4. Oscillations

- Periodic motion → Motion which repeats itself after regular intervals of time
- Oscillatory motion →A body in oscillatory motion moves to and fro about its mean position in a fixed time interval.

- Period (T): It is the interval of time after which a motion is repeated. Its unit is seconds (s).

- Time period → Time required for one complete oscillation

$$T = \frac{1}{\nu}$$

where, v → Frequency

- Frequency : Number of oscillations in one second. The unit is Hertz.

- An oscillatory motion is said to be simple harmonic, when the displacement (x) of the particle from origin varies with time given as,

$$x(t) = A\cos(\omega t + \phi)$$

- Displacement is sinusoidal function of time.
- Displacement − A continuous function of time for SHM

- Non-harmonic oscillation is a combination of two or more harmonic oscillation.

- SHM is defined as the projection of uniform circular motion on the diameter of a circle of reference.
- Amplitude − Maximum displacement on either side of the mean position

- **Displacement** → It is indicated by sinusoidal trigonometric function.

  $x = A\sin wt$ and $w = 2\pi f$

  $x = A\cos wt$

- **Velocity** →If x = Asin (ωt ± f), then $v = \frac{dx}{dt} = \omega A\cos(\omega t \pm \phi)$.

$$v = \omega A\sqrt{1 - \sin^2(\omega t + \phi)}$$

$$= \omega A\sqrt{1 - \left(\frac{x^2}{A^2}\right)} = \omega\sqrt{A^2 - x^2}$$

  - **Acceleration→** $a = \frac{dv}{dt} = -\omega^2 A\sin(\omega t \pm \phi) = -\omega^2 x$

  - **Time period of a pendulum→** $a = \frac{dv}{dt} = -\omega^2 A\sin(\omega t \pm \phi) = -\omega^2 x$

- ***l* is the length of the pendulum.**
- **Restoring force**→Iτ ισ τηε force that is responsible for maintaining SHM.

$$F = -kx$$

Here, k is the force constant.

- A particle of mass m oscillating under the influence of Hooke's law of restoring force given by $F = -kx$ exhibits simple harmonic motion with

$$\omega = \sqrt{\frac{k}{m}} \text{ and}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

- The maximum velocity of the particle in SHM is at mean position and it is given by vmax=±aω.
- The minimum velocity of the particle in SHM is at extreme position and it is 0.
- At mean position, the particle has minimum acceleration and its magnitude is 0.
- At extreme position, the particle has maximum value of acceleration and its magnitude is ω2a.
- The frequency of SHM is given by f=12πkm.
- The period of SHM is given by T=2πax=2πAcceleration per unit displacement.
- The physical quantity that describes the state of oscillation is known as the phase of SHM.
- The physical quantity that describes the state of oscillation of the particle performing SHM at the beginning of the motion is called the epoch of SHM.

Energy in Simple Harmonic Motion

- Potential energy $= \frac{1}{2}m\omega^2x^2 \text{ or } \frac{1}{2}m\omega^2A^2\cos^2\omega t$
- Kinetic energy $= \frac{1}{2}m\omega^2\left(A^2 - x^2\right) \text{ or } \frac{1}{2}m\omega^2A^2\sin^2\omega t$
- Total energy $= \frac{1}{2}m\omega^2A^2\sin^2\omega t + \frac{1}{2}m\omega^2A^2\cos^2\omega t = \frac{1}{2}m\omega^2A^2$

- The instantaneous displacements of two SHMs travelling along the same straight line,with same time period and different amplitudes and phases are $x_1 = a_1\sin(\omega t + \alpha_1)$ and $x_2 = a_2\sin(\omega t + \alpha_2)$.
- The resultant displacement is given by

x = Rsinωt + δ

- R is the resultant amplitude and is given by

R = α12 + α22 + 2α1α2cosα1-α2

- δ represents the resultant phase of the S.H.M and is given by

δ = tan-1a1sinα1 + a2sinα2a1cosα1 + a2cosα2

- **Special cases:**
  - When $\alpha1\text{-}\alpha2 = 0$,
    - $R = \alpha1 + \alpha2$
  - When $\alpha1 \text{-}\alpha2 = \pi$,
    - $R = \alpha1 - \alpha2$
  - When $\alpha1 \text{-}\alpha2 =\pi2$,
    - $R = \alpha12 + \alpha22$

- A simple pendulum is a heavy point mass suspended by a weightless, inextensible, flexible string attached to a rigid support from where it moves freely.
- The periodic motion of a simple pendulum for small displacements is simple harmonic.
- Time period of simple pendulum:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

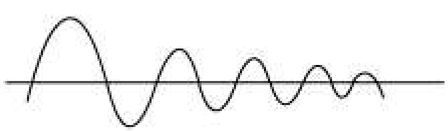
Laws of simple pendulum:

- The time period of the pendulum is directly proportional to the square root of its length.
- The time period of the pendulum is inversely proportional to the square root of the acceleration due to gravity of the place.
- The time period of the pendulum is independent of the mass of the bob.
- The time period of the pendulum does not depend upon its amplitude of oscillations.

Seconds Pendulum

- It is a simple pendulum that has a time period equal to 2 seconds.

- **Damped oscillation** → When the motion of an oscillator is reduced by an external force



Damped oscillation

Angular frequency of the damped oscillation

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

Where, *b* is a damping constant

- Damping force (F_d) depends on the nature of the surrounding medium; it is proportional to the velocity (v) of the bob, and acts opposite to the direction of velocity.

$$F_d \propto -v$$
$$\therefore F_d = -bv$$