CHAPTER

01

NUMBER SYSTEM

(RATIONAL NUMBERS)

In this chapter, we study the various types of numbers, properties of rational numbers, simplification and test for divisibility.

Digits The symbols 0,1, 2, 3, 4, 5, 6, 7, 8, 9 are known as digits in Hindu Arabic System.

Numbers or Numerals A mathematical symbol which represent the digits, are known as numbers or numerals.

Types of Numbers There are some types of numbers, which are given below

(i) **Natural Numbers** Those numbers which are used for counting, are known as natural numbers. These are denoted by N.

e.g. $N = 1, 2, 3, \ldots$

Here 1 is the first and smallest natural number.

(ii) **Whole Numbers** If 0 is included in natural numbers, then these numbers are known as whole numbers. These numbers are denoted by W.

e.g. $W = 0, 1, 2, 3, \ldots$

(iii) **Integers** All whole numbers and their negative numbers are known as integers. These numbers are denoted by I.

$\therefore \ I = 0, \pm 1, \pm 2, \pm 3, \ldots$

Here, 1, 2, 3, … are positive integers, denoted by I^+, and

$-1, -2, -3, \ldots$ are negative integers, denoted by I^-.

Here 0 is neither positive nor negative integer.

(iv) **Rational Numbers** Numbers in the form of $\frac{p}{q}$, where $p, q \in I$ and $q \neq 0$, are known as rational numbers. It is denoted by Q.

e.g. $\frac{2}{3}, \frac{5}{6}, 6, \frac{-4}{5}$, etc.

(v) **Irrational Numbers** Numbers which can not be expressed in the form of $\frac{p}{q}$, where $p, q \in I$ and $q \neq 0$, are known as irrational numbers.

e.g. $\pi, \sqrt{2}, \sqrt{6}, \sqrt[3]{11}$, etc.

(vi) **Real Numbers** Those numbers which are either rational or irrational, are known real numbers. It is denoted by R.

All natural, whole, integer, rational and irrational numbers are real numbers.

e.g. $2, 0, -5, \frac{1}{2}, \sqrt{6}$, etc.

(vii) **Even Numbers** Those numbers which are divisible by 2, are known as even numbers.

e.g., 2, 4, 6, …

Note 2 is smallest even number.

(viii) **Odd Numbers** Those numbers, which are not divisible by 2, are known as odd numbers.

e.g. 1, 3, 5, 7, …

(ix) **Prime Numbers** Those numbers which are divisible by 1 and the number itself, are known as prime numbers.

e.g. 2, 3, 5, 7, 11, 13, …

Here 2 is the only even prime number.

(x) **Composite Numbers** Those numbers which are divisible by at least one number except 1 and the number itself, are known as composite numbers.

e.g. 12, 8 and 15 etc, are composite numbers.

Example 1 Which of the following is not true?

(a) 11 is prime number

(b) 2/5 is rational number

(c) ± 8 is the real number

(d) $\frac{6}{0}$ is rational number

Sol. *(d)* $\frac{6}{0}$ is not true, because $\frac{6}{0}$ is not rational number, as in its denominator, $(p/q$ form) $q = 0$, which is wrong.

Example 2 Sum of first five prime numbers is

(a) 25 (b) 26 (c) 27 (d) 28

Sol. *(d)* We know that, first five prime numbers are 2, 3, 5, 7, 11

$\therefore$ Their sum $= 2 + 3 + 5 + 7 + 11 = 28$

Properties of Rational Numbers

(i) **Closure Property** The sum and multiplication of two rational numbers are rational numbers.

(a) $2 + \frac{2}{7} = \frac{16}{7}$ [rational number]

(b) $2 \times \frac{2}{7} = \frac{4}{7}$ [rational number]

(ii) **Commutative Property** If a and b are any rational numbers, then we have

(a) $a + b = b + a$ [for addition]

(b) $a \times b = b \times a$ [for multiplication]

(iii) **Associative Property** If a, b and c are three any rational numbers, then we have,

(a) $(a + b) + c = a + (b + c)$ [for addition]

(b) $(a \times b) \times c = a \times (b \times c)$ [for multiplication]

(iv) **Distributive Property** If a, b, c are any rational numbers, then we have

$$a(b + c) = ab + ac,$$

which is called distributive property of multiplication over addition.

(v) **Additive Identity** If a is any rational number, then we have

$$a + 0 = a,$$

i.e. adding a number 0 to any number, is equal to the number itself. Therefore, '0' is called additive identity.

(vi) **Multiplicative Identity** If a is any rational number, then we have

$$a \times 1 = a$$

i.e. multiplying a number 1 to any number, is equal to the number itself. Therefore, 1 is called multiplicative identity.

Rational Numbers Between any Two Given Rational Numbers

Between any two given rational numbers, there are countless or infinite rational numbers.

To find rational numbers between any two given rational numbers, we can use the idea of mean.

Thus, we can say that, 'if a and b are two rational numbers, then $\frac{a+b}{2}$ will be a rational number, such that $a < \frac{a+b}{2} < b$.

e.g. A rational number between 2 and 3 is $\frac{2+3}{2} = \frac{5}{2}$.

Here, we find that, $2 < \frac{5}{2} < 3$.

Example 3 A rational number between $\frac{2}{3}$ and $\frac{3}{4}$ is

(a) 15/24 (b) 17/24 (c) 13/24 (d) 5/24

Sol. *(b)* We find the mean of $\frac{2}{3}$ and $\frac{3}{4}$.

i.e. $\left(\frac{2}{3}+\frac{3}{4}\right) \div 2 = \left(\frac{8+9}{12}\right) \div 2$

$= \frac{17}{12} \div 2 = \frac{17}{12} \times \frac{1}{2} = \frac{17}{24}$

Simplification

To solve any expression or to simplify, we have many operations, i.e. Brackets, Addition, Subtraction, Multiplication, Division, etc.

We follow the rule of VBODMAS, i.e.

V $\rightarrow$ Vinculum or bar '–'
B $\rightarrow$ Brackets, i.e. ( ), { }, [ ]
O $\rightarrow$ Of
D $\rightarrow$ Division, i.e. ÷
M $\rightarrow$ Multiplication, i.e. '×'
A $\rightarrow$ Addition, i.e. '+'
S $\rightarrow$ Subtraction, i.e. '–'

To solve brackets, we follow the order

(i) ( ), circular or small bracket
(ii) { }, curly or middle bracket
(iii) [ ], square or big bracket

Note In the absence of any bracket or operation, their is no change in order to solve the expression.

Example 4 $\frac{3}{4}$ of $\frac{2}{7}$ of $\frac{1}{5}$ of 560 = ?

(a) 28 (b) 24 (c) 32 (d) 36

Sol. *(b)* We have, $\frac{3}{4}$ of $\frac{2}{7}$ of $\frac{1}{5}$ of 560

$= \frac{3}{4} \times \frac{2}{7} \times \frac{1}{5} \times 560 = 24$

Example 5 The value of

$1 \div [1 + 1 \div \{1 + 1 \div (1 + 1 \div 2)\}]$ is

(a) $\frac{5}{8}$ (b) $\frac{8}{5}$ (c) $\frac{2}{5}$ (d) $\frac{3}{8}$

Sol. *(a)* We have,

$1 \div [1 + 1 \div \{1 + 1 \div (1 + 1 \div 2)\}]$

$= 1 \div \left[1 + 1 \div \left\{1 + 1 \div \left(1 + \frac{1}{2}\right)\right\}\right]$

$= 1 \div \left[1 + 1 \div \left\{1 + 1 \div \frac{3}{2}\right\}\right]$

$= 1 \div \left[1 + 1 \div \left\{1 + \frac{2}{3}\right\}\right]$

$= 1 \div \left[1 + 1 \div \frac{5}{3}\right] = 1 \div \left[1 + \frac{3}{5}\right] = 1 \div \frac{8}{5} = \frac{5}{8}$

Test for Divisibility

Generally, to check the divisibility of one number by another, we normally do actual division and see whether remainder is zero or not. But sometimes we use direct condition for divisibility, which is as shown below

- **Divisible by 2** When the last digit of a number is either 0 or even, then the number is divisible by 2.

  e.g. 12, 86, 472, 520, 1000 etc. are divisible by 2.

- **Divisible by 3** When the sum of the digits of a number is divisible by 3, then the number is divisible by 3.

  e.g. 1233 as sum of digits

  1 + 2 + 3 + 3 = 9, which is divisible by 3, so 1233 must be divisible by 3.

- **Divisible by 4** When the number made by last two digits of a number is divisible by 4, then that particular number is divisible by 4. Apart from this, the number having two or more zeros at the end, is also divisible by 4.

  e.g. 6428 is divisible by 4 as the number made by its last two digits i.e., 28 is divisible by 4.
- **Divisible by 5** Numbers having 0 or 5 at the end are divisible by 5.

  e.g. 45, 4350, 135, 14850 etc. are divisible by 5 as they have 0 or 5 at the end.
- **Divisible by 6** When a number is divisible by both 3 and 2, then that particular number is divisible by 6 also.

  e.g. 18, 36, 720, 1440 etc. are divisible by 6 as they are divisible by both 3 and 2.
- **Divisible by 7** A number is divisible by 7 when the difference between twice the digit at ones place and the number formed by other digits is either zero or a multiple of 7.

  e.g. 658 is divisible by 7 because $65 - 2 \times 8 = 65 - 16 = 49$. As 49 is divisible by 7, the number 658 is also divisible by 7.
- **Divisible by 8** When the number made by last three digits of a number is divisible by 8, then the number is also divisible by 8. Apart from this, if the last three or more digits of a number are zeros, then the number is divisible by 8.

  e.g. 2256. As 256 (the last three digits of 2256) is divisible by 8, therefore 2256 is also divisible by 8.
- **Divisible by 9** When the sum of all the digits of a number is divisible by 9, then the number is also divisible by 9.

  e.g. 936819 as sum of digits $9 + 3 + 6 + 8 + 1 + 9 = 36$, which is divisible by 9. Therefore, 936819 is also divisible by 9.
- **Divisible by 10** When a number ends with zero, then it is divisible by 10.

  e.g. 20, 40, 150, 123450, 478970 etc. are divisible by 10 as these all end with zero.
- **Divisible by 11** When the sums of digits at odd and even places are equal or differ by a number divisible by 11, then the number is also divisible by 11.

  e.g. 217382 Let us see

  Sum of digits at odd places $= 2 + 7 + 8 = 17$

  Sum of digits at even places $= 1 + 3 + 2 = 6$

  Difference $= 17 - 6 = 11$

  Clearly, 217382 is divisible by 11.
- **Divisible by 12** A number which is divisible by both 4 and 3 is also divisible by 12.

  e.g. 2244 is divisible by both 3 and 4. Therefore, it is also divisible by 12.
- **Divisible by 25** A number is divisible by 25 when its last 2 digits are divisible by 25.

  e.g. 500, 1275, 13550 are divisible by 25 as last 2 digits of these numbers are divisible by 25.

Example 6 Which of the following number is not divisible by 3?

(a) 75 (b) 52 (c) 63 (d) 42

Sol. *(b)* $\because$ Sum of digits, we have

$75 = 7 + 5 = 12$ (divisible by 3)

$52 = 5 + 2 = 7$ (not divisible by 3)

$63 = 6 + 3 = 9$ (divisible by 3)

$42 = 4 + 2 = 6$ (divisible by 3)

$\therefore$ 52 is not divisible by 3, since its sum is not divisible by 3.

PRACTICE EXERCISE

1. A number in the form $\frac{p}{q}$ is said to be a rational number, if

(a) p, q are integers
(b) p, q are integers and $q \neq 0$
(c) p, q are integers and $p \neq 0$
(d) p, q are integers and $p \neq 0$, also $q \neq 0$

2. The numerical expression $\frac{3}{8} + \frac{(-5)}{7} = \frac{-19}{56}$ shows that

(a) rational numbers are closed under addition
(b) rational numbers are not closed under addition
(c) rational numbers are closed under multiplication
(d) addition of rational numbers is not commutative

3. Which of the following is not true?

(a) rational numbers are closed under addition
(b) rational numbers are not closed under subtraction
(c) rational numbers are closed under multiplication
(d) rational numbers are closed under division

4. Between two given rational numbers, we can find

(a) one and only one rational number
(b) only two rational numbers
(c) only ten rational numbers
(d) infinitely rational numbers

5. The product $\frac{3}{4}, \frac{2}{5}$ and $\frac{25}{3}$ is

(a) 5/2 (b) 2/5 (c) 3/5 (d) 5/3

6. Smallest 3-digit prime number is

(a) 103 (b) 107 (c) 101 (d) 109

7. What should be added to $-\frac{5}{7}$ to get $-\frac{3}{2}$?

(a) $-\frac{11}{14}$ (b) $\frac{11}{14}$
(c) $\frac{14}{11}$ (d) $-\frac{14}{11}$

8. Find the additive identity for the rational number

(a) 0 (b) 1 (c) 2 (d) 3

9. Which statement is true?

(a) $-5 + 3 \neq 3 + (-5)$
(b) $\frac{-8}{12} = \frac{10}{-15}$
(c) 2 is not natural number
(d) 17 is not prime number

10. What should be subtracted from $-\frac{3}{4}$ to make $\frac{2}{3}$?

(a) $\frac{-17}{12}$ (b) $\frac{17}{12}$
(c) $\frac{12}{9}$ (d) $\frac{11}{12}$

11. A rational number between $-\frac{3}{5}$ and $\frac{1}{4}$ is

(a) $\frac{7}{40}$ (b) $-\frac{7}{40}$
(c) $\frac{9}{40}$ (d) $-\frac{9}{40}$

12. The value of $\frac{3}{5} + \frac{3}{5} + \ldots$ upto 25 times is

(a) 25 (b) 10 (c) 15 (d) 35

13. $8\frac{1}{4} + 8\frac{1}{2} + ? = 20\frac{1}{8}$

(a) $8\frac{1}{4}$ (b) $3\frac{5}{8}$
(c) $3\frac{3}{8}$ (d) $8\frac{5}{9}$

14. Which of the following is correct?

(a) $a + 0 = b$ (b) $-a \times b = b \times (-a)$
(c) $a - b = b - a$ (d) $\frac{a}{b} = \frac{b}{a}$

15. The value of $2\frac{4}{5} \div 3\frac{1}{2}$ of $\frac{4}{5}$ is

(a) 0 (b) 1
(c) 2 (d) 3

16. By division algorithm, which of the following is correct?
(a) $41 = 7 \times 5 + 6$ (b) $56 = 5 \times 11 + 2$
(c) $30 = 5 \times 8 - 5$ (d) $25 = 5 \times 4 + 4$

17. If $157x234$ is divisible by 3, then the digit at the place of x is
(a) 0 (b) 1 (c) 2 (d) 4

18. By which number, 91476 is not divisible?
(a) 11 (b) 7 (c) 3 (d) 8

19. If a number $573xy$ is divisible by 90, then the value of $x + y$ is
(a) 13 (b) 3 (c) 8 (d) 6

20. Which number is divisible by 5 and 9 both?
(a) 585 (b) 285 (c) 389 (d) 560

21. Which number is not divisible by 6?
(a) 270 (b) 385
(c) 312 (d) 432

22. Which number is divisible by 5 and 25 both?
(a) 2170 (b) 5125
(c) 3107 (d) 4115

23. What least number should be subtracted from 1365 to get a number exactly divisible by 25?
(a) 15 (b) 5 (c) 10 (d) 20

24. Which of the following number is divisible by 9?
(a) 4621 (b) 2834 (c) 9216 (d) 1560

25. By how much $\frac{3}{4}$th of 52 is lesser than $\frac{2}{3}$rd of 99?
(a) 27 (b) 33 (c) 39 (d) 66

26. The value of K, where $31K2$ is divisible by 6, is
(a) 1 (b) 2
(c) 3 (d) 7

Answers

1	(b)	2	(a)	3	(d)	4	(d)	5	(a)	6	(c)	7	(a)	8	(a)	9	(b)	10	(a)
11	(b)	12	(c)	13	(c)	14	(b)	15	(b)	16	(a)	17	(c)	18	(d)	19	(b)	20	(a)
21	(b)	22	(b)	23	(a)	24	(c)	25	(a)	26	(c)								

Hints and Solutions

1. A number in the form $\frac{p}{q}$ is said to be a rational number, if p and q are integers and $q \neq 0$.

2. We have, $\frac{3}{8} + \left(\frac{-5}{7}\right) = \frac{-19}{56}$

$\because \frac{3}{8}$ and $\frac{-5}{7}$ are rational numbers and their addition is $\frac{-19}{56}$, which is also a rational number.

Hence, the rational numbers are closed under addition.

3. Rational numbers are not closed under division.

As, 1 and 0 are the rational numbers but $\frac{1}{0}$ is not defined.

4. We can find infinite rational numbers between any two given rational numbers.

5. We have, $\frac{3}{4} \times \frac{2}{5} \times \frac{25}{3} = \frac{1}{2} \times \frac{1}{5} \times 25 = \frac{5}{2}$

6. 101 is not divisible by any of the prime numbers 2, 3, 5, 7, 11.

$\therefore$ 101 is smallest three-digit prime number.

7. Let x should be added to $-\frac{5}{7}$ to get $-\frac{3}{2}$.

Then, $\frac{-5}{7} + x = \frac{-3}{2}$

$\Rightarrow \quad x = \frac{-3}{2} + \frac{5}{7} = \frac{-21 + 10}{14} = -\frac{11}{14}$

Hence, $-\frac{11}{14}$ should be added.

8. The additive identity for the rational number is 0.

9. By options,

(a) $-5+3=-2$

and $3+(-5)=-2$, which are equal.

(b) $\frac{-8}{12}=\frac{10}{-15} \Rightarrow \frac{-2}{3}=\frac{-2}{3}$, which is true.

(c) 2 is a natural number

(d) 17 is also a prime number.

10. Let x be subtracted.

Then, $\frac{-3}{4}-x=\frac{2}{3}$

$\Rightarrow \quad \frac{-3}{4}-\frac{2}{3}=x$

$\Rightarrow \quad x=\frac{-9-8}{12}=\frac{-17}{12}$

Hence, $-\frac{17}{12}$ should be subtracted.

11. The rational number between $-\frac{3}{5}$ and $\frac{1}{4}$

is $\frac{\frac{-3}{5}+\frac{1}{4}}{2}=\frac{-12+5}{20\times 2}=-\frac{7}{40}$

$\left[\because \text{ A rational number between two rational number} = \frac{\text{Sum of rational number}}{2}\right]$

12. $\frac{3}{5}+\frac{3}{5}+\ldots\ldots$ upto 25 times

$=\frac{3}{5}\times 25=3\times 5=15$

13. $8\frac{1}{4}+8\frac{1}{2}+?=20\frac{1}{8}$

$\Rightarrow \quad ?=20\frac{1}{8}-8\frac{1}{4}-8\frac{1}{2}$

$=(20-8-8)+\left(\frac{1}{8}-\frac{1}{4}-\frac{1}{2}\right)$

$=4+\frac{1-2-4}{8}$

$=4+\frac{-5}{8}=\frac{27}{8}=3\frac{3}{8}$

14. $-a\times b=b\times(-a)$

Because multiplication of two numbers in any order are same.

15. We have, $2\frac{4}{5}\div 3\frac{1}{2}$ of $\frac{4}{5}$

$=\frac{14}{5}\div\frac{7}{2}\times\frac{4}{5}$

$=\frac{14}{5}\div 7\times\frac{2}{5}$

$=\frac{14}{5}\div\frac{14}{5}=1$

16. By option (a),

RHS $=7\times 5+6=35+6=41=$ LHS

$\therefore \quad 41=7\times 5+6$ is correct.

17. $\because$ Sum of digits $=1+5+7+x+2+3+4$

$=22+x$

This addition is divisible by 3, if 2 is at the place of x,

i.e. $\quad 22+x=22+2=24$, divisible by 3.

So, $\quad x=2$.

18. $\because$ We have, to check

divisible by 11 91476 = Sum of odd places digits − Sum of even places digits

$=(9+4+6)-(1+7)$

$=19-8=11$, which is divisible by 11.

Divisible by 7 $=19476=194-2(76)=194-152$

$=42$, which is divisible by 7.

Divisible by 2 given number is even number, hence it is divisible by 2.

Divisible by 8 $91476\rightarrow 476$ is not divisible by 8.

Hence, 91476 is not divisible by 8.

19. We know that, if any number divisible by 90, i.e. divisible by 9 and 10.

So, to make number $573xy$ divisible by 10, we have to put $y=0$ to make unit digit 0, (i.e. 0 unit digit number divisible by 10), i.e. $y=0$.

Now, to also make $573x0$ divisible by 9,

$5+7+3+x+10=15+x=15+\underline{3}=18$,

i.e. divisible by 3.

So, $\quad x=3$

$\therefore \quad x+y=3+0=3$

20. By option (a),

We have, 585, divisible by 5 because unit digit is 5, and sum of digits $=5+8+5=18=1+8=9$ which is also divisible by 9.

21. We know that, if any number divisible by 2 and 3, it is also divisible by 6.

$\therefore$ By option (b), 385 is not divisible by 2. Hence, it is also not divisible by 6.

22. We know that,

Number is divisible by 5, if last digit is 5 or 0.

And divisible by 25, if last two digits is divisible by 25.

Hence, 5125 is only number divisible by 5 and 25 both.

23.
$$\begin{array}{l} 25)1365(54 \\ \quad \underline{125} \\ \quad\ \ 115 \\ \quad\ \ \underline{100} \\ \quad\ \ \ \underline{15} \end{array}$$

$\therefore$ Required number is 15.

24. We have, $4621 = 4 + 6 + 2 + 1 = 13$, not divisible by 9

$2834 = 2 + 8 + 3 + 4 = 17$, not divisible by 9

$9216 = 9 + 2 + 1 + 6 = 18$, divisible by 9

$1560 = 1 + 5 + 6 + 0 = 12$, not divisible by 9

Hence, 9216 is only number divisible by 9.

25. $\therefore$ Required answer

$$= \frac{2}{3} \times 99 - \frac{3}{4} \times 52$$
$$= 2 \times 33 - 3 \times 13$$
$$= 66 - 39 = 27$$

26. The number $31K2$ is divisible by 6, it mean it is divisible by 2 and 3 both.

Here, unit digit is 2 (even), so this number is divisible by 2.

Now, for 3, first we have to add all the digits.

$\therefore \quad 3 + 1 + K + 2 = 6 + K$

$= 6 + 3 = 9,$

for $K = 3$, it is divisible by 3.

Hence, $K = 3$