CHAPTER

09

LINEAR EQUATIONS IN ONE VARIABLE

In this chapter, we study one and two variable types linear equations and various word problems by using linear equations.

Equation

An equation is an equality involving variables and it has an equality sign (=). The expression on the left of the equality sign is the Left Hand Side (LHS) and expression on the right of the equality sign is the Right Hand Side (RHS).

Linear Equations

The equations, in which the highest power of the variable appearing in the expression is 1, are called linear equations.

Linear Equations in One Variable

The expression, which form the equation contains only one variable and the highest power of the variable appearing in the equation is 1, are called linear equations in one variable.

e.g. $2x-5=3$ and $\frac{5}{3}x+3=\frac{1}{2}$

Note The graph of the linear equation is always a straight line.

Properties of Linear Equations

(i) If same number is added or subtracted to both the sides of an equation, the equality remains the same.

(ii) If same number is multiplied to both the sides of an equation, the equality remains the same.

(iii) If both sides are divided by a same non-zero number, the equality remains the same.

Example 1 Solve $2(x-3)-(5-3x) = 3(x+1)-4(2+x)$

(a) 2 (b) 1
(c) 3 (d) 0

Sol. *(b)* Given,

$$2(x-3)-(5-3x)=3(x+1)-4(2+x)$$

$\Rightarrow 2x-6-5+3x=3x+3-8-4x$

$\Rightarrow 5x-11=-x-5 \Rightarrow 6x=6$

$\Rightarrow x=1$

Example 2 If $\frac{2x+1}{3x-1}=\frac{2x+1}{3x+2}$, the value of x is

(a) $-\frac{1}{2}$ (b) 0
(c) $\frac{1}{3}$ (d) $-\frac{2}{3}$

Sol. *(a)* We have, $\frac{2x+1}{3x-1}=\frac{2x+1}{3x+2}$

$\Rightarrow (2x+1)(3x+2)=(2x+1)(3x-1)$

$\Rightarrow 6x^2+4x+3x+2=6x^2-2x+3x-1$

$\Rightarrow 6x^2+7x+2=6x^2+x-1$

$\Rightarrow 7x-x=-1-2 \Rightarrow 6x=-3$

$\therefore x=-\frac{3}{6}=-\frac{1}{2}$

Solving Word Problems by Using Linear Equation

I. Firstly, denote the unknown quantity by any letters, say x, y, z, etc.

II. Translate the statements of the problem into mathematical statements.

III. Using the conditions given in the problem, form the equation.

IV. Solve the equation for the unknown quantity.

Example 3 The digit in the ten's place of a two-digit number is 3 more than the digit in the unit's place. Let the digit at unit's place be b. Then, the number is

(a) $11b+30$ (b) $10b+30$
(c) $11b+3$ (d) $10b+3$

Sol. *(a)* Let digit at unit's place be b.

Then, digit at ten's place $=(3+b)$

$\therefore$ Number $=10(3+b)+b$

$=30+10b+b=11b+30$

Example 4 Thirty one is added 63 to twice a whole number gives. The number is

(a) 15 (b) 16 (c) 17 (d) 18

Sol. *(b)* Let whole number be x.

Then, $31+2x=63$

$\Rightarrow 2x=32 \Rightarrow x=16$

Example 5 A boy is now one-third as old as his father. Twelve years hence, he will be half as old as his father. The present age of the boy is

(a) 12 yr (b) 20 yr (c) 36 yr (d) 25 yr

Sol. *(a)* Let the present age of the father be x yr and that of the son be $\frac{1}{3}x$ yr.

After twelve years,

Age of father $=(x+12)$ yr

and age of son $=\left(\frac{1}{3}x+12\right)$ yr

According to given condition,

$$\frac{1}{3}x+12=\frac{1}{2}(x+12)$$

$\Rightarrow \frac{x+36}{3}=\frac{x+12}{2}$

$\Rightarrow 2x+72=3x+36 \Rightarrow x=36$

$\therefore$ Age of the boy $=\frac{x}{3}=\frac{36}{3}=12$ yr.

PRACTICE EXERCISE

1. Linear equation in one variable has
(a) only one variable with any power
(b) only one term with a variable
(c) only one variable with power 1
(d) only constant term

2. Which of the following is a linear expression?
(a) $x^2 + 1$ (b) $y + y^2$
(c) 4 (d) $1 + z$

3. A linear equation in one variable has
(a) only one solution
(b) two solutions
(c) more than two solutions
(d) no solution

4. If $8x - 3 = 25 + 17x$, then x is
(a) a fraction
(b) an integer
(c) a rational number
(d) cannot be solved

5. If $7x : 63 = 1 : 9$, then x is equal to
(a) 1 (b) 2
(c) 3 (d) -1

6. If $\frac{3x+6}{8} - \frac{11x-8}{24} + \frac{x}{3} = \frac{3x}{4} - \frac{x+7}{24}$, then the value of x is
(a) -3 (b) 3/2
(c) 3 (d) 1/3

7. If $\sqrt{3}x - 2 = 2\sqrt{3} + 4$, then the value of x is
(a) $2(1 - \sqrt{3})$ (b) $2(1 + \sqrt{3})$
(c) $1 + \sqrt{3}$ (d) $1 - \sqrt{3}$

8. If $a(x - a^2) - b(x - b^2) = 0$, then x is equal to
(a) $\frac{(-a+b)(a^2+ab+b^2)}{(a+b)}$ (b) $\frac{a^3+b^3}{(a-b)}$
(c) $\frac{a^3-b^3}{a+b}$ (d) $a^2 + ab + b^2$

9. If one number is thrice the other and their sum is 20, then the numbers are
(a) 5, 15 (b) 4, 12
(c) 3, 9 (d) 6, 18

10. If the sum of five consecutive even numbers is 340, then the smallest numbers will be
(a) 62 (b) 64 (c) 66 (d) 68

11. The sum of 4 consecutive odd numbers is 56. The smallest number is
(a) 12 (b) 11 (c) 13 (d) 14

12. The sum of the two numbers is 11 and their product is 30, then the numbers are
(a) 8, 3 (b) 9, 2 (c) 7, 4 (d) 6, 5

13. Divide 60 into two parts, so that three times the greater may be exceed 100 by as much as 8 times the less falls short of 200. What is the greater part ?
(a) 36 (b) 24 (c) 20 (d) 25

14. The ages of two persons differ by 20 yr. If 5 yr ago, the elder one be 5 times as old as the younger one, their present ages are
(a) 50 yr, 30 yr (b) 28 yr, 5 yr
(c) 20 yr, 10 yr (d) 30 yr, 10 yr

15. Given that $\frac{4p+9q}{p} = \frac{5q}{p-q}$ and p and q are both positive. The value of $\frac{p}{q}$ is
(a) $\frac{5}{3}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{3}{5}$

16. A fraction becomes 4/5 when 5 is added to its numerator and 1 is subtracted from its denominator. It becomes 1/2 when 3 and 5 are subtracted from its numerator and denominator. The numerator of the fraction will be
(a) 4 (b) 11 (c) 10 (d) 9

17. The total cost of 6 books and 4 pencils is ₹ 34 and that of 5 books and 5 pencils is ₹ 30. The cost of each book and pencil (in ₹) respectively is
(a) 1 and 5 (b) 5 and 1
(c) 6 and 1 (d) 1 and 6

18. The age of a man is 4 times that of his son. Five years ago, the man was nine times as old as his son was at that time. The present age of the man is
(a) 28 yr (b) 45 yr (c) 32 yr (d) 48 yr

19. In a class $\frac{3}{5}$ of the students are girls and rest are boys. If $\frac{2}{9}$ of the girls and $\frac{1}{4}$ of the boys are absent. What part of the total number of students are present ?
(a) $\frac{23}{30}$ (b) $\frac{23}{36}$ (c) $\frac{18}{49}$ (d) $\frac{17}{25}$

20. If a sum of ₹ 275 is to be divided between Ram and Shyam so that Ram gets more than 3/4th of what Shyam gets, then the share of Ram will be
(a) ₹ 200 (b) ₹ 175 (c) ₹ 160 (d) ₹ 100

21. Find a number such that if 5, 15 and 35 are added to it, the product of the first and third results may be equal to the square of the second.
(a) 10 (b) 7 (c) 6 (d) 5

22. The length of a rectangle is 8 cm more than its breadth. If the perimeter of the rectangle is 68 cm, its length and breadth are respectively.
(a) 21 cm, 13 cm (b) 21 cm, 32 cm
(c) 20 cm, 10 cm (d) 13 cm, 15 cm

23. In a two digit number, the tens digit is twice the unit digit. When the digits are reversed, the new number formed is 18 less than the original number. The original number is
(a) 45 (b) 42
(c) 43 (d) 41

24. A man has a certain number of chickens and goats. Their head count is 30. If the total number of their legs is 84, what is the ratio between the number of chickens and goats ?
(a) 1 : 2 (b) 2 : 3
(c) 3 : 2 (d) 3 : 4

Answers

1	(c)	2	(d)	3	(a)	4	(c)	5	(a)	6	(c)	7	(b)	8	(d)	9	(a)	10	(b)
11	(b)	12	(d)	13	(a)	14	(d)	15	(c)	16	(b)	17	(b)	18	(c)	19	(a)	20	(b)
21	(d)	22	(a)	23	(b)	24	(c)												

Hints and Solutions

1. Linear equation in one variable has only one variable with power 1.

2. We know that, the algebraic expression in one variable having the highest power of the variable as 1, is known as the linear expression.
Here, $1 + z$ is the only linear expression, as the power of the variable z is 1.

3. A linear equation in one variable has only one solution.

4. Given, $8x - 3 = 25 + 17x$
$\Rightarrow \quad 8x - 17x = 25 + 3$
$\Rightarrow \quad -9x = 28$
$\therefore \quad x = \frac{-28}{9}$
Hence, x is a rational number.

5. $\frac{7x}{63} = \frac{1}{9} \Rightarrow x = \frac{63}{9 \times 7} = 1$

6. Given, $\frac{3x + 6}{8} - \frac{11x - 8}{24} + \frac{x}{3} = \frac{3x}{4} - \frac{x + 7}{24}$
$\Rightarrow \quad \frac{3(3x + 6) - (11x - 8) + 8x}{24} = \frac{6(3x) - (x + 7)}{24}$
$\Rightarrow \quad 9x + 18 - 11x + 8 + 8x = 18x - x - 7$

$\Rightarrow \quad 26 + 6x = 17x - 7$

$\Rightarrow \quad 11x = 33$

$\Rightarrow \quad x = 3$

7. Given, $\sqrt{3}x - 2 = 2\sqrt{3} + 4$

$\therefore \quad \sqrt{3}x = 2\sqrt{3} + 6$

$\Rightarrow \quad x = \frac{2\sqrt{3} + 6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 2(1 + \sqrt{3})$

8. Since, $ax - a^3 - bx + b^3 = 0$

$\therefore \quad (a - b)x - (a^3 - b^3) = 0$

$\Rightarrow (a - b)[x - (a^2 + b^2 + ab)] = 0$

$\Rightarrow \quad x = a^2 + b^2 + ab$

9. Let the one number be x and number be $3x$.

According to the given condition,

$x + 3x = 20 \Rightarrow x = 5$

$\therefore$ Other number is $(3x) = 3 \times 5 = 15$

10. Let the smallest number be x.

Then,

$x + (x + 2) + (x + 4) + (x + 6) + (x + 8) = 340$

$\Rightarrow \quad 5x + 20 = 340$

$\Rightarrow \quad 5x = 320$

$\therefore \quad x = 64$

11. Let four consecutive odd number be

$x+1, x+3, x+5, x+7$.

Then, $(x+1) + (x+3) + (x+5) + (x+7) = 56$

$4x = 40 \Rightarrow x = 10$

$\therefore$ The smaller odd number is $x+1 = 10+1 = 11$

12. Let numbers be x and $11 - x$.

Since, product $x(11 - x) = 30$ [given]

$\Rightarrow \quad x^2 - 11x + 30 = 0$

$\Rightarrow \quad (x - 5)(x - 6) = 0$

$\Rightarrow \quad x = 5, 6$

13. Let two parts be x and $60 - x$.

Then, $3x - 100 = 200 - 8(60 - x)$

$\Rightarrow \quad 3x - 100 = 200 - 480 + 8x$

$\Rightarrow \quad 8x - 3x = 480 - 200 - 100$

$\Rightarrow \quad 5x = 180$

$\therefore \quad x = \frac{180}{5} = 36$

Thus, another number $60 - 36 = 24$.

$\therefore$ Greater number $= 36$

14. Let their ages be x and $(x - 20)$ yr.

5 yr ago, their ages are $(x - 5)$ and $(x - 20 - 5)$ yr.

According to the given condition,

$5(x - 20 - 5) = (x - 5)$

$\Rightarrow \quad 5x - 125 = x - 5$

$\Rightarrow \quad 4x = 120 \Rightarrow x = 30$

Hence, their present ages are 30 yr and 10 yr.

15. Given, $\frac{4p + 9q}{p} = \frac{5q}{p - q}$

$\Rightarrow \quad \frac{4\left(\frac{p}{q}\right) + 9}{\left(\frac{p}{q}\right)} = \frac{5}{\left(\frac{p}{q}\right) - 1}$

[on dividing by q in numerator and denominator]

$\Rightarrow 4\left(\frac{p}{q}\right)^2 + 9\left(\frac{p}{q}\right) - 4\left(\frac{p}{q}\right) - 9 = 5\left(\frac{p}{q}\right)$

$\Rightarrow \quad 4\left(\frac{p}{q}\right)^2 - 9 = 0 \Rightarrow \left(\frac{p}{q}\right)^2 = \left(\frac{3}{2}\right)^2$

$\therefore \quad \frac{p}{q} = \frac{3}{2}$

16. Let the fraction be $\frac{x}{y}$.

Then, $\frac{x + 5}{y - 1} = \frac{4}{5} \Rightarrow 5x - 4y = 29$ ...(i)

and $\frac{x - 3}{y - 5} = \frac{1}{2} \quad \Rightarrow \quad 2x - y = 1$ ...(ii)

On solving Eqs. (i) and (ii), we get $x = 11$

17. Let CP of a book be ₹ x and CP of pencil be ₹y.

Then, $6x + 4y = 34$ ...(i)

and $5x + 5y = 30$ ...(ii)

On solving Eqs. (i) and (ii), we get

$x = 5$ and $y = 1$.

18. Let the son's age be x yr, then man age's be $4x$.

Five years ago, their ages are $(x - 5)$ yr and $(4x - 5)$ yr respectively.

According to the given condition,

$(4x - 5) = 9(x - 5)$

$\Rightarrow \quad 5x = 40 \quad \Rightarrow \quad x = 8$ yr

Hence, the man's age $= 4x = 4 \times 8 = 32$ yr.

19. Let the number of students be x.

Then, number of girls $= \frac{3}{5}x$ and number of boys

$$= \frac{2}{5}x$$

Number of girls present $= \frac{7}{9} \times \frac{3}{5}x = \frac{7x}{15}$ and

Number of boys present $= \frac{3}{4} \times \frac{2}{5}x = \frac{3x}{10}$

$\therefore$ Total students present $= \left(\frac{7x}{15} + \frac{3x}{10}\right) = \frac{23}{30}x$

20. Let Shyam's share be ₹ x.

Then, Ram's share $= \left(x + \frac{3x}{4}\right) =$ ₹ $\frac{7x}{4}$

$\therefore \quad x + \frac{7x}{4} = 275 \Rightarrow x = 100$

$\therefore \quad$ Ram's share $= \frac{7}{4} \times 100 =$ ₹ 175

21. Let the number be x.

Then, first number $= (x + 5)$

second number $= (x + 15)$

and third number $= (x + 35)$

According to the question,

$(x + 5)(x + 35) = (x + 15)^2$

$\Rightarrow \quad 175 + 40x = 225 + 30x$

$\Rightarrow \quad 10x = 50$

$\therefore \quad x = 5$

22. Let breadth of rectangle be x.

Then, its length $= (x + 8)$ cm

$\therefore$ Perimeter of rectangle $= 2[x + (x + 8)]$

$= 2(2x + 8) = 4x + 16$

$\therefore \quad 4x + 16 = 68$ [given]

$\Rightarrow \quad 4x = 52$

$\Rightarrow \quad x = 13$

So, breadth of rectangle $= 13$ cm

and length $= 13 + 8 = 21$ cm

23. Let unit digit $= x$

Then, tens digit $= 2x$

$\therefore$ Original number $= 10 \times 2x + x = 20x + x = 21x$

New Number $= 10 \times x + 2x$

$= 10x + 2x = 12x$

Now, $21x - 12x = 18$

$\Rightarrow \quad 9x = 18$

$\Rightarrow \quad x = 2$

$\therefore$ Unit digit $= 2$ and Tens digit $= 2 \times 2 = 4$

Thus, required number $= 10 \times 4 + 2 = 42$

24. Let the number of chickens and goat be x and

According to the question,

$$x + y = 30 \quad \ldots(i)$$

and $\quad 2x + 4y = 84$

$\Rightarrow \quad x + 2y = 42 \quad \ldots(ii)$

On solving Eqs. (i) and (ii), we get

$x = 18$ and $y = 12$

$\therefore$ Required ratio $= 18 : 12 = 3 : 2$