CHAPTER

11

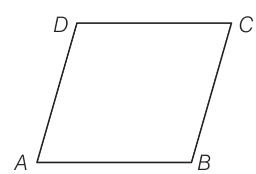
QUADRILATERALS

(PARALLELOGRAM, RHOMBUS, RECTANGLE, SQUARE, KITE)

In this chapter, we study the quadrilateral and their types. Also study the circle and cyclic quadrilaterals with their important results.

Quadrilateral

A plane figure which is made up of joining any four non-collinear points, is called quadrilateral.



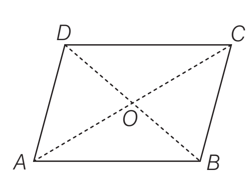
The sum of all angles of a quadrilateral is 360°.
i.e. $\angle A + \angle B + \angle C + \angle D = 360°$.

Types of Quadrilateral

Some types of quadrilateral are as given below

(i) Parallelogram

A quadrilateral is a parallelogram, if its both pairs of opposite sides are parallel.



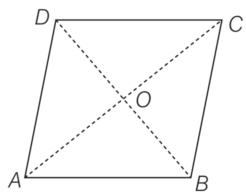
In figure, quadrilateral $ABCD$ is a parallelogram because $AB \parallel DC$ and $AD \parallel BC$.

- Opposite sides are equal, i.e. $AB = CD$ and $AD = BC$.

- Opposite angles are equal i.e. $\angle A = \angle C$ and $\angle B = \angle D$.
- Diagonals bisect each other, i.e. $AO = OC$, $OD = OB$.

(ii) Rhombus

A parallelogram in which all the sides are equal is called a rhombus.

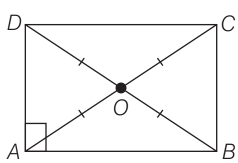


In figure, $ABCD$ is rhombus in which $AB \parallel DC$ and $AD \parallel BC$ and $AB = BC = CD = DA$.

- In rhombus diagonal bisects each other but they are not equal.

(iii) Rectangle

A parallelogram in which each angle is a right angle and opposite sides are equal is called rectangle.

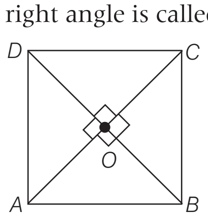


In figure, $ABCD$ is a rectangle in which $AB = DC$ and $AD = BC$, also $\angle A = \angle B = \angle C = \angle D = 90°$.

- In rectangle diagonals bisect each other and they are equal, i.e. $AO = OB = OC = OD$

(iv) Square

A parallelogram having all sides equal and each angle equal to a right angle is called a square.



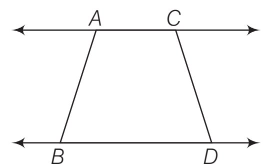
In figure, $ABCD$ is a square in which $AB = BC = CD = DA$ and $\angle A = \angle B = \angle C = \angle D = 90°$.

- In a square diagonals bisect each other at 90°and they are equal, i.e. $AO = OB = OC = OD$.

(v) Trapezium

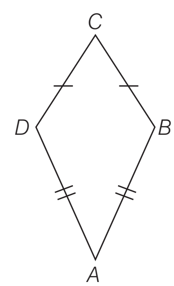
A quadrilateral in which one pair of opposite sides is parallel, is called a trapezium.

In trapezium $ABCD$, sides AC and BD are parallel to each other, but AB and CD neither parallel nor equal.



(vi) Kite

A quadrilateral which has two pairs of equal adjacent sides but unequal opposite sides, is a kite.

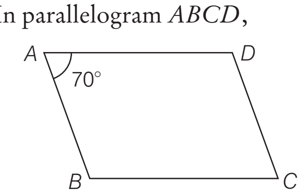


In figure, $ABCD$ is a kite in which $AB = AD$ and $BC = CD$ but $AD \neq BC$ and $AB \neq CD$.

Example 1 $ABCD$ is a parallelogram in which $\angle A = 70°$, the remaining angles of parallelogram are

(a) 110°, 65°, 115° (b) 110°, 70°, 110°
(c) 110°, 50°, 130° (d) None of these

Sol. *(b)* In parallelogram $ABCD$,



$$AD \parallel BC$$

$\therefore$ $\angle A + \angle B = 180°$

$\Rightarrow$ $\angle B = 180° - 70° = 110°$

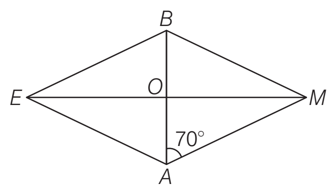
and $\angle C = \angle A$ [opposite angles]

$\therefore$ $\angle C = 70°$

and $\angle D = \angle B = 110°$ [opposite angles]

Hence, the angles B, C, D are $110°, 70°, 110°$ respectively.

Example 2 In rhombus $BEAM$, $\angle AME$ and $\angle AEM$ are



(a) 20°, 20° (b) 30°, 30°
(c) 35°, 35° (d) 40°, 40°

Sol. *(a)* Given, $\angle BAM = 70°$

We know that, in rhombus, diagonals bisect each other at right angles.

$\therefore$ $\angle BOM = \angle BOE = \angle AOM = \angle AOE = 90°$

Now, in ΔAOM,

$\angle AOM + \angle AMO + \angle OAM = 180°$

[angle sum property of triangle]

$\Rightarrow$ $90° + \angle AMO + 70° = 180°$

$\Rightarrow$ $\angle AMO = 180° - 90° - 70°$

$\Rightarrow$ $\angle AMO = 20° = \angle AME$

Also, $AM = BM = BE = EA$

In ΔAME, we have,

$AM = EA$

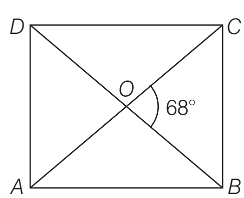
$\therefore$ $\angle AME = \angle AEM = 20°$

[$\because$ equal sides make equal angles]

Example 3 The diagonals of a rectangle $ABCD$ intersect in O, if $\angle BOC = 68°$, find $\angle ODA$.

(a) 55° (b) 56°
(c) 57° (d) 58°

Sol. *(b)* Given, $\angle BOC = 68°$ [given]



Then, $\angle AOD = 68°$ [vertically opposite angles]

$\because$ $OA = OD$

[$\because$ diagonals bisects each other]

$\therefore$ $\angle ODA = \angle OAD$

$\angle ODA + \angle OAD + \angle AOD = 180°$

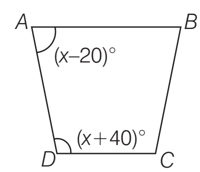
[sum of angles of a ΔAOD]

$2\angle ODA + 68° = 180°$

$\Rightarrow$ $2\angle ODA = 180° - 68° = 112°$

$\Rightarrow$ $\angle ODA = 56°$

Example 4 The value of x in the trapezium $ABCD$ given below is



(a) 80° (b) 82°
(c) 84° (d) 85°

Sol. *(a)* Given, a trapezium $ABCD$ in which

$\angle A = (x - 20)°$, $\angle D = (x + 40)°$

Since, in a trapezium, the angles on either side of the base are supplementary, therefore

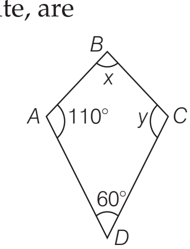
$(x - 20°) + (x + 40°) = 180°$

$\Rightarrow$ $2x + 20° = 180°$

$\Rightarrow$ $2x = (180° - 20°) = 160°$

$\therefore$ $x = 80°$

Example 5 The values of x and y in the following kite, are



(a) 80° (b) 70°
(c) 82° (d) 84°

Sol. *(a)* In a kite, one pair of opposite angles are equal.

$\therefore$ $y = 110°$

Now, by the angle sum property of a quadrilateral, we have

$\angle A + \angle B + \angle C + \angle D = 360°$

$$110° + x + 110° + 60° = 360°$$
$$\Rightarrow \quad x = 360° - 280°$$
$$\therefore \quad x = 80°$$

Polygons

A polygon is a closed plane figure bounded by straight lines.

Types of Polygons

Polygons are as following four types

1. **Convex polygon** A polygon in which none of its interior angles is more than 180°, is called convex polygon.
2. **Concave polygon** A polygon in which atleast one angle is more than 180°, is called concave polygon.
3. **Irregular polygon** A polygon in which all the sides or angles are not of the same measure, is called an irregular polygon.
4. **Regular polygon** A regular polygon has all its sides and angles equal.

(i) Each exterior angle of a regular polygon

$$= \frac{360°}{\text{Number of sides}}$$

(ii) Each interior angle = 180° – Exterior angle

(iii) Sum of all interior angles $= (2n - 4) \times 90°$
$= (n-2) \times 180°$

(iv) Sum of all exterior angles = 360°

(v) Number of diagonals of polygon of n sides

$$= \frac{n(n-3)}{2}$$

Example 6 Each interior angle of a regular polygon is 144°. Find the interior angle of a regular polygon which has double the number of sides as the first polygon.

(a) 170° (b) 160°
(c) 162° (d) 180°

Sol. *(c)* $\because$ Each interior angle of polygon $= 144°$

$$\Rightarrow \quad \frac{(2n-1)90°}{n} = 144°$$
$$\Rightarrow \quad 180n - 360° = 144n$$
$$\Rightarrow \quad 36n = 360°$$
$$\Rightarrow n = 10°$$

According to the given condition,

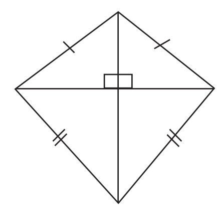
Total sides of a new polygon $= 2 \times 10 = 20$

Each interior angle of new polygon

$$= \frac{(2\times 20 - 4)\times 90°}{20}$$
$$= \frac{36 \times 9}{2} = 162°$$

Kite

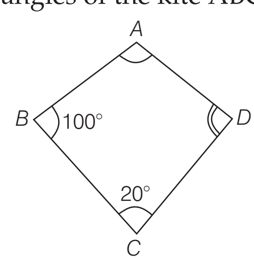
A kite is a quadrilateral whose four sides can be grouped into two pairs of equal- length sides that are adjacent to each other



Properties of Kite

- Its two diagonals are at right angles to each other.
- There is a pair of equal opposite angles.

Example 7 Calculate the measures of the unmarked angles of the kite ABCD.



(a) 170° and 120° (b) 100° and 100°
(c) 100° and 140° (d) 110° and 130°

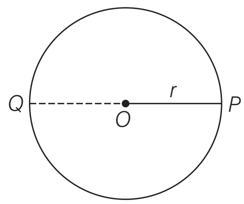
Sol. *(c)* We know that, $\angle ABC = \angle ADC = 100°$

$\because$ Sum of four angles of kite $= 360°$

$$\angle ABC + \angle BCD + \angle ADC + \angle BAD = 360°$$
$$100° + 20° + 100° + \angle BAD = 360°$$
$$\angle BAD = 360° - 220°$$
$$\angle BAD = 140°$$

Circle

A circle is a set of those points in a plane, which are at a given constant distance from a given fixed point in the plane.



- The fixed point O is called the centre of the circle.
- The constant distance r is called the radius of the circle.
- A circle can have many radii measure and all the radii of a circle are of equal length.
- The line PQ is a diameter (d) of a circle and

$$d = PQ = 2 \times \text{Radius} = 2r$$

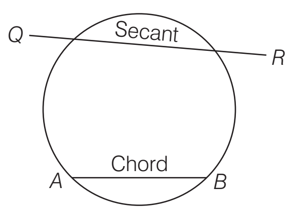
(i) Arc of the Circle

A continuous part of a circle is called an arc of the circle.

(ii) Chord

A line segment joining any two points on the circle is called its chord.

In a figure, AB is a chord.



Note Biggest chord of a circle is its diameters.

(iii) Secant

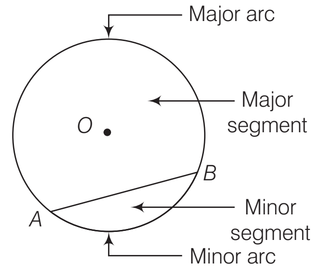
A line which intersect a circle in two distinct points is called a secant of the circle.

In above figure, QR is a secant.

(iv) Semicircle

A diameter divides the circle into two equal arcs, each of these two arcs is called a semicircle.

An arc whose length is less than the arc of a semicircle is called a minor arc, otherwise it is called a major arc.



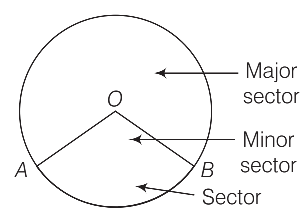
(v) Segment

If AB be a chord of the circle then AB divides the circular region into two parts, each part is called a segment of the circle.

The segment containing the minor arc is called the minor segment and the segment containing the major arc is called the major segment.

(vi) Central Angle

If $C(O, r)$ be any circle, then any angle whose vertex is centre of circle is called a central angle.



The degree measure of an arc is the measure of the central angle containing the arc.

(vii) Sector

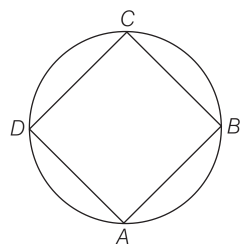
A sector is that region of a circle $C(O, r)$ which lies between an arc and the two radii joining the extremities of the arc to the centre.

(viii) Quadrant

One fourth of a circular region is called a quadrant.

(ix) Cyclic Quadrilateral

If all four vertices of a quadrilateral lie on a circle, then such a quadrilateral is called a cyclic quadrilateral.



The sum of opposite angles of a cyclic quadrilateral is 180°, i.e.,

$$\angle A + \angle C = 180^\circ$$

and $\angle B + \angle D = 180^\circ$

Some Important Results Related to Circles

(i) An infinite number of circles can pass through the two points.

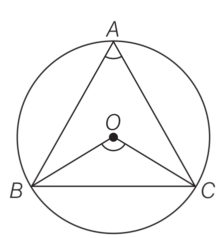
(ii) There is one and only one circle passing through three non-collinear points.

(iii) Angles in the same segment of a circle are equal.

(iv) The perpendicular from the centre to any chord bisects the chord.

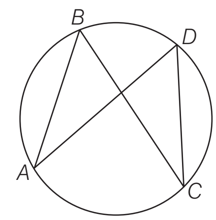
(v) The line joining the centre of the circle to the mid-point of any chord of a circle, is perpendicular to the chord.

(vi) The angle subtended by an arc at the centre of a circle is double the angle subtended by it at any point on the remaining part of the circle.



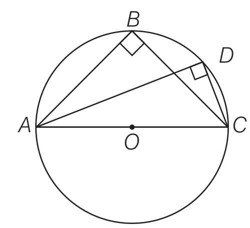
i.e. $\angle BOC = 2\angle BAC$

(vi) Angles in the same segment of a circle are equal.



Here, $\angle ABC = \angle ADC$

(vii) The angle made in a semicircle is always a right angle.

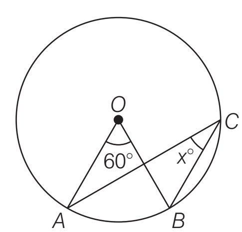


Here, $\angle ABC = 90^\circ$

and $\angle ADC = 90^\circ$

(viii) If the sum of any pair of opposite angles of a quadrilateral is 180°, then the quadrilateral is cyclic.

Example 6 The value of x° in the figure is



(a) 20° (b) 100°
(c) 60° (d) 30°

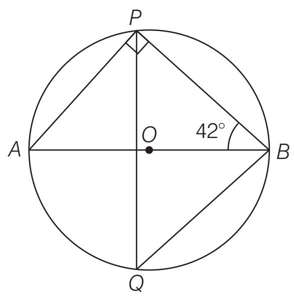
Sol. *(d)* We know that, the angle subtended by an arc at the centre of a circle is double the angle subtended by it any point of the remaining part of the circle.

$\therefore \quad \angle AOB = 2(\angle ACB)$

$\Rightarrow \quad 60^\circ = 2\angle ACB$

$\Rightarrow \quad \angle ACB = 30^\circ$

Example 7 In figure, the value $\angle PQB$, where O is the centre of the circle, is



(a) 47° (b) 48° (c) 49° (d) 50°

Sol. *(b)* In ΔAPB,

$$\angle APB = 90^\circ \quad \text{[angle in a semi-circle]}$$
$$\angle PBA = 42^\circ \quad \text{[given]}$$

Now, $\angle PAB + \angle APB + \angle PBA = 180^\circ$

$\Rightarrow \quad \angle PAB + 90^\circ + 42^\circ = 180^\circ$

$\Rightarrow \quad \angle PAB = 180^\circ - 132^\circ = 48^\circ$

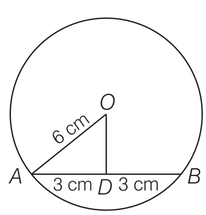
Since, we know that the angle subtended by an arc in the same segment are equal.

$\therefore \quad \angle PQB = \angle PAB = 48^\circ$

Example 8 The radius of a circle is 6 cm and the length of one of its chords is 6 cm. The distance of the chord from the centre is

(a) $3\sqrt{5}$ cm (b) $3\sqrt{2}$ cm
(c) $3\sqrt{3}$ cm (d) $3\sqrt{6}$ cm

Sol. *(c)* Let AB be a chord of a circle with centre O and radius 6 cm such that $AB = 6$ cm.



From O, draw $OD \perp AB$. Join OA

Clearly,

$$AD = \frac{1}{2} AB = 3 \text{ cm and } OA = 6 \text{ cm.}$$

Now, in right angle ΔODA,

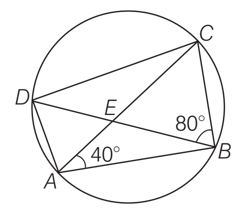
$$OD = \sqrt{OA^2 - AD^2}$$

[using Pythagoras theorem]

$$= \sqrt{6^2 - 3^2} = \sqrt{27} = 3\sqrt{3} \text{ cm}$$

Hence, the distance of the chord from the centre is $3\sqrt{3}$ cm.

Example 9 In figure, if $\angle DBC = 80^\circ$ and $\angle BAC = 40^\circ$, then the value of $\angle BCD$ is



(a) 50° (b) 60°
(c) 70° (d) 80°

Sol. *(b)* Given,

$$\angle DBC = 80^\circ \text{ and } \angle BAC = 40^\circ$$

Consider the chord CD, we find that $\angle CBD$ and $\angle CAD$ are angles in the same segment of the circle.

$\therefore \quad \angle CBD = \angle CAD$

$\Rightarrow \quad 80^\circ = \angle CAD$

$\Rightarrow \quad \angle CAD = 80^\circ$

Now, $\angle BAD = \angle BAC + \angle CAD$

$\Rightarrow \quad \angle BAD = 40^\circ + 80^\circ = 120^\circ \quad$ ...(i)

Since, $ABCD$ is a cyclic quadrilateral.

$\therefore \quad \angle BAD + \angle BCD = 180^\circ$

$\Rightarrow \quad 120^\circ + \angle BCD = 180^\circ \quad$ [using Eq. (i)]

$\Rightarrow \quad \angle BCD = 60^\circ$

PRACTICE EXERCISE

1. A quadrilateral has three acute angles each measuring $75°$, the measure of fourth angle is
(a) $145°$ (b) $135°$ (c) $125°$ (d) $130°$

2. The measures of the four angles of a quadrilateral are in the ratio of $1:2:3:4$. What is the measure of fourth angle?
(a) $144°$ (b) $135°$ (c) $125°$ (d) $150°$

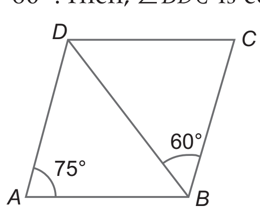
3. The diagonals of a rectangle $ABCD$ cut at O. OAL is an equilateral triangle drawn so that B and L are on the same side of AC. If $\angle ACD = 30°$, then the angles of ΔALB are
(a) $60°$, $60°$ and $60°$
(b) $30°$, $30°$ and $120°$
(c) $30°$, $60°$ and $120°$
(d) Cannot be determined

4. The sum of two opposite angles of a parallelogram is $130°$. All the angles of parallelogram are
(a) $65°, 65°, 115°, 115°$ (b) $145°, 135°, 35°, 45°$
(c) $90°, 130°, 80°, 60°$ (d) $40°, 140°, 80°, 110°$

5. The length of the diagonal of a rectangle whose sides are 12 cm and 5 cm, is
(a) 17 cm (b) 13 cm (c) 25 cm (d) 14 cm

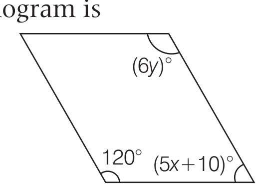
6. In a quadrilateral $ABCD$, if AO and BO be the bisectors of $\angle A$ and $\angle B$ respectively, $\angle C = 70°$ and $\angle D = 30°$, then $\angle AOB$ is
(a) $40°$ (b) $50°$ (c) $80°$ (d) $100°$

7. In the given figure, $ABCD$ is a parallelogram in which $\angle DAB = 75°$ and $\angle DBC = 60°$. Then, $\angle BDC$ is equal to



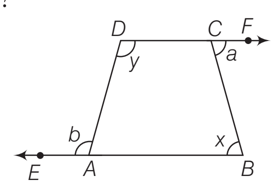
(a) $75°$ (b) $45°$
(c) $60°$ (d) $55°$

8. The values of x and y in the following parallelogram is



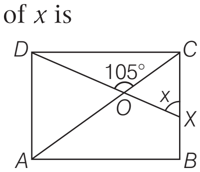
(a) $10°, 20°$ (b) $60°$ $80°$
(c) $90°, 110°$ (d) None of these

9. The sides BA and DC of quadrilateral $ABCD$ are produced as shown in figure. Then, which of the following statement is correct?



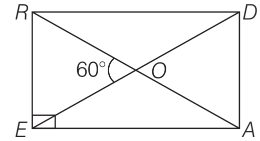
(a) $2x + y = a + b$ (b) $x + \frac{y}{2} = \frac{a+b}{2}$
(c) $x + y = a + b$ (d) $x + a = y + b$

10. In the given figure, $ABCD$ is a square. A line segment DX cuts the side BC at X and the diagonal AC at O such that $\angle COD = 105°$, $\angle OCX = 45°$ and $\angle OXC = x$. The value of x is



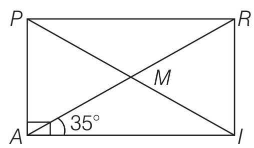
(a) $40°$ (b) $60°$ (c) $80°$ (d) $85°$

11. In rectangle $READ$, the values of $\angle EAR$, $\angle RAD$ and $\angle ROD$ are respectively



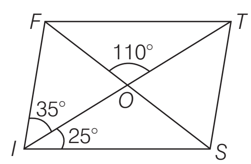
(a) $30°, 60°, 120°$ (b) $40°, 60°, 110°$
(c) $30°, 40°, 110$ (d) None of these

12. In rectangle *PAIR*, the values of $\angle ARI$, $\angle RMI$ and $\angle PMA$ are



(a) 60°, 70°, 70°
(b) 55°, 70°, 70°
(c) 60°, 80°, 80°
(d) None of these

13. In parallelogram *FIST*, the value of $\angle OST$ is

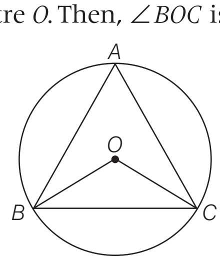


(a) 70° (b) 72°
(c) 75° (d) 80°

14. The external angle of a regular polygon is 45°. Find the sum of all the internal angles of it.
(a) 1082°
(b) 1080°
(c) 1085°
(d) 1090°

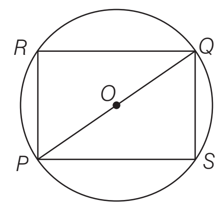
15. Find the number of non overlapping triangles can be formed in 9 sided polygon by joining the vertices.
(a) 5 (b) 7
(c) 6 (d) 4

16. An equilateral ΔABC is inscribed in a circle with centre *O*. Then, $\angle BOC$ is equal to



(a) 120° (b) 75°
(c) 180° (d) 160°

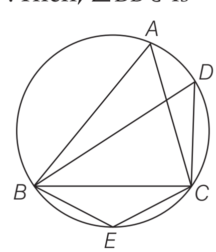
17. In the adjoining figure, *POQ* is the diameter of the circle, *R* and *S* are any two points on the circle. Then,



(a) $\angle PRQ > \angle PSQ$ (b) $\angle PRQ < \angle PSQ$
(c) $\angle PRQ = \angle PSQ$ (d) $\angle PRQ = \frac{1}{2}\angle PSQ$

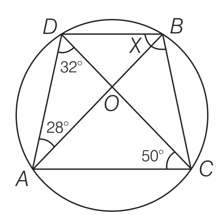
18. In a circle with centre *O* and radius 5 cm, *AB* is a chord of length 8 cm. If $OM \perp AB$, then the length of *OM* is
(a) 4 cm (b) 5 cm
(c) 3 cm (d) 2 cm

19. In the adjoining figure, ΔABC is an isosceles triangle with $AB = AC$ and $\angle ABC = 50°$. Then, $\angle BDC$ is



(a) 110° (b) 90° (c) 80° (d) 70°

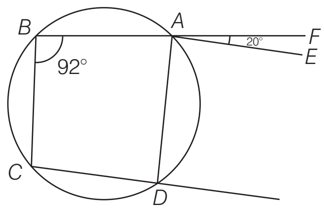
20. If *O* is the centre of the circle, then *x* is



(a) 72° (b) 62° (c) 82° (d) 52°

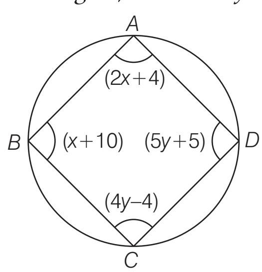
21. In a cyclic quadrilateral *ABCD*, if $\angle B - \angle D = 60°$, then the measure of the smaller of the two is
(a) 60° (b) 40° (c) 38° (d) 30°

22. In the given figure, $ABCD$ is a cyclic quadrilateral. AE is drawn parallel to CD and BA is produced. If $\angle ABC = 92°$ and $\angle FAE = 20°$, then $\angle BCD$ is equal to



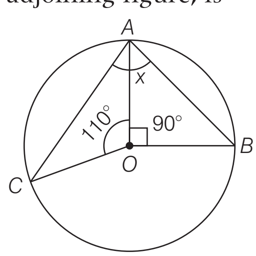
(a) 88° (b) 98° (c) 108° (d) 72°

23. The values of x and y in the figure are measure of angles, then $x + y$ is equal to



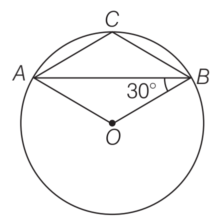
(a) 90° (b) 85° (c) 75° (d) 65°

24. If O is the centre of the circle, the value of x in the adjoining figure, is



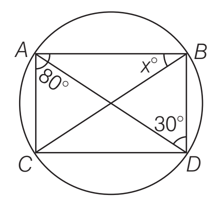
(a) 80° (b) 70° (c) 60° (d) 50°

25. In the given figure, O is centre, then $\angle ACB$ is



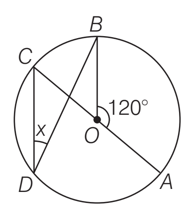
(a) 60° (b) 120°
(c) 75° (d) 90°

26. In the following figure, the value of $x°$ is



(a) 60° (b) 90°
(c) 70° (d) 40°

27. In the figure, O is the centre and AOC is the diameter of the circle. BD is chord and OB and CD are joined D is joined to A. If $\angle AOB = 120°$, then the value of x is



(a) 30° (b) 40°
(c) 50° (d) 60°

Answers

1	(b)	2	(a)	3	(b)	4	(a)	5	(b)	6	(d)	7	(b)	8	(a)	9	(c)	10	(b)
11	(a)	12	(b)	13	(c)	14	(b)	15	(b)	16	(a)	17	(c)	18	(c)	19	(c)	20	(c)
21	(a)	22	(c)	23	(d)	24	(a)	25	(b)	26	(c)	27	(a)						

Hints and Solutions

1. Since, $\angle A + \angle B + \angle C + \angle D = 360^\circ$

$\therefore$ $75^\circ + 75^\circ + 75^\circ + \angle D$

$\Rightarrow$ $225^\circ + \angle D = 360^\circ$

$\Rightarrow$ $\angle D = 360^\circ - 225^\circ = 135^\circ$

2. Let the angles be x, $2x$, $3x$ and $4x$.

$\because$ $x + 2x + 3x + 4x = 360^\circ$

$\Rightarrow$ $10x = 360^\circ$

$\Rightarrow$ $x = 36^\circ$

$\therefore$ Fourth angle $= 4 \times 36^\circ = 144^\circ$

3. $\because$ $OA = OB$

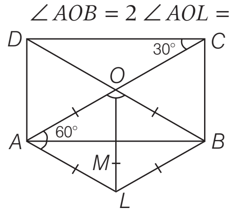
[$\because$ the diagonals of a rectangle bisect each other]

Also, $OA = OL$ $\Rightarrow$ $AOBL$ is a rhombus.

Since, Δ AOL is an equilateral triangle.

$\therefore$ $\angle AOL = 60^\circ$

$\Rightarrow$ $\angle AOB = 2\angle AOL = 120^\circ$



Since, $CD \parallel AB$

$\Rightarrow$ $\angle DCA = \angle CAB = 30^\circ$

and $\angle OAL = 60^\circ$

$\Rightarrow$ $\angle BAL = 60^\circ - 30^\circ = 30^\circ = \angle ABL$

$\therefore$ In Δ ALB,

$\angle ALB = 120^\circ$, $\angle ABL = 30^\circ$, $\angle LAB = 30^\circ$

4. Let $\angle A + \angle C = 130^\circ$, then

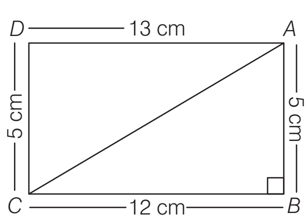
$\angle B + \angle D = 360^\circ - 130^\circ$

$= 230^\circ$

$\therefore$ Angles $= \dfrac{130^\circ}{2}, \dfrac{230^\circ}{2} = 65^\circ, 115^\circ$

Hence, all angles are 65°, 65°, 115°, 115°.

5.



Since, here $\angle ABC = 90^\circ$

So, by pythagoras theorem,

diagonal $AC = \sqrt{AB^2 + BC^2}$

$= \sqrt{5^2 + 12^2}$

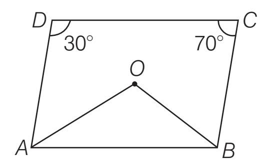
$= \sqrt{25 + 44}$

$= \sqrt{169} = 13$ cm

6. Since, $\angle A + \angle B + \angle C + \angle D = 360^\circ$

$\therefore$ $\angle A + \angle B = 360^\circ - (130^\circ + 70^\circ)$

$= 360^\circ - 200^\circ = 160^\circ$



$\Rightarrow$ $\dfrac{1}{2}(\angle A + \angle B) = 80^\circ$

So, $\angle OAB + \angle ABO = 80^\circ$

$\therefore$ $\angle AOB = 180^\circ - (\angle OAB + \angle ABO)$

$= (180^\circ - 80^\circ) = 100^\circ$

7. $\angle C = \angle A = 75^\circ$

[opposite angles of a parallelogram are equal]

$\therefore$ $\angle BDC = 180^\circ - (60^\circ + 75^\circ)$

$= 180^\circ - 135^\circ = 45^\circ$

8. In a parallelogram, adjacent angles are supplementary.

$\therefore$ $120^\circ + (5x + 10)^\circ = 180^\circ$

$\Rightarrow$ $5x + 10^\circ + 120^\circ = 180^\circ$

$\Rightarrow$ $5x = 180^\circ - 130^\circ$

$\Rightarrow$ $5x = 50^\circ$

$\Rightarrow$ $x = 10^\circ$

Also, opposite angles are equal in a parallelogram.

Therefore, $6y = 120^\circ$ $\Rightarrow$ $y = 20^\circ$

9. Since, $\angle A + b = 180^\circ \Rightarrow \angle A = 180^\circ - b$

Also, $\angle C + a = 180^\circ$ [linear pair]

$\Rightarrow$ $\angle C = 180^\circ - a$

But $\angle A + \angle B + \angle C + \angle D = 360^\circ$

$\Rightarrow$ $(180^\circ - b) + x + (180^\circ - a) + y = 360^\circ$

$\therefore$ $x + y = a + b$

10. Given, $\angle COD = 105^\circ$ and $\angle OCX = 45^\circ$

$$\angle COD + \angle COX = 180^\circ$$

$\Rightarrow$ $\angle COX = 180^\circ - \angle COD$

$= 180^\circ - 105^\circ = 75^\circ$

In ΔOCX, $\angle OCX + \angle COX + \angle OXC = 180^\circ$

$\Rightarrow$ $45^\circ + 75^\circ + x = 180^\circ$

$\therefore$ $x = 180^\circ - 120^\circ = 60^\circ$

11. Given, a rectangle *READ*, in which

$\angle ROE = 60^\circ$

$\therefore$ $\angle EOA = 180^\circ - 60^\circ = 120^\circ$ [linear pair]

Now, in ΔEOA, $\angle OEA = \angle OAE = 30^\circ$

[$\because$ $OE = OA$ and equal sides make equal angles]

$\therefore$ $\angle EAR = 30^\circ$, $\angle RAD = 90^\circ - \angle EAR = 60^\circ$

and $\angle ROD = \angle EOA = 120^\circ$

12. Given, $\angle RAI = 35^\circ$

$\therefore$ $\angle PRA = 35^\circ$

[$PR \parallel AI$ and AR is transversal]

Now, $\angle ARI = 90^\circ - \angle PRA = 90^\circ - 35^\circ = 55^\circ$

$\because$ $AM = IM$, $\angle MIA = \angle MAI = 35^\circ$

In ΔAMI, $\angle RMI = \angle MAI + \angle MIA = 70^\circ$

[exterior angle]

Also, $\angle RMI = \angle PMA$

$\Rightarrow$ $\angle PMA = 70^\circ$ [vertically opposite angles]

13. Given, $\angle FIS = 60^\circ$

Now, $\angle FTS = \angle FIS = 60^\circ$

[$\because$ opposite angles of a parallelogram are equal]

Now, $FT \parallel IS$ and TI is a transversal, therefore

$\angle FTO = \angle SIO = 25^\circ$ [alternate angles]

$\therefore$ $\angle STO = \angle FTS - \angle FTO = 60^\circ - 25^\circ = 35^\circ$

Also, $\angle FOT + \angle SOT = 180^\circ$ [linear pair]

$\Rightarrow$ $110^\circ + \angle SOT = 180^\circ$

$\Rightarrow$ $\angle SOT = 180^\circ - 110^\circ = 70^\circ$

In ΔTOS, $\angle TSO + \angle OTS + \angle TOS = 180^\circ$

[angle sum property of triangle]

$\therefore$ $\angle OST = 180^\circ - (70^\circ + 35^\circ) = 75^\circ$

14. External angle of any polygen

$$\frac{360^\circ}{n} = 45^\circ \Rightarrow n = \frac{360^\circ}{45^\circ} \Rightarrow n = 8^\circ$$

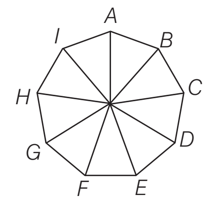
$\therefore$ Every interior angle of regular polygon

$= 180^\circ -$ External angle

$= 180^\circ - 45^\circ = 135^\circ$

$\therefore$ Sum of interior angles of it $= 8 \times 135^\circ = 1080^\circ$

15. Hence total number of non overlapping triangles can be formed in 9 sided polygon is 7.



16. We know that, the angle subtended by an arc at the centre of a circle is double the angle subtended by it any point on the remaning part of the circle.

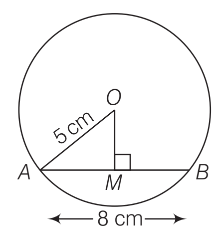
$\angle BOC = 2\angle A$

$= 2 \times 60^\circ$

$\therefore$ $\angle BOC = 120^\circ$

17. $\angle PRQ = \angle PSQ = 90^\circ$ [each angle in semi-circle]

18. $\because$ $OA = 5$ cm



and $AM = \frac{1}{2} AB = \frac{1}{2} \times 8 = 4$ cm

$\therefore$ $OM = \sqrt{OA^2 - AM^2}$

$= \sqrt{5^2 - 4^2} = 3$ cm

19. Since, $AB = AC$

$\Rightarrow$ $\angle ACB = \angle ABC = 50^\circ$

In ΔABC,

$\angle BAC = 180^\circ - (50^\circ + 50^\circ)$

$= 80^\circ$

$\therefore$ $\angle BDC = \angle BAC = 80^\circ$

[angle in the same segment]

20. In ΔDAC, $\angle ADC + \angle DCA + \angle CAD = 180^\circ$

$\Rightarrow$ $\angle CAD = 180^\circ - 32^\circ - 50^\circ = 98^\circ$

Now, $\angle CAD + \angle CBD = 180^\circ$

[opposite angles of a quadrilateral]

$\therefore$ $x = 180^\circ - 98^\circ = 82^\circ$

21. Since, $\angle B + \angle D = 180^\circ$

[sum of opposite angles of a cyclic quadrilateral]

$\angle B - \angle D = 60^\circ$ [given]

$\Rightarrow$ $\angle B = 120^\circ$ and $\angle D = 60^\circ$

$\therefore$ Required smaller angle $\angle D = 60^\circ$

22. $\angle B + \angle D = 180^\circ$

$\Rightarrow$ $\angle D = 180^\circ - 92^\circ = 88^\circ$

Now, $\angle DAE = \angle D = 88^\circ$ [$\because AE \parallel CD$]

$\Rightarrow$ $\angle FAD = 88^\circ + 20^\circ = 108^\circ$

$\therefore$ $\angle BCD = \angle FAD = 108^\circ$

23. Since, $ABCD$ is a cyclic quadrilateral.

$\angle B + \angle D = 180^\circ$

and $\angle A + \angle C = 180^\circ$

$\Rightarrow$ $x + 10 + 5y + 5 = 180^\circ$

$x + 5y = 165^\circ$ ...(i)

and $2x + 4 + 4y - 4 = 180^\circ$

$\Rightarrow$ $2x + 4y = 180^\circ$ ...(ii)

Solving Eqs.(i) and (ii), we get

$x = 40^\circ$ and $y = 25^\circ$

$\therefore$ $x + y = 40^\circ + 25^\circ = 65^\circ$

24. $\angle COB = 360^\circ - (\angle COA + \angle BOA)$

$= 360^\circ - (110^\circ + 90^\circ)$

$= 160^\circ$

$\therefore$ $x = \frac{1}{2} \times \angle COB$ [by theorem]

$= \frac{1}{2} \times 160^\circ = 80^\circ$

25. In the given figure, $OA = OB$ (radius of circle)

$\Rightarrow$ $\angle OAB = \angle OBA = 30^\circ$

$\therefore$ $\angle AOB = 180^\circ - 60^\circ = 120^\circ$

$\therefore$ Major $\angle AOB = 360^\circ - 120^\circ = 240^\circ$

$\Rightarrow$ $\angle ACB = \frac{1}{2} \times 240^\circ = 120^\circ$

[angle subtended in the arc is half of that subtended at the centre.]

26. In a given figure,

$\angle ADB = \angle ACB = 30^\circ$

[angle subtended in the same segment]

In ΔABC, $\angle x^\circ = 180^\circ - (\angle ACB + \angle CAB)$

$= 180^\circ - (30^\circ + 80^\circ) = 70^\circ$

27. In a given figure,

$\angle COB = 180^\circ - 120^\circ = 60^\circ$ [$\because COA$ is a line]

$\therefore$ $x = \frac{1}{2} \angle COB$

[angle subtended in the arc is half of that subtended at the centre.]

$= \frac{1}{2} \times 60^\circ = 30^\circ$