Signal processing in Digital Holography

COMP.SGN.240

Advanced Signal Processing Laboratory

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Abstract

The goal is to learn how digital signal processing is applied to off-axis digital holographic 2D data to obtain 3D reconstructions of an object. Laboratory work involves 2D Fourier filtering and angular spectrum wavefront propagation.

1 Introduction

Holography is a class of methods for recording and reconstructing three-dimensional images based on interference phenomena. Holographic images are called holograms. Unlike normal photographic images, they do not use a mapping of individual object points to individual points in the hologram; in that sense, they are not images. Digital holography refers to acquiring and processing holograms with a digital sensor array, typically a CMOS camera or a similar device. Reconstruction of object data is performed numerically from digitized interferograms [1].

Digital holography has a huge number of practical applications, such as microscopy imaging, information encryption, object recognition data storage, measurements of thermal fields, measuring statistics of hydrodynamics facilities, testing vibrations, investigations of strained surfaces, and object displacements. There are specific problems in digital holography, such as zero-order and conjugate images, sampling, and sensitivity to vibration of optical components. Many different techniques have been proposed that help avoid some of these problems. For dynamic event investigations, off-axis digital holography is applied. As it is well known optical wavefront that goes though an object is described by a complex-domain equation: $U_0(x,y) = E_0(x,y)\exp[i\phi_0(x,y)]$, here $E_0(x,y)$ is an objects' amplitude, and $\phi_0(x,y)$ is an objects' phase, x and y are spatial coordinates. A hologram is interference of two wavefronts: one that goes from an object $U_0(x,y)$ and another one that goes unperturbed $U_r(x,y)$, named reference wavefront, $U_r(x,y) = E_r(x,y)\exp[i\phi_r(x,y)]$, see Fig. 1. Since we may

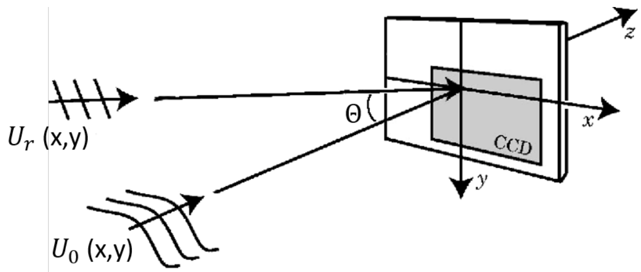

Figure 1: Off-axis configuration of the object $U_0(x, y)$ and reference $U_r(x, y)$ wavefronts.

register only the squared absolute value of the interferogram, the equation for hologram is as follows:

$$H(x, y) = E_0(x, y)^2 + E_r(x, y)^2 + U_0(x, y)U_r^*(x, y) + U_0^*(x, y)U_r(x, y). \quad (1)$$

1.1 2D Fourier filtering

Your task is to recover object wavefront, $U_0(x, y)$, from the hologram. For this purpose, you will employ spatial frequency separation by use of the 2D Fourier transform of the hologram $H(x, y)$:

$$\begin{aligned} \mathcal{F}[H(x, y)] &= \mathcal{F}\left[E_0(x, y)^2 + E_r(x, y)^2\right] + \\ &+ \mathcal{F}\left[U_0(x, y)E_r \exp(i2\pi\eta)\right] + \mathcal{F}\left[U_0^*(x, y)E_r \exp(-i2\pi\eta)\right], \end{aligned} \quad (2)$$

Here $\mathcal{F}[\,]$ is the Fourier transform, the symbol '*' is for the complex conjugation, and η is a shift between the wavefronts in the frequency domain, $\eta = \frac{\sin(\Theta)}{\lambda}$, its' value is dictated by the angle Θ between the object and reference wavefronts and by the light wavelength utilized, λ. The second and third terms of the Equation 2 are calculated as:

$$\mathcal{F}\left[U_0(x, y)E_r \exp(i2\pi\eta)\right] = \mathcal{F}\left[U_0\right] E_r \otimes \delta(f - \eta) \quad (3)$$

$$\mathcal{F}\left[U_0^*(x, y)E_r \exp(-i2\pi\eta)\right] = \mathcal{F}\left[U_0^*\right] E_r \otimes \delta(f + \eta) \quad (4)$$

Here $\otimes$ is the convolution operator, and δ is the Kronecker delta. It is seen from Equations (3,4) that Fourier orders are shifted in the frequency domain, and the shift governs the spatial distribution of the orders of Equation 2. The first term is localized in the zero- and low-frequency region and the second and third are localized symmetrically near frequencies $\pm\eta$. The object's wavefront is located in the second term (Eq.3), therefore, we should filter out it using spatial information. It is done by zeroing all other orders in the frequency domain. The second term is then placed in the center of the frequency domain, and its inverse Fourier transform produces the object wavefront $U_0(x, y)$ [2]. Figure 2 illustrates the reconstruction processing line.

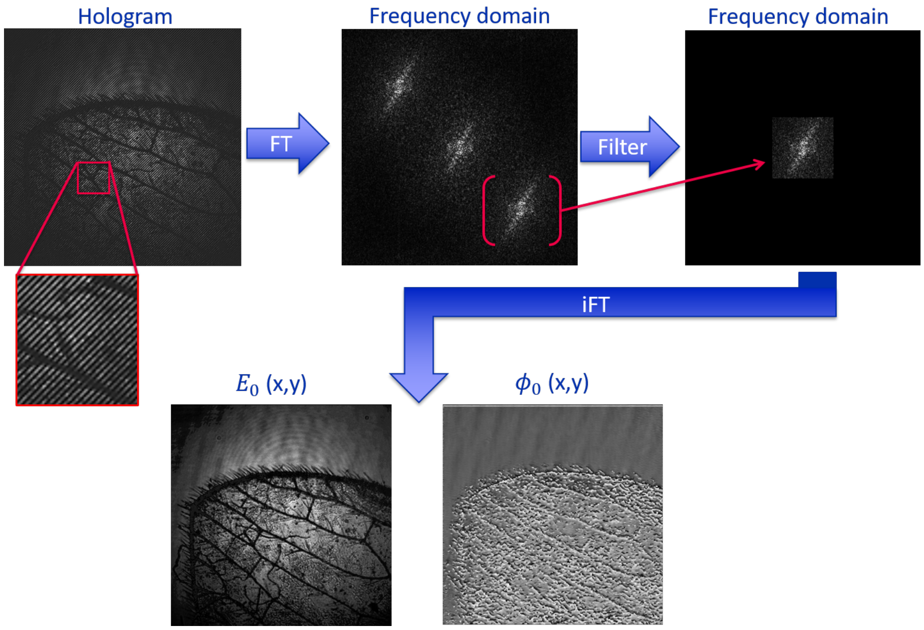

Figure 2: 2D Fourier filtering scheme.

1.2 Wavefront propagation

2D Fourier filtering reconstructs the object wavefront in the detector plane. To get a focused image of the object, it is necessary to produce back-propagation of the reconstructed wavefront to the object plane. For this purpose, you will use the Rayleigh-Sommerfeld model with the transfer function defined through the angular spectrum (AS) ([3], Eq. (3-74)):

$$U_0(x,y,z) = \mathcal{F}^{-1}\Big[H(f_x,f_y,z)\cdot\mathcal{F}\big[U_0(x,y,0)\big]\Big] \tag{5}$$

$$H(f_x,f_y,z) = \begin{cases}\exp\left[i\frac{2\pi}{\lambda}z\sqrt{1-\lambda^2(f_x^2+f_y^2)}\right], f_x^2+f_y^2\leq\frac{1}{\lambda^2},\\ 0,\ \text{otherwise},\end{cases} \tag{6}$$

where the operators $\mathcal{F}$ and $\mathcal{F}^{-1}$ stay for the Fourier transform (FT) and inverse Fourier transforms, respectively; $U_0(x,y,z)$ is a wavefront propagated at the distance z from the initial position $U_0(x,y,0)$; $H(f_x,f_y,z)$ is the angular spectrum transfer function; f_x, f_y are spatial frequencies; x, y denote spatial coordinates, and λ is the wavelength.

As can be seen from Equation 5, propagation also involves Fourier transforms, therefore, for the realization of propagation, we may include AS transfer function multiplication into the operation of spatial filtration described in Section 1.1.

2 Laboratory equipment and setup

The laboratory SC102a contains the equipment needed for this assignment. The setup is depicted in Figure 3. It is a traditional Mach-Zehnder interferometer, which works in a transmissive regime with transparent objects. The setup consists of 2 mirrors, 2 beamsplitters, a coherent light source (laser), and a hologram-detecting array (CMOS camera).

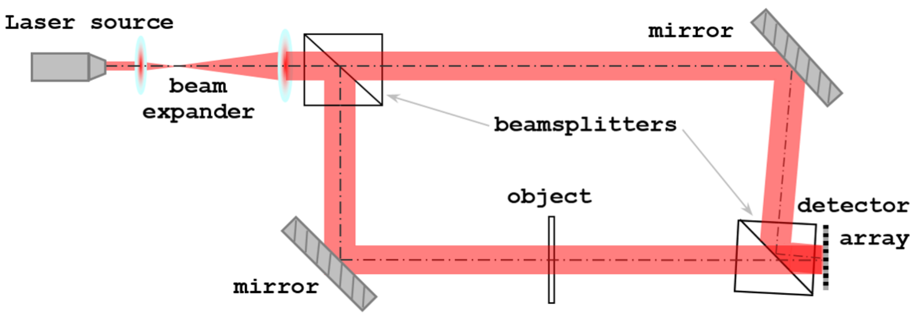

Figure 3: Mach-Zehnder interferometer.

3 Tasks

1. Carefully read the safety instructions and sign the instruction list.
2. Turn on the laser and check the alignment of the optical system by observing the hologram fringes on a monitor. Adjust angles between the object and reference wavefronts so that the width of the fringes becomes about 12 pixels of the camera.
3. Record hologram without the object.
4. Place the object in the object arm of the interferometer and record the object's hologram.
5. Change the angle between the object and reference wavefronts so that the width of the fringes becomes about 5 pixels of the camera.
6. Produce steps 3 and 4 with the new width of the fringes.
7. Write Matlab script for the wavefront reconstruction as it is described in the introduction section. Reconstruct the object wavefront $U_0(x, y)$. Additional useful information for script writing is in the book [4]. For successful reconstructions, request information about the wavelength used, pixel size of the camera, number of pixels, and distance between the camera and object.

8. Make a comparison of two reconstructions of the object. Plot cross-sections of the amplitudes and phases of the reconstructed wavefronts. Describe the difference and reasons for it.

9. Prepare report.

4 Report

Submit your report to Moodle in *.pdf format by the date given on the Moodle course page. Include in the report the measurement data and the phase images you obtained. Discuss also what are the possible sources of errors. Attach your commented Matlab function(s).

References

[1] Wikipedia, *Digital holography* (https://en.wikipedia.org/wiki/Digital_holography).

[2] U. Schnars and W. Jüptner, "Direct recording of holograms by a ccd target and numerical reconstruction," Applied optics **33**, 179–181 (1994).

[3] J. W. Goodman, *Introduction to Fourier optics* (Roberts and Company Publishers, 2005).

[4] T.-C. Poon, *Optical scanning holography with MATLAB*, vol. 21 (Springer, 2007).