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Signal processing in digital holography - Report

Faculty of Information Technology and Communication Sciences (ITC)
Laboratory Report

Abstract

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Laboratory Report
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This report documents the work done in the Signal processing in digital holography assingment as a part of the Advanced signal processing laboratory course. In this assignment we familiazired ourselves with basics of intereference based holography.
Keywords: Hologram, Mach-Zehnder interferometer, Wave propagation.

Contents

1 Introduction

In this report we describe our work done in the 'Signal processing in digital holography' laboratory assignment for the Advanced Signal Processing Laboratory course. In this assignment we used a Mach-Zehnder interferometer to record holographic images and Matlab to reconstruct the object from the recorded wavefront.

2 Methodology

This section describes the methods and equations we used to capture the holograms and reconstruct the objects.

2.1 Experiment setup

Holographic imagery relies on the phenomena of light wave superpositions. In this assignment the object is captured using a Mach-Zehnder interferometer, where a monochromatic laser is used to produce two wavefronts. The first wavefront passes trough the object which we want to record, and the second wavefront is used as reference wavefront. The wavefront are the combined to a single wavefront via the interference phenomena, so that one of the wavefronts is combined with the other wavefront in a slight angle, causing a phase difference. The resulting total wave is then captured with a CMOS camera to produce a image file. The phase difference is detectable as fringes in the captured images. The complete setup for the experiment is shown in figure 2.1.

2.2 Object reconstruction

The object can be extracted from the captured hologram, using a two step method, Fourier filtering and wavefront propagation.

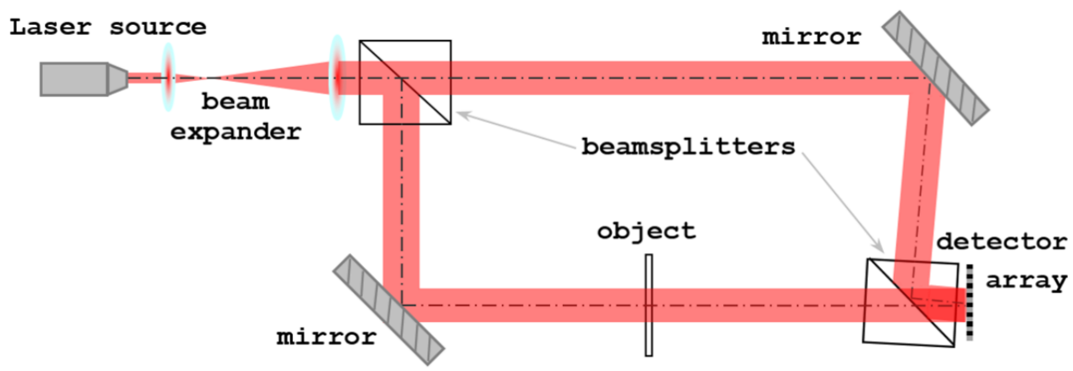


Figure 2.1 *Experiment was done using a Mach-Zehnder interferometer. Taken from [***assignment***]*

2.2.1 Fourier filtering

First step is to use Fourier filtering to extract the object wave. Since we know that the hologram is defined by the following equation:

$$H(x,y) = E_0(x,y)^2 + E_r(x,y)^2 + U_0(x,y)U_r^*(x,y) + U_0^*(x,y)U_r(x,y) \tag{2.1}$$

Where $H(x,y)$ is the hologram wavefront in given position, E_0 is the objects amplitude, E_r is the reference wave's amplitude, U_r is the reference wavefront and U_0 is the object wavefront which we want to extract. The extraction can the be accomplished by performing frequency filtering to the Fourier transformation of the hologram, which is shown in equation 2.2. From the Fourier transformation we can see that the second term of the hologram contains the object wavefront U_0. Equations 2.3 and 2.4 show the Fourier transformations for the second and third term respectively.

$$\begin{aligned}\mathcal{F}[H(x,y)] = \mathcal{F}\left[E_0(x,y)^2 + E_r(x,y)^2\right] + \mathcal{F}\left[U_0(x,y)E_r \exp(i2\pi\eta)\right] \\ + \mathcal{F}\left[U_0^*(x,y)E_r \exp(-i2\pi\eta)\right]\end{aligned} \tag{2.2}$$

$$F[U_0(x,y)E_r \exp(i2\pi\eta)] = F[U_0]E_r \otimes \delta(f-\eta) \tag{2.3}$$

$$F[U_0^*(x,y)E_r \exp(-i2\pi\eta)] = F[U_0^*]E_r \otimes \delta(f+\eta) \tag{2.4}$$

By removing all the other terms from the Fourier transformation of the hologram, we are left with only the Fourier transformation of the object wave. The object wave can the be brought back to spatial domain with inverse Fourier transformation. The figure 2.2 shows how the second term is isolated from the rest of the hologram.

2.2.2 Wave propagation

The wavefront alone isn't that usable. The object needs to be projected to an object plane using wave propagation. Using Rayleigh-Sommerfield holographic model with the angular spectrum transfer function defined in equation 2.5 the object is projected on the object plane as defined by the equation 2.6, where z is the propagation distance.

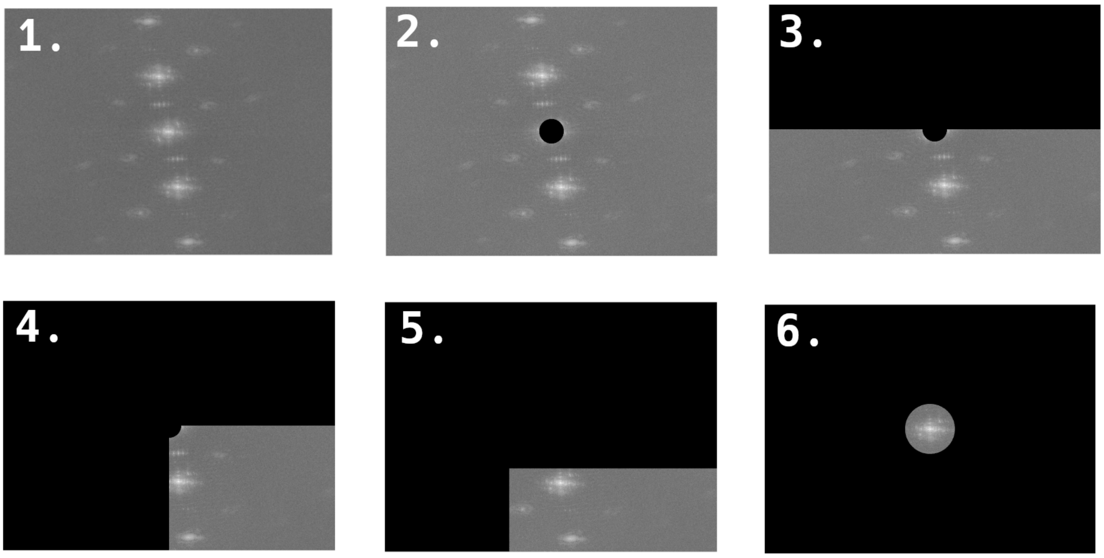


Figure 2.2 *Extracting the second term from Fourier transformation of the hologram*

$$H(f_x, f_y, z) = \begin{cases} \exp[i2\pi z\sqrt{1-\lambda^2(f_x^2+f_y^2)}], & f_x^2+f_y^2 \leq 1/\lambda^2 \\ 0, & \text{otherwise} \end{cases} \tag{2.5}$$

$$U_0(x, y, z) = F^{-1}[H(f_x, f_y, z) \cdot F[U_0(x, y, 0)]] \tag{2.6}$$

To reconstruct an object that is in focus, we need to determine the optimal distance for the wave to be propagated. This can be accomplished with an autofocusing algorithm or manually by comparing reconstruction results with different values of z by eye.

3 Results

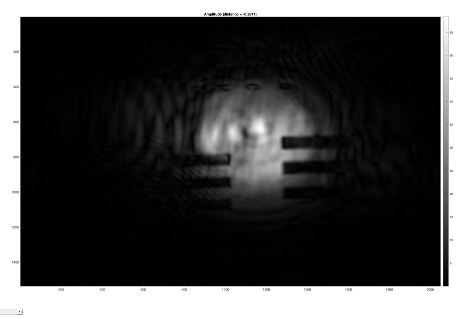
This section of report presents the results of the wavefront reconstruction, and the analysis of those results. The object wavefront reconstruction process described in the section 2 was implemented as a Matlab script, using a provided implementation of the wave propagation algorithm.

3.1 Wavefront Reconstruction

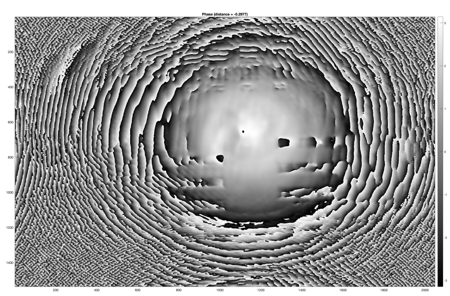
In the laboratory we captured to hologram containing the same object with two different fringe widths. The distance between ray and camera was 11 cm, wavelength was $532 * 10^{-9}$ m, and a number of pixels was $3,45 * 10^{-6}$ m. The first hologram had fringes of 5 pixels and the second had fringes of 12 pixels. The figures 3.1 and 3.2 show the amplitude and phase images of the reconstructed object wavefronts. To determine the optimal propagation distance (z) for our figures, we implemented a "slider" to dynamically adjust the distance value. With this tool, we fine-tuned the reconstruction to get the sharpest phase image. In 5 pixel fringe hologram, the best propagation distance was 29,77 cm. In 12 pixel fringe hologram, the sharpest details of phase was determined to be 29,86 cm.

3.2 Analysis

From the images we can see that the picture with smaller fringes had a more clear reconstruction, and the image with larger fringes was more blurry even after fine tuning the focus. This is because the smaller fringes enable for finer phase information to be captured. This is how higher resolution cameras can capture more information than lower resoultion cameras, creating sharper images when capturing objects in focus.

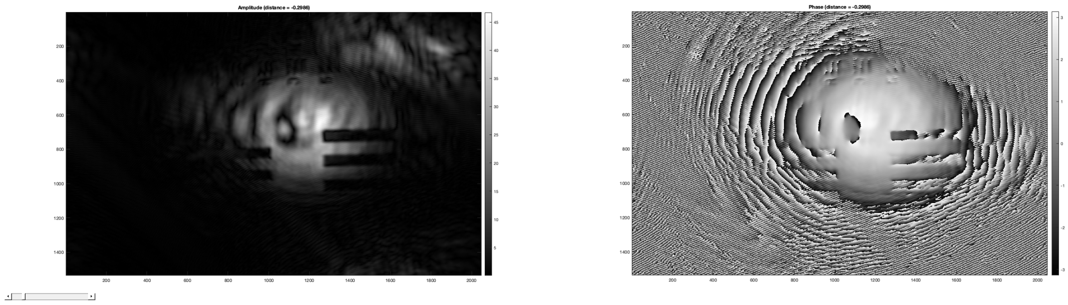


(a) Amplitude



(b) Phase

Figure 3.1 *Object wavefront reconstruction of the 5px fringe image*



(a) Amplitude (b) Phase

Figure* *3.2 *Object wavefront reconstruction of the 12px fringe image*

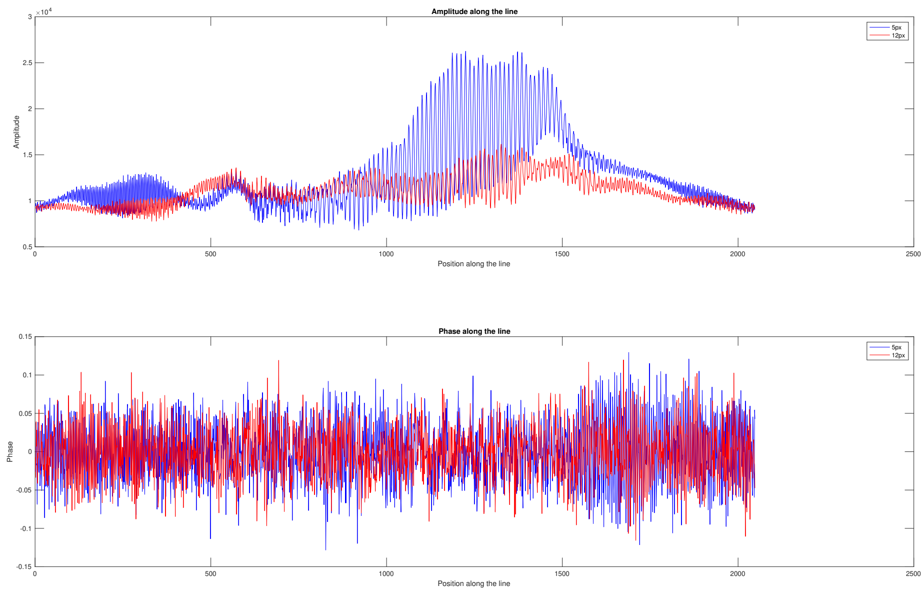


Figure* *3.3 *Amplitude and phase graphs for objects with 5px and 12px fringes*

Figure 3.3 compares the mean amplitudes and phases of between the two captured object wavefronts. From the figure we can see that wavefront with 5px fringes has noticebly higher amplitude along the part of the image where the object is. This indicates that more light is present at that part in comparision to the image with 12px fringes. [TODO: Lue kirjasta tälle selitys]

The phase comparison doesn't tell more than that the wavefronts are in different phases. This makes sense since the angle of the mirrors is different between the two images, causing interference in different phase.

4 Conclusions

The goal of this assignment was to learn about signal processing in digital holography by capturing and reconstructing an object from a holographic image, by methods of Fourier filtering and wavepropagation. The hologram was created and captured with a Mach-Zehnder interferometer, and the reconstruction was succesfully implemented with a Matlab script.

APPENDIX

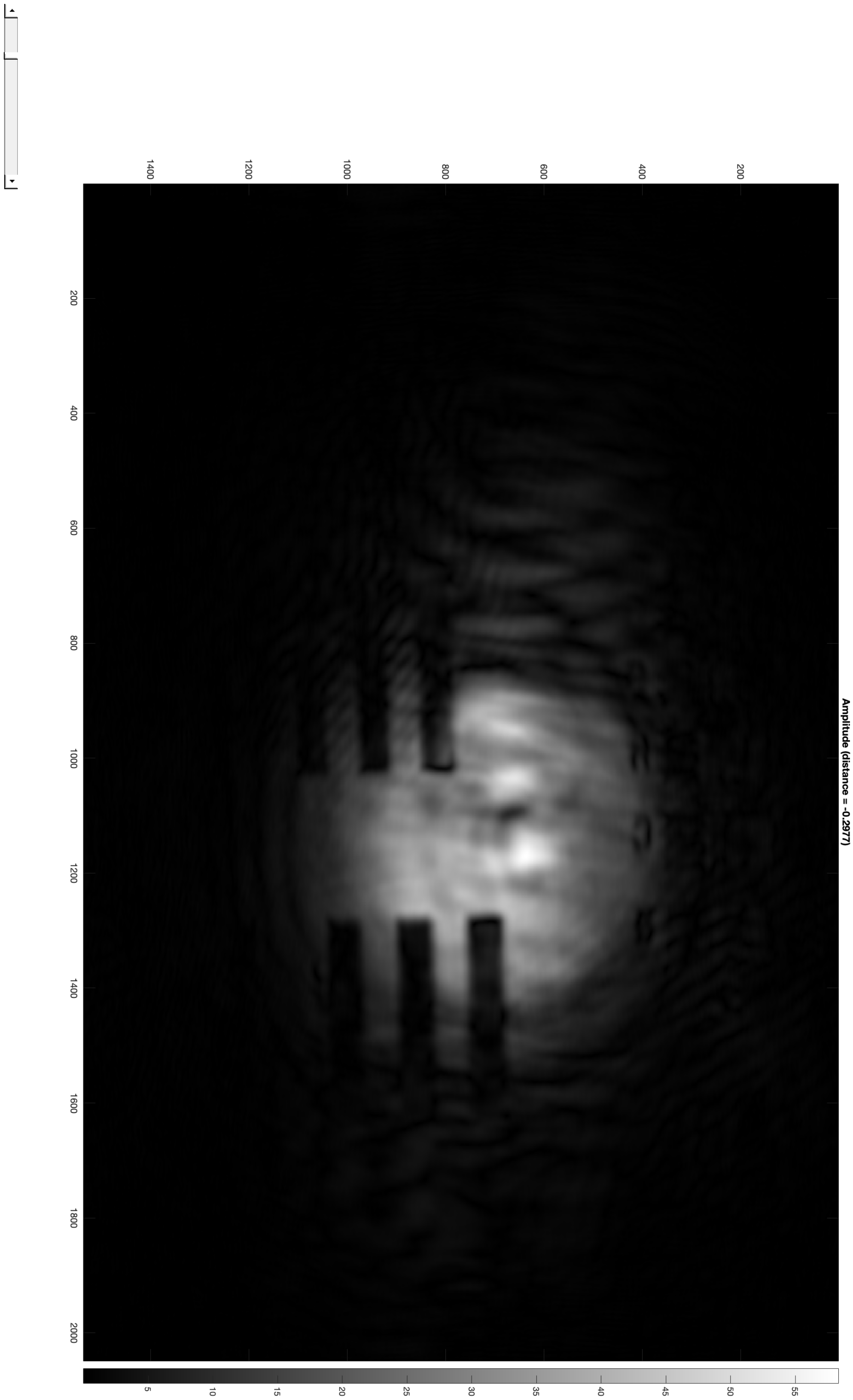


Figure 1 *Amplitude of 5px image*

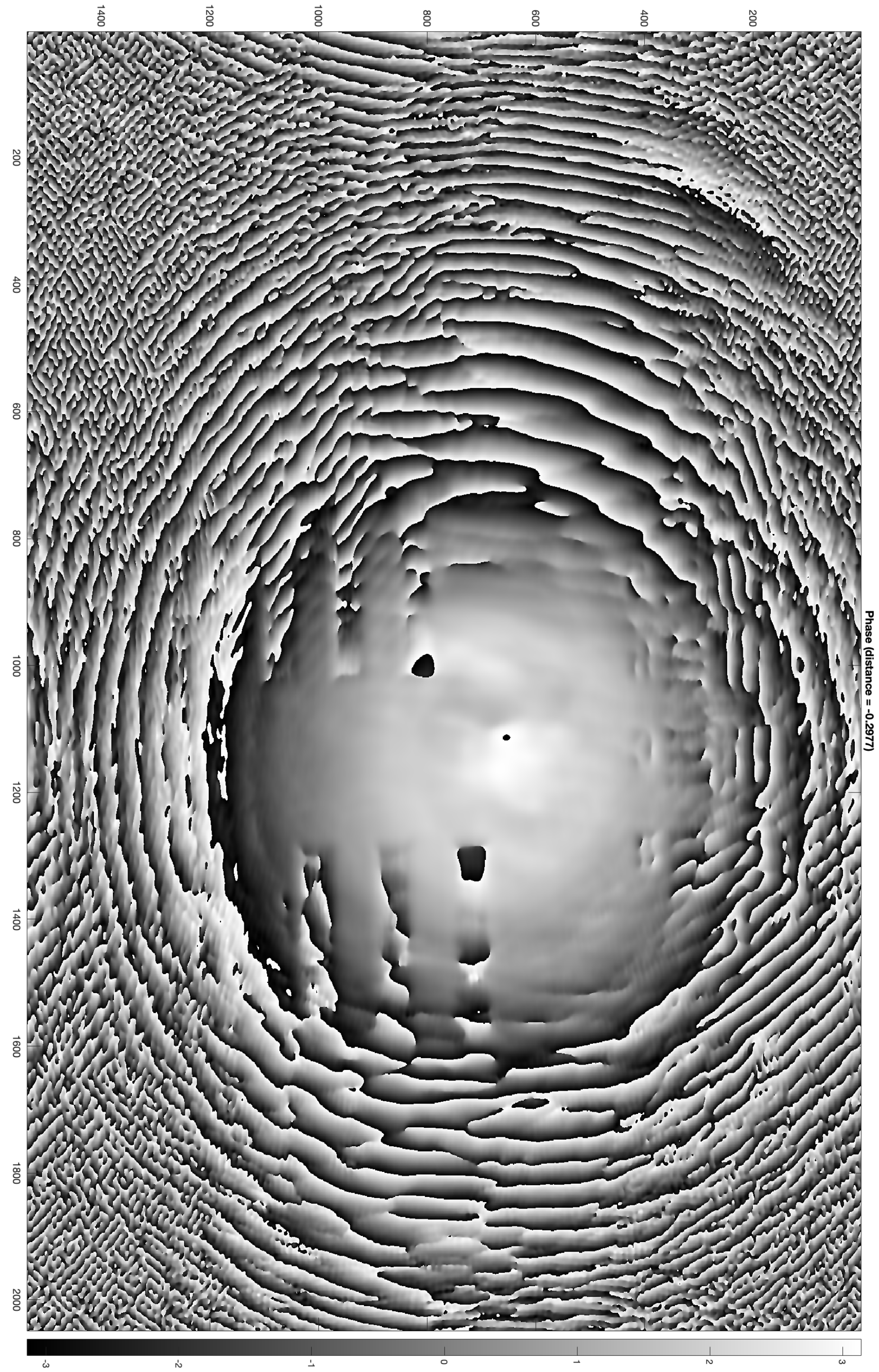


Figure **2** *Phase of 5px image*

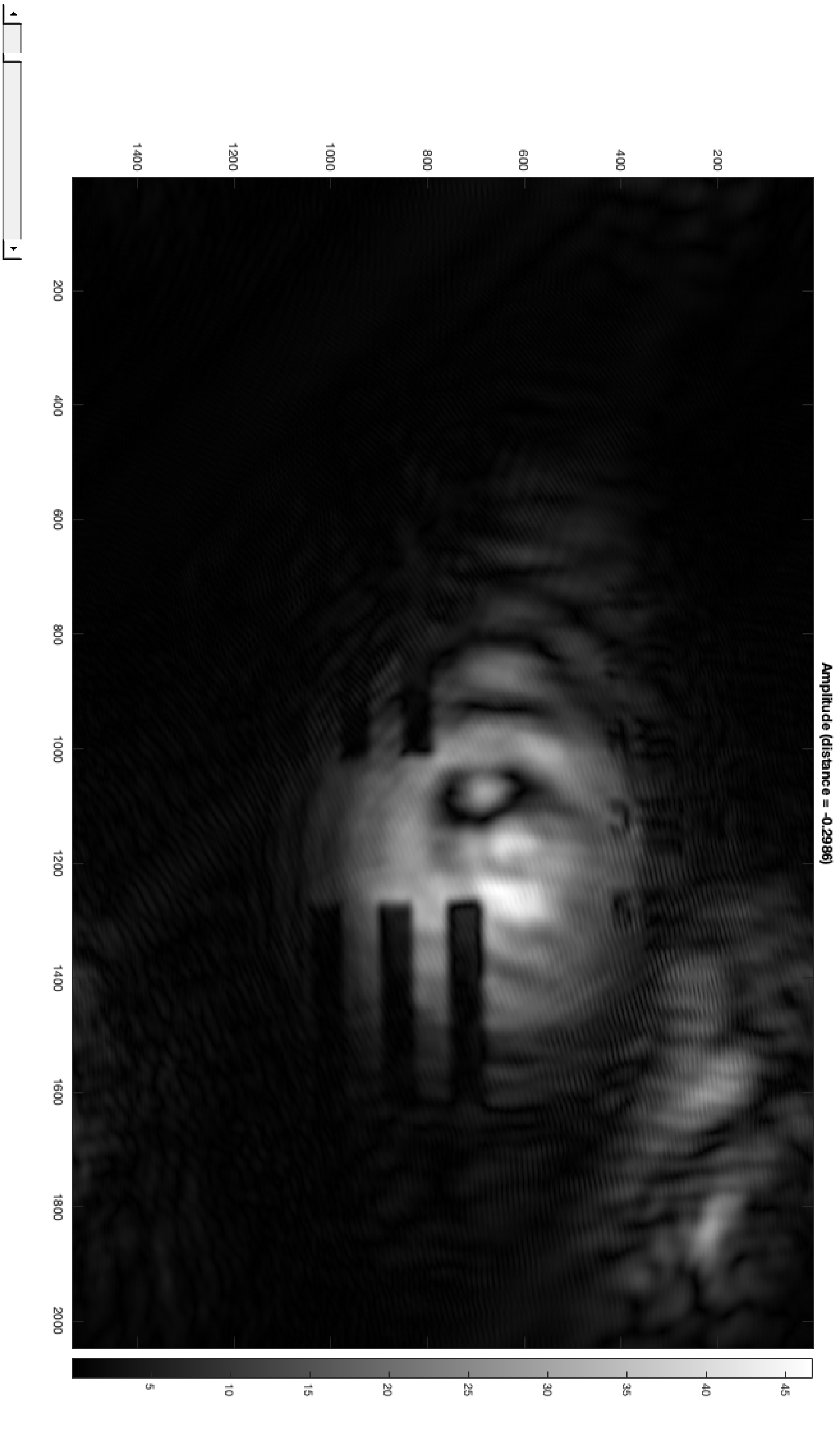


Figure 3 *Amplitude of 12px image*

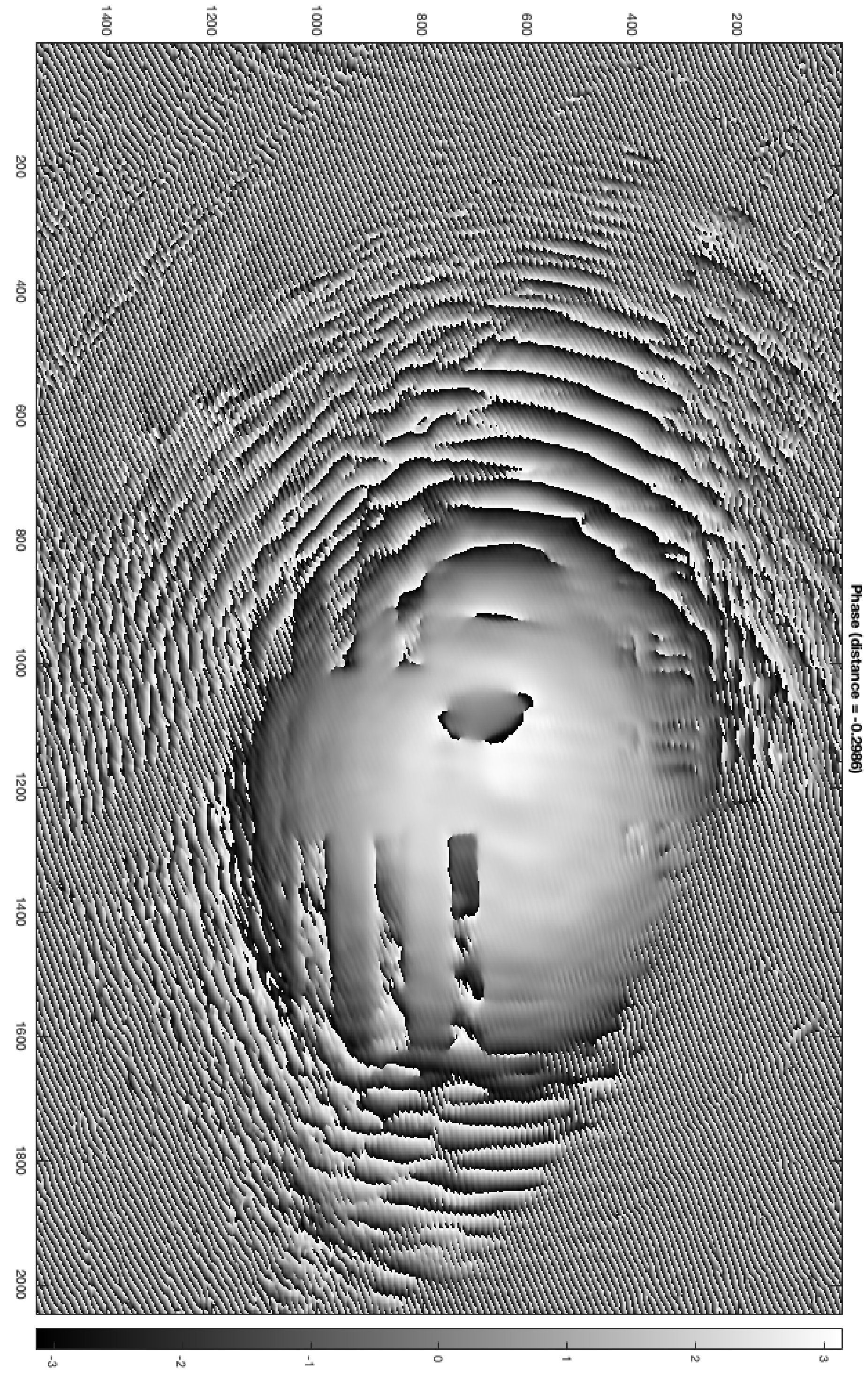


Figure 4 *Phase of 12px image*

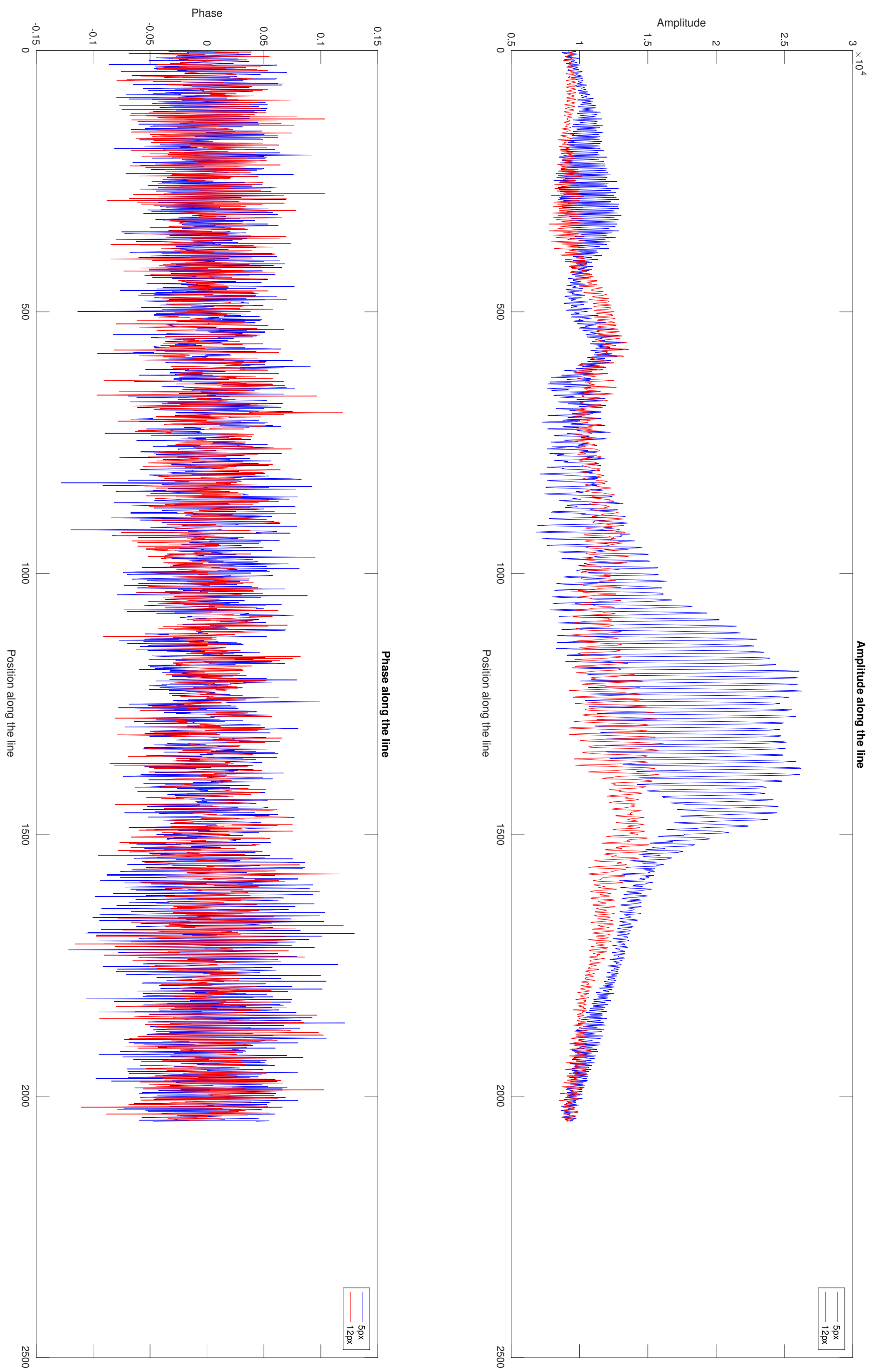


Figure 5 *Mean amplitudes and phases between 5px and 12px wavefronts*