

2023 III 03

1100

J-262

(E)

MATHEMATICS & STATISTICS (40)
(ARTS & SCIENCE)

Time : 3 Hrs.

(8 Pages)

Max. Marks : 80

General instructions :

The question paper is divided into **FOUR** sections.

- (1) **Section A:** Q. 1 contains **Eight** multiple choice type of questions, each carrying **Two** marks.
Q. 2 contains **Four** very short answer type questions, each carrying **one** mark.
- (2) **Section B:** Q. 3 to Q. 14 contain **Twelve** short answer type questions, each carrying **Two** marks. (Attempt any **Eight**)
- (3) **Section C:** Q. 15 to Q. 26 contain **Twelve** short answer type questions, each carrying **Three** marks. (Attempt any **Eight**)
- (4) **Section D:** Q. 27 to Q. 34 contain **Eight** long answer type questions, each carrying **Four** marks. (Attempt any **Five**)
- (5) Use of log table is allowed. Use of calculator is not allowed.
- (6) Figures to the right indicate full marks.
- (7) Use of graph paper is not necessary. Only rough sketch of graph is expected.
- (8) For each multiple choice type of question, it is mandatory to write the correct answer along with its alphabet, e.g., (a) / (b) / (c) / (d)....., etc. No marks shall be given, if ONLY the correct answer or the alphabet of correct answer is written. Only the first attempt will be considered for evaluation.
- (9) Start answer to each section on a new page.

Q. 1. Select and write the correct answer for the following multiple choice type of questions :

[16]

(i) If $p \wedge q$ is F, $p \rightarrow q$ is F then the truth values of p and q are _____ respectively.

(a) T, T

(b) T, F

(c) F, T

(d) F, F

(2)

(ii) In $\triangle ABC$, if $c^2 + a^2 - b^2 = ac$, then $\angle B =$ _____.

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$

(2)

(iii) The area of the triangle with vertices $(1, 2, 0)$, $(1, 0, 2)$ and $(0, 3, 1)$ in sq. unit is _____.

(a) $\sqrt{5}$ (b) $\sqrt{7}$ (c) $\sqrt{6}$ (d) $\sqrt{3}$

(2)

(iv) If the corner points of the feasible solution are $(0, 10)$, $(2, 2)$ and $(4, 0)$ then the point of minimum $z = 3x + 2y$ is _____.

(a) $(2, 2)$ (b) $(0, 10)$ (c) $(4, 0)$ (d) $(3, 4)$

(2)

(v) If y is a function of x and $\log(x+y) = 2xy$, then the value of $y'(0) =$ _____.

(a) 2

(b) 0

(c) -1

(d) 1

(2)

(vi) $\int \cos^3 x dx = \underline{\hspace{2cm}}$.

(a) $\frac{1}{12} \sin 3x + \frac{3}{4} \sin x + c$

(b) $\frac{1}{12} \sin 3x + \frac{1}{4} \sin x + c$

(c) $\frac{1}{12} \sin 3x - \frac{3}{4} \sin x + c$

(d) $\frac{1}{12} \sin 3x - \frac{1}{4} \sin x + c$ (2)

(vii) The solution of the differential equation $\frac{dx}{dt} = \frac{x \log x}{t}$ is

(a) $x = e^{ct}$

(b) $x - e^{ct} = 0$

(c) $x = e^t + t$

(d) $x e^{-t} = 0$ (2)

(viii) Let the probability mass function (p.m.f.) of a random

variable X be $P(X = x) = {}^4C_x \left(\frac{5}{9}\right)^x \times \left(\frac{4}{9}\right)^{4-x}$, for $x = 0, 1, 2, 3, 4$ then $E(X)$ is equal to _____

(a) $\frac{20}{9}$

(b) $\frac{9}{20}$

(c) $\frac{12}{9}$

(d) $\frac{9}{25}$ (2)

Q. 2. Answer the following questions :

(i) Write the joint equation of co-ordinate axes. (1)

(ii) Find the values of c which satisfy $|c \vec{u}| = 3$ where

$\vec{u} = \hat{i} + 2\hat{j} + 3\hat{k}$. (1)

(iii) Write $\int \cot x dx$. (1)

(iv) Write the degree of the differential equation

$$e^x \frac{dy}{dx} + \frac{dy}{dx} = x \quad (1)$$

SECTION – B

Attempt any EIGHT of the following questions :

[16]

Q. 3. Write inverse and contrapositive of the following statement :

If $x < y$ then $x^2 < y^2$ (2)

Q. 4. If $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ is a non singular matrix, then find A^{-1} by elementary row transformations.

Hence write the inverse of $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (2)

Q. 5. Find the cartesian co-ordinates of the point whose polar co-

ordinates are $\left(\sqrt{2}, \frac{\pi}{4}\right)$. (2)

Q. 6. If $ax^2 + 2hxy + by^2 = 0$ represents a pair of lines and $h^2 = ab \neq 0$ then find the ratio of their slopes. (2)

Q. 7. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the points A, B, C respectively and $5\vec{a} + 3\vec{b} - 8\vec{c} = \vec{0}$ then find the ratio in which the point C divides the line segment AB. (2)

Q. 8. Solve the following inequations graphically and write the corner points of the feasible region :

$2x + 3y \leq 6, x + y \geq 2, x \geq 0, y \geq 0$ (2)

Q. 9. Show that the function $f(x) = x^3 + 10x + 7$, $x \in R$ is strictly increasing. (2)

Q. 10. Evaluate : $\int_0^{\frac{\pi}{2}} \sqrt{1 - \cos 4x} \, dx$ (2)

Q. 11. Find the area of the region bounded by the curve $y^2 = 4x$, the X-axis and the lines $x = 1$, $x = 4$ for $y \geq 0$. (2)

Q. 12. Solve the differential equation $\cos x \cos y \, dy - \sin x \sin y \, dx = 0$ (2)

Q. 13. Find the mean of number randomly selected from 1 to 15. (2)

Q. 14. Find the area of the region bounded by the curve $y = x^2$ and the line $y = 4$. (2)

SECTION - C

Attempt any EIGHT of the following questions : [24]

Q. 15. Find the general solution of $\sin \theta + \sin 3\theta + \sin 5\theta = 0$ (3)

Q. 16. If $-1 \leq x \leq 1$, then prove that $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ (3)

Q. 17. If θ is the acute angle between the lines represented by

$$ax^2 + 2hxy + by^2 = 0 \text{ then prove that } \tan \theta = \frac{2\sqrt{h^2 - ab}}{a+b} \quad (3)$$

Q. 18. Find the direction ratios of a vector perpendicular to the two lines whose direction ratios are $-2, 1, -1$ and $-3, -4, 1$. (3)

Q. 19. Find the shortest distance between lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}, \text{ and } \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5} \quad (3)$$

Q. 20. Lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ and $\vec{r} = (4\hat{i} - 3\hat{j} + 2\hat{k}) + \mu(\hat{i} - 2\hat{j} + 2\hat{k})$ are coplanar. Find the equation of the plane determined by them. (3)

Q. 21. If $y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + \dots + \infty}}}$, then

$$\text{show that } \frac{dy}{dx} = \frac{\sec^2 x}{2y-1}.$$

$$\text{Find } \frac{dy}{dx} \text{ at } x = 0. \quad (3)$$

Q. 22. Find the approximate value of $\sin(30^\circ 30')$.

$$\text{Given that } 1^\circ = 0.0175^c \text{ and } \cos 30^\circ = 0.866 \quad (3)$$

Q. 23. Evaluate $\int x \tan^{-1} x \, dx$ (3)

Q. 24. Find the particular solution of the differential equation

$$\frac{dy}{dx} = e^{2y} \cos x, \text{ when } x = \frac{\pi}{6}, y = 0. \quad (3)$$

Q. 25. For the following probability density function of a random variable X , find (a) $P(X < 1)$ and (b) $P(|X| < 1)$.

$$f(x) = \frac{x+2}{18}; \text{ for } -2 < x < 4$$
$$= 0, \text{ otherwise} \quad (3)$$

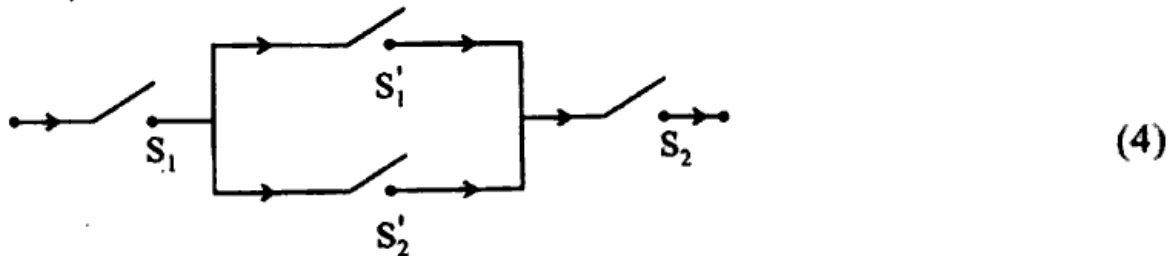
Q. 26. A die is thrown 6 times. If 'getting an odd number' is a success, find the probability of at least 5 successes. (3)

SECTION – D

Attempt any FIVE of the following questions :

[20]

Q. 27. Simplify the given circuit by writing its logical expression. Also write your conclusion.



Q. 28. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ verify that

$$A(\text{adj}A) = (\text{adj}A)A = |A|I \quad (4)$$

Q. 29. Prove that the volume of a tetrahedron with coterminus edges

$$\bar{a}, \bar{b}, \text{ and } \bar{c} \text{ is } \frac{1}{6} [\bar{a} \bar{b} \bar{c}].$$

Hence, find the volume of tetrahedron whose coterminus edges

$$\text{are } \bar{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \bar{b} = -\hat{i} + \hat{j} + 2\hat{k} \text{ and } \bar{c} = 2\hat{i} + \hat{j} + 4\hat{k}. \quad (4)$$

Q. 30. Find the length of the perpendicular drawn from the point $P(3, 2, 1)$ to the line

$$\bar{r} = (7\hat{i} + 7\hat{j} + 6\hat{k}) + \lambda(-2\hat{i} + 2\hat{j} + 3\hat{k}) \quad (4)$$

Q. 31. If $y = \cos(m \cos^{-1} x)$ then show that

$$(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + m^2 y = 0 \quad (4)$$

Q. 32. Verify Lagrange's mean value theorem for the function

$$f(x) = \sqrt{x+4} \text{ on the interval } [0, 5]. \quad (4)$$

Q. 33. Evaluate :

$$\int \frac{2x^2 - 3}{(x^2 - 5)(x^2 + 4)} dx \quad (4)$$

Q. 34. Prove that :

$$\int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_0^a f(2a - x) dx \quad (4)$$

