdot 5}{\sqrt{55}\cdot 2} = \dfrac{Y - 275}{\sqrt{220}}$

$$P(Y \leq 290) = P\left(Z \leq \frac{290 - 275}{\sqrt{220}}\right) \approx P(Z \leq 1.01)$$

$P(Z \leq 1.01) \approx \underline{0.8438.}$

b) Let X be the difference of group A's and group B's size

The mean of X size $= 55\cdot 5 - 45\cdot 5 = 50$

$Var(A) = 4 \quad Var(B) = 4$

$Var(A-B) = Var(A) + Var(B) = 8 \qquad n = 100$

$$Z = \frac{X - 50}{\sqrt{100\cdot 8}} = \frac{X - 50}{\sqrt{800}}$$

$$P(X > 0) = P\left(Z > \frac{0 - 50}{\sqrt{800}}\right) \approx P(Z > -1{,}77)$$

$P(Z < 1{,}77) \approx \underline{0{,}9616}$