CHAPTER

07

SQUARE-SQUARE ROOT AND CUBE-CUBE ROOT

Square

When a number is multiplied by itself, the number thus obtained is called square of given number.

e.g. $2^2 = 2 \times 2 = 4$, $3^2 = 3 \times 3 = 9$,
$4^2 = 4 \times 4 = 16$; $n^2 = n \times n$

The numbers 4, 9 and 16 are the squares of 2, 3 and 4 and 4, 9, 16 are called perfect squares.

Properties of Square

- A number ending in 2, 3, 7 or 8 is never a perfect square.
- The number of zeros in the end of a perfect square is never odd.
- Squares of even numbers are always even.
- Squares of odd numbers are always odd.

Square and Square Root of Some Standard Numbers

Square	Square Root	Square	Square Root
$1^2 = 1$	$\sqrt{1} = 1$	$11^2 = 121$	$\sqrt{121} = 11$
$2^2 = 4$	$\sqrt{4} = 2$	$12^2 = 144$	$\sqrt{144} = 12$
$3^2 = 9$	$\sqrt{9} = 3$	$13^2 = 169$	$\sqrt{169} = 13$
$4^2 = 16$	$\sqrt{16} = 4$	$14^2 = 196$	$\sqrt{196} = 14$
$5^2 = 25$	$\sqrt{25} = 5$	$15^2 = 225$	$\sqrt{225} = 15$
$6^2 = 36$	$\sqrt{36} = 6$	$16^2 = 256$	$\sqrt{256} = 16$
$7^2 = 49$	$\sqrt{49} = 7$	$17^2 = 289$	$\sqrt{289} = 17$
$8^2 = 64$	$\sqrt{64} = 8$	$18^2 = 324$	$\sqrt{324} = 18$
$9^2 = 81$	$\sqrt{81} = 9$	$19^2 = 361$	$\sqrt{361} = 19$
$10^2 = 100$	$\sqrt{100} = 10$	$20^2 = 400$	$\sqrt{400} = 20$

Square Root

The square root of a number is that factor of the number which, when multiplied by itself, will give that number.

The square root of a number is indicated by the sign $\sqrt{\ }$.

e.g. The square root of 25 is written as $\sqrt{25}$.

Thus, $\sqrt{25} = \sqrt{5 \times 5} = 5$

Methods of Finding Square Root

Factorisation Method

Following steps are to be followed find the square root by factorization method

Step I Write the given number as product of prime factors. *e.g.* $\sqrt{144}$

Step II Make pairs of prime factor and take the product by choosing one digit from each paird
Eq. $\sqrt{144} = \sqrt{\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3}} = 2 \times 2 \times 3 = 12$

Example **1.** The value of $\sqrt{1764}$ is equal to

(1) 40 (2) 32 (3) 52 (4) 42

Sol. (4) $1764 = \underbrace{2 \times 2} \times \underbrace{3 \times 3} \times \underbrace{7 \times 7}$

$\sqrt{1764} = 2 \times 3 \times 7 = 42$

Example **2.** The value of $\sqrt{48}$ is equal to

(1) 6.289 (2) 6.829 (3) 6.928 (4) 7.729

Sol. (3) $48 = \underbrace{2 \times 2} \times \underbrace{2 \times 2} \times 3$

$\sqrt{48} = 2 \times 2 \times \sqrt{3} = 4\sqrt{3}$ [We know that, $\sqrt{3} = 1.732$]

$= 4 \times 1.732 = 6.928$

Division Method

The steps of this method can be easily understood with the help of following example.

Example **3.** Find the square root of 18769.

(1) 133 (2) 137 (3) 135 (4) 134

Sol. (2)

Step I In the given number, mark off the digits in pairs starting from the unit digit. Each pair and the remaining one digit (if any) is called a period.

	$\overline{1}\,\overline{87}\,\overline{69}$

Step II Choose a number whose square is less than or equal to 1. Here, $1^2 = 1$, on subtracting, we get 0 (zero) as remainder.

	1
1	$\overline{1}\,\overline{87}\,\overline{69}$
	1

Step III Bring down the next period, i.e. 87. Now, the trial divisor is $1 \times 2 = 2$ and trial dividend is 87. So, we take 23 as divisor and put 3 as quotient. The remainder is 18 now.

	13
1	$\overline{1}\,\overline{87}\,\overline{69}$
	1
23	87
	69
	18

Step IV Bring down the next period, which is 69. Now, trial divisor is $13 \times 2 = 26$ and trial dividend is 1869. So, we take 267 as dividend and 7 as quotient. The remainder is 0.

	137
1	$\overline{1}\,\overline{87}\,\overline{69}$
	1
23	87
	69

267	1869
	1869
	×

Step V The process (processes like III and IV) goes on till all the periods (pairs) come to an end and we get remainder as 0 (zero) now.

Hence, the required square root = 137

Square Root of a Decimal Number

To make periods in decimal number, make pair near decimal point, number before decimal point will be paired starting from left of decimal and number after decimal will be paired starting from right of decimal.

Example **4.** Find the square root of 232.5625.

(1) 10.50 (2) 15.25 (3) 14.50 (4) 17.25

Sol. (2)

	15.25
1	2 32 . 56 25
	1
25	132
5	125
302	756
2	604
3045	15225
5	15225
	×

$\therefore \quad \sqrt{232.5625} = 15.25$

Square Root of Fractions

If denominator of the fraction is perfect square, then find the square root of numerator and denominator separately. If denominator of the fraction is not a perfect square, then make it a perfect square by multiply a number.

Example **5.** Find the square root of 7/5.

(1) 1.1832 (2) 1.2437 (3) 1.1932 (4) 1.2071

Sol. (1) Since, 5 is not a perfect square.

$\therefore \quad \frac{7 \times 5}{5 \times 5} = \frac{35}{25} = \sqrt{\frac{35}{25}}$

Now, $\sqrt{25} = 5$

↗ Now we will calculate the square root of 35.

	5.916
5	35 . 00 00 00
	25
109	1000
9	981
1181	1900
1	1181
11826	71900
6	70956

$\therefore$ Square root of 35 at three places of decimal
$= 5.916$

Now, $\sqrt{\frac{7}{5}} = \frac{5.916}{5} = 1.1832$

↗ *Square root of fraction can be find after convert it into decimal number.*

Example **6.** Find the value of $\sqrt{\frac{0.081 \times 0.484}{0.0064 \times 6.25}}$.

(1) 7.64 (2) 1.84 (3) 0.99 (4) 2.46

Sol. (3) $\sqrt{\frac{0.081 \times 0.484}{0.0064 \times 6.25}} = \sqrt{\frac{81 \times 484}{64 \times 625}}$

$= \sqrt{\frac{9 \times 9 \times 22 \times 22}{8 \times 8 \times 25 \times 25}} = \frac{9 \times 22}{8 \times 25} = \frac{198}{200} = \frac{99}{100} = 0.99$

Cube

If a number is multiplied two times with itself, then the result of this multiplication is called the cube of that number. *e.g.,* cube of $6 = 6 \times 6 \times 6 = 216$

Cube Root

The cube root of a number is that number in which we multiply thricely, it gives the given number.
The cube root is denoted by the symbol '$\sqrt[3]{\ }$'.
e.g., $\sqrt[3]{8} = \sqrt[3]{2 \times 2 \times 2} = 2$ 944

Method of Finding Cube Root

Prime Factorisation Method

This method has following steps

Step I Express the given number as the product of prime factors.

Step II Keep these factors in a group of three.

Step III Take the product of these prime factors picking one out of every group (group of three) of the same primes. This product gives us the cube root of given number.

Example **7.** Find the cube root of 9261.

(1) 22 (2) 21
(3) 23 (4) 24

Sol. (2) Prime factors of 9261
$= (3 \times 3 \times 3) \times (7 \times 7 \times 7)$
$\sqrt[3]{9261} = \sqrt[3]{3 \times 3 \times 3 \times 7 \times 7 \times 7}$
Now, taking one number from each group of three, we get $\sqrt[3]{9261} = 3 \times 7 = 21$

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

Entrance Corner

1. The product of two numbers is 18.75. If one number is thrice of another. Find the larger number. **[JNV 2018]**
(1) 2.5 (2) 9.5 (3) 4.5 (4) 7.5

2. A school collected ₹ 2304 as fees from its students. If each student paid as many paise as there were students in the school, how many students were there in the school? **[JNV 2017]**
(1) 240 (2) 460 (3) 480 (4) 440

3. Square root of 4096 is **[JNV 2016]**
(1) 74 (2) 64 (3) 66 (4) 63

4. What is the square root of $\frac{1}{4}$? **[JNV 2012]**
(1) $\frac{1}{16}$ (2) $\frac{1}{2}$ (3) 1 (4) 0

5. Simplify $(256)^{3/4}$. **[JNV 2012]**
(1) 52 (2) 62 (3) 84 (4) 64

6. A man plants his orchard with 729 trees and arranges them 50 that there are as many rows as there are trees in a row. How many rows are there? **[JNV 2000]**
(1) 25 (2) 26 (3) 27 (4) 28

7. Square root of 25 is **[JNV 2000]**
(1) 2 (2) 3 (3) 4 (4) 5

8. Simplify $\sqrt{169} + \sqrt{144} - \sqrt{196}$. **[JNV 1999]**
(1) 11 (2) 12 (3) 13 (4) 14

9. $\sqrt{72}$ is equal to **[JNV 1999]**
(1) $6\sqrt{2}$ (2) $4\sqrt{6}$ (3) $9\sqrt{6}$ (4) $2\sqrt{6}$

10. $\frac{\sqrt{28} \times \sqrt{24}}{\sqrt{42} \times \sqrt{8}}$ is equal to **[JNV 1998]**
(1) 2 (2) 21 (3) $\sqrt{2}$ (4) $\frac{1}{\sqrt{2}}$

11. Square root of 289 is **[JNV 1998]**
(1) 15 (2) 16 (3) 17 (4) 18

12. Simplify $\sqrt{\frac{36}{?}} = \frac{6}{7}$. **[JNV 1998]**

(1) 49 (2) 64 (3) 81 (4) 121

13. By what least number, 720 be multiplied so that the resulting number is a perfect square? **[JNV 1997]**

(1) 3 (2) 4 (3) 5 (4) 6

14. Find the least number which must be subtracted to make 175 a perfect square. **[JNV 1997]**

(1) 2 (2) 3 (3) 6 (4) 7

15. Square root of 0.09 is **[JNV 1997]**

(1) 0.03 (2) 0.13
(3) 0.3 (4) 0.5

16. Simplify $\sqrt{8464} + \sqrt{?} = 102$. **[JNV 1997]**

(1) 100 (2) 225 (3) 400 (4) 625

17. Simplify $\sqrt{\frac{1694}{?}} + 14 = 25$. **[JNV 1996]**

(1) 11 (2) 12
(3) 14 (4) 22

18. A general wishing to draw up his 16160 soldiers in the form of a solid square, found that he had 31 soldiers over. Find the number of men in the front line. **[JNV 1996]**

(1) 127 (2) 133 (3) 160 (4) 172

19. A gardener plants 3600 trees in garden, he arranges them so that there are as many rows as there are trees in a row, how many rows are there? **[JNV 1996]**

(1) 45 (2) 55 (3) 57 (4) 60

20. The square root of $\frac{289}{225}$ is equal to **[JNV 1995]**

(1) $\frac{15}{17}$ (2) $\frac{16}{17}$ (3) $\frac{17}{15}$ (4) $\frac{15}{18}$

21. 27×243 is equal to **[JNV 1995]**

(1) 3^8 (2) 3^9 (3) 3^{10} (4) 3^{11}

Answers

1. (4)	**2.** (3)	**3.** (2)	**4.** (2)	**5.** (4)	**6.** (3)	**7.** (4)	**8.** (1)	**9.** (1)	**10.** (3)
11. (3)	**12.** (1)	**13.** (3)	**14.** (3)	**15.** (3)	**16.** (1)	**17.** (3)	**18.** (1)	**19.** (4)	**20.** (3)
21. (1)									

Hints and Solutions

1. Let smaller number = x
Larger number = $3x$
According to the question

$$x \times 3x = 18.75 \Rightarrow 3x^2 = 18.75$$
$$x^2 = 6.25 \Rightarrow x = \sqrt{6.25} = 2.5$$

$\therefore$ Larger number = $3x = 3 \times 2.5 = 7.5$

2. Total money collected = ₹ 2304 = 230400 paise
As number of students = Money paid by students
$\therefore$ Number of students in school = $\sqrt{230400} = 480$

3.

	64
6	40 96
6	36
124	496
4	496
	x

$\therefore$ Square root of 4096 = 64

4. Square root of $\frac{1}{4} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

5. $(256)^{3/4} = (4^4)^{3/4} = (4)^{4 \times \frac{3}{4}} = 4^3 = 64$

6. According to the given condition number of rows in the orchard is equal to the number of trees in each row. Therefore, number of rows will be equal to the square root of 729.

	27
2	729
2	4
47	329
7	329
	×

$\therefore$ There are 27 rows in the orchard.

7.

	5
5	25
5	25
	×

8. $\sqrt{169}+\sqrt{144}-\sqrt{196}=13+12-14$

$=25-14=11$

9.

2	72
2	36
2	18
3	9
	3

$\sqrt{72}=2\times2\times2\times3\times3=6\sqrt{2}$

10. $\dfrac{\sqrt{28}\times\sqrt{24}}{\sqrt{42}\times\sqrt{8}}=\dfrac{2\sqrt{7}\times2\sqrt{6}}{\sqrt{2}\times\sqrt{3}\times\sqrt{7}\times2\sqrt{2}}$

$=\dfrac{4\times\sqrt{7}\times\sqrt{3}\times\sqrt{2}}{4\times\sqrt{3}\times\sqrt{7}}=\sqrt{2}$

11.

	17
1	289
1	1
27	189
7	189
	×

∴ Square root of 289 is 17.

12. Let ? = $x\Rightarrow\sqrt{\dfrac{36}{x}}=\dfrac{6}{7}\Rightarrow\dfrac{6}{\sqrt{x}}=\dfrac{6}{7}\Rightarrow6\sqrt{x}=42$

$\Rightarrow\quad\sqrt{x}=7\ \Rightarrow x=49$

13. Prime factors of 720 are

$2\times2\times2\times2\times3\times3\times5$

∴ Required number = 5

14.

	13
1	175
1	1
23	75
3	69
	6

Extracting the square root we get a remainder 6. Hence, 6 is the least number which ought to be subtracted to make it a perfect square.

15. $0.09=\dfrac{9}{100}$

Now, $\sqrt{9}=3$

$\sqrt{100}=10$

∴ $\sqrt{\dfrac{9}{100}}=\dfrac{3}{10}=0.3$

16. Let ? = x; $\sqrt{8464}+\sqrt{x}=102\Rightarrow92+\sqrt{x}=102$

$\Rightarrow\sqrt{x}=102-92=10\therefore x=10\times10=100$

17. Let ? = x, then $\sqrt{\dfrac{1649}{x}}+14=25$

$\Rightarrow\sqrt{\dfrac{1694}{x}}=25-14=11$

$\Rightarrow\quad\dfrac{1694}{x}=121\Rightarrow\ x=\dfrac{1694}{121}=14$

18. $16160-31=16129$

	127
1	161 29
1	1
22	61
2	44
247	1729
7	1729
	×

∴ Number of men in the front line = 127

19.

	60
6	36 00
6	36
120	00
0	00
	×

∴ Required rows = 60

20. Now,

	17
1	2 89
1	1
27	189
7	189
	×

and

	15
1	2 25
1	1
25	125
5	125
	×

$\therefore \quad \sqrt{\frac{289}{225}} = \frac{17}{15}$

21. $27 \times 243 = [3 \times 3 \times 3] \times [3 \times 3 \times 3 \times 3 \times 3]$
$= 3^3 \times 3^5 = 3^8$

Practice Exercise

1. Find the value of $\frac{112}{\sqrt{196}} \times \frac{\sqrt{576}}{12}$.
(1) 8 (2) 12 (3) 16 (4) 18

2. If $\sqrt{4096} = 64$, then the value of $\sqrt{4096} + \sqrt{0.4096} + \sqrt{0.004096} + \sqrt{0.00004096}$ is
(1) 7.09 (2) 7.1014
(3) 7.1104 (4) 7.12

3. Simplify $\sqrt{256\sqrt{16 \div ?}} = 16$.
(1) 8 (2) 16 (3) 4 (4) 256

4. What is the square root of 2^8?
(1) 64 (2) 48 (3) 32 (4) 16

5. The value of $\sqrt{0.01} + \sqrt{0.81} + \sqrt{1.21} + \sqrt{0.0009}$ is
(1) 2.03 (2) 2.1 (3) 2.11 (4) 2.13

6. The square root of 6561 by means of factors is
(1) 81 (2) 64 (3) 96 (4) 24

7. The least number which must be subtracted to 4931 to make it a perfect square, is
(1) 100 (2) 31 (3) 140 (4) 110

8. The least number which must be subtracted from 2361 to make it a perfect square, is
(1) 48 (2) 88 (3) 57 (4) 40

9. The smallest number by which 9408 must be divided so that it becomes a perfect square. Also, the square root of the perfect square so obtained, is
(1) 3, 56 (2) 4, 56 (3) 21, 56 (4) 42, 56

10. 5929 students are sitting in an auditorium in such a manner that there are as many students in a row as there are rows in auditorium. How many rows are there in the auditorium?
(1) 66 (2) 7 (3) 11 (4) 77

11. A General arranges his soldiers in rows to form a perfect square. He find that in doing so 60 soldiers are leftout. If the total number of soldiers be 8160. The number of soldiers in each row, are
(1) 90 (2) 80 (3) 70 (4) 40

12. If $\sqrt[3]{185193} = 57$, then the value of $\sqrt[3]{185193} + \sqrt[3]{185.193} + \sqrt[3]{0.000185193}$ is
(1) 6.327 (2) 63.275
(3) 632.75 (4) 62.757

13. $\sqrt[3]{1 - \frac{127}{343}}$ is equal to
(1) $\frac{5}{9}$ (2) $1 - \frac{1}{7}$ (3) $\frac{4}{7}$ (4) $1 - \frac{2}{7}$

14. What is the smallest number by which 3600 must be divided to make it a perfect cube?
(1) 9 (2) 50 (3) 300 (4) 450

Answers

1. (3)	**2.** (3)	**3.** (2)	**4.** (4)	**5.** (4)	**6.** (1)	**7.** (2)	**8.** (3)	**9.** (1)	**10.** (4)
11. (1)	**12.** (4)	**13.** (2)	**14.** (4)						

Hints and Solutions

1. $\frac{112}{\sqrt{196}} \times \frac{\sqrt{576}}{12} = \frac{112}{14} \times \frac{24}{12} = 16$

2. $\sqrt{40.96} + \sqrt{0.4096} + \sqrt{0.004096} + \sqrt{0.00004096}$
$= \sqrt{\frac{4096}{100}} + \sqrt{\frac{4096}{10000}} + \sqrt{\frac{4096}{1000000}} + \sqrt{\frac{4096}{100000000}}$
$= \frac{64}{10} + \frac{64}{100} + \frac{64}{1000} + \frac{64}{10000}$
$= 6.4 + 0.64 + 0.064 + 0.0064 = 7.1104$

3. Let $? = x$
Then, $\sqrt{256\sqrt{16 \div x}} = 16$

$\Rightarrow \sqrt{16\times 16\sqrt{4\times 4\div x}} = 16$

$\Rightarrow \quad \sqrt{16\times 16\sqrt{\frac{4\times 4}{x}}} = 16$

On squaring both sides, we get

$16\times 16\sqrt{\frac{4\times 4}{x}} = (16)^2$

$\sqrt{\frac{4\times 4}{x}} = 1 \Rightarrow \frac{4}{\sqrt{x}} = 1 \Rightarrow \sqrt{x} = 4 \quad \Rightarrow x = 16$

4. Required square root

$= \sqrt{2\times 2\times 2\times 2\times 2\times 2\times 2\times 2} = \sqrt{256} = 16$

5. $\sqrt{0.01} + \sqrt{0.81} + \sqrt{1.21} + \sqrt{0.009}$

$= \sqrt{0.1\times 0.1} + \sqrt{0.9\times 0.9} + \sqrt{1.1\times 1.1}$

$+\sqrt{0.03\times 0.03} = 0.1 + 0.9 + 1.1 + 0.03 = 2.13$

6.

3	6561
3	2187
3	729
3	243
3	81
3	27
3	9
3	3
	1

Thus, $6561 = \underline{3\times 3}\times\underline{3\times 3}\times\underline{3\times 3}\times\underline{3\times 3}$

$\therefore \sqrt{6561} = 3\times 3\times 3\times 3 = 81$

7.

	70
7	49 31
	49
	31

$\therefore$ Required number to be subtracted = 31

8.

	48
4	23 61
	16
88	761
× 8	704
	57

Hence, the required number to be subtracted from 2361 to make it a perfect square = 57

9.

2	9408
2	4704
2	2352
2	1176
2	588
2	294
3	147
7	49
7	7
	1

Thus, $9408 = \underline{2\times 2}\times\underline{2\times 2}\times\underline{2\times 2}\times 3\times\underline{7\times 7}$

Hence, required number = 3

$\therefore$ Required square root $= 2\times 2\times 2\times 7 = 56$

10.

7	5929
7	847
11	121
11	11
	1

Thus, $5929 = 7\times 7\times 11\times 11$

$\therefore \quad \sqrt{5929} = 7\times 11 = 77$

Hence, the number of rows in the auditorium = 77

11. Number of soldiers arranged in rows

$= 8160 - 60 = 8100$

	90
9	81 00
	81
180	00
	00
	×

$\therefore$ Number of soldiers in each row

$= \sqrt{8100} = 90$

12. $\therefore \sqrt[3]{185193} = 57$

$\therefore \sqrt[3]{185193} + \sqrt[3]{185.193} + \sqrt[3]{0.000185193}$

$= 57 + 5.7 + 0.057 = 62.757$

13. $\sqrt[3]{1-\frac{127}{343}} = \sqrt[3]{\frac{343-127}{343}} = \sqrt[3]{\frac{216}{343}}$

$= \frac{6}{7} = 1 - \frac{1}{7}$

14.

2	3600
2	1800
2	900
2	450
3	225
3	75
5	25
5	5
	1

$\therefore$ Prime factors of $3600 = 2^3\times 2\times 3^2\times 5^2$

To make it a perfect cube, it must be divided by

$2\times 3^2\times 5^2 = 450$

Self Practice

1. The value of $\sqrt{1\frac{7}{9}}$ is

(1) 3 (2) 2 (3) 9 (4) $1\frac{1}{3}$

2. The square root of $1\frac{15}{49}$ is

(1) $\frac{1}{7}$ (2) $1\frac{1}{7}$ (3) $\frac{9}{7}$ (4) $\frac{4}{7}$

3. The square root of $\frac{400}{169}$ is

(1) $\frac{20}{23}$ (2) $1\frac{7}{13}$ (3) $\frac{10}{13}$ (4) $\frac{20}{7}$

4. What should be added to 79 to make it a perfect square?

(1) 3 (2) 5 (3) 2 (4) 4

5. By what least number must 21600 be multiplied to make it a perfect cube?

(1) 6 (2) 10 (3) 30 (4) 60

6. The number of trees is equal to number of their rows. Then, number of total trees is 5625. What is the number of the rows?

(1) 85 (2) 70 (3) 75 (4) 65

7. $\sqrt{144} \times \sqrt{81} \times \sqrt{9}$ is equal to

(1) 729 (2) 224 (3) 108 (4) 324

8. $(64)^2 \div \sqrt[3]{32768}$ is equal to

(1) 128 (2) 132 (3) 142 (4) 104

9. By which of the numbers 450 to be multiplied to make complete square.

(1) 2 (2) 5 (3) 4 (4) 3

10. $\sqrt{169} + \sqrt{100} - \sqrt{121}$ is equal to

(1) 12 (2) 11 (3) 13 (4) 34

11. What is the least number of 4 digits, which is perfect square ?

(1) 1024 (2) 1065 (3) 1000 (4) 1020

12. Find the least number by which 175760 be multiplied to make it a perfect cube.

(1) 125 (2) 150 (3) 100 (4) 200

Answers

1. (4)	**2.** (2)	**3.** (2)	**4.** (3)	**5.** (1)	**6.** (3)	**7.** (4)	**8.** (1)	**9.** (1)	**10.** (1)
11. (1)	**12.** (3)								