7. Vectors

- Let a1→, a2→, a3→, ..., an→ be *n* vectors. Let the linear combination of these vectors be denoted by L→. Then:

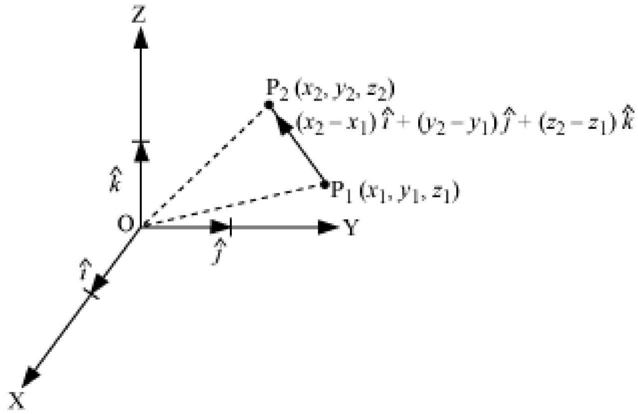
  L→=x1 a1→+x2 a2→+x3 a3→, ...+xn an→ , where x1, x2 , x3 , ..., xn∈R

- If x1 a1→+x2 a2→+x3 a3→, ...+xn an→=0 such that not all x1, x2 , x3 , ..., xn∈R are zero, then it can be said that a1→, a2→, a3→, ..., an→ are linearly dependent vectors.
- If x1 a1→+x2 a2→+x3 a3→, ...+xn an→=0⇒ a1→= a2→ =a3→ ... an→=0, then a1→, a2→, a3→, ..., an→ are linearly independent vectors.
- Let a→, b→ be two vectors and there exist a scalar x∈R such that a→=x b→. Then we can say that the two vectors a→, b→ are collinear.
- Let a1→, a2→, a3→ be three vectors and there exist three scalars x1, x2 , x3∈R, not all zero such that x1 a1→+x2 a2→+x3 a3→=0, where $x_1 + x_2 + x_3 = 0$. Then we can say that the three vectors a1→, a2→,a3→ are collinear.
- Let *A, B, C* be three collinear points. Then each pair of the vectors AB→, BC→; AB→,AC;→ and BC→, AC→ is a pair of collinear vectors. Thus, to check the collinearity of three points, we can check the collinearity of any two vectors obtained with the help of three points.
- Three points with position vectors a→, b→, c→ are collinear, only if there exist three scalars x, y, z, not all zero simultaneously such that xa→ +yb→ +zc→ = 0→, together with x + y + z = 0.
- Let a1→, a2→, a3→, a4→ be three vectors and there exist three scalars x1, x2 , x3, x4∈R, not all zero such that x1 a1→+x2 a2→+x3 a3→+x4 a4→=0 , where $x_1 + x_2 + x_3 + x_4 = 0$. Then we say that the three vectors a1→, a2→, a3→, a4→ are coplanar.

Vector Joining Two Points

The vector joining two points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$, represented as $\overrightarrow{P_1P_2}$, is calculated as

$$\overrightarrow{P_1P_2} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$



The magnitude of $\overrightarrow{P_1P_2}$ is given by $\left|\overrightarrow{P_1P_2}\right| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Section Formula

If point *R* (position vector $\vec{r}$) lies on the vector $\overrightarrow{PQ}$ joining two points *P* (position vector $\vec{a}$) and *Q* (position vector $\vec{b}$) such that *R* divides $\overrightarrow{PQ}$ in the ratio *m: n* $\left[\text{i.e.} \frac{\overrightarrow{PR}}{\overrightarrow{RQ}} = \frac{m}{n}\right]$

Internally, then $\vec{r} = \dfrac{m\vec{b} + n\vec{a}}{m+n}$

Externally, then $\vec{r} = \dfrac{m\vec{b} - n\vec{a}}{m-n}$

- **Scalar Triple Product**

The scalar triple product of the three vectors a→, b→, c→, defined by a→· b→× c→ is a scalar quantity.

a→·b→× c→=a1a2a3b1b2b3c1c2c3

The scalar triple product, a→· b→× c→ can be denoted by a→ b→ c→**.**

Remarks :

1. a→ b→ c→ =b→ c→ a→ =c→ a→ b
2. a→ b→ c→ =-b→ a→ c→ =-a→ c→ b→
3. a→+b→ c→ d→=a→ c→ d→+b→ c→ d→
4. a→ b→ c→=0 if a→=b→ or b→=c→ or c→=a→ or atleast one of the vector is a null vector.

5. Three vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar if and only if $\vec{a}\ \ \vec{b}\ \ \vec{c} = 0$.

6. $l\vec{a}\ \ m\vec{b}\ \ n\vec{c} = lmn\vec{a}\ \ \vec{b}\ \ \vec{c}$, where *l*, *m* and *n* are scalars.

- Volume of the parallelopiped whose concurrent edges are $\vec{a}, \vec{b}, \vec{c}$ is $\vec{a}\cdot \vec{b}\times \vec{c} = \vec{a}\ \ \vec{b}\ \ \vec{c}$.

- If $\vec{a}, \vec{b}, \vec{c}$ represents three adjacent edges of a tetrahedron, then its volume V is given by $V=16\vec{a}\ \ \vec{b}\ \ \vec{c}$.

- The vector product of $\vec{a}$ with $\vec{b}\times\vec{c}$ is the vector triple product of the vectors $\vec{a}, \vec{b}, \vec{c}$ and is defined by $\vec{a}\times \vec{b}\times \vec{c}$. This is vector in the plane of $\vec{b}$ and $\vec{c}$ and perpendicular to $\vec{a}$.

  $\vec{a}\times \vec{b}\times \vec{c} = \vec{a}\cdot \vec{c}\vec{b} - \vec{a}\cdot \vec{b}\vec{c}$.

- If the vectors $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular i.e., $\vec{a}\cdot \vec{c}=0$, $\vec{a}\cdot \vec{b}=0$, $\vec{b}\cdot \vec{c}=0$, then $\vec{a}\times \vec{b}\times \vec{c}=0$.

- If the vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar then $\vec{a}\times \vec{b}\times \vec{c}=0$.

- Let $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ be four vectors then scalar product of these vectors is defined as $\vec{a}\times \vec{b}\cdot \vec{c}\times \vec{d}$.

$$\vec{a}\times \vec{b}\cdot \vec{c}\times \vec{d} = \vec{a}\cdot \vec{c}\ \vec{b}\cdot \vec{d} - \vec{a}\cdot \vec{d}\cdot \vec{b}\cdot \vec{c} = \vec{a}\cdot \vec{c}\ \ \vec{a}\cdot \vec{d}\,\vec{b}\cdot \vec{c}\ \ \vec{b}\cdot \vec{d}$$

- Let $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ be four vectors then vector product of these vectors is defined as $\vec{a}\times \vec{b}\times \vec{c}\times \vec{d}$.

$$\vec{a}\times \vec{b}\times \vec{c}\times \vec{d} = \vec{a}\times \vec{b}\cdot\vec{d}\,\vec{c} - \vec{a}\times \vec{b}\cdot\vec{c}\,\vec{d}$$

$$\Rightarrow \vec{a}\times \vec{b}\times \vec{c}\times \vec{d} = \vec{a}\ \ \vec{b}\ \ \vec{d}\,\vec{c} - \vec{a}\ \ \vec{b}\ \ \vec{c}\,\vec{d}$$