CHAPTER

03

CUBE AND CUBE ROOTS

In this chapter, we study the cubes of numbers and their properties, and cube roots by prime factorisation method.

Cube

The cube of a number is the product of the number itself thricely.

e.g. If x is a non-zero number, then $x \times x \times x = x^3$ is called cube of x.

The cube of rational number is the cube of the numerator divided by the cube of denominator. e.g. Cube of $\frac{4}{5}$ is $\frac{64}{125}$.

Perfect Cube

A natural number n is said to be a perfect cube if there is an integer m such that $n = m \times m \times m$.

Cubes from 1 to 15 numbers

Numbers	Cubes
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512

Numbers	Cubes
9	729
10	1000
11	1331
12	1728
13	2197
14	2744
15	3375

Properties of Cube of Numbers

(i) Cubes of all even natural numbers are always even.

(ii) Cubes of all odd natural numbers are always odd.

(iii) Cubes of negative integers are always negative.

(iv) For any rational number $\frac{a}{b}$, we have

$$\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$$

Example 1 Find the value of $(11)^3 + 4(7)^3 - 5$.

(a) 2697 (b) 2698 (c) 2699 (d) 3000

Sol. *(b)* $(11)^3 + 4(7)^3 - 5 = 1331 + 4(343) - 5$

$= 1331 + 1372 - 5 = 2698$

Example 2 Which one of the following will have odd unit digit

(a) $(24)^3$ (b) $(64)^3$

(c) $(27)^3$ (d) $(52)^3$

Sol. *(c)* We knwo cube of odd number is odd number.

$\therefore$ Number $(27)^3$ have odd unit digit

Cube Root

If n is perfect cube for any integer m i.e. $n = m^3$, then m is called the cube root of n and it is denoted by $m = \sqrt[3]{n}$.

Cube Root of a Perfect Cube by Prime Factorisation

The following steps are given below.

I. Factorise the given number into prime factors.

II. Make triples of similar factors or arrange them in group of three equal factors at a time.

III. Choose one prime from each pair and multiply all primes.

Example 3 Find the cube root of 74088.

(a) 40 (b) 47 (c) 42 (d) 45

Sol. *(c)* Resolving the given number, we get

2	74088
2	37044
2	18522
3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

$$74088 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{7 \times 7 \times 7}$$

$$\therefore \quad \sqrt[3]{74088} = 2 \times 3 \times 7 = 42$$

Cube Root of a Negative Cube

If a is a positive integer, then $-a$ is a negative integer.

We know that, $(-a)^3 = -a^3$

So, $\sqrt[3]{-a^3} = -a$

In general, we have $\sqrt[3]{-x} = -\sqrt[3]{x}$

Cube Root of Product of Numbers and Rational Number

(i) The cube root of product of integers is the cube root of integer by taking separately.

For any two integer a and b, we have

$$\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b}$$

(ii) Cube Root of Rational Number $\frac{a}{b}$ is $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$

Example 4 The simplified form of $\sqrt[3]{125 \times 64}$ is

(a) 20 (b) 40 (c) 60 (d) 80

Sol. *(a)* $125 \times 64 = \underline{5 \times 5 \times 5} \times \underline{4 \times 4 \times 4}$

$\therefore$ LHS $= \sqrt[3]{125 \times 64} = \sqrt[3]{(5 \times 4)^3} = (5 \times 4) = 20$

Now, $\sqrt[3]{125} = \sqrt[3]{5 \times 5 \times 5} = 5$

and $\sqrt[3]{64} = \sqrt[3]{4 \times 4 \times 4} = 4$

$\therefore$ RHS $= \sqrt[3]{125} \times \sqrt[3]{64} = (5 \times 4) = 20$

$\Rightarrow$ LHS = RHS

Hence, $\sqrt[3]{125 \times 64} = \sqrt[3]{125} \times \sqrt[3]{64}$

Example 5 Find the cube root of 4.096.

(a) 1.6 (b) 165

(c) 1.75 (d) None of these

Sol. (a) $\sqrt[3]{4.096} = \sqrt[3]{\frac{4096}{1000}} = \frac{\sqrt[3]{4096}}{\sqrt[3]{1000}}$

$\therefore \quad 4096 = \underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{2\times2\times2}$

$\therefore \quad \sqrt[3]{4096} = 2\times2\times2\times2 = 16$

Also, $\sqrt[3]{1000} = \sqrt[3]{10\times10\times10} = 10$

So, $\frac{\sqrt[3]{4096}}{\sqrt[3]{1000}} = \frac{16}{10} = 1.6$

Hence, $\sqrt[3]{4.096} = 1.6$

Important Points

(i) If 1, 4, 5, 6 and 9 in the unit place, then cube of that number given the same digit in the unit place.

(ii) 3 in the unit place have cube with 7 in the unit place.

(iii) 7 in the unit place have cube with 3 in the unit place.

(iv) 2 in the unit place have cube with 8 in the unit place.

(v) 8 in the unit place have cube with 2 in the unit place.

Example 6 Difference of two perfect cubes is 189. If the cube root of the smaller of the two numbers is 3, then find the cube root of the larger number.

(a) 5 (b) 6 (c) 7 (d) 8

Sol. (b) Given difference of two perfect cube = 189 and cube root of the smaller number = 3

$\therefore$ Cube of smaller number = $(3)^3 = 27$

Let cube root of the larger number be x.

Then, cube of larger number = x^3

According to the question,

$$x^3 - 27 = 189$$

$\Rightarrow \quad x^3 = 189 + 27 \Rightarrow x^3 = 216$

$\Rightarrow \quad x = \sqrt[3]{216} = \sqrt[3]{6\times6\times6}$

$\therefore \quad x = 6$

Hence, the cube root of the larger number is 6.

PRACTICE EXERCISE

1. Which of the following is not a perfect cube ?

(a) 1000000

(b) 216

(c) 10000

(d) None of the above

2. Which of the following perfect cube is the cube of an even number ?

(a) 343 (b) 2197

(c) 216 (d) 1331

3. Which of the following perfect cube is the cube of an odd number ?

(a) 1728 (b) 512

(c) 729 (d) 1000

4. If a number is increased by four times, then cube of the number will increase

(a) 64 times (b) 69 times

(c) 729 times (d) 1000 times

5. The value of $\sqrt[3]{\frac{27}{125}}$ is

(a) $\frac{3}{5}$ (b) $\frac{3}{25}$

(c) $\frac{9}{25}$ (d) not appropriate data

6. The unit place of cube roots of 117649 is

(a) 4 (b) 9

(c) 49 (d) None of these

7. The smallest number by which 3087 may be multiplied so that the product is a perfect cube, is
   (a) 6 (b) 5
   (c) 4 (d) 3
8. The smallest by which 392 may be divided so that the quotient is a perfect cube, is
   (a) 50 (b) 51
   (c) 49 (d) 62
9. In a number pattern, 8, 27, 64, x, the value of x will be
   (a) 125 (b) 216 (c) 100 (d) 115
10. The cube of 31.2 is
    (a) 3037.1328 (b) 3037.1381
    (c) 30371.328 (d) 30371328
11. The sum of the cubes of first three natural number is equal to
    (a) $(1+2+3)^3$ (b) $(1+2+3)^2$
    (c) 3^3 (d) 3^2
12. The cube root of $\frac{-343}{1331}$ is
    (a) $\frac{7}{11}$ (b) $\frac{-7}{11}$ (c) $\frac{11}{7}$ (d) $\frac{-11}{7}$
13. If one side of a cube is 33 m, then the volume of the cube is
    (a) 35937 (b) 35936
    (c) 3934 (d) None of these
14. Three numbers are in the ratio of $1:2:3$. The sum of their cubes is 121500. The numbers are
    (a) 15, 30, 45 (b) 30, 45, 15
    (c) 7, 14, 21 (d) None of these
15. If the volume of a cubical box is $35.937\ \text{m}^3$, what is the length of its one side ?
    (a) 3.3 m (b) 6.6 m
    (c) 3.6 m (d) None of these
16. $(-216 \times 729)^{1/3}$
    (a) 54 (b) −54 (c) −45 (d) 45
17. The value of $\sqrt[3]{0.064} + \sqrt[3]{27} - \sqrt[3]{729}$ is
    (a) 12.4 (b) −2 (c) 5.6 (d) −5.6
18. The value of $\frac{\sqrt[3]{8}}{\sqrt{16}} \div \sqrt{\frac{100}{49}} \times \sqrt[3]{125}$ is
    (a) 7 (b) $1\frac{3}{4}$ (c) $\frac{7}{100}$ (d) $\frac{4}{7}$
19. The value of $\sqrt{\sqrt[3]{0.004096}}$ is
    (a) 0.4 (b) 0.04
    (c) 0.16 (d) 4
20. The value of $(27 \times -2744)^{1/3}$ is
    (a) − 40 (b) − 42
    (c) − 22 (d) − 32
21. The value of $\frac{(73)^3 + (53)^3}{73 \times 73 - 73 \times 53 + 53 \times 53}$ is
    (a) 126 (b) 216 (c) 162 (d) − 126
22. What is the smallest number in which we multiply by 392, we get the factors in the form of cube ?
    (a) 5 (b) 6
    (c) 7 (d) 8
23. The value of $\sqrt[3]{\frac{27}{64}}$ is
    (a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{5}{4}$ (d) $\frac{5}{3}$
24. If $\sqrt{\frac{x}{0.0064}} = \sqrt[3]{0\ .008}$, then the value of x is
    (a) 0.256 (b) 0.0256
    (c) 0.000256 (d) 0.00256
25. The smallest number in which we multiply by 1800, we get the cube. Then, the sum of digit is
    (a) 2 (b) 3 (c) 6 (d) 8
26. If cube root of 175616 is 56, then the value of $\sqrt[3]{175.616} + \sqrt[3]{0.175616} + \sqrt[3]{0.000175616}$ is
    (a) 0.168 (b) 6.216
    (c) 6.116 (d) 62.16
27. The cube root of 0.000001 is
    (a) 0.1 (b) 0.01
    (c) 0.316 (d) 0.031

Answers

1	*(c)*	2	*(c)*	3	*(c)*	4	*(a)*	5	*(a)*	6	*(b)*	7	*(d)*	8	*(c)*	9	*(a)*	10	*(c)*
11	*(b)*	12	*(b)*	13	*(a)*	14	*(a)*	15	*(a)*	16	*(b)*	17	*(d)*	18	*(b)*	19	*(a)*	20	*(b)*
21	*(a)*	22	*(c)*	23	*(a)*	24	*(c)*	25	*(c)*	26	*(b)*	27	*(b)*						

Hints and Solutions

1. $\sqrt[3]{1000000} = 100$ is a perfect cube.

$\sqrt[3]{216} = 6$ is a perfect cube.

$\sqrt[3]{10000} =$ not a perfect cube.

2. 216 is the cube of an even number because cube of an even number is always even.

3. 729 is the cube of an odd number because the cube of odd number is always odd.

4. Let the number be x.
After increasing 4 times the number $= 4x$
Cube of the number $= (4x)^3 = 64x^3$
$\therefore$ The cube of a number increases by 64 times.

5. $\sqrt[3]{\dfrac{27}{125}} = \sqrt[3]{\dfrac{3\times3\times3}{5\times5\times5}} = \dfrac{3}{5}$

6. $\sqrt[3]{117649} = 49$
$\therefore$ Unit place is 9.

7. Writing 3087 as a product of a prime factors, we have

$$\begin{array}{r|l} 3 & 3087 \\ \hline 3 & 1029 \\ \hline 7 & 343 \\ \hline 7 & 49 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$\therefore \quad 3087 = 3 \times 3 \times \overline{7 \times 7 \times 7}$

Clearly, to make it a perfect cube it must be multiplied by 3.

8. Writing 392 as a product of prime factors, we have

$$\begin{array}{r|l} 2 & 392 \\ \hline 2 & 196 \\ \hline 2 & 98 \\ \hline 7 & 49 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$\therefore \quad 392 = 2 \times 2 \times 2 \times 7 \times 7$

Clearly, to make it perfect cube it must be divided by (7×7) i.e., 49.

9. Here, we see that $8 = 2^3$, $27 = 3^3$, $64 = 4^3$.

It means given pattern is a cube of consecutive natural number.

$\therefore \quad x = 5^3 = 125$

10. $31.2 \times 31.2 \times 31.2 = 30371.328$

11. $1^3 + 2^3 + 3^3 = 1 + 8 + 27$

$= 36 = (6)^2 = (1 + 2 + 3)^2$

12. $\sqrt[3]{\dfrac{-343}{1331}} = \dfrac{\sqrt[3]{-343}}{\sqrt[3]{1331}} = \dfrac{\sqrt[3]{-7\times-7\times-7}}{\sqrt[3]{11\times11\times11}} = \dfrac{-7}{11}$

13. Volume of cube $= (33)^3 = 35937$

14. Let the number be x, $2x$ and $3x$.

$$\begin{aligned} \therefore \quad (x)^3 + (2x)^3 + (3x)^3 &= 121500 \\ \Rightarrow \quad 1x^3 + 8x^3 + 27x^3 &= 121500 \\ \Rightarrow \quad 36x^3 &= 121500 \\ \Rightarrow \quad x^3 &= \frac{121500}{36} \\ \Rightarrow \quad x^3 &= 3375 \\ \Rightarrow \quad x &= \sqrt[3]{3375} \\ \Rightarrow \quad x &= \sqrt[3]{15\times15\times15} \\ \Rightarrow \quad x &= 15 \end{aligned}$$

$\therefore$ The numbers are $x = 15$, $2x = 30$, $3x = 45$.

15. $\because$ Volume of a cube $= (\text{side})^3$

$$\begin{aligned} (\text{side})^3 &= 35.937 \\ \Rightarrow \quad \text{side} &= \sqrt[3]{35.937} \\ \Rightarrow \quad \text{side} &= \sqrt[3]{3.3\times3.3\times3.3} \\ \Rightarrow \quad \text{side} &= 3.3 \text{ m} \end{aligned}$$

16. $(-216\times 729)^{1/3} = (-216)^{1/3}\times(729)^{1/3}$

$= -(\underline{6\times 6\times 6})^{1/3}\times(\underline{3\times 3\times 3}\times\underline{3\times 3\times 3})^{1/3}$

$= -6\times 3\times 3 = -54$

17. $\sqrt[3]{0.064}+\sqrt[3]{27}-\sqrt[3]{729}$

$\because \quad \sqrt[3]{0.064} = \sqrt[3]{0.4\times 0.4\times 0.4} = 0.4$

$\sqrt[3]{27} = \sqrt[3]{3\times 3\times 3} = 3$

$\sqrt[3]{729} = \sqrt[3]{\underline{3\times 3\times 3}\times\underline{3\times 3\times 3}}$

$= 3\times 3 = 9$

$\therefore \quad \sqrt[3]{0.064}+\sqrt[3]{27}-\sqrt[3]{729} = 0.4+3-9 = -5.6$

18. $\dfrac{\sqrt[3]{8}}{\sqrt{16}}\div\sqrt{\dfrac{100}{49}}\times\sqrt[3]{125} = \dfrac{2}{4}\times\dfrac{7}{10}\times 5$

$= \dfrac{7}{4} = 1\dfrac{3}{4}$

19. $\sqrt{\sqrt[3]{0.004096}} = \sqrt{\sqrt[3]{(0.16)^3}} = \sqrt{0.16} = 0.4$

20. $(27\times -2744)^{1/3} = (27)^{1/3}\times(-2744)^{1/3}$

$= 3\times -14 = -42$

21. $\dfrac{(73)^3+(53)^3}{73\times 73-73\times 53+53\times 53}$

$= \dfrac{(73+53)\,(73\times 73-73\times 53+53\times 53)}{73\times 73-73\times 53+53\times 53}$

$= 73+53 = 126$

22.

2	392
2	196
2	98
7	49
7	7
	1

The divisible number of $392 = 2\times 2\times 2\times 7\times 7$. It is clear that on multiplying by 7, we get 392 cube number.

23. $\sqrt[3]{\dfrac{27}{64}} = \sqrt[3]{\left(\dfrac{3}{4}\right)^3} = \dfrac{3}{4}$

24. $\sqrt{x} = \sqrt{0.0064}\times\sqrt[3]{0.008}$

$\Rightarrow \sqrt{x} = 0.08\times 0.2 = 0.016$

$\Rightarrow \quad x = 0.000256$

25. $1800 = 2^3\times 3^2\times 5^2$

It is clear that we have to multiply by 15 to get the sum of digit is 6.

26. $\sqrt[3]{175.616}+\sqrt[3]{0.175616}+\sqrt[3]{0.000175616}$

$= 5.6+0.56+0.056 = 6.216$

27. $(0.000001)^{1/3} = \sqrt[3]{0.000001}$

$= \sqrt[3]{\dfrac{1}{1000000}} = \dfrac{1}{100}$

$= 0.01$