Physics

Academic Year: 2012-2013 Marks: 70
Date: October 2012

Question 1 | Attempt Any One **[7]**

Question 1.1: Answer in brief: **[7]**
Derive an expression for the period of motion of a simple pendulum. On which factors does it depend?

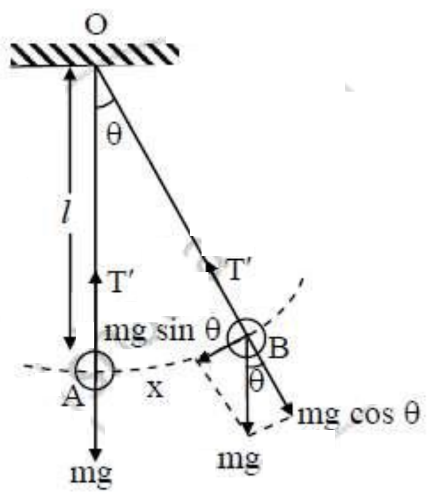
Solution: a) Consider a simple pendulum of mass ‘m’ and length ‘L’.

L = l + r,

Where l = length of string

r = radius of bob

b) Let OA be the initial position of pendulum and OB, its instantaneous position when the string makes an angle θ with the vertical.



In the displaced position, two forces are acting on the bob:

- Gravitational force (weight) ‘mg’ in a downward direction
- Tension T' in the string.

c) Weight ‘mg’ can be resolved into two rectangular components:

- Radial component mg cos θ along OB and
- Tangential component mg sin θ perpendicular to OB and directed towards mean position.

d) mg cos θ is balanced by tension T' in the string, while mg sin θ provides restoring force

$\therefore F = -mg\sin\theta$

Where a negative sign shows that force and angular displacement are oppositely directed.
Hence, restoring force is proportional to sin θ instead of θ. So, the resulting motion is not S.H.M.

e) If θ is very small then,

$$\sin\theta \approx \theta = \frac{x}{L}$$

$$\therefore F = -mg\frac{x}{L}$$

$$\therefore \frac{F}{m} = -g\frac{x}{L}$$

$$\therefore \frac{ma}{m} = -g\frac{x}{L}$$

∴ "a" = - "g"/"L" "x" ....(i)

$\therefore a \alpha - x$ .... $\left[\therefore \frac{g}{L} = \text{constant}\right]$

f) In S.H.M,

$a = -\omega^2 x$ ....(ii)

Comparing equations (i) and (ii), we get,

$$\omega^2 = \frac{g}{L}$$

But, $\omega = \frac{2\pi}{T}$

$$\therefore \left(\frac{2\pi}{T}\right)^2 = \frac{g}{L}$$

$$\therefore \frac{2\pi}{T} = \sqrt{\frac{g}{L}}$$

$$\therefore T = 2\pi\sqrt{\frac{L}{g}} \quad ....(iii)$$

Equation (iii) represents time period of simple pendulum.

g) Thus period of a simple pendulum depends on the length of the pendulum and acceleration due to gravity.

Question 1.1: A ballet dancer spins about a vertical axis at 2.5Π rad/s with his both arms outstretched. With the arms folded, the moment of inertia about the same axis of rotation changes by 25%. Calculate the new speed of rotation in r.p.m.

Solution: $\omega_1 = 2.5\,\pi$ rad/s $= 2\pi n_1$

$$\therefore I_2 = \frac{3}{4} I_1, n_2\,(r.p.m) = ?$$

By conservation of angular momentum,

$I_2\omega_2 = I_1\omega_1$ or $I_2\,(2\pi n_2) = I_1\,(2\pi n_1)$

$$\therefore \quad n_2 = \frac{I_1 n_1}{I_2} = \frac{I_1 \times 1.25}{\frac{3}{4} I_1} = \frac{1.25 \times 4}{3}$$

$$\therefore \quad n_2 = \frac{5}{3}\ \text{r.p.s.}$$

$$= \frac{5}{3} \times 60 = 100\ \text{r.p.m}$$

The new speed of rotation in r.p.m is 100 r.p.m

Question 1.2: Discuss different modes of vibrations in an air column of a pipe open at both the ends. **[7]**

Solution: First mode or fundamental mode:

In this mode of vibration, there is one node at the centre of the pipe and two antinodes, one at each open end as shown in figure (a).

Let,

v = wave velocity in air

n = fundamental frequency

λ = wavelength

L = length of air column

$v = n\lambda$ ….(1)

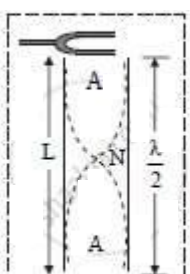


Figure (a)

Also $L = \frac{\lambda}{2}$

$\therefore \lambda = 2L$ ....(2)

From Equation 1 and 2

$v = n2L$

$n = \frac{v}{2L}$.......(3)

Equation (3) represents fundamental frequency or lowest frequency of vibration

ii. Second mode or first overtone:
In this mode of vibration, there are two nodes and three antinodes as shown in figure (b).

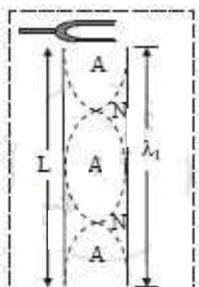


Figure (b)

Let,
v = wave velocity in air [As the medium is same, wave velocity remains same]
n1 = next higher frequency
λ1 = corresponding wavelength
L = length of tube
Velocity of wave is given by,

$v = n_1\lambda_1$ ....(4)

Also $L = \lambda_1$ ....(5)

From equation (4) and (5),

$v = n_1 L$

$n_1 = \frac{v}{L}$

$n_1 = 2 \times \frac{v}{2L}$.................(6)

From equation (3) and (6)

$n_1 = 2n$ ....(7)

Thus, frequency of first overtone (second harmonic) is twice the fundamental frequency

iii. Third mode or second overtone:
In this mode of vibration, there are 3 nodes and 4 antinodes as shown in figure (C).
Let,
v = wave velocity in air (As the medium is same, wave velocity remains same)
n2 = next higher frequency
λ2 = corresponding wavelength
L = length of tube
Velocity of wave is given by,

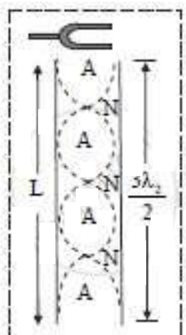


Figure (c)

$v = n_2\lambda_2$ ....(8)

Also $L = \frac{3\lambda_2}{2}$

$\lambda_2 = \frac{2L}{3}$

From equation (8) and (9),

$v = n_2\frac{2L}{3}$

$n_2 = \frac{3v}{2}L$..............(10)

From equation (3) and (10),

$n_2 = 3n$

Thus, frequency of second overtone is thrice the fundamental frequency.

Question 1.2: Draw neat labelled diagrams for modes of vibration of an air column in a pipe when it is closed at one end.
Hence derive an expression for fundamental frequency in each case.

Solution: Modes of vibration of an air column in a pipe when it is closed at one end:

1) **First mode or Fundamental mode**: In this mode of vibration, there is one node at the closed end and one antinode at the open end as shown in figure (a).

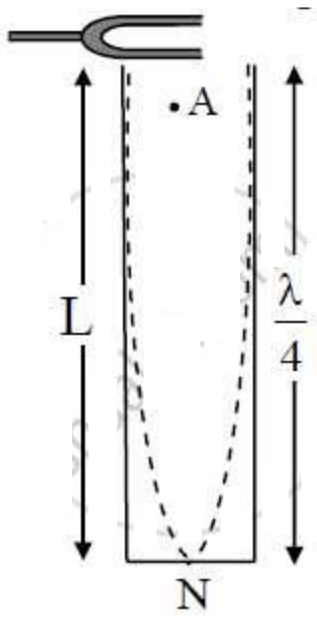


Fig. (a)

Let,
L = length of air column

v = wave velocity in air

n = fundamental frequency

λ = wavelength

Velocity of wave in a tube is given by,

v = nλ ....(i)

But $L = \frac{\lambda}{4}$

From equation (i),

v = 4nL

$\therefore n = \frac{v}{4L}$ .....(2)

Equation (ii) represents fundamental frequency or first mode of vibration.

2) Second mode or first overtone:

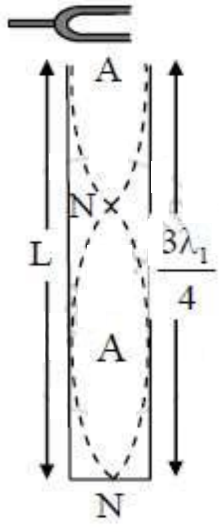


Fig. (b)

3) Third mode or second overtone:

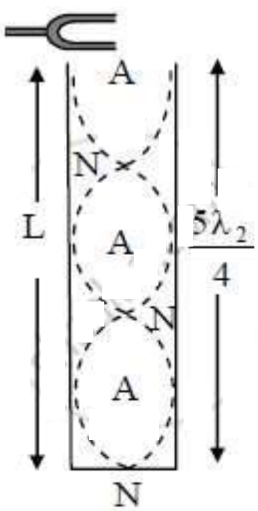


Fig. (c)

Question 1.2: A soap bubble of radius 12 cm is blown. Surface tension of soap solution is 30 dyne/cm. Calculate the work done in blowing the soap bubble.

Solution: r = 12 cm, T = 30 dyne/cm, W = ?

Initial surface area of soap bubble = 0

Final surface area $\Delta A = 2 \times 4\pi r^2$

$\therefore$ Increase in surface area $= 2 \times 4\pi r^2$

$\therefore$ Work done $W = T \times \Delta A$

$= 30 \times 8 \times 3.14 \times (12)^2$

$= 108518.4$ erg

$= 108518.4 \times 10^{-7}$

$= 1.085 \times 10^{-2}$ J

The work done in blowing the soap bubble is 1.085×10^{-2} J.

Question 2 | Attempt any THREE: [9]

Question 2.1: In a conical pendulum, a string of length 120 cm is fixed at rigid support and carries a mass of 150 g at its free end. If the mass is revolved in a horizontal circle of radius 0.2 m around a vertical axis, calculate tension in the string ($g = 9.8\ m/s^2$) [3]

Solution: l = 120 cm = 1.2 m,

r = 0.2 m,

m = 150 g = 150 × 10−3 kg = 0.15 kg

Tension in the supporting thread (T)

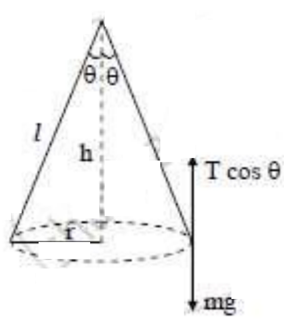
By Pythagoras theorem,

$l^2 = r^2 + h^2$

$h^2 = l^2 - r^2$

$h^2 = 1.44 - 0.04 = 1.4$

∴ h = 1.1 83 m



The weight of bob is balanced by vertical component of tension T

∴ T cos θ = mg

$$\cos\theta = \frac{h}{l} = \frac{1.183}{1.2} = 0.9858$$

$$\therefore \quad T = \frac{mg}{\cos\theta} = \frac{0.15 \times 9.8}{0.9858} = [\log(0.15) + \log(9.8) - \log(0.9858)]$$

$$= [\bar{1}.1761 + 0.9912 - \bar{1}.9938]$$

$$= \text{antilog}\,[0.1735]$$

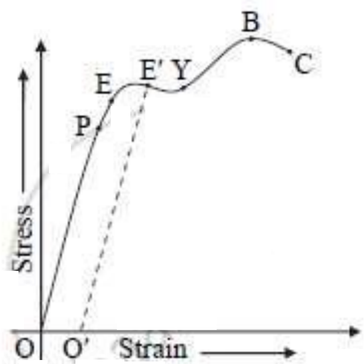
$$= 1.491\ \text{N}$$

$\therefore$ T = 1.491 N

The tension in the string is 1.491 N.

Question 2.2: Discuss the behaviour of wire under increasing load. **[3]**

Solution: Stress v/s strain graph:-



P : Proportional limit
E : Elastic limit
Y : Yield point
B : Breaking stress
C : Breaking point
OO′ : Permanent set

The behaviour of wire under increasing load can be explained with the help of stress v/s strain graph.

a. Proportional limit: The initial portion OP of the graph is a straight line indicating stress is directly proportional to strain. Thus Hooke’s law is completely obeyed in the region OP. Point P is called point of proportional limit.

b. Elastic limit: Beyond the point P, the stress-strain variation is not a straight line as indicated by the part PE of the graph.

If the wire is unloaded at point E, the graph between stress and strain follows the reverse path EPO, then the point E is called elastic limit. The portion between O and E is called elastic region.

c. Permanent set: If the load is increased so that stress becomes greater than that corresponding to the point E, the graph is no longer a straight line and the wire does not obey Hooke's law. If the wire is strained upto E′ beyond point E and then the load is removed, the wire does not regain its original length and there is a permanent increase in length. A small strain corresponding to OO' is set up permanently in the wire, called permanent set. However, the wire is still elastic and if loaded again, gives a linear relation shown by the dotted line O'E'.

d. Yield point: As the stress is increased beyond the elastic limit the graph is a curve and reaches a point Y where the tangent to the curve is parallel to the strain axis. This shows that for the stress corresponding to point Y the strain increases even without any increase in the stress. This is known
as plastic flow.

Point Y on the curve is called yield point.

The value of stress corresponding to yield point is called yield stress.

e. Breaking stress: When the wire begins to flow, its cross-section decreases uniformly and hence, the stress increases steadily. Later a neck or constriction begins to form at a weak point. The maximum stress corresponding to the point B is breaking stress.

f. Breaking point: Once the neck is formed, the wire goes on stretching even if the load is reduced, until the breaking point C is reached when the wire breaks.

Elastic limit:-

The maximum stress to which an elastic body can be subjected without causing permanent deformation is called as elastic limit.

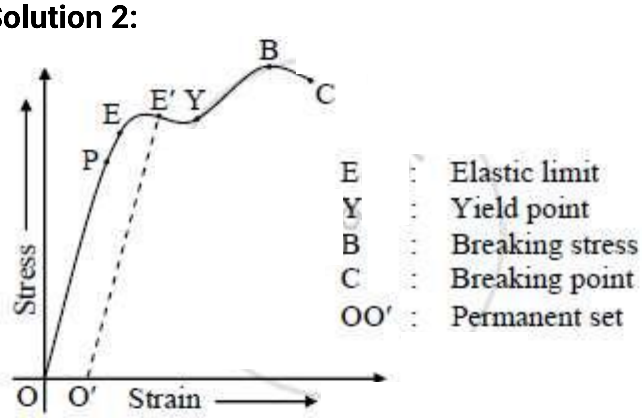
Yield point:-

The point on stress-strain curve at which the strain begins to increase without any increase in the stress is called yield point.

Breaking point:-

The point on stress-strain curve at which the wire breaks, is known as the breaking point.

Solution 2:



a) **Elastic limit**: The initial portion OE of the graph is a straight line, which indicates that upto the point E stress is directly proportional to strain. Hence, Hooke’s law is obeyed upto point E. In this region, wire is perfectly elastic and it completely regains its original length when the load is removed. Point E represents limit of proportionality between stress and strain.

b) **Permanent set**: If the load is increased so that stress becomes greater than that corresponding to the point E, the graph is no longer a straight line and the wire does not obey Hooke's law.

If the wire is strained upto E' beyond point E and then the load is removed, the wire does not regain its original length and there is a permanent increase in length. A small strain corresponding to OO' is set up permanently in the wire, called permanent set. However, the wire is still elastic and if loaded again, gives a linear relation shown by the dotted line O'E'.

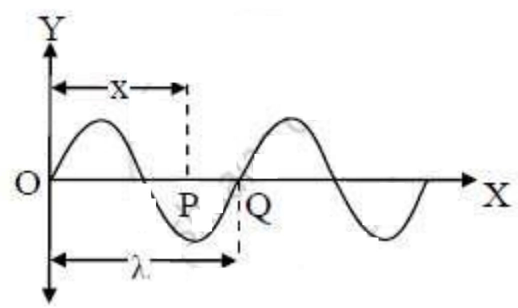
c) **Yield point**: As the stress is increased beyond the elastic limit the graph is a curve and reaches a point Y where the tangent to the curve is parallel to the strain axis. This shows that for the stress corresponding to point Y the strain increases even without any increase in the stress. This is known as plastic flow.
Point Y on the curve is called yield point.

The value of stress corresponding to yield point is called yield stress

e) **Breaking point**: Once the neck is formed, the wire goes on stretching even if the load is reduced, until the breaking point C is reached when the wire breaks.

Question 2.3: Derive an expression for one dimensional simple harmonic progressive wave travelling in the direction of positive X-axis. Express it in 'two' different forms. **[3]**

Solution: Equation of simple harmonic progressive wave:



a) Consider a simple harmonic progressive wave travelling in the direction of the positive X-axis. The vibrations of the particles of the medium is parallel to the Y axis.

b) At time t = 0, particle is at origin O i.e., the mean position. At instant 't', displacement of the particle is,

$y = A \sin \omega t$ ….(i)

Where, A = amplitude,

ω = angular velocity

c) Consider a particle P situated at a distance ‘x’ from O. If ‘δ’ be the phase difference between the particle P and O, then displacement of the particle of the medium at P in instant ‘t’ is given by,

$y = A \sin(\omega t - \delta)$ ....(ii)

d) A path difference of λ between two particles of a medium corresponds to a phase difference of 2π between them (e.g. particle O and Q). Since the path difference between P and O is ‘x’, so the phase difference ‘δ’ between them is given by,

$$\delta = \frac{2\pi x}{\lambda} \quad ....(iii)$$

Substituting equation (iii) in equation (ii) we have

$$y = A \sin\left[\omega t - \frac{2\pi x}{\lambda}\right] \quad ....iv$$

This equation gives the displacement of any particle of the medium at any instant. Hence, it is called equation of a simple harmonic progressive wave.

e) Since

$$T = \frac{2\pi}{\omega} \text{ i.e. } \omega = \frac{2\pi}{T}$$

From equation (iv),

$$y = A \sin\left[\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right]$$

$$\therefore \quad y = A \sin 2\pi\left[\frac{t}{T} - \frac{x}{\lambda}\right] \quad ....(v)$$

Equation of simple harmonic progressive wave in other forms:

a. Since $T = \frac{1}{n}$ i.e. $n = \frac{1}{T}$

$\therefore$ From equation (v),

$$y = A \sin 2\pi\left[nt - \frac{x}{\lambda}\right] \quad ....(vi)$$

b. Also, velocity of the wave is given by,

$$v = n\lambda \text{ i.e. } n = \frac{v}{\lambda}$$

$\therefore$ From equation (vi),

$$y = A \sin 2\pi\left[\frac{vt}{\lambda} - \frac{x}{\lambda}\right]$$

$$\therefore \quad y = A \sin \frac{2\pi}{\lambda}[vt - x] \qquad \text{....(vii)}$$

c. Since $v = n\lambda$ i.e. $\lambda = \frac{v}{n}$

$\therefore$ From equation (vi),

$$y = A \sin 2\pi\left[nt - \frac{nx}{v}\right]$$

$$\therefore \quad y = A \sin 2\pi n\left[t - \frac{x}{v}\right] \qquad \text{....(viii)}$$

All the above equations represent one-dimensional simple harmonic progressive wave, travelling in the direction of the positive X-axis.

Question 2.4: The kinetic energy of nitrogen per unit mass at 300 K is 2.5 × 106 J/kg. Find the kinetic energy of 4 kg oxygen at 600 K. (Molecular weight of nitrogen = 28, Molecular weight of oxygen = 32) **[3]**

Solution: m_1 = 1 kg, T_1 = 300 K, K_1 = 2.5 × 106 J

m_2 = 4 kg, T_2 = 600 K, M_1 = 28, M_2 = 32, K_2 = ?

$$K = \frac{3}{2}\frac{mRT}{M}$$

$$K_1 = \frac{3}{2}\frac{300}{28}$$

$$K_2 = \frac{3}{2}\frac{4 \times 600}{32}$$

$$\therefore \quad \frac{K_1}{K_2} = \frac{4 \times 600}{32} \times \frac{28}{300}$$

$$\therefore \quad K_2 = \frac{224}{32} \times 2.5 \times 10^6$$

$$\therefore \quad K_2 = 7 \times 2.5 \times 10^6$$

$$\therefore \quad K_2 = 17.5 \times 10^6 \text{ J}$$

The kinetic energy of 4 kg oxygen at 600 K is 17.5×10^6 J.

Question 3 | Attempt any SIX: **[12]**

Question 3.1: A racing car completes 5 rounds of a circular track in 2 minutes. Find the radius of the track if the car has uniform centripetal acceleration of Π^2 m/s^2. **[2]**

Solution: 5 rounds = 2πr(5), t = 2 minutes = 120 s,

Radius (R) = ?

$a_{cp} = \omega^2 r$

$\therefore \pi^2 = \frac{v^2}{r}$

But v = $\frac{2\pi r(5)}{t} = \frac{10\pi r}{t}$

$\therefore \pi^2 = \frac{100\pi^2 r^2}{rt^2}$

$\therefore$ r = $\frac{120 \times 120}{100} = 144m$

The radius of the track is 144 m.

Question 3.2: A body weighs 4.0 kg-wt on the surface of the Earth. What will be its weight on the surface of a plant whose mass is 1/8th of the mass of the Earth and radius half (1/2) of that of the Earth? **[2]**

Solution: W_e = 4.0 kg-wt.,

$\frac{M_p}{M_e} = \frac{1}{8}, \frac{R_p}{R_e} = \frac{1}{2}$

Weight (W_p) = ?

$\frac{W_p}{W_e} = \frac{M_p}{M_e} \times \frac{R_e^2}{R_p^2}$

$\frac{W_p}{4} = \frac{1}{8} \times \left(\frac{2}{1}\right)^2 = \frac{1}{2}$

$W_p = \frac{1}{2} \times 4 = 2$ kg-wt.

Weight of the body on the surface of a planet will be 2 kg-wt

Question 3.3: Define radius of gyration **[2]**

Solution: Radius of gyration of a body is defined as the distance between the axis of rotation and a point at which the whole mass of the body is supposed to be concentrated, so as to possess the same moment of inertia as that of body.

Question 3.3: Explain the physical significance of radius of gyration

Solution: Physical significance of K

a. Radius of gyration is a measure of distribution of mass about the given axis of rotation.

b. If the particles of the body are distributed close to the axis of rotation, the radius of gyration is less.

c. If the particles are distributed away from the axis of rotation, the radius of gyration is more.

d. The knowledge of mass and radius of gyration of the body about a given axis of rotation gives the value of its moment of inertia about the same axis, even if we do not know the actual shape of the body.

Question 3.4: A body of mass 1 kg is made to oscillate on a spring of force constant 16 N/m. Calculate: **[2]**
a) Angular frequency
b) frequency of vibration.

Solution: m = 1 kg, K = 16 N/m

ω = ?, n = ?

We have, for S.H.M.

$$\omega = \sqrt{\frac{K}{M}}$$

$$= \sqrt{\frac{16}{1}}$$

$$\therefore \omega = 4 \text{ rad/s}$$

Also, $\omega = 2\pi n$

$$\therefore n = \frac{\omega}{2\pi}$$

$$= \frac{4}{2\pi} = \frac{2}{\pi} Hz$$

$$\therefore n = \frac{2}{3.14} = [\log(2) - \log(3.14)]$$

= [0.3010 − 0.4969]

= antilog [1.8041]

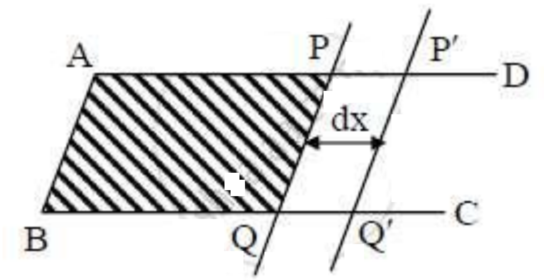
= 0.6369

= 0.6369 Hz

The angular frequency of the body is 4 rad/s and the frequency of vibration is 0.6369 Hz.

Question 3.5: Show that the surface tension of a liquid is numerically equal to the surface energy per unit area. [2]

Solution: a) Let ABCD be an open rectangular frame of wire on which a wire PQ can slide without
friction



b) The frame held in horizontal position is dipped into soap solution and taken out so that
a soap film APQB is formed. Due to surface tension of soap solution, a force 'F' will act on the wire PQ which tends to pull it towards AB.

c) Magnitude of force due to surface tension is, F = 2Tl. [∵ T = F/l]

(A factor of 2 appears because soap film has two surfaces which are in contact with wire.)

d) Let the wire PQ be pulled outwards through a small distance 'dx' to the position P'Q', by applying an external force F' equal and opposite to F. Work done by this force,

$\Delta W = F'dx = 2Tldx$.

e) But, $2ldx = \Delta A$ = increase in area of two surfaces of film

$\therefore \Delta W = T\,\Delta A$

This work done is stored in the form of potential energy (surface energy).

∴ Surface energy, $E = T\Delta A$

$$\therefore \frac{E}{\Delta A} = T$$

Hence, surface tension = surface energy per unit area

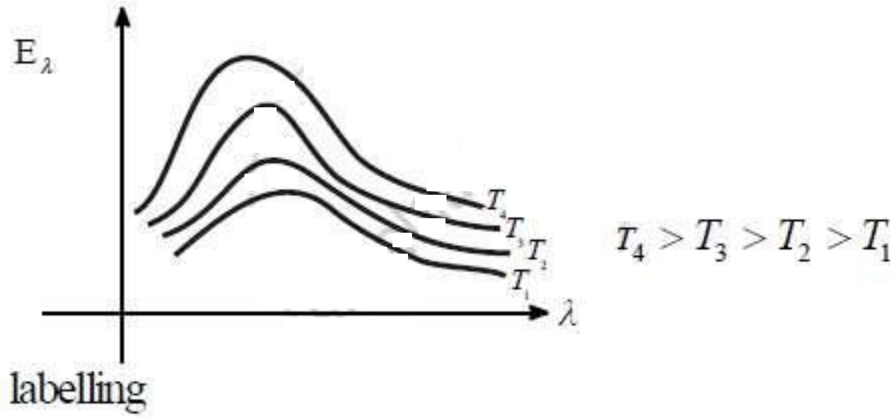
f) Thus, surface tension is equal to the mechanical work done per unit surface area of the liquid, which is also called as surface energy.

Question 3.6: Show graphical representation of energy distribution spectrum of perfectly black body. [2]

Solution: E_λ be emissive power of black body for wavelength λ.

$E_\lambda d\lambda$ be energy density between wavelength λ and λ + dλ

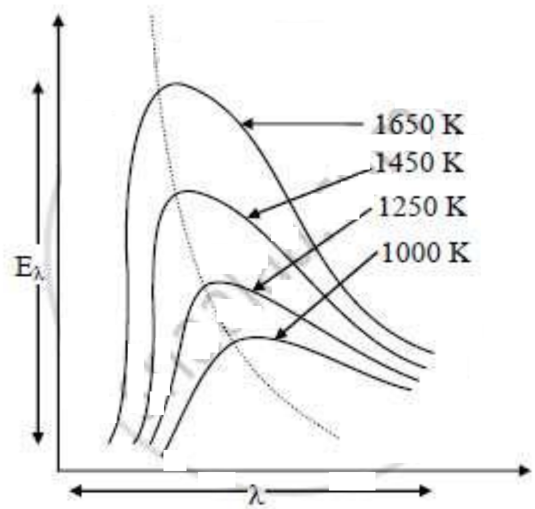
The graph of $E_\lambda d\lambda$ vs λ is called black body spectrum.



Question 3.6: Explain black body radiation spectrum in terms of wavelength

Solution: A perfectly black body emits radiation of all possible wavelength. The radiation emitted by black body depends upon its temperature. The graph between energy emitted and wavelength is called spectrum of black body radiation.

The graph between intensity of radiation Eλ vs λ for various temperature represents black body spectra. The spectra of black body radiation is as shown in the figure.



Observation:

a) Intensity of emitted radiation increases with increase of wavelength.

b) The intensity of emitted radiation is maximum for a particular wavelength (λ_{max}) and then intensity decreases with further increase in wavelength

c) Area under the curve (E_λ) versus (λ) represents total energy emitted per second per unit area by the black body including all the wavelength

Conclusion:

a) At a given temperature of black body:

- Energy distribution curve continues to be non-uniform.
- Energy emitted is maximum corresponding to specific wavelength (λmax) and falls on either side of it
- Total energy (E) emitted per second per unit area corresponding to all wavelengths is represented by the area under the curve

b) With the rise in temperature of black body:

- Total energy emitted increases rapidly for a given wavelength
- Peak of E_λ versus λ curve shifts towards left indicating decrease in value of λ_{max}.
- The area enclosed by each curve with λ-axis increases.

Question 3.7: 'g' is the acceleration due to gravity on the surface of the Earth and 'R' is the radius of the Earth. **[2]**
Show that acceleration due to gravity at height 'h' above the surface of the Earth is

$$gh = g\left(\frac{R}{R+H}\right)^2$$

Solution: a) At the surface of the earth acceleration due to gravity is given by,

$$g = \frac{GM}{R^2} \ldots(i)$$

b) Let acceleration due to gravity at a height h above the surface of the earth be g_h

c) At height h,

Weight of the object = Gravitational force

$$\therefore \quad mg_h = \frac{GMm}{(R+h)^2}$$

$$\therefore \quad g_h = \frac{GM}{(R+h)^2} \qquad \ldots.(ii)$$

d) Dividing equation (ii) by equation (i), we have,

$$\frac{g_h}{g} = \frac{R^2}{(R+h)^2} = \left(\frac{R}{R+h}\right)^2$$

$$\therefore \quad g_h = g\left(\frac{R}{R+h}\right)^2$$

Question 3.8: In Melde's experiment, the number of loops on a string changes from 7 to 5 by the addition of 0.015 kgwt. Find the initial tension applied to the string. **[2]**

Solution: p1 = 7, p_2 = 5,

T_2 = (T_1 + 0.015) kgwt, T_1 = ?

$$\text{we have, } T_1p_1^2 = T_2p_2^2$$

$$\frac{T_2}{T_1} = \left(\frac{p_1}{p_2}\right)^2$$

$$\frac{T_1 + 0.015}{T_1} = \left(\frac{7}{5}\right)^2$$

$$1 + \frac{0.015}{T_1} = \frac{49}{25} \text{ or } \frac{0.015}{T_1} = \frac{24}{25}$$

$$T_1 = \frac{25 \times 0.015}{24} = [\log(25) + \log(0.015) - \log(24)]$$

$$= [1.3979 + \bar{2}.1761 - 1.3802]$$

$$= \text{antilog}\,[\bar{2}.1938]$$

$$= 0.01563 \text{ kgwt}$$

The initial tension applied to the string is 0.01563 kgwt.

Question 4 | Select and write the most appropriate answer from the given alternatives for each sub-questions: **[7]**

Question 4.1: **[1]**

A planet is revolving around a star in a circular orbit of radius R with a period T. If the gravitational force between the planet and the star is proportional to $R^{-\frac{3}{2}}$ then

A) $T^2 \propto R^{\frac{5}{2}}$

B) $T^2 \propto R^{\frac{-7}{2}}$

C) $T^2 \propto R^{\frac{3}{2}}$

D) $T^2 \propto R^4$

Solution:

$$F = \frac{Gmm}{R^{\frac{3}{2}}} \Rightarrow \frac{mv_c^2}{R_2} = \frac{Gmm}{R^{\frac{3}{2}}}$$

$$\therefore \quad v_c^2 = \frac{Gm}{R^{\frac{1}{2}}} \qquad \therefore \left(\frac{2\pi R}{T}\right)^2 = \frac{Gm}{R^{\frac{1}{2}}}$$

$$\therefore \quad \frac{4\pi^2 R^2}{T^2} = \frac{GM}{R^{\frac{1}{2}}} \Rightarrow T^2 = \frac{4\pi^2}{GM} \times R^2 \times R^{\frac{1}{2}}$$

$$\therefore \quad T^2 = \left(\frac{4\pi^2}{GM}\right) \times R^{\frac{5}{2}} \text{ or } T^2 \propto R^{\frac{5}{2}}$$

Question 4.2: If 'L' is the angular momentum and 'I' is the moment of inertia of a rotating body, then $L^2/2I$ represents its _____ **[1]**

(A) rotational P.E.
(B) total energy
(C) rotational K.E.
(D) translational K.E

Solution: Rotational K.E.

Question 4.3: A particle executing linear S.H.M. has velocities v_1 and v2 at distances x_1 and x_2 respectively from the mean position. The angular velocity of the particle is _______ **[1]**

$$\sqrt{\frac{x_1^2 - x_2^2}{v_2^2 - v_1^2}}$$

$$\sqrt{\frac{v_2^2 - v_1^2}{x_1^2 - x_2^2}}$$

$$\sqrt{\frac{x_1^2 + x_2^2}{v_2^2 + v_1^2}}$$

$$\sqrt{\frac{v_2^2 + v_1^2}{x_2^2 + x_1^2}}$$

Solution:

$$\sqrt{\frac{v_2^2 - v_1^2}{x_1^2 - x_2^2}}$$

Question 4.4: A metal rod having coefficient of linear expansion (α) and Young's modulus (Y) is heated to raise the temperature by ΔΘ. The stress exerted by the rod is _______ **[1]**

A) $\frac{Y\alpha}{\Delta\Theta}$
B) $\frac{Y\Delta\Theta}{\alpha}$
C) $Y\alpha\Delta\Theta$
D) $\frac{\alpha\Delta\Theta}{Y}$

Solution: $Y\alpha\Delta\Theta$

Question 4.5: A big drop of radius R is formed from 1000 droplets of water. The radius of a droplet will be _______ **[1]**
A) 10 R
B) R/10
C) R/100
D) R/1000

Solution: R/10

Question 4.6: Apparent frequency of the sound heard by a listener is less than the actual frequency of sound emitted by source. In this case _______. **[1]**
(A) listener moves towards source
(B) source moves towards listener
(C) listener moves away from the source.
(D) source and listener move towards each other.

Solution: listener moves away from the source.

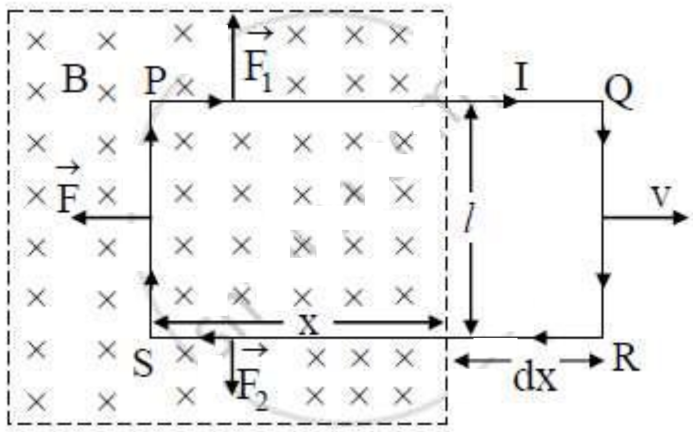
Question 4.7: The substance which allows heat radiations to pass through is _______.
[1]
(A) iron
(B) water vapour
(C) wood
(D) dry air

Solution: dry air

Question 5: Attempt Any One **[7]**

Question 5.1: Obtain an expression for the induced e.m.f. in a coil rotating with uniform angular velocity in uniform magnetic field. Plot a graph of variation of induced e.m.f. against phase ($\Theta = \omega t$) over one cycle. **[7]**

Solution: a) Consider a rectangular loop of conducting wire 'PQRS' partly placed in uniform magnetic field of induction 'B' as shown in figure.



b) Let ‘l’ be the length of the side PS and ‘x’ be the length of the loop within the field.

∴ A = lx = area of the loop, which lies inside the field.

c) The magnetic flux (φ) through the area A at certain time ‘t’ is Φ = BA = Blx

d) The loop is pulled out of the magnetic field of induction ‘B’ to the right with a uniform velocity ‘v’.

e) The rate of change of magnetic flux is given by, $\frac{d\phi}{dt} = \frac{d}{dt}(Blx)$

$$\therefore \quad \frac{d\phi}{dt} = Bl\left(\frac{dx}{dt}\right)$$

$$\text{But, } \left(\frac{dx}{dt}\right) = v$$

$$\therefore \quad \frac{d\phi}{dt} = Blv \qquad \text{....(i)}$$

f) Due to change in magnetic flux, induced current is set up in the coil. The direction of this current is clockwise according to Lenz’s law. Due to this, the sides of the coil experience the forces, F1, F2 and F as shown in figure. The directions of these forces is given by Fleming’s left hand rule.

g) The magnitude of force ‘F’ acting on the side PS is given by, F = BIl.

h) The force $\vec{F}_1$ and $\vec{F}_2$ are equal in magnitude and opposite in direction, therefore they cancel out. The only unbalanced force which opposes the motion of the coil is $\vec{F}$ Hence, work must be done against this force in order to pull the coil.

i) The work done in time ‘dt’ during the small displacement ‘dx’ is given by, dW = −Fdx
− ve sign shows that F and ‘dx’ are opposite to each other.

∴ dW = − (BIl) dx ….(ii)

j) Mechanical power is given by,

$$P = \frac{dW}{dt} = BIl\left(\frac{dx}{dt}\right)$$

$$\therefore \quad P = BIlv \qquad \left[\because \frac{dx}{dt} = v\right]$$

k) This external work provides the energy needed to maintain the induced current I through the loop (coil).

l. If ‘e’ is the e.m.f induced then,

electric power = $\frac{dW}{dt} = eI$

$\therefore$ dW = eIdt ....(iii)

m) From equations (ii) and (iii), we have, eIdt = − BIl dx

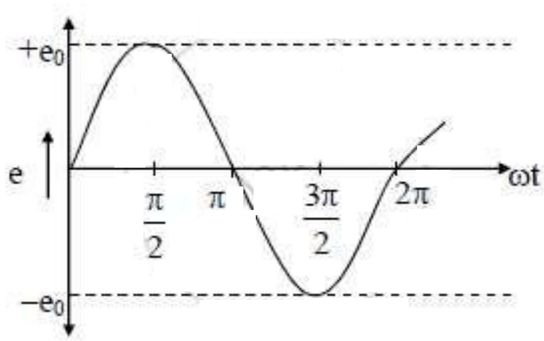
$\therefore e = -Bl\frac{dx}{dt}$

$\therefore$ e = - Blv ...(iv)

n. From equation (i) and (iv), we have,

$$e = -\frac{d\phi}{dt}$$

o) Graph of variation of induced e.m.f. against phase (θ = ωt) over one cycle



Question 5.1: The energy density at a point in a medium of dielectric constant 6 is 26.55 × 10^6 J/m^3. Calculate electric field intensity at that point. (ε_0 = 8.85 × 10^{-12} SI units). **[7]**

Solution: k = 6, u = 26.55 × 10^6 J/m^3

$$u = \frac{1}{2}\varepsilon_0 k E^2$$

$$E = \left(\frac{2u}{\varepsilon_0 k}\right)^{1/2}$$

$$= \frac{2 \times 26.55 \times 10^6}{8.85 \times 10^{-12} \times 6} = \left(10^{18}\right)^{1/2}$$

$$E = 10^9 \text{ N/C}$$

Electric field intensity at the point in the medium is 10^9 N/C.

Question 5.2: Write notes on Nuclear fission **[7]**

Solution: Nuclear fission:

1) The process of splitting a heavy nucleus into two lighter nuclei after bombardment with neutrons is called nuclear fission.

2) The process of nuclear fission was first discovered by German scientists Otto Hahn and Strassman in 1939.

3) Example of nuclear fission:

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{236}_{92}U \rightarrow ^{144}_{56}Ba + ^{89}_{36}Kr + 3\,^{1}_{0}n$$

In the above reaction, when $^{235}_{92}U$ is bombarded by neutron, it breaks up into two intermediate fragments which emit β-particles to achieve stable end products.

4) The energy released in fission first appears as K.E which gets converted into heat in surrounding

5) Fission energy is being used in nuclear power projects for generation of electricity.

6) The uncontrolled fission process is used in atom bomb

Question 5.2: A galvanometer has a resistance of 16Ω. It shows full scale deflection, when a current of 20 mA is passed through it. The only shunt resistance available is 0.06 which is not appropriate to convert a galvanometer into an ammeter. How much resistance should be connected in series with the coil of galvanometer, so that the range of ammeter is 8 A?

Solution: Let 'X' be the resistance connected in series with galvanometer.
Since S is not sufficient for I = 10 A
G = 16 Ω,

$$\frac{I_g}{I} = \frac{S}{(G+X)+S}$$

$$\therefore (G+X)+S = \frac{I}{I_g}S$$

$$\therefore (16 + X + 0.06) = \frac{8}{2 \times 10^{-2}} \times 0.06$$

$\therefore$ (16 + X + 0.06) = 24

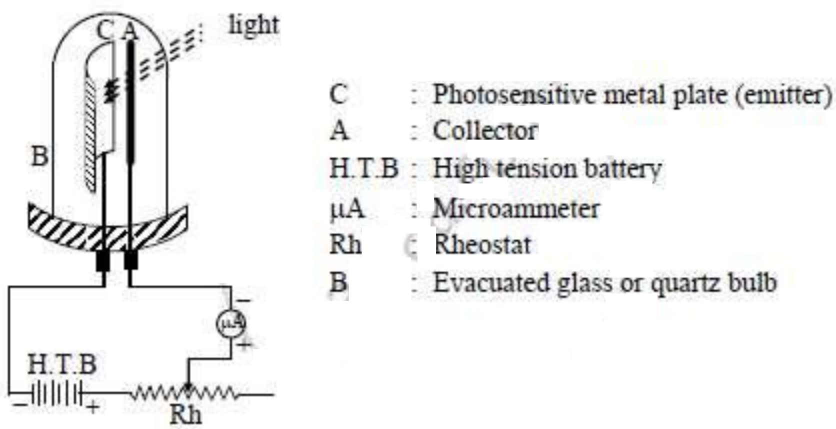
$\therefore$ X = 24 - 16.06

$\therefore$ X = 7.94

Question 6 | Attempt any THREE [9]

Question 6.1: Draw a well labelled diagram of photoelectric cell. [3]

Solution:



Photoelectric cell

Question 6.1: Explain the observations made by Hertz and Lenard about the phenomenon of photoelectric emission.

Solution: Observations made by Hertz and Lenard:

a) The phenomenon of photoelectric emission was discovered in 1887 by Heinrich Hertz.

b) During his experimental investigation on the production of electromagnetic waves by means of a spark discharge, Hertz observed that high voltage sparks across the detector loop were enhanced when the emitter plate was illuminated by ultraviolet light from an arc lamp.

c) Lenard observed that when ultraviolet radiations were allowed to fall on the emitter plate of an evacuated glass tube enclosing two electrodes (metal plates), current flows in the circuit.

d) As soon as the ultraviolet radiations were stopped, the current flow also stopped. These observations indicated that when ultraviolet radiations fall on the emitter plate C, electrons are ejected from it which are attracted towards the positive collector plate A by the electric field.

e) The electron flow through the surface of emitter caused current in the external circuit.

f) Hallwachs and Lenard studied how this photocurrent varied with collector plate potential, frequency and intensity of incident light.

Question 6.2: Explain the working of transistor as a switch. [3]

Solution: Working of transistor as a switch:

a) A base-biased N-P-N common emitter transistor is as shown in figure (a).

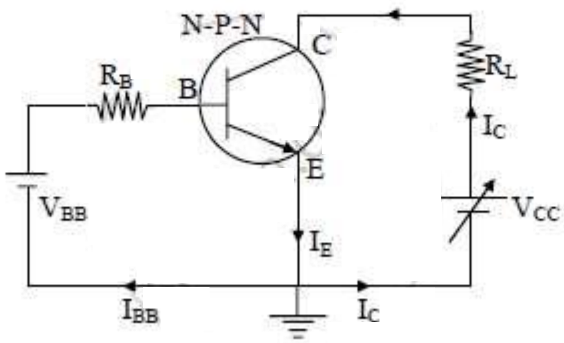


Figure (a)

b) On applying Kirchhoff's law in input circuit, we get,

$V_{BB} - I_B R_B = V_{BE}$

$\therefore V_{BB} = V_{BE} + I_B R_B$

c) Considering VBB as the d.c input voltage 'Vi' then the above equation may be written as $V_i = V_{BE} + I_B R_B$ ….(i)

Applying Kirchhoff's law in output circuit, we get,

$V_{CE} = V_{CC} - I_C R_L$

d) Considering V_{CC} as the dc output voltage 'Vo' then the above equation may be written as,

$V_o = V_{CC} - I_C R_C$ ….(ii)

As Vi is increased from 0 to 0.6 V (for silicon transistor), $I_C = 0$ and the transistor is said to be in the cut-off state.

$\therefore I_C = 0$

From equation (ii),

$V_o = V_{CC}$ = constant

f) As 'V_i' is increased from 0.6 V to 1 V, transistor will be in active state therefore IC increases linearly. From equation (ii) 'V_o' decreases linearly. The transistor in this range is called in active state.

g) If 'V_i' is increased further, 'V_o' becomes non-linear and decreases further and tends to become zero, though its value nearly reaches zero. The transistor in this range is said to be in saturation state. The above characteristics are shown in figure (b).

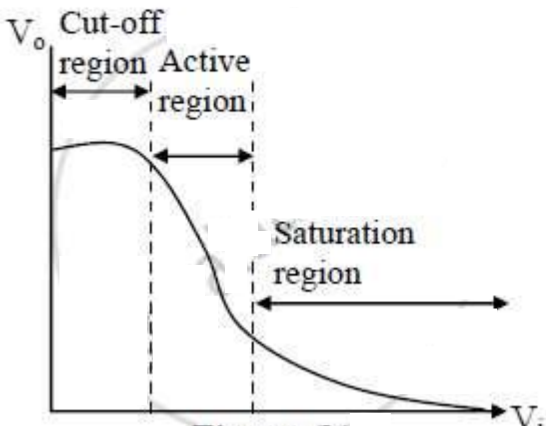


Figure (b)

h) When V_i is low(< 0.6 V), V_o is high and when V_i is high (> 1 V), V_o is low. By defining low voltage state as cut off state and high voltage level as saturation state, transistor can be used as a switch.

i) A low voltage input keeps the transistor in cut-off region (non-working) and the transistor is said to be switched off.

j) A high voltage input keeps the transistor in saturation state (working) and the transistor is said to be switched on.

Question 6.3: The refractive indices of water for red and violet colours are 1.325 and 1.334 respectively. Find the difference between the velocities of rays for these two colours in water. ($c = 3 \times 10^8$ m/s) **[3]**

$$\mu_r = 1.325,\ \mu_v = 1.334,\ v_r - v_v = ?$$

$$\mu = \frac{c}{v} \Rightarrow v_r = \frac{c}{\mu_r} \text{ and } v_v = \frac{c}{\mu_v}$$

$$\therefore \quad v_r - v_v = c\left[\frac{1}{\mu_r} - \frac{1}{\mu_v}\right]$$

$$= 3 \times 10^8 \left[\frac{1.334 - 1.325}{1.325 \times 1.334}\right]$$

$$= \frac{3 \times 10^8 \times 9 \times 10^{-3}}{1.325 \times 1.334}$$

$$= \frac{27}{1.325 \times 1.334} \times 10^5$$

Solution:

$= [\log (27) - \log (1.325) - \log (1.334)] \times 10^5$

= [1.4314 − 0.1222 − 0.1252] × 10^5

= [antilog (1.184)] × 10^5

= 15.28 × 105 = 1.528 × 10^6 m/s.

The difference between the velocities of rays for red and violet colours in water is 1.528 × 10^6 m/s.

Question 6.4: In Young's experiment, the ratio of intensity at the maxima and minima in an interference pattern is 36 : 9. What will be the ratio of the intensities of two interfering waves? **[3]**

Solution:

$$\frac{I_{max}}{I_{min}} = \frac{36}{9} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$$

$$\Rightarrow \frac{a_1 + a_2}{a_1 - a_2} = \frac{6}{3} = \frac{2}{1}$$

$$\therefore \quad a_1 + a_2 = 2a_1 - 2a_2$$

$$\therefore \quad a_1 = 3a_2$$

$$\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{(3a)^2}{a_2^2} = \frac{9}{1}$$

The ratio of the intensities of the two interfering waves is 9 : 1.

Question 7 | Attempt any SIX: **[12]**

Question 7.1: Explain the principle of potentiometer. **[2]**

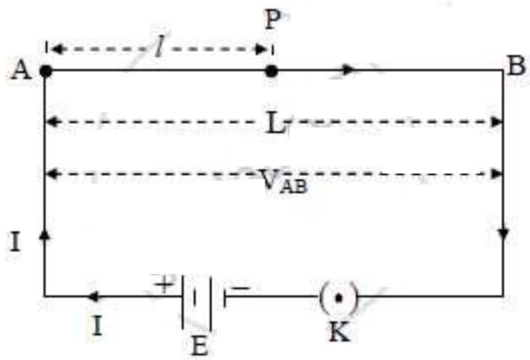
Solution: Principle:

The potential difference between any two points of the potentiometer wire is directly proportional to the length of wire between these two points.

The fall of potential per unit length of potentiometer wire (potential gradient of wire) is constant.

Explanation:

a) Suppose a potentiometer wire AB of length L and resistance R is stretched on the rectangular wooden board. The source of e.m.f E and negligible internal resistance is connected to the wire AB through a key K as shown in figure.



b) Resistance per unit length of wire AB is given by, σ = R/L

∴ R = σL

c) Let 'V_{AB}' be the P.D across the wire. There is uniform fall of potential along the wire from A to B.

d) By Ohm's law, the current 'I' passing through the wire is given by,

$$I = \frac{V_{AB}}{R} = \frac{V_{AB}}{\sigma L}$$

e) Let 'P' be any point on the wire, such that AP = 'l'. The resistance of wire AP of length 'l' is R_{AP} = σl. So the potential difference 'V'_{AP}' between points A and P is given by,

$$V_{AP} = IR_{AP} = I\sigma l = \frac{V_{AB}}{\sigma L} \times \sigma\, l$$

$$V_{AP} = \left(\frac{V_{AB}}{L}\right).l$$

f) But, V_{AB} and L are constant,

∴ $V_{AP} \propto L$

$$\therefore \frac{V_{AP}}{L} = K = \text{constant}$$

Where, K is called potential gradient of wire.

Question 7.2: Define Magnetic intensity. **[2]**

Solution 1: Magnetic intensity:- Magnetic intensity is a quantity used in describing magnetic phenomenon in terms of the magnetic field. The strength of the magnetic field at a point can be given in terms of a vector quantity called magnetic intensity (H).

Solution 2: Magnetic intensity: The ratio of the strength of magnetising field to the permeability of free space is called as magnetic intensity.

The strength of magnetic field at a point can be given in terms of vector quantity called as magnetic intensity (H).

$$H = \frac{B_0}{\mu_0} \text{ or } B_0 = \mu_0 H$$

The SI unit of magnetic intensity is Am^{-1}.

Question 7.3: What do you mean by polar molecules and non-polar molecules? Give ‘one’ example each. **[2]**

Solution: Polar molecules: The molecules in which “centre of gravity” of positive nuclei and revolving electrons do not coincide are known as polar molecules.

Example: HCl, H2O, N2O etc.

Polar substances behave as a tiny electric dipole because polar molecules have a permanent electric dipole moment.

Non-polar molecules: The molecules in which “centre of gravity” of positive nuclei and revolving electrons coincide are known as non-polar molecules.

Example: O2, H2, CO2, polyethylene, polystyrene etc.

Non-polar molecules do not have permanent electric dipole moment because of their symmetry.

Question 7.4: The minimum angular separation between two stars is 4×10^{-6} rad, if telescope is used to observe them with an objective of aperture 16 cm. Find the wavelength of light used. **[2]**

Solution: $d\theta = 4 \times 10^{-6}$ rad, $a = 16$ cm $= 16 \times 10^{-2}$ m

$$d\theta = \frac{1.22\lambda}{a}$$

$$\begin{aligned}
\lambda &= \frac{d\theta \times a}{1.22} \\
&= \frac{4 \times 16 \times 10^{-8}}{1.22} \\
&= \frac{64 \times 10^{-8}}{1.22} \\
&= [\log(64) - \log(1.22)] \times 10^{-8} \\
&= [1.8062 - 0.0864] \times 10^{-8} \\
&= [\text{antilog}\,(1.7198)] \times 10^{-8} \\
&= 52.46 \times 10^{-8}\ \text{m} \\
&= 5246\ \text{Å}
\end{aligned}$$

The wavelength of light used is 5246 Å.

Question 7.5: Explain the need for modulation related to the size of antenna (aerial).

Solution: 1) For transmitting a signal, we need an antenna or an aerial. This antenna should have length comparable to the quarter wavelength of the signal ($\lambda/4$) so that the antenna properly senses the time variation of signal **[2]**

2) For an electromagnetic wave of frequency 20 kHz, wavelength λ is 15 km. Thus, vertical antenna of this size required at audio frequencies (< 20 KHZ) is impracticable. Hence, direct transmission of such baseband signals is not practical.

Question 7.6: Four resistances 4Ω, 8Ω, XΩ, and 6Ω are connected in a series so as to form Wheatstone's network. If the network is balanced, find the value of 'X'. **[2]**

Solution: $R_1 = 4\Omega$, $R_2 = 8\Omega$, $R_3 = X\Omega$, $R_4 = 6\Omega$

When Wheatstone's network is balanced,

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

$$\frac{4}{8} = \frac{X}{6}$$

$\therefore X = 3\ \Omega$

The value of 'X' is 3Ω.

Question 7.7: The magnetic susceptibility of annealed iron at saturation is 4224. Find the permeability of annealed iron at saturation. ($\mu_0 = 4\Pi \times 10^{-7}$ SI unit) **[2]**

Solution: $\chi = 4224$

$\therefore \mu = \mu_0 (1 + \chi)$

$= 4\pi \times 10^{-7} (1 + 4224)$

$= 16900\ \pi \times 10^{-7}$

$= 169 \times 3.142 \times 10^{-5}$

$= [\log (169) + \log (3.142)] \times 10^{-5}$

$= [2.2279 + 0.4972] \times 10^{-5}$

$= \text{antilog } (2.7251)] \times 10^{-5}$

$= 531 \times 10^{-5}$

$= 5.31 \times 10^{-3}\ Hm^{-1}$

The permeability of annealed iron at saturation is $5.31 \times 10^{-3}\ Hm^{-1}$.

Question 8 | Select and write the most appropriate answer from the given alternatives for each subquestions: **[7]**

Question 8.1: A ray of light passes from a vacuum to a medium of refractive index (μ). The angle of incidence is found to be twice the angle of refraction. The angle of incidence is ______. **[1]**

A) $\cos^{-1}\left(\frac{\mu}{2}\right)$

B) $\cos^{-1}(\mu)$

C) $2\cos^{-1}\left(\frac{\mu}{2}\right)$

D) $2\sin^{-1}\left(\frac{\mu}{2}\right)$

Solution: For i = 2r

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin i}{\sin\left(\frac{i}{2}\right)} = \frac{2\sin\left(\frac{i}{2}\right)\cos\left(\frac{i}{2}\right)}{\sin\left(\frac{i}{2}\right)}$$

$$\therefore \quad \mu = 2\cos\left(\frac{i}{2}\right)$$

$$\Rightarrow i = 2\cos^{-1}\left(\frac{\mu}{2}\right)$$

Question 8.2: The fringes produced in diffraction pattern are of ______. **[1]**
(A) equal width with same intensity
(B) unequal width with varying intensity
(C) equal intensity
(D) equal width with varying intensity

Solution: Unequal width with varying intensity

Question 8.3: If 'R' is the radius of dees and 'B' be the magnetic field of induction in which positive charges (q) of mass (m) escape from the cyclotron, then its maximum speed (vmax) is ______. **[1]**

A) $\frac{qR}{Bm}$

B) $\frac{qm}{Br}$

C) $\frac{qBR}{m}$

D) $\frac{m}{qBR}$

Solution:

$$\frac{qBR}{m}$$

Question 8.4: The number of photoelectrons emitted ______. **[1]**

(A) varies inversely with frequency
(B) varies directly with frequency
(C) varies inversely with intensity
(D) varies directly with intensity

Solution: Varies directly with intensity

Question 8.5: The width of depletion region of p-n junction diode is ______. **[1]**
(A) 0.5 nm to 1 nm
(B) 5 nm to 10 nm
(C) 50 nm to 500 nm
(D) 500 nm to 1000 nm

Solution: 500 nm to 1000 nm

Question 8.6: Any device that converts one form of energy into another is termed as _____. **[1]**
(A) amplifier
(B) transducer
(C) receiver
(D) demodulator

Solution: transducer

Question 8.7: A transformer converts 240 V AC to 60 V AC. The secondary has 75 turns. The number of turns in primary are ______. **[1]**
(A) 600
(B) 500
(C) 400
(D) 300

Solution:

$$\frac{e_p}{e_s} = \frac{N_p}{N_s}$$

$$\frac{240}{60} = \frac{N_p}{75}$$

$\therefore N_p = 300$