CHAPTER

02

SQUARE AND SQUARE ROOTS

In this chapter, we study the squares of number and their properties. Also finding the square root of numbers and decimal numbers by prime factorisation and long division method.

Square

The square of a number is the product of the number itself, i.e. $a \times a = a^2$.

Perfect Square

A given number is said to be a perfect square, if it can be expressed as the product of two equal factors. A natural number 'n' is a perfect square if $n = m^2$ for any natural number m.

e.g. $4 = 2^2$ or 2×2 and $25 = 5^2$ is a perfect square.

Squares from 1 to 20 numbers

Numbers	Squares	Numbers	Squares
1	1	11	121
2	4	12	144
3	9	13	169
4	16	14	196
5	25	15	225
6	36	16	256
7	49	17	289
8	64	18	324
9	81	19	361
10	100	20	400

Properties of Square

(i) The number of zeroes at the end of a perfect square is always even.

(ii) A perfect square leaves a remainder 0 or 1 on division by 3.

(iii) The number ending with an odd number of zeroes is never a perfect square.

(iv) Squares of even numbers are always even.

(v) Squares of odd numbers are always odd.

Example 1 Which are of the following will have odd unit digit?

(a) $(32)^2$ (b) $(35)^2$ (c) $(64)^2$ (d) $(68)^2$

Sol. (b) We know that that square of odd number in an odd number

The number $(35)^2$ have odd unit digit.

Pythagorean Triplet

In a triplet (m, n, p) of three natural numbers m, n and p is called a Pythagorean triplet, if $m^2 + n^2 = p^2$. It is easy to prove that for any natural number m greater than 1, $(2m, m^2 - 1, m^2 + 1)$ is a Pythagorean triplet.

Example 2 Find the value of $(13)^2 + 3 \times 7$.

(a) 190 (b) 189 (c) 191 (d) 192

Sol. (a) $(13)^2 + 3 \times 7 = 169 + 21 = 190$

Square Root

The square root of the number x is the number which multiplied by itself gives x as the product. It is denoted by the symbol $\sqrt[2]{\ }$ or $\sqrt{\ }$.

e.g. If $y = x^2$, then we call, x is a square root of y,

i.e. $x = \pm \sqrt{y}$.

Properties of Square Root

(i) If the unit digit of a number is 2, 3, 7 or 8, then it does not have a square root.

(ii) Square root of even number is even.

(iii) Square root of odd number is odd.

Square Root of a Perfect Square by the Prime Factorisation Method

The following steps are given below.

I. Resolve the given number into prime factors.

II. Make pairs of similar factors.

III. Choose one prime from each pair and multiply all primes.

Thus, the product obtained is the square root of given number.

Example 3 The square root of 1764 is

(a) 41 (b) 43 (c) 42 (d) 40

Sol. (c) By prime factorisation method,

2	1764
2	882
3	441
3	147
7	49
7	7
	1

$$\therefore \quad \sqrt{1764} = \sqrt{\overline{2 \times 2} \times \overline{3 \times 3} \times \overline{7 \times 7}}$$
$$= 2 \times 3 \times 7 = 42$$

Square Root of a Perfect Square by Long Division Method

If it is not easy to evaluate square root using prime factorisation method, then we use division method.

The steps of this method can be easily understood with the help of following examples.

e.g. Find the square root of 18769.

Step **I** In the given number, mark off the digits in pairs starting from the unit digit. Each pair and the remaining one digit (if any) is called a period.

Step **II** Choose a number whose square is less than or equal to 1. Here, $1^2 = 1$, on subtracting, we get 0 (zero) as remainder.

Step **III** Bring down the next period, i.e. 87. Now, the trial divisor is $1 \times 2 = 2$ and trial dividend is 87. So, we take 23 as divisor and put 3 as quotient. The remainder is 18 now.

	137
1	$\overline{1}\,\overline{87}\,\overline{69}$
	1
23	87
	69
267	1869
	1869
	×

Step **IV** Bring down the next period, which is 69. Now, trial divisor is $13 \times 2 = 26$ and trial dividend is 1869. So, we take 267 as dividend and 7 as quotient. The remainder is 0.

Step **V** The process (processes like III and IV) goes on till all the periods (pairs) come to an end and we get remainder as 0 (zero) now.

Hence, the required square root = 137

Example 4 2025 plants are to be planted in a garden in such a way that each row contains as many plants as the number of rows. Find the number of rows and the number of plants in each now.

(a) 55, 45 (b) 45, 45 (c) 35, 35 (d) 36, 36

Sol. *(b)* Let the number of rows be x.

Then, number of plants in a row = x

So, number of plants to be planted in a garden

$$x \times x = x^2$$

According to the question,

Total number of plants to be planted = 2025

$\therefore$ $x^2 = 2025$

3	2025
3	675
3	225
3	75
5	25
5	2
	1

$\Rightarrow$ $x = \sqrt{2025} = \sqrt{3 \times 3 \times 3 \times 3 \times 5 \times 5}$

$= 3 \times 3 \times 5 = 45$

Hence, the number of rows is 45 and the number of plants in each row is 45.

Formula for Finding the Number of Digits in the Square Root of a Perfect Square

If any perfect square number contains 'n' digits. then, its square root will contain $\frac{n}{2}$ digits, when n is even and $\frac{n+1}{2}$ digits, when n is odd.

e.g. Square root of 64 is 8. [$\because n = 2$ i.e. even]

Also, square root of 144 is 12. [$\because n = 3$ i.e. odd]

Square Root of Product of Numbers and Rational Number

(i) The square root of product of integers is the square root of integer by taking separately, for any integer a and b, we have

$$\sqrt[2]{ab} = \sqrt[2]{a}\ \sqrt[2]{b}$$

(ii) The square root of rational number $\frac{a}{b}$ is

$$\sqrt[2]{\frac{a}{b}} = \frac{\sqrt[2]{a}}{\sqrt[2]{b}}$$

Square Root of Number in Decimal Form

Make the number of decimal places even by affixing zero, if necessary. Now, mark bars and find out the square root by the long division method. Put the decimal point in the square root as soon as the integral part is completed.

Example 5 Find the square root of 176.252176.

(a) 13.276 (b) 13.801 (c) 13.295 (d) 13.218

Sol. *(a)* By long division method,

	13.276
1	$\overline{1}\,\overline{76.25}\,\overline{21}\,\overline{76}$
	1
23	76
	69
262	725
	524
2647	20121
	18529
26546	159276
	159276
	×

$\therefore$ $\sqrt{176.252176} = 13.276$

PRACTICE EXERCISE

1. The value of $(15)^2 + (8)^2 + 2$ is
(a) 289 (b) 291 (c) 293 (d) 295

2. Which of the following cannot be a perfect square?
(a) 841 (b) 529
(c) 198 (d) All of these

3. Which of the following cannot be a digit in the unit place of a perfect square ?
(a) 0 (b) 1 (c) 5 (d) 7

4. The square root of 73.96 is
(a) 8.6 (b) 86
(c) 0.86 (d) None of these

5. If $x = \sqrt{3018 + \sqrt{36 + \sqrt{169}}}$, then the value of x is
(a) 55 (b) 44 (c) 63 (d) 42

6. $\sqrt{12} + \sqrt{24}$ is equal to
(a) $2\sqrt{3} + 3\sqrt{2}$ (b) $4\sqrt{3} + \sqrt{6}$
(c) $\sqrt{7} + 2\sqrt{3}$ (d) $2\sqrt{6} + 2\sqrt{3}$

7. Which one of the following will have even unit digit?
(a) $(43)^2$ (b) $(37)^2$
(c) $(63)^2$ (d) $(34)^2$

8. If the area of an equilateral triangle is $24\sqrt{3}\ m^2$, then its perimeter is
(a) $12\sqrt{6}$ m (b) $9\sqrt{6}$ m
(c) $8\sqrt{3}$ m (d) $4\sqrt{3}$ m

9. The value of $(301)^2 - (300)^2$ is
(a) 1 (b) 601
(c) 106 (d) 100

10. A General arranges his soldiers in rows to form a perfect square. He finds that in doing so, 60 soldiers are left out. If the total number of soldiers be 8160. Then, the number of soldiers in each row is
(a) 90 (b) 91
(c) 92 (d) 80

11. The greatest six digit number which is a perfect square is
(a) 998004 (b) 998006
(c) 998049 (d) 998001

12. What is that fraction which when multiplied by itself gives 227.798649 ?
(a) 15.093 (b) 15.099
(c) 14.093 (d) 9.0019

13. In a triplet $(6, a, 10)$ what value of 'a' will make it a Pythagorean triplet ?
(a) 4 (b) 16
(c) 8 (d) 5

14. If a number is increased by two times, then the square of the number will increase
(a) two times (b) three times
(c) four times (d) five times

15. Two numbers are in the ratio of 9 : 7. If the difference of their square is 288, then the smaller of the number is
(a) 21 (b) 24
(c) 27 (d) 28

16. The number of digits in the square root of 298116 is
(a) 4 (b) 5
(c) 3 (d) 6

17. If $\sqrt{2401} = \sqrt{7^x}$, then the value of x is
(a) 3 (b) 4
(c) 5 (d) 6

18. The least number to be added to 269 to make it a perfect square is
(a) 31 (b) 16
(c) 17 (d) 20

19. If $\sqrt{18225} = 135$, then the value of $\sqrt{18225} + \sqrt{182.25} + \sqrt{1.8225} + \sqrt{0.018225}$ is
(a) 1.49985 (b) 14.985
(c) 149.985 (d) 1499.85

20. The expression
$$\sqrt{\frac{0.85\times(0.105+0.024-0.008)}{0.022\times0.25\times1.7}}$$
simplifies to
(a) $\sqrt{11}$ (b) $\sqrt{1.1}$
(c) 11 (d) $\sqrt{0.011}$

21. The value of $\sqrt{\frac{16}{36}+\frac{1}{4}}$ is
(a) $\frac{2}{5}$ (b) $\frac{1}{3}$ (c) $\frac{5}{3}$ (d) $\frac{5}{6}$

22. If $\frac{52}{x}=\sqrt{\frac{169}{289}}$, then the value of x is
(a) 52 (b) 58 (c) 62 (d) 68

23. A square board has an area of 144 sq units. How long is each side of the board?
(a) 11 units (b) 12 units
(c) 13 units (d) 14 units

24. If $\sqrt{1+\frac{25}{144}}=1+\frac{x}{12}$, then x is equal to
(a) 1 (b) 2 (c) 5 (d) 9

25. The value of $\frac{\sqrt{80}-\sqrt{112}}{\sqrt{45}-\sqrt{63}}$ is
(a) $\frac{3}{4}$ (b) $1\frac{3}{4}$
(c) $1\frac{1}{3}$ (d) $1\frac{7}{9}$

26. The least number which is added to 17420 will make it a perfect square is
(a) 3 (b) 5 (c) 9 (d) 4

27. If $\sqrt{0.09\times0.09\times x}=0.09\times0.09\times\sqrt{z}$, then the value of $\frac{x}{z}$
(a) 0.0081 (b) 0.810
(c) 0.801 (d) 8.09

Answers

1	(b)	2	(c)	3	(d)	4	(a)	5	(a)	6	(d)	7	(d)	8	(a)	9	(b)	10	(a)
11	(d)	12	(a)	13	(c)	14	(c)	15	(a)	16	(c)	17	(b)	18	(d)	19	(c)	20	(a)
21	(d)	22	(d)	23	(b)	24	(a)	25	(c)	26	(d)	27	(a)						

Hints and Solutions

1. $(15)^2+(8)^2+2=225+64+2=291$

2. We know that, a number ending with digits 2, 3, 7 or 8 can never be a perfect square. So, 198 cannot be written in the form of a perfect square.

3. Digit 7 cannot be a place of a perfect square.

4. By using long division method,

	8.6
8	$\overline{73}.\overline{96}$
+8	64
166	996
	996
	×

Hence, $\sqrt{73.96}=8.6$

5. According to question,
$$\sqrt{3018+\sqrt{36+\sqrt{169}}}=\sqrt{3018+\sqrt{36+13}}$$
$$=\sqrt{3018+7}=\sqrt{3025}=55$$

6. $\sqrt{12}+\sqrt{24}=\sqrt{2\times2\times3}+\sqrt{2\times2\times6}$
$$=2\sqrt{3}+2\sqrt{6}$$

7. We know that, the square of even number is even.
$\therefore$ The number $(34)^2$ has even unit digit.

8. Let the side of an equilateral triangle be a m.
Since, the area of an equilateral triangle
$$=24\sqrt{3}\,m^2$$

$\Rightarrow \quad \frac{\sqrt{3}}{4}a^2 = 24\sqrt{3} \quad \Rightarrow \quad a^2 = 96$

$\Rightarrow \quad a = 4\sqrt{6}$ m

9. $(301)^2 - (300)^2$

$= (301 + 300)(301 - 300)$

$[\because a^2 - b^2 = (a + b)(a - b)]$

$= (601) \times 1$

$= 601$

10. Total number of soldiers arranged

$= 8160 - 60 = 8100$

Since, the number of soldiers in each row is equal to number of rows.

$\therefore$ Number of soldiers in each row

$= \sqrt{8100}$

$= \sqrt{9 \times 9 \times 10 \times 10} = 90$

11. The greatest six digit number = 999999

	999
9	99 99 99
	81
189	1899
	1701
1989	19899
	17901
	1998

×

$\therefore$ The greatest number of six digit which is a perfect square

$= 999999 - 1998$

$= 998001$

12. Let the fraction be x.

Then, $x^2 = 227.798649$

$\therefore \quad x = \sqrt{227.798649} = 15.093$

$[\because$ using long divison method$]$

13. $6^2 = 36$, $a^2 = a^2, 10^2 = 100$

By Pythagoraem triplet, $6^2 + a^2 = 10^2$

$\Rightarrow \quad a^2 = 10^2 - 6^2$

$\Rightarrow \quad a^2 = 100 - 36 = 64$

$\Rightarrow \quad a = \sqrt{64} = 8$

14. Let the number be y.

If the number is increased by two times it becomes $2y$.

Square of the number $= (2y)^2 = 4y^2$

$\therefore$ The number will be increased by four times.

15. Let the number be $9x$ and $7x$.

According to the question,

$81x^2 - 49x^2 = 288$

$\Rightarrow \quad x^2 = \frac{288}{32} \quad \Rightarrow x^2 = 9 \quad \Rightarrow x = 3$

$\therefore$ The smaller number is 21.

16.

2	298116
2	149058
3	74529
3	24843
7	8281
7	1183
13	169
13	13
	1

$\Rightarrow \quad \sqrt{298116} = 2 \times 3 \times 7 \times 13$

$= 546$

$\therefore$ Number of digits = 3

17. $\sqrt{2401} = \sqrt{7^x}$

$\Rightarrow \quad 2401 = 7^x$

$\Rightarrow \quad 7^4 = 7^x$

$\therefore \quad x = 4$

18. We know, $256 < 269 < 289$

$\Rightarrow (16)^2 < 269 < (17)^2$

$\therefore$ Number to be added $= (17)^2 - 269$

$= 289 - 269 = 20$

19. $\sqrt{18225} + \sqrt{182.25} + \sqrt{1.8225} + \sqrt{0.018225}$

$= \sqrt{18225} + \sqrt{\frac{18225}{100}} + \sqrt{\frac{18225}{10000}} + \sqrt{\frac{18225}{1000000}}$

$= 135 + \frac{135}{10} + \frac{135}{100} + \frac{135}{1000}$

$= 135 + 13.5 + 1.35 + 0.135$

$= 149.985$

20. $\sqrt{\dfrac{0.85\times(0.105+0.024-0.008)}{0.022\times0.25\times1.7}}$

$=\sqrt{\dfrac{0.85\times0.121}{0.022\times0.25\times1.7}}$

$=\sqrt{\dfrac{85\times121\times10}{22\times25\times17}}=\sqrt{11}$

21. $\sqrt{\dfrac{16}{36}+\dfrac{1}{4}}=\sqrt{\dfrac{16+9}{36}}$

$=\sqrt{\dfrac{25}{36}}=\dfrac{5}{6}$

22. Given, $\dfrac{52}{x}=\sqrt{\dfrac{169}{289}}$

$\Rightarrow \quad x=\dfrac{52\times17}{13}=68$

23. Given, area of square board = 144 sq units

$\therefore \quad (\text{side})^2=144 \quad [\because \text{area of square} = (\text{side})^2]$

$\Rightarrow \quad (\text{side})^2=12$

$\Rightarrow \quad \text{side}=12 \text{ units}$

Hence, the length of each side of the board is 12 units.

24. Given, $\sqrt{1+\dfrac{25}{144}}=1+\dfrac{x}{12}$

$\Rightarrow \quad \sqrt{\dfrac{169}{144}}=1+\dfrac{x}{12}$

$\Rightarrow \quad \dfrac{13}{12}=1+\dfrac{x}{12}$

$\Rightarrow \quad \dfrac{x}{12}=\dfrac{1}{12}\Rightarrow x=1$

25. $\dfrac{4\sqrt{5}-4\sqrt{7}}{3\sqrt{5}-3\sqrt{7}}=\dfrac{4(\sqrt{5}-\sqrt{7})}{3(\sqrt{5}-\sqrt{7})}=\dfrac{4}{3}=1\dfrac{1}{3}$

26. Since, 17420 lies between 131^2 and 132^2.

Now, $(132)^2=17424$

Hence, it should be 4 added.

27. $\sqrt{0.09\times0.09\times x}=0.09\times0.09\times\sqrt{z}$

$\Rightarrow \quad 0.09\times\sqrt{x}=0.09\times0.09\times\sqrt{z}$

$\Rightarrow \quad \dfrac{\sqrt{x}}{\sqrt{z}}=0.09$

$\Rightarrow \quad \dfrac{x}{z}=(0.09)^2 \text{ (squaring both sides)}$

$\Rightarrow \quad \dfrac{x}{z}=0.0081$