DAY —— 09 SEAT NUMBER | | | | | | |

| 2024 | III | 02 | 1100 | J-862 | (E) |
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# MATHEMATICS & STATISTICS (40)
## (ARTS & SCIENCE)

**Time : 3 Hrs.** **(8 Pages)** **Max. Marks : 80**

***General instructions :***

*The question paper is divided into **FOUR** sections.*

*(1)* ***Section A :*** *Q. 1 contains **Eight** multiple choice type of questions, each carrying **Two** marks.*
*Q. 2 contains **Four** very short answer type questions, each carrying **One** mark.*

*(2)* ***Section B :*** *Q. 3 to Q. 14 contain **Twelve** short answer type questions, each carrying **Two** marks. (Attempt any **Eight**)*

*(3)* ***Section C :*** *Q. 15 to Q. 26 contain **Twelve** short answer type questions, each carrying **Three** marks. (Attempt any **Eight**)*

*(4)* ***Section D :*** *Q. 27 to Q. 34 contain **Eight** long answer type questions, each carrying **Four** marks. (Attempt any **Five**)*

*(5) Use of log table is allowed. Use of calculator is not allowed.*

*(6) Figures to the right indicate full marks.*

*(7) Use of graph paper is <u>not</u> necessary. Only rough sketch of graph is expected.*

*(8) For each multiple choice type of question, only the first attempt will be considered for evaluation.*

*(9) Start answer to each section on a new page.*

# SECTION – A

**Q. 1. Select and write the correct answer for the following multiple choice type of questions :** [16]

(i) The dual of statement $t \vee (p \vee q)$ is _____.

(a) $c \wedge (p \vee q)$ (b) $c \wedge (p \wedge q)$

(c) $t \wedge (p \wedge q)$ (d) $t \wedge (p \vee q)$ (2)

(ii) The principle solutions of the equation $\cos\theta = \frac{1}{2}$ are _____.

(a) $\frac{\pi}{6}, \frac{5\pi}{6}$ (b) $\frac{\pi}{3}, \frac{5\pi}{3}$

(c) $\frac{\pi}{6}, \frac{7\pi}{6}$ (d) $\frac{\pi}{3}, \frac{2\pi}{3}$ (2)

(iii) If $\alpha, \beta, \gamma$ are direction angles of a line and $\alpha = 60°, \beta = 45°$, then $\gamma =$ _____.

(a) 30° or 90° (b) 45° or 60°

(c) 90° or 130° (d) 60° or 120° (2)

(iv) The perpendicular distance of the plane $\bar{r} \cdot (3\hat{i} + 4\hat{j} + 12\hat{k}) = 78$, from the origin is _____.

(a) 4 (b) 5

(c) 6 (d) 8 (2)

(v) The slope of the tangent to the curve $x = \sin\theta$ and $y = \cos 2\theta$ at $\theta = \frac{\pi}{6}$ is _____.

(a) $-2\sqrt{3}$ (b) $\frac{-2}{\sqrt{3}}$

(c) $-2$ (d) $-\frac{1}{2}$ (2)

**(vi)** If $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} x^3.\sin^4 x\,dx = k$ then $k =$ _____.

(a) 1 (b) 2

(c) 4 (d) 0 (2)

**(vii)** The integrating factor of linear differential equation $x\frac{dy}{dx}+2y=x^2\log x$ is _____.

(a) $x$ (b) $\frac{1}{x}$

(c) $x^2$ (d) $\frac{1}{x^2}$ (2)

**(viii)** If the mean and variance of a binomial distribution are 18 and 12 respectively, then the value of $n$ is _____.

(a) 36 (b) 54

(c) 18 (d) 27 (2)

**Q. 2. Answer the following questions :** **[4]**

**(i)** Write the compound statement 'Nagpur is in Maharashtra and Chennai is in Tamilnadu' symbolically. (1)

**(ii)** If the vectors $2\hat{i}-3\hat{j}+4\hat{k}$ and $p\hat{i}+6\hat{j}-8\hat{k}$ are collinear, then find the value of $p$. (1)

**(iii)** Evaluate : $\int\frac{1}{x^2+25}dx$ (1)

**(iv)** A particle is moving along X-axis. Its acceleration at time $t$ is proportional to its velocity at that time. Find the differential equation of the motion of the particle. (1)

# SECTION – B

**Attempt any EIGHT of the following questions :** **[16]**

**Q. 3.** Construct the truth table for the statement pattern : (2)

$[(p \rightarrow q) \wedge q] \rightarrow p$

**Q. 4.** Check whether the matrix $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ is invertible or not. (2)

**Q. 5.** In $\Delta ABC$, if $a = 18$, $b = 24$ and $c = 30$ then find the value of

$\sin\left(\frac{A}{2}\right)$. (2)

**Q. 6.** Find $k$, if the sum of the slopes of the lines represented by $x^2 + kxy - 3y^2 = 0$ is twice their product. (2)

**Q. 7.** If $\bar{a}, \bar{b}, \bar{c}$ are the position vectors of the points $A, B, C$ respectively and $5\bar{a} - 3\bar{b} - 2\bar{c} = \bar{0}$, then find the ratio in which the point $C$ divides the line segment $BA$. (2)

**Q. 8.** Find the vector equation of the line passing through the point having position vector $4\hat{i} - \hat{j} + 2\hat{k}$ and parallel to the vector $-2\hat{i} - \hat{j} + \hat{k}$. (2)

**Q. 9.** Find $\frac{dy}{dx}$, if $y = (\log x)^x$. (2)

**Q. 10.** Evaluate : $\int \log x \, dx$. (2)

**Q. 11.** Evaluate : $\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$ (2)

Q. 12. Find the area of the region bounded by the curve $y = x^2$, and the lines $x = 1$, $x = 2$ and $y = 0$. (2)

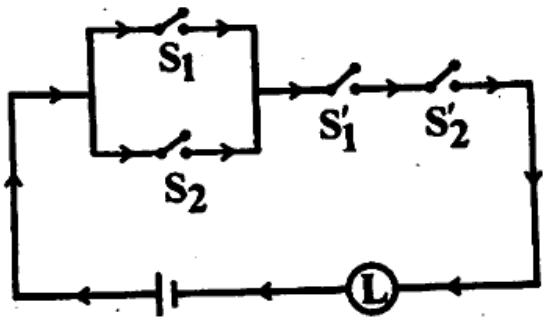
Q. 13. Solve : $1 + \frac{dy}{dx} = \text{cosec}(x + y)$; put $x + y = u$. (2)

Q. 14. If two coins are tossed simultaneously, write the probability distribution of the number of heads. (2)

## SECTION – C

**Attempt any EIGHT of the following questions :** [24]

Q. 15. Express the following switching circuit in the symbolic form of logic. Construct the switching table :




(3)

Q. 16. Prove that : $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$ (3)

Q. 17. In $\Delta ABC$, prove that : $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{a^2 + b^2 + c^2}{2abc}$. (3)

Q. 18. Prove by vector method, the angle subtended on a semicircle is a right angle. (3)

Q. 19. Find the shortest distance between the lines $\bar{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$ and $\bar{r} = (\hat{i} - \hat{j} - 2\hat{k}) + \mu(\hat{i} + 4\hat{j} - 5\hat{k})$ (3)

Q. 20. Find the angle between the line $\bar{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} + \hat{j} + \hat{k})$ and the plane $\bar{r} \cdot (2\hat{i} + \hat{j} + \hat{k}) = 8$. (3)

Q. 21. If $y = \sin^{-1} x$, then show that : $(1 - x^2)\frac{d^2y}{dx^2} - x \cdot \frac{dy}{dx} = 0$. (3)

Q. 22. Find the approximate value of $\tan^{-1}(1.002)$.

[Given : $\pi = 3.1416$] (3)

Q. 23. Prove that : $\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log\left(\frac{a+x}{a-x}\right) + c$. (3)

Q. 24. Solve the differential equation :

$$x \cdot \frac{dy}{dx} - y + x \cdot \sin\left(\frac{y}{x}\right) = 0.$$ (3)

Q. 25. Find $k$, if

$f(x) = kx^2(1 - x)$, for $0 < x < 1$,

$= 0$ otherwise

is the p.d.f. of random variable $X$. (3)

Q. 26. A die is thrown 6 times, if 'getting an odd number' is success, find the probability of 5 successes. (3)

# SECTION – D

**Attempt any FIVE of the following questions :** [20]

**Q. 27.** Solve the following system of equations by the method of reduction : $x+y+z=6,\ y+3z=11,\ x+z=2y.$ (4)

**Q. 28.** Prove that the acute angle $\theta$ between the lines represented by the equation $ax^2+2hxy+by^2=0$ is $\tan\theta=\left|\frac{2\sqrt{h^2-ab}}{a+b}\right|$.

Hence find the condition that the lines are coincident. (4)

**Q. 29.** Find the volume of the parallelopiped whose vertices are $A(3,2,-1)$, $B(-2,2,-3)$, $C(3,5,-2)$ and $D(-2,5,4)$. (4)

**Q. 30.** Solve the following L.P.P. by graphical method :

Maximize : $z=10x+25y$

Subject to : $0\le x\le 3,$

$0\le y\le 3,$

$x+y\le 5.$

Also find the maximum value of $z$. (4)

**Q. 31.** If $x=f(t)$ and $y=g(t)$ are differentiable functions of $t$, so that $y$ is function of $x$ and $\frac{dx}{dt}\neq 0$ then prove that $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$.

Hence find $\frac{dy}{dx}$, if $x=at^2,\ y=2at.$ (4)

**Q. 32.** A box with a square base is to have an open top. The surface area of box is 147 sq.cm. What should be its dimensions in order that the volume is largest? (4)

**Q. 33.** Evaluate : $\int \frac{5e^x}{(e^x+1)(e^{2x}+9)}\,dx$ (4)

**Q. 34.** Prove that :

$$\int_0^{2a} f(x)\,dx = \int_0^{a} f(x)\,dx + \int_0^{a} f(2a-x)\,dx$$

Hence show that :

$$\int_0^{\pi} \sin x\,dx = 2\int_0^{\frac{\pi}{2}} \sin x\,dx$$ (4)