

# Physics

Academic Year: 2014-2015

Marks: 70

Date & Time: 26th February 2015, 11:00 am

Duration: 3h

**Question 1: Select and write the most appropriate answer from the given alternatives for each sub-question :** [7]

**Question 1.1:** The period of a conical pendulum in terms of its length ( $l$ ), semi-vertical angle ( $\theta$ ) and acceleration due to gravity ( $g$ ) is: [1]

$$\begin{aligned} & \frac{1}{2\pi} \sqrt{\frac{l \cos \theta}{g}} \\ & \frac{1}{2\pi} \sqrt{\frac{l \sin \theta}{g}} \\ & 4\pi \sqrt{\frac{l \cos \theta}{4g}} \\ & 4\pi \sqrt{\frac{l \tan \theta}{g}} \end{aligned}$$

**Solution:** The time period of a conical pendulum is

$$T = 4\pi \sqrt{\frac{l \cos \theta}{4g}}$$

**Question 1.2:** The kinetic energy of a rotating body depends upon..... [1]

- a. distribution of mass only.
- b. angular speed only.
- c. distribution of mass and angular speed.
- d. angular acceleration only.

**Solution:** distribution of mass and angular speed

**Question 1.3:** If the metal bob of a simple pendulum is replaced by a wooden bob of the same size, then its time period will..... [1]

- a. increase
- b. remain same
- c. decrease
- d. first increase and then decrease.

**Solution:** (b) remain same

The time period of a simple pendulum is

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Where  $l$  = length of the pendulum

$g$  = acceleration due to gravity

Therefore, from the given equation, we know that the periodic time of the pendulum does not depend on the mass of the bob, and so, it does not matter of what material the bob is made of, and hence, its time period remains the same.

**Question 1.4:** The graph between applied force and change in the length of wire within elastic limit is a..... [1]

- a. straight line with positive slope.
- b. straight line with negative slope.
- c. curve with positive slope.
- d. curve with negative slope.

**Solution:** (a) straight line with positive slope.

Stress is directly proportional to strain and the elastic limit is

$$\frac{F}{A} \propto \frac{\Delta L}{L}$$

Since,  $A$  and  $L$  are constants.

$$\therefore F \propto \Delta L$$

Thus, the graph between applied force and change in length is a straight line with a positive slope.

**Question 1.5:** When longitudinal wave is incident at the boundary of denser medium, then..... [1]

- a. compression reflects as a compression.
- b. compression reflects as a rarefaction.
- c. rarefaction reflects as a compression.
- d. longitudinal wave reflects as transverse wave.

**Solution:** (a) compression reflects as a compression

A compression is reflected as a compression at the boundary of a denser medium, but it is reflected as a rarefaction at the boundary of a rarer medium.

**Question 1.6:** The dimensions of universal gravitational constant are..... [1]

- a.  $[L^1 M^0 T^0]$
- b.  $[L^2 M^1 T^0]$
- c.  $[L^{-1} M^1 T^{-2}]$
- d.  $[L^3 M^{-1} T^{-2}]$

**Solution:** (d)  $[L^3 M^{-1} T^{-2}]$

$$G = \frac{Fr^2}{Mm}$$

$$\therefore [G] = \frac{[L^1 M^1 T^{-2}][L^2]}{[M^2]} = [L^3 M^{-1} T^{-2}]$$

**Question 1.7:** Two copper spheres of radii 6 cm and 12 cm respectively are suspended in an evacuated enclosure. Each of them are at a temperature  $15^\circ\text{C}$  above the surroundings. The ratio of their rate of loss of heat is..... [1]

- a. 2:1
- b. 1:4
- c. 1:8
- d. 8:1

**Solution:** (b) 1:4

$$r_1 = 6 \text{ cm}$$

$$r_2 = 12 \text{ cm}$$

$$T_1 = T_2 = 15^\circ\text{C}$$

Ratio of loss of heat is

$$\frac{R_1}{R_2} = \frac{A_1}{A_2} \times \frac{T_1^4}{T_2^4}$$

$$\frac{R_1}{R_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{6^2}{12^2} = \frac{1}{4}$$

**Question 2 | Attempt any Six**

[12]

**Question 2.1:** In circular motion, assuming  $\vec{v} = \vec{\omega} \times \vec{r}$ , obtain an expression for the resultant acceleration of a particle in terms of tangential and radial component. [2]

**Solution:** Acceleration of a particle,

$$a = \lim_{\delta t \rightarrow 0} \left( \frac{\delta v}{\delta t} \right) \dots \delta t \rightarrow 0; \delta t \neq 0$$

$$\therefore a = \frac{dv}{dt}$$

$$\text{But, } v = r\omega$$

$$\therefore a = \frac{d}{dt}(r\omega)$$

$$= r \frac{d\omega}{dt} + \omega \frac{dr}{dt}$$

$$\because r \text{ is constant}$$

$$\frac{dr}{dt} = 0$$

$$\therefore a = r \frac{d\omega}{dt}$$

$$\because \frac{d\omega}{dt} = \alpha$$

$$a = r\alpha$$

Given that:

$$\vec{v} = \vec{\omega} \times \vec{r}$$

Differentiating w.r.t. time

$$\frac{d\vec{v}}{dt} = \frac{d}{dt}(\vec{\omega} \times \vec{r})$$

$$\frac{d\vec{v}}{dt} = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}$$

$$\frac{d\vec{v}}{dt} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$$

$$\therefore \vec{a} = \vec{a}_T + \vec{a}_r$$

Where,

$a$  = Linear acceleration

$a_T$  = Tangential component of linear acceleration

$a_r$  = Radial component of linear acceleration

**Question 2.2:** Explain why an astronaut in an orbiting satellite has a feeling of weightlessness. [2]

**Solution:** When an astronaut is in an orbiting satellite, the astronaut and satellite are attracted towards the centre of the Earth and both will fall towards the Earth with the same acceleration. This acceleration is the same as 'g' at the satellite. Thus, the astronaut is unable to exert weight on the floor of the satellite. Because of this, the satellite does not provide a normal reaction on the astronaut, and hence, the astronaut feels weightlessness.

**Question 2.3.1:** State the theorem of parallel axes about moment of inertia. [2]

**Solution: Definition of moment of inertia :**

A measure of the resistance of a body to angular acceleration about a given axis that is equal to the sum of the products of each element of mass in the body and the square of the element's distance from the axis.

**Theorem of parallel axes:-**

The moment of inertia of a body about any axis is equal to the sums of its moment of inertia about a parallel axis passing through its centre of mass and the product of its mass and the square of the perpendicular distance between the two parallel axes.

Mathematically,  $I_o = I_c + Mh^2$

Where  $I_o = M \cdot I$  of the body about any axis passing through centre O.

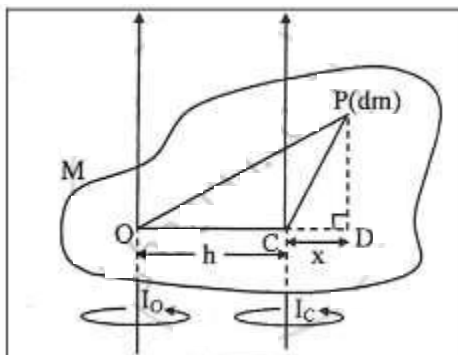
$I_c = M \cdot I$  of the body about parallel axis passing through centre of mass.

$h$  = distance between two parallel axes.

**Proof :**

i) Consider a rigid body of mass  $M$  rotating about an axis passing through a point  $O$  as shown in the following figure.

Let  $C$  be the centre of mass of the body, situated at distance  $h$  from the axis of rotation.



ii) Consider a small element of mass  $dm$  of the body, situated at a point  $P$ .

iii) Join  $PO$  and  $PC$  and draw  $PD$  perpendicular to  $OC$  when produced.

iv) M.I of the element  $dm$  about the axis through  $O$  is  $(OP)^2 dm$

∴ M.I of the body about the axis through O is given by

$$I_o = \int (OP)^2 dm \quad \dots\dots(1)$$

v) M.I of the element dm about the axis through c is  $CP^2 dm$

∴ M.I of the body about the axis through C

$$I_c = \int (CP)^2 dm \quad \dots\dots(2)$$

vi) From the figure,

$$OP^2 = OD^2 + PD^2$$

$$= (OC + CD)^2 + PD^2$$

$$= OC^2 + 2OC \cdot CD + CD^2 + PD^2$$

$$\therefore CP^2 = CD^2 + PD^2$$

$$\therefore OP^2 = OC^2 + 2 OC \cdot CD + CP^2 \quad \dots\dots(3)$$

vii) From equation (1)

$$I_o = \int (OP)^2 dm$$

From equation (3)

$$I_o = \int (OC^2 + 2OC \cdot CD + CP^2) dm$$

$$\therefore I_o = \int (h^2 + 2hx + CP^2) dm$$

$$= \int h^2 dm + \int 2h \cdot x dm + \int CP^2 dm$$

$$= h^2 \int dm + 2h \int x dm + \int CP^2 dm$$

$$I_o = h^2 \int dm + 2h \int x dm$$

[From equation (2)]

$$\therefore I_o = I_c + h^2 \int dm + 2h \int x dm \quad \dots\dots(4)$$

viii) Since  $\int dm = M$  and  $\int x dm = 0$  and

Algebraic sum of the moments of the masses of its individual particles about the centre of mass is zero for body in equilibrium.

∴ Equation (4) becomes

$$I_o = I_c + Mh^2$$

Hence proved.

**Question 2.3.2:** State the theorem of perpendicular axes about moment of inertia. [2]

**Solution: Theorem of perpendicular axes:-** The moment of inertia of a plane lamina about an axis perpendicular to its plane is equal to the sum of its moment of inertia about two mutually perpendicular axes concurrent with the perpendicular axis and lying in the plane of the laminar body.

**Question 2.4:** State Wein's displacement law [2]

**Solution:** The wavelength for which the emissive power of a black body is maximum is inversely proportional to the absolute temperature of the black body.

**Question 2.4.1:** State Wein's displacement law [1]

**Solution:** The wavelength for which the emissive power of a black body is maximum is inversely proportional to the absolute temperature of the black body.

**Question 2.4.2:** State the First law of thermodynamics. [1]

**Solution: The first law of thermodynamics:**

The First Law of Thermodynamics states that heat is a form of energy, and thermodynamic processes are therefore subject to the principle of conservation of energy. This means that heat energy cannot be created or destroyed. It can, however, be transferred from one location to another and converted to and from other forms of energy.

The equation for the first law of thermodynamics is given as;

$$\Delta U = q + W$$

Where,

- $\Delta U$  = change in internal energy of the system.
- $q$  = algebraic sum of heat transfer between system and surroundings.
- $W$  = work interaction of the system with its surroundings.

**Question 2.5:** A particle in S.H.M. has a period of 2 seconds and amplitude of 10 cm. Calculate the acceleration when it is at 4 cm from its positive extreme position. [2]

**Solution:** A particle in SHM has a period of 2 seconds and an amplitude of 10 cm. When it is at 4 cm from its positive extreme position, the displacement ( $x$ ) of the particle is  $10 - 4 = 6$  cm.

Acceleration =  $\omega^2 \cdot x$  (in magnitude)

$$\begin{aligned}
 &= \left( \frac{2\pi}{T} \right)^2 x \\
 &= \frac{4\pi^2}{(2)^2} \times 6 \\
 &= 59.16 \text{ cm/s}^2
 \end{aligned}$$

Thus, the acceleration of the particle at 4 cm from its positive extreme position is 59.16 cm/s<sup>2</sup>.

**Question 2.6:** The surface tension of water at 0°C is 75.5 dyne/cm. Calculate surface tension of water at 25°C. ( $\alpha$  for water =  $2.7 \times 10^{-3}/^\circ\text{C}$ ) [2]

**Solution: Given:**

$$T_0 = 75.5 \text{ dyne/cm}$$

$$\alpha_{\text{water}} = 2.7 \times 10^{-3}/^\circ\text{C}$$

**To find:**

Surface tension of water at 25°C

**Formula:**

$$T_1 = T_0(1 - \alpha\Delta t)$$

**Solution:**

$$T_{25} = T_0(1 - \alpha\Delta t)$$

$$T_{25} = T_0(1 - \alpha(25 - 0))$$

$$T_{25} = 75.5(1 - 2.7 \times 10^{-3} \times 25)$$

$$T_{25} = 75.5(1 - 0.0675)$$

$$T_{25} = 70.4 \text{ dyne/cm}$$

**The surface tension of water at 25°C is 70.4 dyne/cm.**

**Question 2.7:** The spin dryer of a washing machine rotating at 15 r.p.s. slows down to 5 r.p.s. after making 50 revolutions. Find its angular acceleration. [2]

**Solution: Given:**

$$n_1 = 15 \text{ r.p.s}$$

$$n_2 = 5 \text{ r.p.s}$$

$$\text{No. of revolution} = 50$$

$$\text{As, } \omega = 2\pi n$$



$$\omega = 2\pi \times 5 = 10\pi$$

$$\text{Also, } \omega_0 = 2\pi n_0 = 2\pi \times 15 = 30\pi$$

$$\text{Angular displacement in/revolution} = 2\pi$$

$$\text{Angular displacement in 50 revolution} = 2 \times 50\pi \\ = 100\pi$$

$$\text{Now, } \omega^2 = \omega_0^2 + 2\alpha\theta$$

$$(10\pi)^2 = (30\pi)^2 + 2\alpha(100\pi)$$

$$900\pi^2 - 100\pi^2$$

$$= -\alpha = \frac{\quad}{200\pi}$$

$$\alpha = -4\pi \text{ rad/s}^2$$

$$\text{Also, } \alpha = -12.56 \text{ rad/s}^2$$

**Question 2.8:** Calculate the period of revolution of Jupiter around the Sun. The ratio of the radius of Jupiter's orbit to that of the Earth's orbit is 5. (Period of revolution of the Earth is 1 year) [2]

**Solution:**

$$\text{Given that } \frac{r_J^3}{r_E^3} = 5$$

$$\frac{T_J^2}{T_E^2} = \frac{r_J^3}{r_E^3}$$

$$\text{Or, } \frac{T_J}{T_E} = \left( \frac{r_J}{r_E} \right)^{3/2}$$

$$\frac{T_J}{1} = (5)^{3/2} = 11.18 \text{ years}$$

Thus, the period of revolution of Jupiter is 11.18 years.

**Question 3 | Attempt any Three**

[9]

**Question 3.1:** Derive an expression for excess pressure inside a drop of liquid. [3]

**Solution:** Consider a liquid drop of radius R and surface tension T.

Due to surface tension, the molecules on the surface film experience the net force in the inward direction normal to the surface.

Therefore there is more pressure inside than outside.

Let  $p_1$  be the pressure inside the liquid drop and  $p_0$  be the pressure outside the drop.

Therefore excess of pressure inside the liquid drop is,

$$p = p_1 - p_0$$

Due to excess pressure inside the liquid drop the free surface of the drop will experience the net force in outward direction due to which the drop will expand.

Let the free surface displace by  $dR$  under isothermal conditions.

Therefore excess of pressure does the work in displacing the surface and that work will be stored in the form of potential energy.

The work done by an excess of pressure in displacing the surface is,

$$dW = \text{Force} \times \text{displacement}$$

$$= (\text{Excess of pressure} \times \text{surface area}) \times \text{displacement of the surface}$$

$$= p \times 4\pi R^2 \times dR \quad \dots(1)$$

Increase in the potential energy is,

$$dU = \text{surface tension} \times \text{increase in area of the free surface}$$

$$= T[4\pi(R + dR)^2 - 4\pi R^2]$$

$$= T[4\pi(2RdR)] \quad \dots(2)$$

From (1) and (2)

$$p \times 4\pi R^2 \times dR = T[4\pi(2RdR)]$$

$$\Rightarrow p = \frac{2T}{R}$$

The above expression gives us the pressure inside a liquid drop.

**Question 3.2:** Explain what is Doppler effect in sound [3]

**Solution:** The apparent change in the frequency of sound emitted by a source as heard by the observer when there is a relative motion between the source of the sound and an observer is called **Doppler Effect**.

**Question 3.2:** State any four applications of Doppler effect

**Solution: Applications of Doppler Effect:-**

- In colour Doppler sonography – To provide information about the rate of flow of various fluids, including blood.
- For the determination of the speed of rotation of the Sun and the speed of stars
- In RADAR
- For speed detection on highways

**Question 3.3:** Calculate the average molecular kinetic energy : [3]

(a) Per kilo mole, (b) per kilogram, of oxygen at 27°C.

( $R = 8320 \text{ J/k mole K}$ , Avogadro's number =  $6.03 \times 10^{26} \text{ molecules/K mole}$ )

**Solution:** Given that

Temperature =  $27^\circ\text{C} = (273 + 27) \text{ K} = 300 \text{ K}$

$$\begin{aligned} \text{(a) Average kinetic energy per kilo mole} &= \frac{3}{2} RT \\ &= \frac{3}{2} \times 8320 \frac{\text{J}}{\text{K}} \text{ mole} \times 300 \text{ K} \\ &= 3.744 \times 10^6 \text{ J} \end{aligned}$$

$$\text{(b) Kinetic energy per kilogram} = \frac{\text{Kinetic energy per kilo mole}}{\text{Molecular weight}}$$

Molecular weight of oxygen = 32

$$\text{Kinetic energy per kilogram} = \frac{3.744 \times 10^6 \text{ J}}{32} = 0.117 \times 10^6 \text{ J}$$

**Question 3.4:** A uniform steel rod of  $5 \text{ mm}^2$  cross section is heated from  $0^\circ\text{C}$  to  $25^\circ\text{C}$ . Calculate the force which must be exerted to prevent it from expanding. Also calculate strain. ( $\alpha$  for steel =  $12 \times 10^{-6}/^\circ\text{C}$  and  $\gamma$  for steel =  $20 \times 10^{10} \text{ N/m}^2$ ) [3]

**Solution:** Given that  $\alpha_{\text{steel}} = 12 \times 10^{-6} / ^\circ\text{C}$ ,  $\gamma_{\text{steel}} = 20 \times 10^{10} \text{ N/m}^2$

Area of cross-section of the rod,  $A = 5 \text{ mm}^2 = 5 \times 10^{-6} \text{ m}^2$

Change in temperature ( $\Delta T$ ),  $(T_2 - T_1) = 25^\circ\text{C}$

Force exerted by the rod due to heating = Thermal stress  $\times$  Area

Thermal stress =  $\gamma \times \text{Strain}$

$$= \gamma_{\text{steel}} \times \alpha_{\text{steel}} \times \Delta T$$

Therefore, the force exerted by the rod due to heating is

$$\begin{aligned} &= \gamma_{\text{steel}} \times \alpha_{\text{steel}} \times \Delta T \times A \\ &= 20 \times 10^{10} \times 12 \times 10^{-6} \times 25 \times 5 \times 10^{-6} \\ &= 300 \text{ N} \end{aligned}$$

$$\text{Strain} = \frac{\text{Change in length}}{\text{Original length}} = \alpha \Delta T$$

$$\text{Strain} = \alpha_{\text{steel}} \Delta T$$

$$\begin{aligned} &= 12 \times 10^{-6} \times 25 \\ &= 3 \times 10^{-4} \end{aligned}$$

**Question 4: Attempt any One****[7]****Question 4.1: What are the forced vibrations and resonance?****[7]**

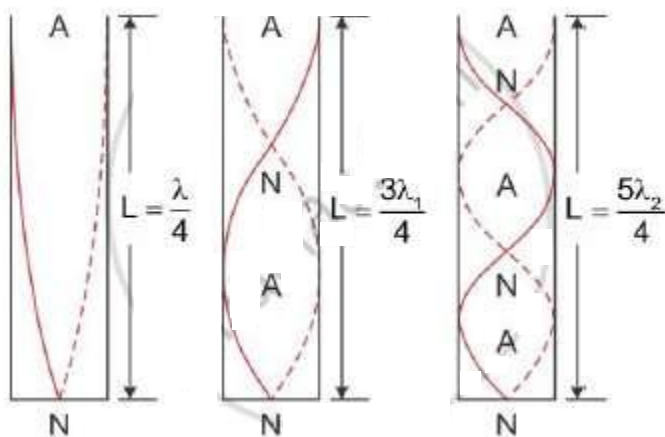
**Solution:** The vibrations of a body under the action of an external periodic force in which the body vibrates with a frequency equal to the frequency of the external periodic force, other than its natural frequency, are called **forced vibrations**.

Resonance is the phenomenon in which the body vibrates under the action of an external periodic force whose frequency is equal to the natural frequency of the driven body so that its amplitude becomes maximum. Resonance is a special case of forced vibrations.

**Question 4.1:** Show that only odd harmonics are present in an air column vibrating in a pipe closed at one end.

**Solution:** Consider the different modes of vibration of an air column within a pipe closed at one end. Let  $L$  be the length of the pipe.

Stationary waves are formed within the air column when the time taken by the sound waves to produce a compression and rarefaction becomes equal to the time taken by the wave to travel twice the length of the tube. The standing waves are formed only for certain discrete frequencies.



In the first mode of vibration of the air column, there is one node and one antinode as shown in the figure above.

If  $\lambda$  is the length of the wave in the fundamental mode of vibration,

Then, Length of the air column,  $L = \lambda/4$

$$\Rightarrow \lambda = 4L \dots\dots\dots (1)$$

As velocity of the wave,  $v = n\lambda$

$$n = v/\lambda$$

Substituting (1) we get:

$$n = v/4L; \text{ Frequency of the fundamental mode}$$

In the **second mode of vibration** of the air column, two nodes and two antinodes are formed.

In this case:

$$\text{The length of the air column, } L = 3\lambda_1/4$$

Where  $\lambda_1$  is the wavelength of the wave in the second mode of vibration,

$$\Rightarrow \lambda = 4L/3 \dots\dots\dots(1)$$

$$v = n_1\lambda_1$$

$$n_1 = v/\lambda_1$$

Substituting (1), we get

$$n_1 = \frac{v}{\frac{4L}{3}} = \frac{3v}{4L} = 3n$$

This frequency is called the third harmonics or first overtone

Similarly, during the third mode of vibration of the air column, three nodes and three antinodes are formed.

Here,

$$\text{The length of the air column, } L = 5\lambda_2/4$$

Where  $\lambda_2$  is the wavelength of the wave in third mode of vibration,

$$\Rightarrow \lambda_2 = 4L/5 \dots\dots\dots(1)$$

$$v = n_2\lambda_2$$

$$n_2 = v/\lambda_2$$

Substituting (1), we get

$$n_2 = \frac{v}{\frac{4L}{5}} = \frac{5v}{4L} = 5n$$

This frequency is called the fifth harmonic or second overtone.

Thus, we see that the frequencies of the modes of vibrations are in the ratio  $n:n_1:n_2 = 1:3:5$ . This shows that only odd harmonics are present in the modes of vibrations of the air column closed at one end.

**Question 4.1:** A stretched wire emits a fundamental note of frequency 256 Hz. Keeping the stretching force constant and reducing the length of wire by 10 cm, the frequency becomes 320 Hz. Calculate the original length of wire. [3]

**Solution:** Let 'l' be the length of the wire which emits a fundamental note of frequency 256 Hz. When length = (l - 10) cm, fundamental frequency n = 320 Hz.

We know that the fundamental frequency n of a stretched string is given by

$$n = \frac{1}{2l} \sqrt{\frac{T}{m}}$$

where 'T' is the tension and 'm' the linear density of the string.

When length = l, n = 256 Hz

i.e.

$$256 = \frac{1}{2l} \sqrt{\frac{T}{m}} \dots\dots\dots(1)$$

When length = (l - 10) cm, n = 320 Hz

$$320 = \frac{1}{2(l - 10)} \sqrt{\frac{T}{m}} \dots\dots\dots(2)$$

Dividing (1) by (2) gives

$$\frac{256}{320} = \frac{2(l - 10)}{2l}$$

$$\frac{4}{5} = \frac{(l - 10)}{l}$$

$$l \text{ 50 cm} = 0.5 \text{ m}$$

Therefore, the original length of the wire is 50 cm.

**Question 4.2:** Obtain an expression for potential energy of a particle performing simple harmonic motion. Hence evaluate the potential energy [3]

- at mean position and
- at extreme position.

**Solution:** An oscillating particle possess both types of energies : Potential as well as kinetic. It possesses potential energy on account of its displacement from the equilibrium position.

Potential energy : Consider a particle of mass  $m$  executing S.H.M. Let  $x$  be its displacement from the equilibrium position at any instant  $t$ . Since in S.H.M. the force  $F$  acting upon the particle is proportional to and opposite to the displacement  $x$ , we have  $F = -kx$

Where, the constant  $k$  gives the force per unit displacement  $x$ . The force can also be expressed in terms of potential energy (P.E) of the particle as :

$$F = -\frac{dw}{dx}$$

$$\text{Thus, } \frac{dw}{dx} = kx$$

On integrating, we get

$$P.E = \frac{1}{2}kx^2 + C$$

Where  $C$  is a constant. If we assume the potential energy zero in the equilibrium position, i.e., If  $P.E = 0$  at  $x = 0$ , then  $C = 0$

$$\text{Therefore, } P.E = \frac{1}{2}kx^2$$

$$P.E = \frac{1}{2}mw^2x^2$$

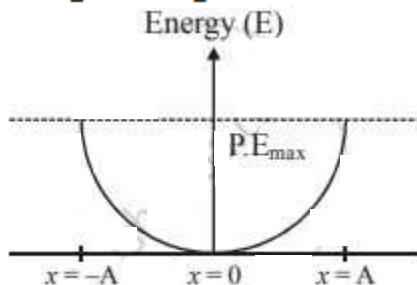
Hence from here it is clear that the potential energy of a particle doing S.H.M. is directly proportional to the square of the displacement ( $P.E \propto x^2$ )

(a) Potential energy at mean position : At mean position, velocity of a particle executing S.H.M. is maximum and displacement is minimum i.e.,  $x = 0$

$$P.E = \frac{1}{2}mw^2x^2 = 0$$

(b) Potential energy at extreme position: At extreme position, velocity of a particle executing S.H.M. is minimum and displacement is maximum, i.e.,  $x = \pm a$

$$P.E = \frac{1}{2}ka^2 = \frac{1}{2}mw^2a^2$$



**Question 5.1:** Electric field intensity in free space at a distance ' $r$ ' outside the charged conducting sphere of radius ' $R$ ' in terms of surface charge density ' $\sigma$ ' is..... [1]

- (a)  $\frac{\sigma}{\epsilon_0} \left[ \frac{R}{r} \right]^2$   
 (b)  $\frac{\epsilon_0}{\sigma} \left[ \frac{R}{r} \right]^2$   
 (c)  $\frac{R}{r} \left[ \frac{\sigma}{\epsilon_0} \right]^2$   
 (d)  $\frac{R}{\sigma} \left[ \frac{r}{\epsilon_0} \right]^2$

**Solution:**

(a)  $\frac{\sigma}{\epsilon_0} \left[ \frac{R}{r} \right]^2$

Electric field intensity in the free space outside the charged conducting sphere in terms of surface charged density is given as

$$E = \frac{\sigma}{\epsilon_0} \left[ \frac{R}{r} \right]^2$$

**Question 5.2:** Instrument which can measure terminal potential difference as well as electro motive force (e.m. f.) is..... [1]

- Wheatstone's meter bridge
- Voltmeter
- Potentiometer
- Galvanometer

**Solution:** (c) Potentiometer

A potentiometer is an instrument used to compare the e.m.f. of two cells or any two sources of e.m.f. If the e.m.f. of one of the sources is known, then the absolute value of the other source can be determined. The potentiometer can also be used to determine the internal resistance of a cell, to measure current, to calibrate an ammeter or voltmeter, to compare resistances and for measurement of thermoelectric e.m.f.

**Question 5.3:** If the frequency of incident light falling on a photosensitive material is doubled, then the kinetic energy of the emitted photoelectron will be..... [1]

- same as its initial value  
 two times its initial value  
 more than two times its initial value  
 less than two times its initial value

**Solution:** More than two times its initial value

$$E = W_0 + K_{\max} \Rightarrow K_{\max} = E - W_0 = h\nu - W_0 \Rightarrow K_1 = h\nu - W_0 \text{ and } K_2 = 2h\nu - W_0 \Rightarrow K_2 > 2K_1$$



**Question 5.4:** Linear momentum of an electron in Bohr orbit of H-atom (principal quantum number n) is proportional to..... [3]

- a.  $1/n^2$
- b.  $1/n$
- c.  $n$
- d.  $n^2$

**Solution:** (b)  $1/n$

Linear momentum,  $p = mv$

Velocity of electron in Bohr's orbit is given as

$$v = \frac{e^2}{2\epsilon_0 n h}$$

$$\therefore mv = \frac{e^2}{2\epsilon_0 n h}$$

$$\Rightarrow p = \frac{me^2}{2\epsilon_0 n h}$$

$$\Rightarrow p \propto \frac{1}{n}$$

**Question 5.5:** In a semiconductor, acceptor impurity is..... [1]

- a. antimony
- b. indium
- c. phosphorous
- d. arsenic

**Solution:** (b) indium

Indium is an acceptor impurity, whereas antimony, phosphorus and arsenic are donor impurities.

**Question 5.6:** The power radiated by linear antenna of length T is proportional to..... [1]

( $\lambda$  = wavelength)

a.  $\frac{\lambda}{l}$

b.  $\left[\frac{\lambda}{l}\right]^2$

c.  $\frac{l}{\lambda}$

d.  $\left[\frac{l}{\lambda}\right]^2$

**Solution:** (d)  $(l/\lambda)^2$

Effective power radiated by the antenna would be too small, because the power radiated by a linear antenna of length 'l' into space is  $(l/\lambda)^2$ .

As high powers are needed for good transmission, so higher frequency is required which can be achieved by modulation.

**Question 5.7:** The numerical aperture of objective of a microscope is 0.12. The limit of resolution, when light of wavelength 6000Å is used to view an object is.....

[1]

- a.  $0.25 \times 10^{-7}\text{m}$
- b.  $2.5 \times 10^{-7}\text{m}$
- c.  $25 \times 10^{-7}\text{m}$
- d.  $250 \times 10^{-7}\text{m}$

**Solution:** (c)  $25 \times 10^{-7}\text{m}$

The minimum resolvable linear distance between two nearby objects is given by

$$x = \frac{\lambda}{2NA} = \frac{6000 \times 10^{-10}}{2 \times 0.12} = 2.5 \times 10^{-6}\text{m} = 25 \times 10^{-7}\text{m}$$

**Question 6: Attempt any Six**

[12]

**Question 6.1:** What is a Polaroid?

[2]

**Solution:** A Polaroid is a material which polarises light. The phenomenon of selective absorption is made use of in the construction of polaroids. There are different types of polaroids.

A Polaroid consists of micro crystals of herapathite (an iodosulphate of quinine). Each crystal is a doubly refracting medium, which absorbs the ordinary ray and transmits only the extra ordinary ray. The modern Polaroid consists of a large number of ultra-microscopic crystals of herapathite embedded with their optic axes, parallel, in a matrix of nitro - cellulose.

Recently, new types of Polaroid are prepared in which thin film of polyvinyl alcohol is used. These are colourless crystals which transmit more light, and give better polarisation.

**Question 6.1:** State two uses of Polaroid.

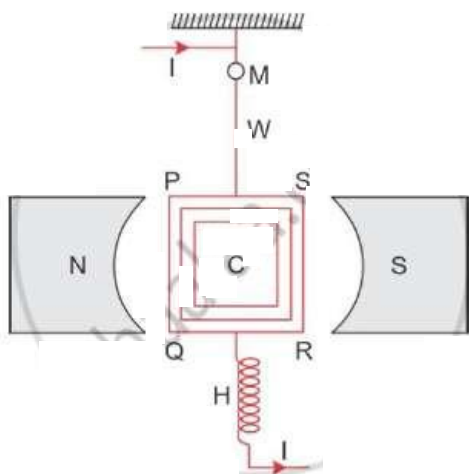
**Solution: Uses of Polaroid:-**

- (1) In motor car head lights – To remove headlight glare
- (2) To improve color contrast in old oil paintings

**Question 6.2:** Draw a neat and labelled diagram of suspended coil type moving coil galvanometer.

[2]

**Solution:** Suspended type of moving coil galvanometer:-



PQRS = Rectangular coil

W = Thin phosphor bronze wire suspension

M = Plane mirror

H = Helical spring

C = Soft iron cylinder

I = Current through the coil

**Question 6.3:** Define Magnetization

[2]

**Solution: Magnetization:-** The net magnetic dipole moment per unit volume is magnetization of the sample.

$$\text{Magnetization} = \frac{\text{Net magnetic moment}}{\text{Volume}}$$

**Question 6.3:** Define Magnetic intensity.

[2]

**Solution 1: Magnetic intensity:-** Magnetic intensity is a quantity used in describing magnetic phenomenon in terms of the magnetic field. The strength of the magnetic field at a point can be given in terms of a vector quantity called magnetic intensity (H).

**Solution 2: Magnetic intensity:** The ratio of the strength of magnetising field to the permeability of free space is called as magnetic intensity.

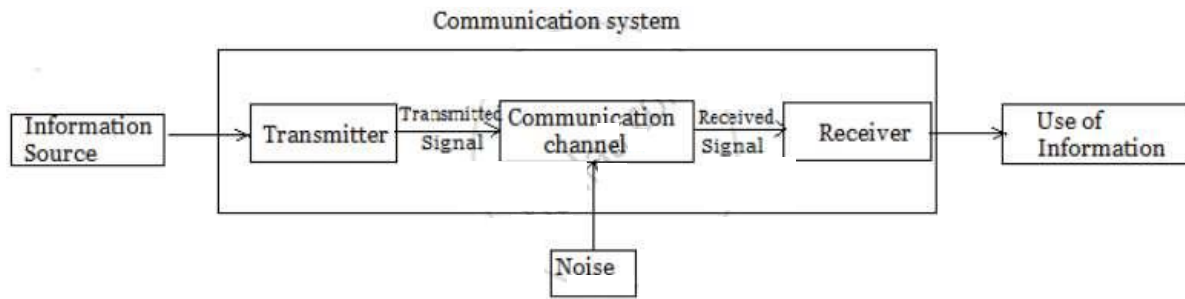
The strength of magnetic field at a point can be given in terms of vector quantity called as magnetic intensity (H).

$$H = \frac{B_0}{\mu_0} \text{ or } B_0 = \mu_0 H$$

The SI unit of magnetic intensity is  $\text{Am}^{-1}$ .

**Question 6.4:** Draw a block diagram of generalized communication system. [2]

**Solution: Block diagram of generalization of the communication system:-**



**Question 6.5:** A solenoid 3.142m long and 5.0 cm in diameter has two layers of windings of 500 turns each and carries a current of 5A. Calculate the magnetic induction at its centre along the axis. [2]

**Solution:** Given:

Length of solenoid,  $l = 3.142\text{m}$

Diameter,  $d = 5.0\text{cm} = 5 \times 10^{-2}\text{m}$

Radius,  $r = 2.5\text{cm} = 2.5 \times 10^{-2}\text{m}$

No. of turns,  $N = 2 \times 500 = 1000$

Current,  $I = 5\text{A}$

To calculate:

The magnetic induction at its centre along the axis.

Magnetic Induction,  $B = ?$

Formula:

$$B = \mu_0 n I$$

Solution:

$$B = \mu_0 \left( \frac{N}{l} \right) I$$

$$\therefore B = \frac{4\pi \times 10^{-7} \times 1000 \times 5}{3.142}$$

$$\therefore B = 2 \times 10^{-3}\text{T}$$

Hence, the magnetic induction at the centre of the circular loop along its axis is  $2 \times 10^{-3} \text{ T}$ .

**Question 6.6:** A circular coil of 300 turns and average area  $5 \times 10^{-3} \text{ m}^2$  carries a current of 15A. Calculate the magnitude of magnetic moment associated with the coil. [2]

**Solution:** No. of turns,  $n = 300$

$$A = 5 \times 10^{-3} \text{ m}^2$$

$$I = 15 \text{ A}$$

For  $n$  turns,  $M = nIA$

$$M = 300 \times 15 \times 5 \times 10^{-3}$$

$$M = 22.5 \text{ Am}^2$$

Hence, the magnitude of magnetic induction associated with the coil is  $22.5 \text{ Am}^2$ .

**Question 6.7:** The magnetic flux through a loop varies according to the relation  $\Phi = 8t^2 + 6t + C$ , where 'C' is constant, ' $\Phi$ ' is in millimeter and 't' is in second. What is the magnitude of induced e.m.f. in the loop at  $t = 2$  seconds. [2]

**Solution:** Magnetic flux through the coil is given by relation

$$\Phi = 8t^2 + 6t + c \text{ (where } c \text{ is constant)} \quad (i)$$

To find the magnitude of the induced e.m.f. 'e' in the loop at  $t = 2$  seconds, we know that

$$e = \left| \frac{d\Phi}{dt} \right|$$

Differentiating equation (i) w.r.t.  $t$  we get,

$$e = 16t + 6$$

$$e = 16 \times (2) + 6$$

$$e = 38 \text{ millivolt} = 0.038 \text{ volt}$$

Hence, the magnitude of induced e.m.f. is 0.038 volt.

**Question 6.8:** An electron is orbiting in 5<sup>th</sup> Bohr orbit. Calculate ionisation energy for this atom, if the ground state energy is -13.6 eV. [2]

**Solution:** Ground state energy  $E_1 = -13.6 \text{ eV}$

$$E_5 = ?$$

Energy of electron in Bohr's orbit is inversely proportional to the square of the principal quantum number.

$$\therefore \frac{E_5}{E_1} = \frac{n_1^2}{n_5^2}$$

$$\therefore \frac{E_5}{E_1} = \frac{1^2}{5^2}$$

$$\therefore E_5 = \frac{-13.6}{25} = -0.544 \text{ eV}$$

The ionization energy =  $E_\infty - E_5 = 0 - (-0.544) = 0.544 \text{ eV}$

Hence, the ionization energy in the 5<sup>th</sup> orbit is 0.544 eV.

### Question 7 | Attempt any Three

[9]

**Question 7.1:** Obtain an expression for the radius of Bohr orbit for H-atom. [3]

**Solution:** Let us consider an electron revolving around the nucleus in a circular orbit of radius 'r'.

According to Bohr's first postulate, the centripetal force is equal to the electrostatic force of attraction. That is

$$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0} \times \frac{e^2}{r^2}$$

$$\text{Or, } v^2 = \frac{e^2}{4\pi\epsilon_0 mr} \text{ -----(1)}$$

According to the Bohr's second postulate:

$$\text{Angular momentum} = n \frac{h}{2\pi}$$

$$mvr = n \frac{h}{2\pi}$$

$$\text{Or, } v = \frac{nh}{2\pi mr} \text{ -----(2)}$$

$$\text{Or, } v^2 = \frac{n^2 h^2}{4\pi^2 m^2 r^2} \text{ -----(3)}$$

Comparing eqn (1) and eqn (3), we get

$$\frac{e^2}{4\pi\epsilon_0 mr} = \frac{n^2 h^2}{4\pi^2 m^2 r^2}$$

$$\text{Or, } r = \left( \frac{h^2 \epsilon_0}{\pi m e^2} \right) n^2 \text{ -----(4)}$$

This equation gives the radius of the n<sup>th</sup> Bohr orbit.

$$\text{For } n = 1, r_1 = \left( \frac{h^2 \epsilon_0}{\pi m e^2} \right) = 0.537 \text{ \AA} \text{ -----(5)}$$

$$\text{In general, } r_n = \left( \frac{h^2 \epsilon_0}{\pi m e^2} \right) n^2$$

The above equation gives the radius of Bohr orbit.

**Question 7.2:** What are  $\alpha$  and  $\beta$  parameters for a transistor? Obtain a relation between them. [3]

**Solution:** Alpha ( $\alpha_{dc}$ ): It is defined as the ratio of collector current to emitter current.

$$\alpha_{dc} = \frac{I_c}{I_E} \text{ -----(Equation 1)}$$

Beta ( $\beta_{dc}$ ): It is the current gain defined as the ratio of collector current to the base current.

$$\beta_{dc} = \frac{I_c}{I_B} \text{ -----(Equation 2)}$$

Relation between  $\alpha_{dc}$  and  $\beta_{dc}$ :

For a transistor,

$$I_E = I_B + I_C \text{ (with } I_C \cong I_E \text{) -----(Equation 3)}$$

Dividing both the side of eqn.(3) with  $I_C$ , we get :

$$\frac{I_E}{I_C} = \frac{I_B}{I_C} + 1$$

$$\frac{1}{\alpha_{dc}} = \frac{1}{\beta_{dc}} + 1$$

$$\alpha_{dc} = \frac{\beta_{dc}}{1 + \beta_{dc}} \text{ -----(Equation 4)}$$

$$\text{Or, } \beta_{dc} = \frac{\alpha_{dc}}{1 - \alpha_{dc}} \text{ -----(Equation 5)}$$

Equations (4) and (5) give the relation between  $\alpha_{dc}$  and  $\beta_{dc}$  of a transistor.

**Question 7.3:** Two metal spheres having charge densities  $5\mu\text{C}/\text{m}^2$  and  $-2\mu\text{C}/\text{m}^2$  with radii 2mm and 1mm respectively are kept in a hypothetical closed surface. Calculate total normal electric induction over the closed surface. [3]

**Solution:**  $\sigma_1 = 5 \mu\text{C}/\text{m}^2$

$$\sigma_2 = -2 \mu\text{C}/\text{m}^2$$

$$r_1 = 2 \text{ mm}$$

$$r_2 = 1 \text{ mm}$$

$$q_1 = \sigma_1 4\pi r_1^2$$

$$q_2 = \sigma_2 4\pi r_2^2$$

$$\text{T.N.E.I.} = q_1 + q_2$$

$$\therefore \text{T.N.E.I.} = 4\pi(\sigma_1 r_1^2 + \sigma_2 r_2^2)$$

$$\therefore \text{T.N.E.I.} = 4\pi(5 \times 10^{-6} \times 4 \times 10^{-6} + (-2) \times 10^{-6} \times 1 \times 10^{-6})$$

$$\therefore \text{T.N.E.I.} = 72\pi \times 10^{-12} \text{ C}$$

$$\therefore \text{T.N.E.I.} = 226.08 \times 10^{-12} \text{ C}$$

Hence, the total normal electric induction over the closed surface is  $226.08 \times 10^{-12} \text{ C}$ .

**Question 7.4:** The threshold wavelength of silver is  $3800\text{\AA}$ . Calculate the maximum kinetic energy in eV of photoelectrons emitted, when ultraviolet light of wavelength  $2600\text{\AA}$  falls on it. [3]

(Planck's constant,  $h = 6.63 \times 10^{-34} \text{ J.s.}$ ,

Velocity of light in air,  $c = 3 \times 10^8 \text{ m / s}$ )

**Solution:** Wavelength of silver,  $\lambda_1 = 3800 \text{ \AA}$

Wavelength of ultraviolet light,  $\lambda_2 = 2600 \text{ \AA}$

$$H = 6.63 \times 10^{-34} \text{ Js}$$

Velocity of light in air,  $c = 3 \times 10^8 \text{ m/s}$

To calculate kinetic energy,  $\text{K.E.} = h\nu - h\nu_0$

$$\text{K.E.} = hc \left[ \frac{1}{\lambda} - \frac{1}{\lambda_0} \right]$$

$$\text{K.E.} = 19.89 \times 10^{-19} \left[ \frac{1}{2.6} - \frac{1}{3.8} \right]$$

$$\text{K.E.} = \frac{2.416 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$$\text{K.E.} = 1.51 \text{ eV}$$

Hence, the maximum kinetic energy emitted by the photoelectron is **1.51 eV**.

**Question 8 | Attempt any One**

[7]

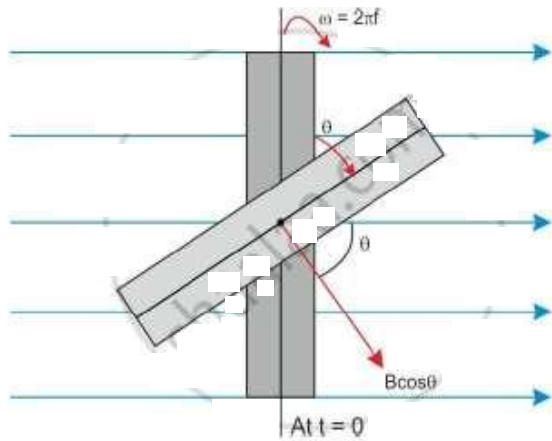
**Question 8.1:** Obtain an expression for e.m.f. induced in a coil rotating with uniform angular velocity in a uniform magnetic field. Show graphically the variation of e.m.f. with time (t). [7]



Resistance of a potentiometer wire is  $0.1\Omega/\text{cm}$ . A cell of e.m.f  $1.5\text{V}$  is balanced at  $300\text{ cm}$  on this potentiometer wire. Calculate the current and balancing length for another cell of e.m.f.  $1.4\text{V}$  on the same potentiometer wire.

**Solution:** Coil rotating in uniform magnetic field:

Consider a coil of ' $N$ ' turns with effective area  $NA$  placed in uniform magnetic induction  $B$  as shown in the figure below.



The coil is rotated continuously with constant angular velocity  $\omega$ . The axis of rotation is in the plane of the coil and normal to the magnetic induction  $B$ .

At  $t = 0$ , the plane of the coil is perpendicular to the magnetic induction  $B$ .

The magnetic flux passing through the coil is  $NAB$ .

After  $t$  seconds, the plane of the coil is at an angle  $\theta$ .

Thus, the magnetic flux  $\Phi$  through the coil at time  $t$  is given by

$$\Phi = NAB \cos \theta = NAB \cos \omega t$$

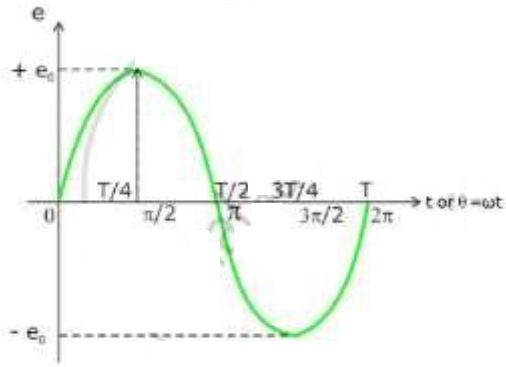
As time changes, the magnetic flux goes on changing. Hence, the e.m.f. generated in the coil is given by

$$e = -\frac{d\Phi}{dt} = -\frac{d}{dt}(NAB \cos \omega t)$$

$$e = NAB \omega \sin \omega t$$

$$e = 2\pi f NAB \sin \omega t$$

This is the expression for induced e.m.f. generated in the coil at any instant  $t$ . It is known as instantaneous e.m.f.



Given that

$$R/L = \sigma = 0.1 \Omega/\text{cm} = 0.1 \times 100 = 10 \Omega/\text{m}$$

$$l_1 = 300 \text{ cm} = 3 \text{ m}, E_1 = 1.5 \text{ V}, E_2 = 1.4 \text{ V}$$

We know that

$$E_1 = iR_1 = i\sigma l_1$$

$$\therefore i = \frac{E_1}{\sigma l_1} = \frac{1.5}{10 \times 3} = 0.05 \text{ A}$$

Hence, the current for the other cell is 0.05 A.

For potentiometer, the balancing condition is

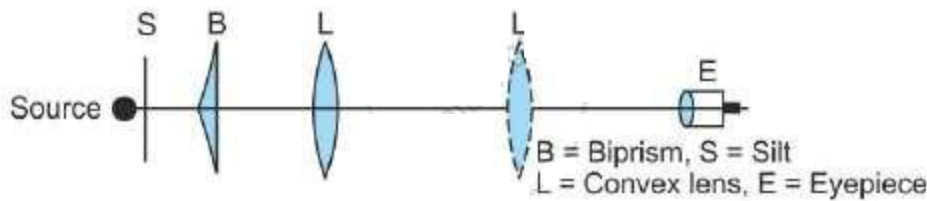
$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

$$\therefore l_2 = l_1 \frac{E_2}{E_1} = 3 \times \frac{1.4}{1.5} = 2.8 \text{ m}$$

So, the balancing length for the other cell is 2.8 m.

**Question 8.2:** Describe biprism experiment to calculate the wavelength of a monochromatic light. Draw the necessary ray diagram. [7]

**Solution:** To measure the wavelength of light, an optical bench is used. It is about one and a half metre long, and a scale is marked along its length. Four adjustable stands carrying the slit (S), biprism (B), lens (L) and micrometer eyepiece (E) are mounted on the optical bench.



Initially, the slit, biprism and eyepiece are kept at the same height such that their centres are in the same line. The slit is made narrow and is illuminated by a sodium vapour lamp. The biprism is now rotated slowly about a horizontal axis so that its refracting edge becomes parallel to the slit.

When the refracting edge of the biprism becomes exactly parallel, the interference pattern consisting of alternate bright and dark bands appear in the field of view of the eyepiece. The formula to be used is

$$\lambda = \frac{Xd}{D}$$

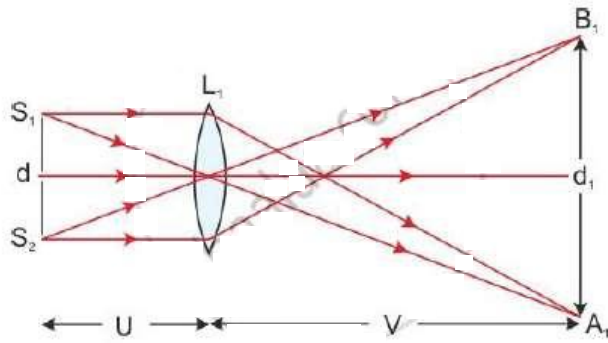
To determine the wavelength, the following steps are taken:

- (1) The distance between the slit and the eyepiece  $D$  can be easily measured from the scale marked on the optical bench.
- (2) The bandwidth  $X$  is measured with the help of the micrometer eyepiece. The vertical crosswire in the eyepiece is adjusted at the centre of the bright fringe. The micrometer eyepiece reading is noted.

Now, the eyepiece is moved horizontally until the crosswire has moved over a known number  $N$  of bright fringes. Again the reading of the micrometer eyepiece is noted. The difference between the two readings of the micrometer eyepiece gives the distance  $x$  through which the eyepiece is moved. Thus, the average distance between two adjacent bright fringes is

$$X = \frac{x}{N}$$

- (3) The distance ' $d$ ' between two coherent sources cannot be measured directly because the sources are virtual. Hence, the method of conjugate foci is used. In this method, the object and image distances get interchanged in two adjustments.



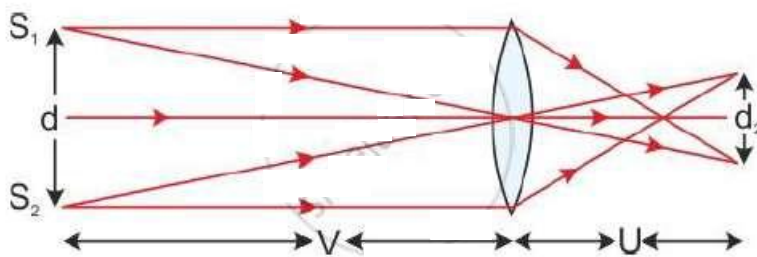
The convex lens of short focal length is introduced between the biprism and the eyepiece. Without disturbing the slit and biprism, the eyepiece is moved back so that its distance from the slit becomes greater than four times the focal length of the lens.

The lens is moved towards the slit and its position  $L_1$  is so adjusted that two magnified images  $A_1$  and  $B_1$  of  $S_1$  and  $S_2$  are formed in the focal plane of the eyepiece. The distance  $d_1$  between  $A_1$  and  $B_1$  is measured by the micrometre eyepiece.

From the figure, we get

$$\frac{\text{Size of image}}{\text{Size of object}} = \frac{\text{Distance of image}}{\text{Distance of object}}$$

$$\frac{d_1}{d} = \frac{v}{u} \text{ -----(1)}$$



The lens is now moved towards the eyepiece to the position  $L_2$  where two diminished images  $A_2$  and  $B_2$  of  $S_1$  and  $S_2$  are formed in the focal plane of the eyepiece.

The distance  $d_2$  between  $A_2$  and  $B_2$  is measured by the micrometer eyepiece. Then by the principle of conjugate foci, we can write

$$\frac{d_2}{d} = \frac{u}{v} \text{-----(2)}$$

Multiplying equations (1) and (2), we get

$$\frac{d_1 d_2}{d_2} = \frac{v}{u} \times \frac{u}{v} = 1$$

$$\therefore d^2 = d_1 d_2$$

$$\therefore d = \sqrt{d_1 d_2}$$

Thus, knowing D, X and d, we can calculate the wavelength  $\lambda$  of monochromatic light by using the equation  $\lambda = Xd/D$

The critical angle is given as

$$\sin \theta_c = \frac{1}{n}$$

It is given that

$$\theta_c = \sin^{-1}\left(\frac{3}{5}\right)$$

$$\therefore \frac{1}{n} = \frac{3}{5}$$

$$\therefore n = \frac{5}{3}$$

Now, the polarising angle is given as

$$\theta_p = \tan^{-1} n = \tan^{-1}\left(\frac{5}{3}\right)$$

**Question 8.2:** If the critical angle of a medium is  $\sin^{-1}(3/5)$ , find the polarising angle. [7]

**Solution:** Given :

The critical angle is given as

$$\sin \theta_c = 1/n$$

$$\therefore \theta_c = \sin^{-1}\left(\frac{1}{n}\right) \dots(1)$$

It is given that

$$\theta_c = \sin^{-1}\left(\frac{3}{5}\right)$$

$$\therefore \sin^{-1}\left(\frac{1}{n}\right) = \sin^{-1}\left(\frac{3}{5}\right) \dots\dots\dots\text{form (1)}$$

$$\therefore \frac{1}{n} = \frac{3}{5}$$

$$\therefore n = \frac{5}{3}$$

Now, the polarising angle is given as

$$\theta_p = \tan^{-1} n$$

$$\theta_p = \tan^{-1}\left(\frac{5}{3}\right)$$

$$\theta_p = \tan^{-1}(1.667)$$

$$\theta_p = 59^\circ 2'$$

the polarising angle is  $59^\circ 2'$