Physics [Set 1]

Academic Year: 2018-2019 Marks: 80
Date & Time: 25th February 2019, 11:00 am
Duration: 3h

SECTION – A

Question 1: When a sparingly soluble substance like alcohol is dissolved in water, surface tension of water **[1]**

increases
decreases
remains constant
becomes infinite

Solution: Decreases
When / sparingly soluble substances are added surface tension decreases.

Question 2: **[1]**

The specific heat capacity of water is

8R

$\frac{7}{8}R$

9R

$\frac{9}{7}R$

Solution: 9R

One molecule of water is having 3 atoms.

Water can be considered at solid as its volume is constant.

Specific heat capacity of one atom 8

Solid is 3R

for 3 atoms

3 x 3R = 9R

Question 3: The electric field intensity outside the charged conducting sphere of radius ‘R’, placed in a medium of permittivity ∈ at a distance ‘r’ from the centre of the sphere in terms of surface charge density σ is **[1]**

$\frac{\sigma}{\in}\left(\frac{R}{r}\right)^2$
$\frac{\sigma}{\in}\left(\frac{r}{R}\right)^2$
$\frac{\sigma}{\in}\left(\frac{R^2}{r^2}\right)^2$
$\frac{\sigma}{\in}\left(\frac{r^2}{R^2}\right)^2$

Solution:

$$\frac{\sigma}{\in}\left(\frac{R}{r}\right)^2$$

By using Gauss theorem we know that

Question 4: An electron of energy 150 eV has wavelength of 10^{-10}m. The wavelength of a 0.60 keV electron is? **[1]**
0.50 Å
0.75 Å
1.2 Å
1.5 Å

Solution:

0.50 Å

$$\lambda = \frac{h}{\sqrt{meV}}$$

$$\lambda_1 \alpha \frac{1}{\sqrt{v_1}}$$

$$\lambda_2 \alpha \frac{1}{\sqrt{v_2}}$$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{v_2}{v_1}}$$

$$\frac{10^{-10}}{\lambda_2} = \frac{\sqrt{0.60 \times 10^3}}{150}$$

$$= \sqrt{\frac{600}{150}} = 2$$

$$\frac{10^{-10}}{\lambda_2} = \frac{10^{-10}}{2}$$

$$= 0.5 \times 10^{-10} m$$

$$\lambda_2 = 0.5 \text{Å}$$

Question 5: What is the value of tangential acceleration in U.C.M.? [1]

Solution: The concept of tangential acceleration is used to measure the change in the tangential velocity of a point with a specific radius with the change in time. The linear and tangential accelerations are the same but in the tangential direction which leads to circular motion. Tangential acceleration is defined as the rate of change of tangential velocity of the matter in the circular path.

Tangential Acceleration Formula:

$$a_t = \frac{\triangle v}{\triangle t}$$

Formula for Tangential Acceleration In Terms Of Distance

$$a_t = \frac{d^2 s}{dt^2}$$ or

$$a_t = v.\frac{dv}{ds}$$

Question 6: What happens to a ferromagnetic substance heated above Curie temperature? [1]

Solution: It loses its magnetism and becomes paramagnetic substance.

Question 7: At which position of the plane of the rotating coil with the direction of magnetic field, the e.m.f. induced in the coil is maximum? [1]

Solution: We know that

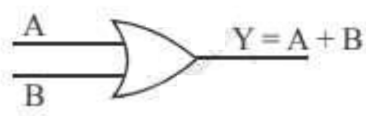
$$e = e_0 \sin \omega t$$

Where e_0 is peak value for e to be maximum.
Sin $\omega t = 1$
$\omega t = \sin^{-1} [1] = 90^o$
$\omega t = 90^o$
At 90^o induced e.m.f. will be maximum

Question 8: Name the logic gate which generated high output when at least one input is high. [1]

Solution: OR Gate :

Y = A + B



A	B	Y = A + B
0	0	0
0	1	1
1	0	1
1	1	1

SECTION – B

Question 9: In Young's experiment interference bands were produced on a screen placed at 150 cm from two slits, 0.15 mm apart and illuminated by the light of wavelength 6500 Å. Calculate the fringe width. **[2]**

Solution: Given:
D = 150 cm = 1.5 m,
d = 0.15 mm = 1.5×10^{-4} m,
λ = 6500 Å = 6.5×10^{-7} m

To find:
Fringe width (X)

Formula:

$$X = \frac{\lambda D}{d}$$

Calculation:

From formula,

$$X = \frac{6.5 \times 10^{-7} \times 1.5}{1.5 \times 10^{4}}$$

$X = 6.5 \times 10^{-3}$ m

X = 6.5 mm

The fringe width is 6.5 mm

Question 10: The susceptibility of magnesium at 300 K is 1.2×10^{-5}. What will be its susceptibility at 200 K. **[2]**

Solution:

$T_1 = 300k$

$x_1 = 1.2 \times 10^{-5}$

$T_2 = 200k$

$x_2 = ?$

Formula:

$\therefore x \; \alpha \frac{1}{T}$

$\therefore \frac{x_1}{x_2} \; \alpha \frac{T_2}{T_1}$

$\therefore x_2 = \frac{x_1 \times T_1}{T_2}$

$= \frac{1.2 \times 10^{-5} \times 300}{200}$

$= \frac{1.2 \times 3 \times 10^{-5}}{2}$

$\therefore x_2 = 1.8 \times 10^{-5}$

Question 11 | Attempt any one:

Question 11.1: The length of the second's pendulum in a clock is increased to 4 times its initial length. Calculate the number of oscillations completed by the new pendulum in one minute. [2]

Solution: Time period of second's pendulum = 2 seconds

Let the length of the pendulum by L

So, T = $2\pi\sqrt{\frac{L}{9}}$

so, 2 = $2\pi\sqrt{\frac{L}{9}}$

Now ,length becomes 4 times.

so,new time period be T

so, T = $2\pi\sqrt{\frac{4L}{9}}$

$= \left(2\pi\sqrt{\frac{L}{9}}\right) \times 2$

= T × 2 = 2 × 2 = 4 seconds

So, one oscillation is completed in 4 second

So, for 60 seconds or 1 minutes number of oscillations will be $= \frac{1}{4} \times 60 = 15$

OR

Question 11.2: A body of mass 1 kg is made to oscillate on a spring of force constant 16 N/m. Calculate (a) Angular frequency, (b) Frequency of vibrations. **[2]**

Solution: Given:

m = 1kg

k= 16N lm

ω = ?

n = ?

Formula:

$k=m\omega^2$

$$\therefore \omega = \frac{\sqrt{k}}{m}$$

$$\omega = \frac{\sqrt{16}}{1}$$

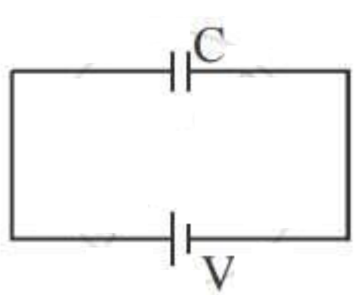
$$\therefore \omega = 4rad/s$$

$$\therefore \omega = 2\pi n$$

$$\therefore n = \frac{\omega}{2\pi} = \frac{4}{2 \times 3.14} = \frac{2}{3.14} = 0.636Hz$$

Question 12: Define the capacitance of a capacitor and its SI unit. **[2]**

Solution: Capacitance



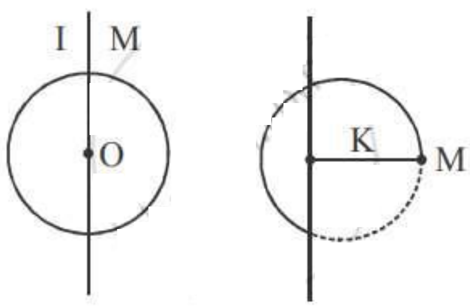
Capacitor is a device which is used to store charge. The capacity of a capacitor to store charge is called capacitance.

C = Q/V

SI unit of capacitor is farad.

Question 13: Define radius of gyration. Write its physical significance. **[2]**

Solution: Radius of gyration or gyradius refers to the distribution of the component of object around the axis in terms of the mass moment of inertia, As it is the perpendicular distance from the axis of rotation to a point mass m that gives an equivalent inertia to the original object m The nature of the object does not affect the concept, which applies equally to a surface bulk mass. Mathematically the radius of gyration is the root mean square distance of the object's parts from either its center of mass or the given axis, depending on the relevant application.



$$I = MK^2$$

$$K^2 = I/M$$

$$K = \sqrt{I/M}$$

Physical Signification:
It signifies distribution of mass about axis of rotation.

Question 14: Distinguish between p-type and n-type semiconductors. **[2]**

Solution: p-typed
(i) Number of holes are more than the number of electrons
(ii) Trivalent impurities are used for doping
(iii) These are called as acceptors
(iv) Holes are majority charge carriers.

n-typed
(i) Number of electrons are more than the number of holes.
(ii) Pentavalent impurities are used for doping.
(iii) These are called as doners
(iv) Electrons are majority charge carriers.

Question 15: Explain the terms (a) Transducer and (b) Attenuation in communication system. **[2]**

Solution: (a) **Transducer**: The device which converts one form of energy into another is called Transducer.
Example: LED, Microphone, Speaker.

(b) **Attenuation**: Loss of strength of signal while propagation through medium is called attenuation.

SECTION – C

Question 16: Obtain expressions of energy of a particle at different positions in the vertical circular motion. **[3]**

Solution: Lowest point of circular motion:

Total energy = KE

$$= \frac{1}{2}mv^2$$

Substitute v = $\sqrt{5rg}$

Total energy = $\frac{1}{2}m\left[\sqrt{5rg}\right]^2$

$$= \frac{5mrg}{2}$$

Highest point of circular motion:

Total enery = PE + KE

= mgh + KE

$$= mg[2r] + \frac{1}{2}mv^2$$

Substitute v = $\sqrt{rg}$

Total energy = 2mgr $+\frac{1}{2}m[\sqrt{rg}]^2$

$$= 2mgr + \frac{mgr}{2}$$

$$= \frac{5mrg}{2}$$

Question 17: Define binding energy and obtain an expression for binding energy of a satellite revolving in a circular orbit round the earth. **[3]**

Solution: Let the mass of satellite is m and r is the radius of the circular orbit. a satellite moves in the circumference of circular orbit around the earth.

Now, at equilibrium condition,
centripetal force = gravitational force

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

or $mv^2 = \frac{GMm}{r}$

or, $\frac{1}{2}mv^2 = \frac{GM}{2r}$

or K.E. = $\frac{GMm}{2r}$ ....(1)

Now, potential energy between satellite and the earth is given by

P.E. = $-\frac{GMm}{r}$

[Here negative sign indicates force acts between satellite and earth is attractive]

So, T.E = P.E + K.E

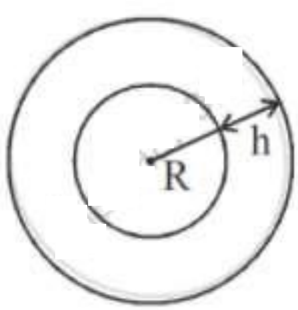
or, T.E = $-\frac{GMm}{r} + \frac{GMm}{2r}$

or, T.E = $-\frac{GMm}{2r}$

or $E_r = -\frac{GMm}{2r}$

Here negative sign indicates that the satellite is bound to the earth by attractive force and cannot leave it on its own. To move the satellite to infinity, we have to supply energy from outside to satellite - planet system.
So, binding energy of a satellite revolving in a circular orbit round the earth is

E = $\frac{GMm}{2r}$



Question 18: State Hooke's law. Define the elastic limit and modulus of elasticity. **[3]**

Solution: Hooke's law: Provided the elastic limit is not exceeded, the deformation of a material is proportional to the force applied to it.

$$\Rightarrow \frac{\text{stress}}{\text{strain}} = \text{k}$$

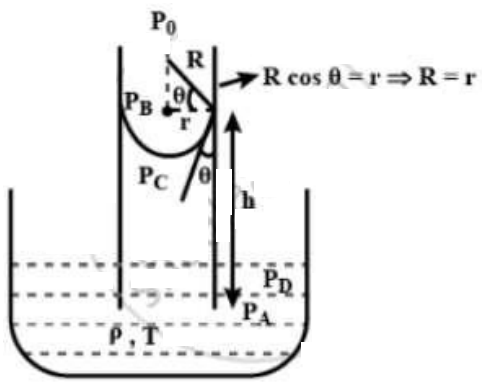
k is known as modulus of elasticity.

Elastic limit: The maximum stress within which the body
regains its original size and shape after removing of deforming force is called elastic li mit.

Modulus of Elasticity: The modulus of elasticity or coefficient of elasticity of the body is defined as the ratio of stress to the corresponding strain within the elastic limit.

Question 19: Obtain an expression for the rise of a liquid in a capillary tube. [3]

Solution:



Lets say capillary of radius r is placed in a beaker containing a liquid of density & and water rises in the capillary the height h,

$$\mathrm{P_B} - \mathrm{P_C} = \frac{2\mathrm{T}}{\mathrm{R}} \quad ......(1) \;\&$$

$$\mathrm{P_B} = \mathrm{P_O} \quad .....(3)$$

$$\mathrm{P_A} = \mathrm{P_0} \quad .....(4)$$

$$\mathrm{P_C} + \rho\mathrm{gh} = \mathrm{P_A} \quad ....(2)$$

From (1),(2)&(3),(4) press are gradient

$$\mathrm{P_0} - \frac{2\mathrm{T}}{\mathrm{R}} = \mathrm{P}\rho\mathrm{gh}$$

$$\frac{2\mathrm{T}}{\mathrm{R}} = \rho\mathrm{gh} \Rightarrow \mathrm{h} = \frac{2\mathrm{T}}{\mathrm{R}\rho\mathrm{g}} = \frac{2\mathrm{T}\cos\theta}{\rho\mathrm{gr}}$$

Question 20: Explain the reflection of transverse and longitudinal waves from a denser medium and a rared medium. [3]

Solution: ● Transverse waves -
- Transverse waves consists of crests and troughs.

- In denser medium after reflection, phase difference is π. Therefore, crest travels as trough and trough travels as crest.
- In rarer medium after refection, phase difference is 0. Therefore, crest travels as crest and trough travels as trough.

● **Longitudinal waves -**
- Longitudinal waves consist of compression and rarefaction.
- In denser medium after reflection, phase difference is π. Therefore, compression becomes compression and rarefaction as rarefaction.
- In rarer medium after reflection, phase difference is 0. Therefore, compression becomes rarefaction and rarefaction as compression.

Question 21: What is photoelectric effect? Define (i) Stopping potential (ii) Photoelectric work function. **[3]**

Solution: The phenomenon of emission of electron by certain substance (metal), when it is exposed to radiation of suitable frequencies is called photo electric effect. Thus in photoelectric effect emitted electrons are called
photoelectrons.

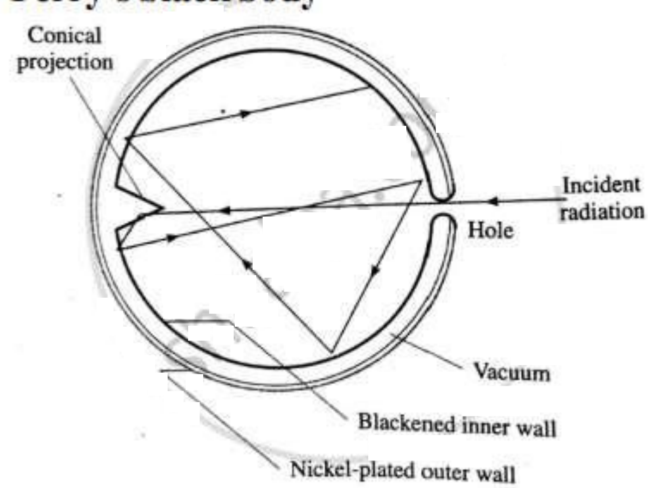
(i) Stopping -potential: The minimum negative potential given to the plate for which photo electric current stop or become zero.

(ii) Photo electric work function: The minimum energy required to remove electron from a given surface is called photo electric work function

Question 22: What is perfectly black body ? Explain Ferry's black body.

Solution: A perfectly black body which absorbs all the energy incident on it. The coefficient of absorption of lamp black is 98%.

Ferry's black body



(i) It consist of double walled hollow metal sphere having small aperture.
(ii) Inner surface is coated with lamp black.

(iii) Outer surface is Nickel polished.
(iv) The space between sphere is evacuated.
(v) The radiant energy entering suffers multiple reflection and get 98% absorbed at each reflection.
(vi) Thus the aperture acts as perfectly black body.

Question 23: When a resistor of 5Ω is connected across the cell, its terminal potential difference is balanced by 150 cm of potentiometer wire and when a resistance of 10 Ω is connected across the cell, the terminal potential difference is balanced by 175 cm same potentiometer wire. Find the balancing length when the cell is in open circuit and the internal resistance of the cell.

Solution:

$$r = 5\left(\frac{l_1}{150} - 1\right) \ldots\ldots\ldots\ldots.(1)$$

$$r = 10\left(\frac{l_1}{175} - 1\right) \ldots\ldots\ldots\ldots.(2)$$

$$5\left(\frac{l_1}{150} - 1\right) = 10\left(\frac{l_1}{175} - 1\right)$$

$$\frac{l_1}{150} - 1 = 2\frac{l_1}{175} - 2$$

$$-1 + 2 = 2\frac{l_1}{175} - \frac{l_1}{150}$$

$$1 = l_1\left(\frac{2}{175} - \frac{1}{150}\right)$$

$$= l_1\left(\frac{300 - 175}{175 \times 150}\right)$$

$$= l_1\frac{125}{175 \times 150}$$

$$l_1 = 210cm$$

$$r = R\left(\frac{L_1}{L_2} - 1\right)$$

$$= 5\left(\frac{210}{150} - 1\right)$$

$$r = 2\Omega$$

Question 24: A cyclotron is used to accelerate protons to a kinetic energy of 5 MeV. If the strength of magnetic field in the cyclotron is 2T, find the radius and the frequency needed for the applied alternating voltage of the cyclotron. (Given: Velocity of proton= 3 × 10^7 m/s) **[3]**

Solution:

$K.E = 5MeV$

$B = 2T$

$r = ?$

$f = ?$

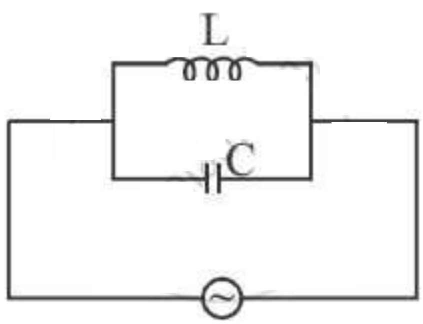
$v = 3 \times 10^7 m/s$

$$r = \frac{mv}{Bq} = \frac{1.67 \times 10^{-27} \times 3 \times 10^7}{2 \times 1.6 \times 10^{-19}} = 0.1565m$$

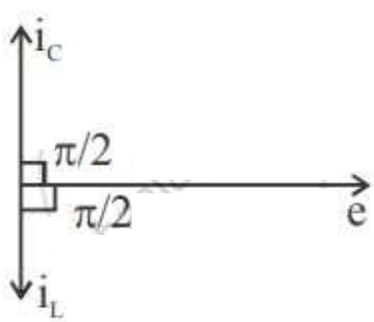
$$f = \frac{qB}{2\pi m} = \frac{1.6 \times 10^{-19} \times 2}{2 \times 3.14 \times 1.67 \times 10^{-27}} = 3.05 \times 10^7 Hz$$

Question 25: Assuming expression for impedance in a parallel resonant circuit, state the conditions for parallel resonance. Define resonant frequency and obtain an expression for it. **[3]**

Solution:



$i = i_0 \sin \omega t$



$$i = i_c - i_L$$

$$\frac{e}{z} = \frac{e}{X_c} - \frac{e}{X_L}$$

$$\frac{1}{z} = \frac{1}{X_c} - \frac{1}{X_L}$$

For parallel resonance

$$X_L = X_C$$

$$\frac{1}{z} = 0$$

$$z = \alpha$$

Resonant frequency (fr) :

The frequency at which impedance (L) because infinity and current becomes zero is called resonant frequency.

$$\omega L = \frac{1}{\omega L}$$

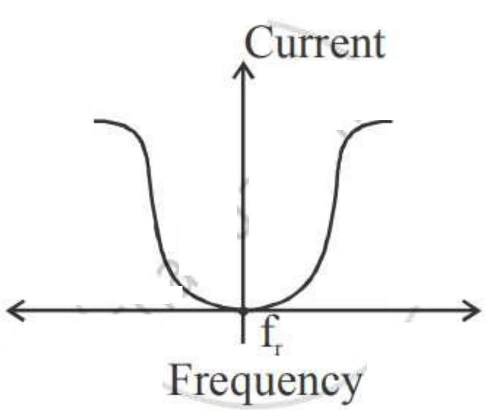
$$\omega^2 LC = 1$$

$$\omega^2 = \frac{1}{LC}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$2\pi f_r = \frac{1}{\sqrt{LC}}$$

$$f_r = \frac{1}{2\pi} \frac{1}{\sqrt{LC}}$$



Question 26 | Attempt any one:

Question 26.1: Using an expression for energy of electron, obtain the Bohr's formula for hydrogen spectral lines. **[3]**

Solution: Suppose an electron jumps from n^{th} higher orbit to p^{th} lower orbit.

Let and E_n E_p be the energies of electron in n^{th} and p^{th} orbit respectively.

$\therefore$ Energy of electron in n^{th} orbit is, $E_n = -\left(\frac{me^4}{8\varepsilon_0^2h^2}\right) = \frac{1}{2}$ .... (1)

$\therefore$ Energy of electron in p^{th} orbit , $E_p = -\left(\frac{me^4}{8\varepsilon_0^2h^2}\right)\frac{1}{P^2}$ .... (2)

$\therefore$ According to Bohr's 3^{rd} postulate, energy emitted is given by,

$\therefore$ energy emitted, $h\nu = E_n - E_p$

$$\therefore h\nu = \frac{me^4}{8\varepsilon_0^2h^2}\left(\frac{1}{p^2} - \frac{1}{n^2}\right)$$

$$\text{Or } \nu = \frac{me^4}{8\varepsilon_0^2h^3}\left(\frac{1}{p^2} - \frac{1}{n^2}\right)$$

$$\therefore \frac{1}{\lambda} = \frac{me^4}{8\varepsilon_0^2h^3C}\left(\frac{1}{p^2} - \frac{1}{n^2}\right) \quad \left[\because \nu = \frac{C}{\lambda}\right]$$

$$\therefore \frac{1}{\lambda} = \mathrm{R}\left(\frac{1}{p^2} - \frac{1}{n^2}\right)$$

Where, $\mathrm{R} = \frac{me^4}{8\varepsilon_0^2ch_3}$ is called Rydberg's constant.

Question 26.2: State the law of radioactive decay. Hence derive the relation $N = Noe^{-\lambda t}$. Represent it graphically. **[3]**

Solution: Radioactive decay law :
The number of nuclei undergoing the decay per unit time is proportional to the number of unchanged nuclei present at that instant.
Proof:

- Let 'N' be the number of nuclei present at any instant 't'.
- Let 'dN' be the number of nucleic that disintegrated in short interval of time 'dt'.
- According to law

$$\frac{-\mathrm{dN}}{\mathrm{dt}} \propto \mathrm{N}$$

$\therefore \frac{\mathrm{dN}}{\mathrm{dt}} = -\lambda\mathrm{N}$........................(1)

Where λ - decayconst

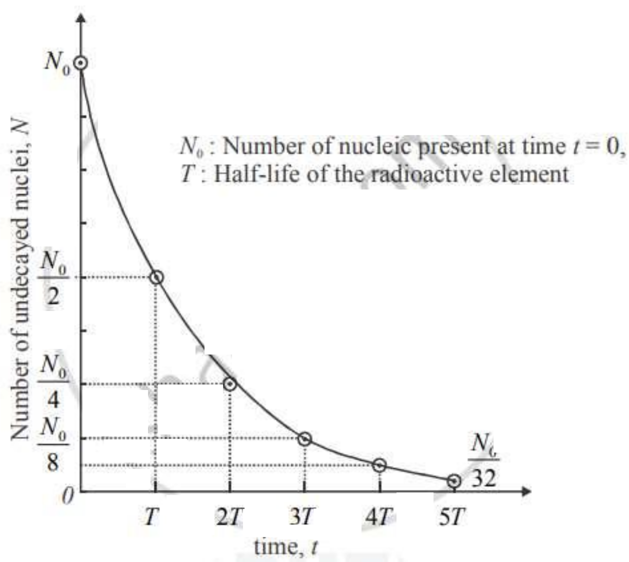
$$\therefore \frac{\mathrm{dN}}{\mathrm{N}} = -\lambda \mathrm{dt}$$

$\therefore$ integrating both sides

$$\int \frac{\mathrm{dN}}{\mathrm{N}} = \int -\lambda \, \mathrm{dt}$$

$[ell]\mathrm{N} = \lambda t + C\ell$"N"= λ t+C `.................................(2)

Where C → integration consta



$\therefore$ at t = 0 , N = N_0

$\therefore \ell\, nN_0$ = C ....(3)

$\therefore$ Frome equation (2) and (3)

$$lnN = -\lambda t + \ell\, nN_0$$

$$\lambda t = In\ N - In\ N_0$$

-λT= In $\left(\frac{N}{N_0}\right)$

$$\frac{N}{N_0} = e^{-(\lambda t)}$$

$$\therefore N = N_0 e^{-(\lambda t)}$$

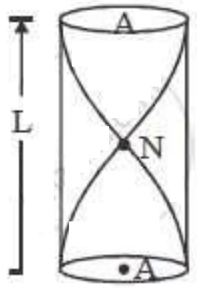
SECTION – D

Question 27 | Attempt any two: **[10]**

Question 27.1: **[10]**

Question 27.1.1: Show that even as well as odd harmonics are present as overtones in the case of an air column vibrating in a pipe open at both the ends. **[3]**

Solution: Mode 1:



$$L = \frac{\lambda}{2}$$

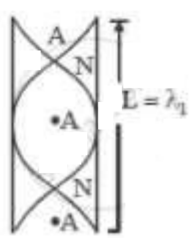
$$\lambda = 2L$$

$$n = \frac{v}{\lambda}$$

$$n = \frac{v}{2L}$$

Fundamental frequency $n = \left[\frac{v}{2L}\right]$

Mode 2 :



$$v = n_1\lambda_1$$

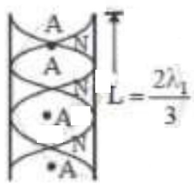
$$n_1 = \frac{V}{\lambda_1}$$

$$\lambda_1 = L$$

$$n_1 = \frac{V}{L}$$

$$h_1 = 2\left(\frac{V}{2L}\right)[n_1 = 2n]$$

Mode 3 :



$$V = n_2\lambda_2$$

$$n_2 = \frac{V}{\lambda_2}$$

$$n_2 = \frac{V}{\lambda_2} = 3\left[\frac{V}{2L}\right]$$

$$n_2 = 3n$$

In general,

$$n_p = (P+1)n$$

$$n_p = (P+1)\frac{V}{2L}$$

P = 0,1,2........

From the above expression, it is clear that in a tube which is open at both the ends all the harmonics are present

Question 27.1.2: A wheel of moment of inertia 1 kg.m^2 is rotating at a speed of 30 rad/s. Due to friction on the axis, it comes to rest in 10 minutes. Calculate the average torque of the friction. **[2]**

Solution: Given
I = 1Kg m^2

ω_0 =30 rad/sec ω = 0

t = 10 min = 10×60 = 600 sec

$$\alpha = \frac{\omega - \omega_0}{600}$$
$$= \frac{0-30}{600} = -\frac{30}{600} = -\frac{1}{20}$$

α = -0.05 rad/sec^2

$\tau = I\alpha$ = 0.05 × I = -0.05 Nm.

$$\theta = \omega t + \frac{1}{2}\alpha t^2$$

$$\theta = 30 \times 600 - \frac{1}{2} \times 0.5 \times 300 \times 600$$

= 18000 - 180000 × 0.05

=18000 - 9000

θ = 9000 rad

$\omega = \tau.\theta$

= 0.05×9000

ω = - 450 J

OR

Question 27.2: **[5]**

Question 27.2.1: Explain the formation of stationary waves by analytical method. Show that nodes and antinodes are equally spaced in stationary waves. **[3]**

Solution: Analytical treatment of Stationary Waves : Consider two simple harmonic progressive waves having same amplitude (a), frequency (n) and period (T) travelling along same medium in opposite direction. The wave along positive direction of X-axis is given by:

$$Y_1 = \alpha \sin 2\pi\left(nt - \frac{x}{\lambda}\right) \quad \text{..............................(1)}$$

The wave along negative direction of x-axis is given by:

$$Y_2 = \alpha \sin 2\pi\left(nt + \frac{x}{\lambda}\right) \quad \text{..............................(2)}$$

By principle of superposition, the resultant displacement of stationary wave is given by:

$$Y = Y_1 + Y_2$$

$$\therefore Y = \alpha \sin 2\pi \left(nt - \frac{x}{\lambda}\right) + \alpha \sin 2\pi \left(nt + \frac{x}{\lambda}\right)$$

$$\therefore Y = 2\alpha \sin \frac{2\pi}{2} \left(nt - \frac{x}{\lambda} + nt + \frac{x}{\lambda}\right). \cos \frac{2\pi}{2} \left(nt - \frac{x}{\lambda} - nt \frac{x}{\lambda}\right)$$

Using sin C+sin D = 2 sin $\left(\frac{C+D}{2}\right) \cos\left(\frac{C-D}{2}\right)$ and $\cos(-\theta) = \cos\theta$)

$$\therefore Y = 2a \sin 2\pi nt. \cos 2\pi \left(\frac{-x}{\lambda}\right)$$

$$\therefore Y = 2a \left[\sin(2\pi nt). \cos \left(\frac{2\pi x}{\lambda}\right)\right]$$

$$\therefore Y = \left[2a \cos\left(\frac{2\pi x}{\lambda}\right)\right] \sin 2\pi\, nt \qquad \text{....(3)}$$

$$\therefore Y = A \sin(2\pi\, nt)$$

where A = 2a cos $\frac{2\pi x}{\lambda}$ where A is the amplitude of the resutant stationary wave.

The above expression shows that resultant wave is a simple harmonic motion having same period but new amplitude. The absence of the term (x) in the sine function shows that the resultant waves do not move forward or backward. Such waves are called stationary waves.

Conditions for antinodes:
The points of medium which vibrate with maximum amplitude are called antinodes. Antinode is formed when A is maximum.

$$A = \pm 2a$$

$$\therefore 2a \cos \frac{2\pi\, x}{\lambda} = \pm 2a$$

$$\therefore \cos \frac{2\pi x}{\lambda} = \pm 1$$

$$\therefore \frac{2\pi x}{\lambda} = 0, \pi, 2\pi$$

$$\therefore \frac{2\pi x}{\lambda} = p\pi$$

$\therefore x = \frac{\lambda P}{2}$ Where P = 0,1,2.......

When $\frac{2\pi x}{\lambda} = 0$, then x = 0 .

When $\frac{2\pi x}{\lambda} = \pi$ then x = λ / 2.

When $\frac{2\pi x}{\lambda} = 2\pi$ then x = λ.

$\therefore x = 0, \frac{\lambda}{2}, \lambda.....$

The particles at these points vibrate with minimum amplitude. Such points are called antinodes. The distance between two successive antinodes is λ/2.

Conditions for nodes:
The points of medium which vibrate with minimum amplitudes are called nodes. Amplitude at nodes is zero.

A = 0

$\therefore 2a \cos\left(\frac{2\pi x}{\lambda}\right) = 0$

$\therefore \cos \frac{2\pi x}{\lambda} = 0$

$\therefore \frac{2\pi x}{\lambda} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

$x = \left(\frac{2P-1}{4}\right)$ where p = 1,2,.........

When $\frac{2\pi x}{\lambda} = \frac{\pi}{2}$, then x = λ/4.

When $\frac{2\pi x}{\lambda} = \frac{3\pi}{2}$, then x = 3λ/4.

When $\frac{2\pi x}{\lambda} = \frac{5\pi}{2}$, then x = 5λ/4.

$\therefore x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, ...$

The particles at these points vibrate with minimum or zero amplitude. The distance between any two successive nodes is λ/2.
Thus the distance between two successive antinodes and nodes is equal to λ/2.
Therefore antinodes and nodes are equally spaced in a stationary wave.

Question 27.2.2: The radius of gyration of a body about an axis, at a distance of 0.4 m from its centre of mass is 0.5 m. Find its radius of gyration about a parallel axis passing through its centre of mass. **[2]**

Solution: Given

$h = 0.4m$

$K = 0.5m$

$K_{cm} = ?$

By theorem of parallel axes

$I = I_c + Mh^2$

$I = MK^2 \qquad I_c = MK_{cm}^2$

$MK^2 = MK_{cm}^2 + Mh^2$

$K^2 = K_{cm}^2 + h^2$

$(0.5)^2 = K_{cm}^2 + (0.4)^4$

$K_{cm}^2 = (0.5)^2 - (0.4)^2$

$= (0.5 + 0.4)(0.5 - 0.4)$

$= 0.9 \times 0.1$

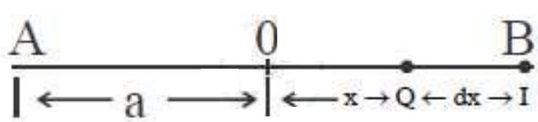
$K_{cm}^2 = 0.09$

$K_{cm} = 0.3m$

Question: 28 | Attempt any one: **[10]**

Question 28.1.1: Obtain an expression for potential energy of a particle performing S.H.M. What is the value of potential energy at (i) Mean position, and (ii) Extreme position? **[3]**

Solution: Potential Energy :



→ consider a particle of mass ‘m’ performing S.H.M along AB about mean position ‘O’.
→ OB = OA = a.
∴ When particle of mass ‘m’ performs S.H.M. is at distance x from its mean position.
→ Let ‘F = k x’ be the restoring force acting on the particle where k → force constant.
→ Let the particle is displaced by infinitesimal small distance 'dx'.
∴ Workdone,

dw = - fdx= -(-kx) dx= kxdx.

∴ total work done is ,

$$\int dw = \int_{x=0}^{x} kxdx = k \int_{x=0}^{x} xdx$$

$$W = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2 \qquad \left[\because \omega^2 = \frac{k}{m}\right]$$

∴ this workdone is stored in the form of P.E.

$$\therefore P.E = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2$$

Cases :

(i) at Mean position : - x = 0

∴ P.E = 0

(ii) at extreme position : x = ± a

∴ magnitude of P.E. is,

$$\therefore P.E = \frac{1}{2}m\omega^2a^2$$

Question 28.1.2: A stretched sonometer wire is in unison with a tuning fork. When the length of the wire is increased by 5%, the number of beats heard per second is 10. Find the frequency of the tuning fork. **[2]**

Solution: Given :

$$\frac{L_1}{L_2} = \frac{100}{105}$$

$$\therefore L_2 = 1.05L_1$$

$$\therefore L_2 > L_1,\ n_2 < n_1$$

$$\therefore n_1 - n_2 = 10, N = n_1 \cong ?$$

→ By the law of length of vibrating string,

$$n_1L_1 = n_2L_2$$

$$\therefore n_1L_1 = n_2L_2$$

$$\therefore n_1 = 1.05n_2 - 10.5$$

$$\therefore n_1 = 210Hz$$

OR

Question 28.2.1: From differential equation of linear S.H.M., obtain an expression for acceleration, velocity and displacement of a particle performing S.H.M. **[3]**

Solution: The differential equation of linear SHM is

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

Where m = mass of the particle performing SHM,

$\frac{d^2x}{dt^2} =$ Acceleration of the particle when its displacement from the mean position is x and k = force constant.

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

Let $\frac{k}{m} = \omega^2$

$$\therefore \frac{d^2x}{dt^2} + \omega^2 x = 0$$

$\therefore$ Acceleration, $a = \frac{d^2x}{dt^2} = -\omega^2 x$..................(1)

The minus sign shows that the acceleration and the displacement have opposite directions. Writing v = dx/dt as the velocity of the particle.

$$a = \frac{d^2x}{dt^2} = \frac{d\nu}{dt} = \frac{d\nu}{dx}.\frac{dx}{dt} = \frac{d\nu}{dx}.\nu = \nu\frac{d\nu}{dx}$$

Hence, Eq.(1) can be written as

$$\nu\frac{d\nu}{dx} = -\omega^2 x$$

$$\therefore \nu d\nu = -\omega^2 x dx$$

Integrating this expression, we get

$$\frac{\nu^2}{2} = \frac{-\omega^2 x^2}{2} + c$$

Where the constant of integration C is found from a boundary condition.
At an extreme position (a turning point of the motion), the velocity of the particle is zero.
Thus, v = 0 when x = ± A. Where A is the amplitude.

$$\therefore 0 = \frac{-\omega^2 A^2}{2} + C \quad \therefore C = \frac{\omega^2 A^2}{2}$$

$$\therefore \frac{\nu^2}{2} = \frac{-\omega^2 x^2}{2} + \frac{\omega^2 A^2}{2}$$

$$\therefore \nu^2 = \omega^2 \left(A^2 - x^2\right)$$

$$\therefore \nu = \pm \omega \sqrt{A^2 - X^2}$$

This equation gives the velocity of the particle in terms of the displacement, x. The velocity towards right is taken to be positive and toward left as negative. Since, v = dx/dt we can write Eq. (2) as follows:

$$\frac{dx}{dt} = \omega \sqrt{A^2 - X^2}$$

$$\therefore \frac{dx}{\sqrt{A^2 - X^2}} = \omega\, dt$$

Integrating this expression, we get,

$$\sin^{-1}\left(\frac{x}{A}\right) = \omega t + \alpha \text{..................(3)}$$

Where the constant of integration, α, is found from the initial conditions, i.e., the displacement and the velocity of the particle at time t = 0

From equation (3), we have

$$\frac{x}{A} = \sin(\omega t + \alpha)$$

∴ Displacement as a function of time is,

$$x = A \sin(\omega t + \alpha)$$

Question 28.2.2: A sonometer wire 1 metre long weighing 2 g is in resonance with a tuning fork of frequency 300 Hz. Find tension in the sonometer wire. **[2]**

Solution: A sonometer wire 1 metre long weighing 2 g is in resonance with a tuning fork of frequency 300 Hz. Find tension in the sonometer wire.

$$= \frac{0.002}{I} = 0.002 kg/m$$

$$n = \frac{1}{2L}\sqrt{\frac{T}{m}}$$

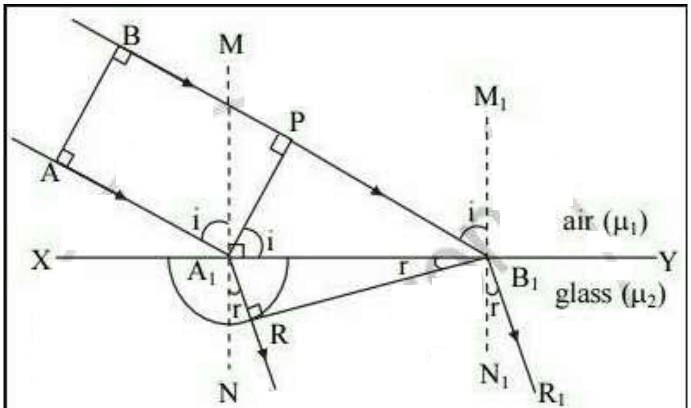
$$\therefore \text{Tension}, \ T = 4n^2 L^2 m = 4 \times (300)^2 \times (1)^2 \times 0.002 = 720N$$

Question 29: Attempt any one of the following **[10]**

Question 29.1.1: Explain refraction of light on the basis of wave theory. Hence prove the laws of refraction **[3]**

Solution: <u>Laws of refraction :</u>
The ratio of the velocity of light in rarer medium to the velocity of light in a denser medium is a constant called the refractive index of denser medium w.r.t. rarer medium.

The incident rays, refracted rays, and normal lie in the same plane. Incident ray and refracted ray lie on opposite sides of normal.



Refraction of light

XY : plane refracting surface
AB : incident plane wavefront
B_1R : refracted wavefront
AA_1, BB_1 : incident rays
A_1R, B_1R_1 : refracted rays
$\angle AA_1M = \angle BB_1M_1 = \angle i$: angle of incidence
$\angle RA_1N = \angle R_1B_1N_1 = \angle r$: angle of refraction

<u>Explanation:</u> Phenomenon of refraction can be explained on the basis of wave theory of light.

1. Let XY be the plane refracting surface separating two media air and glass of respectively. indices μ_1 and μ_2 refractive.

2. A plane wave front AB is advancing obliquely towards XY from the air. It is bounded by rays AA_1 and BB_1 which are incident rays.

3. When 'A' reaches 'A_1', then 'B' will be at 'P'. It still has to cover distance PB1 to reach XY.

4. According to Huygens' principle, secondary wavelets will originate from A_1 and will spread over a hemisphere in the glass.

5. All the rays between AA_1 and BB_1 will reach XY and spread over the hemispheres of increasing radii in the glass. The surface of the tangency of all such hemispheres is RB_1. This gives rise to refracted wave front B_1R in the glass.

6. A_1R and B_1R_1 are refracted rays.

7. Let c1 and c_2 be the velocities of light in air and glass respectively.

8. At any instant of time 't', distance covered by incident wave front from P to B_1 = PB_1 = c_1t Distance covered by a secondary wave from A_1 to R = A_1R = c_2t.

Question 29.1.2: Two coherent sources of light having intensity ratio 81 : 1 produce interference fringes. Calculate the ratio of intensities at the maxima and minima in the interference pattern. **[2]**

Solution: $I_1 : I_2$ 81:1

If A_1 and A_2 are the amplitudes of the interfering waves, the ratio of the intensity maximum to the intensity minimum in the fringe system is

$$\frac{I_{\max}}{I_{\max}} = \left(\frac{A_1 + A_2}{A_1 - A_2}\right)^2 = \left(\frac{r+1}{r-1}\right)^2$$

where $r = \dfrac{A_1}{A_2}$ Since the intensity of a wave is directly proportional to the square of its amplitude,

$$\frac{I_1}{I_2} = \left(\frac{A_1}{A_2}\right)^2 = r^2$$

$$\therefore r = \sqrt{\frac{I_1}{I_2}} = \sqrt{81} = 9$$

$$\therefore \frac{I_{\max}}{I_{\max}} = \left(\frac{9+1}{9-1}\right)^2 = \left(\frac{10}{8}\right)^2 = \left(\frac{5}{4}\right)^2 = \frac{25}{16}$$

$\therefore$ The ratio of the intensities of maxima and minima in the fringe system is 25 : 16.

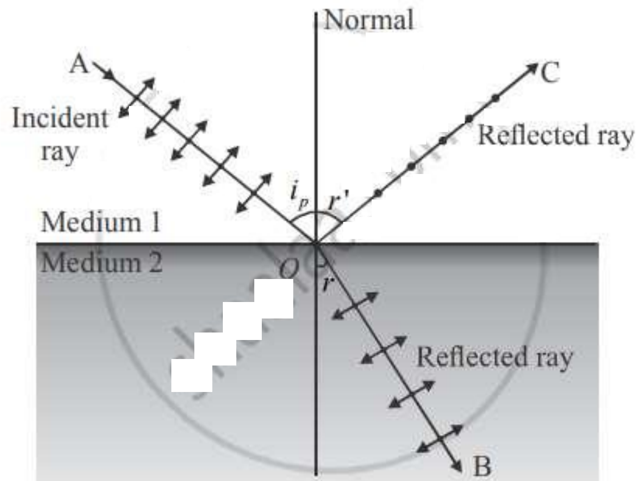
OR

Question 29.2.1: State Brewster's law and show that when light is incident at polarizing angle the reflected and refracted rays are mutually perpendicular to each other. **[3]**

Solution: Brewster's law: The tangent of polarising angle is equal to the refractive index of the reflecting medium with respect to the surrounding (${}_1n_2$). If i_P is the polarising angle.

$$\tan i_p = {}_1n_2 = \frac{n_2}{n_1} \text{ ....(1)}$$

Where n_1 is the absolute refractive index of the surrounding and n_2 is that of the reflecting medium. Figure shows a ray AO of unpolarised light incident on the interface separating two mediums. The degree of polarisation of the reflected ray OC varies with the angle of incidence and is a maximum for the angle of incidence equal to the polarising angle i_p of the pair of mediums. For all angle of incidence, the refracted ray OB is only partially polarised.



In figure, the angle of incidence is i_p , the angle of reflection is r' and the angle of refraction is r. By Snell's law,

$$\frac{n_2}{n_1} = \frac{\sin i_p}{\sin r} \qquad ......(2)$$

From, Eqs. (1) and (2)

$$\tan i_p = \frac{\sin i_p}{\cos i_p =} \frac{\sin i_p}{\sin r}$$

$\therefore \cos i_p = \sin r$

Now, $r' = i_p$

$\therefore \cos r' = i_p$

$\therefore$ cos r' = sir r

$\therefore \sin(90° - r') = \sin r$

$\therefore r = 90° - r'$ or, $r' + r = 90°$

$\therefore \angle COB = 90°$

This indicates that, for complete polarisation of the reflected ray at the polarising angle, the reflected and the refracted rays are mutually perpendicular.

Question 29.2.2: Monochromatic light of wavelength 4300 Å falls on a slit of width ‘a’. For what value of ‘a’ the first maximum falls at 30°? **[2]**

Solution:

$\lambda = 4300\text{Å}$

$= 4300 \times 10^{-10} m$

a = ?

n = 1

$\sin\theta = \frac{L}{a}$

$\therefore a = \frac{\lambda}{\sin 30} = \frac{4300 \times 10^{-10}}{\frac{1}{2}}$

$a = 4300 \times 10^{-10} \times 2$

$= 8600 \times 10^{-10} m$

for first maximum

$\sin\theta = \left(n + \frac{1}{2}\right)\frac{\lambda}{a}$

$= \frac{3}{2} \times \frac{4300 \times 10^{-10}}{\frac{1}{2}} = 1.29\mu\text{m}$