CHAPTER

04

EXPONENTS /POWERS AND SURD

In this chapter, we study the positive and negative exponents with their laws of exponents and also surds with their laws of exponent.

Exponential Form

The repeated multiplication of the same non-zero rational number a with itself in the form of a^n {i.e., $a \times a \times$ .... $\times\, a \times$ (n times) $= a^n$}, where a is called the base and n is an integer called the exponent or index. This type of representation of a number is called the exponential form of the given number. e.g. $6 \times 6 \times 6 = 6^3$

Here, 6 is the base and 3 is the exponent and we read it as ''6 raised to the power of 3''.

Rational Exponents

A rational exponents represent both an integer exponent and nth root.

e.g. $a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

Negative Integral Exponents

For any non-zero integer a, we have

$$a^{-n} = \frac{1}{n} \text{ or } a^{-n} \times a^n = 1$$

So, a^{-n} is the multiplicative inverse or reciprocal of a^n and *vice-versa*.

e.g. $(5)^{-2} = \dfrac{1}{5^2}$

Example 1 Find the multiplicative inverse of 10^{-5}.

(a) 10^4 (b) 10^5

(c) 10^6 (d) None of these

Sol. *(b)* We have, $10^{-5} = \frac{1}{10^5}$

Reciprocal of $\frac{1}{10^5} = 10^5$

$\therefore$ Multiplicative inverse of 10^{-5} is 10^5

$[\because\ 10^{-5} \times 10^5 = 10^0 = 1]$

Laws of Exponent

I. If a and b be any real numbers and m, n be positive integers, then

(i) $a^m \times a^n = a^{m+n}$

(ii) $a^m \div a^n = a^{m-n}, a \neq 0$

(iii) $(a^m)^n = a^{mn}$ (iv) $(ab)^n = a^n b^n$

(v) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ (vi) $(a)^0 = 1,\ a \neq 0$

II. If a and b be any real numbers and m, n be negative integers, then

(i) $a^{-m} \times a^{-n} = \frac{1}{a^m} \times \frac{1}{a^n}$

$$= \frac{1}{a^m \times a^n} = \frac{1}{a^{m+n}} = a^{-(m+n)}$$

(ii) $a^{-m} \div a^{-n} = \frac{1}{a^m} \div \frac{1}{a^n} = \left(\frac{1}{a^m} \times \frac{a^n}{1}\right)$

$$= \frac{a^n}{a^m} = a^{n-m} = a^{-m-(-n)}$$

(iii) $(a^{-m})^{-n} = \left[\frac{1}{(a^{-m})}\right]^n$

$$= (a^m)^n = a^{mn} = a^{(-m)(-n)}$$

(iv) $(ab)^{-n} = \frac{1}{(ab)^n} = \frac{1}{a^n \times b^n}$

$$= \frac{1}{a^n} \times \frac{1}{b^n} = a^{-n} \times b^{-n}$$

(v) $\left(\frac{a}{b}\right)^{-n} = \frac{1}{\left(\frac{a}{b}\right)^n} = \frac{1}{\frac{a^n}{b^n}} = \frac{b^n}{a^n} = \frac{a^{-n}}{b^{-n}}$

Example 2 $\left(\frac{5}{7}\right)^8 \div \left(\frac{4}{5}\right)^8$ is equal to

(a) $\left(\frac{5}{7} / \frac{4}{5}\right)^8$ (b) $\left(\frac{5}{7} \times \frac{4}{5}\right)^8$

(c) $\left(\frac{5}{7} / \frac{4}{5}\right)^0$ (d) None of these

Sol. *(a)* $\frac{(5/7)^8}{(4/5)^8} = \left(\frac{5/7}{4/5}\right)^8$ $\left[\because \frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m\right]$

$$= \left(\frac{5}{7} / \frac{4}{5}\right)^8$$

Example 3 The value of

$$(7^{-1} - 8^{-1})^{-1} - (3^{-1} - 4^{-1})^{-1} \text{ is}$$

(a) 44 (b) 56 (c) 68 (d) 12

Sol. *(a)* Using law of exponents, $a^{-m} = \frac{1}{a^m}$

[$\because a$ is non-zero integer]

$\therefore$ $(7^{-1} - 8^{-1})^{-1} - (3^{-1} - 4^{-1})^{-1}$

$$= \left(\frac{1}{7} - \frac{1}{8}\right)^{-1} - \left(\frac{1}{3} - \frac{1}{4}\right)^{-1} = \left(\frac{1}{56}\right)^{-1} - \left(\frac{1}{12}\right)^{-1}$$

$$= 56 - 12 = 44$$

Example 4 Evaluate $\left(\frac{625}{81}\right)^{-1/4}$

(a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) $\frac{1}{5}$ (d) $\frac{5}{2}$

Sol. *(a)* $\left(\frac{625}{81}\right)^{-1/4} = \left(\frac{81}{625}\right)^{1/4} = \left(\frac{3^4}{5^4}\right)^{1/4}$

$$= \left[\left(\frac{3}{5}\right)^4\right]^{1/4} = \frac{3}{5}$$

Example 5 Simplify $\left(\frac{x^b}{x^c}\right)^a \times \left(\frac{x^c}{x^a}\right)^b \times \left(\frac{x^a}{x^b}\right)^c$

(a) 0 (b) 1 (c) -1 (d) 2

Sol. *(b)* Given expression

$$= (x^{b-c})^a \times (x^{c-a})^b \times (x^{a-b})^c$$

$$= x^{a(b-c) + b(c-a) + c(a-b)}$$

$$= x^0 = 1$$

Surd or Radicals

If $\sqrt[n]{a}$ is irrational, where a is a rational number and n is a positive integer, then $\sqrt[n]{a}$ or $a^{1/n}$ is called a surd or radical of order n and a is called the radicand.

- A surd of order 2 is called a quadratic or square surd.
- A surd of order 3 is called a cubic surd.
- A surd of order 4 is called a biquadratic surd.

Surd in Simplest Form

A surd in its simplest form has

(i) the smallest possible index of this radical.
(ii) no fraction under the radical sign.
(iii) no factor of the form b^n, where b is rational, under the radical sign of index n.

Note Let n be a positive integer and a be a real number.

(i) If a is irrational, then $\sqrt[n]{a}$ is not a surd.
(ii) If a is rational, then $\sqrt[n]{a}$ is a surd.

Laws of Surd

As surds can be expressed with fractional exponent, the laws of indices are therefore, applicable to surd.

(i) $(\sqrt[n]{a})^n = a$ (ii) $\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$

(iii) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$

(iv) $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

(v) $(\sqrt[n]{a})^m = \sqrt[n]{a^m}$

Example 6 The index form of $\sqrt[9]{\left(\frac{4}{5}\right)^3}$ is

(a) $\left(\frac{4}{5}\right)^{1/3}$ (b) $\left(\frac{4}{5}\right)^{3}$ (c) $\left(\frac{4}{5}\right)^{1/2}$ (d) $\left(\frac{4}{5}\right)^{1/27}$

Sol. *(a)* $\sqrt[9]{\left(\frac{4}{5}\right)^3} = \left[\left(\frac{4}{5}\right)^3\right]^{1/9} = \left(\frac{4}{5}\right)^{3/9} = \left(\frac{4}{5}\right)^{1/3}$

$[\because (a^m)^n] = a^{mn}$

Example 7 If $3^x = 5^y = 75^z$, then the value of z is

(a) $\dfrac{xy}{(2x+y)}$ (b) $\dfrac{xy}{x+2y}$

(c) $\dfrac{xy}{x-y}$ (d) $\dfrac{xy}{x-2y}$

Sol. *(a)* Let $3^x = 5^y = (75)^z = k$

Then, $3 = k^{1/x}, 5 = k^{1/y}$ and $75 = k^{1/z}$

Now, $75 = 3 \times 5^2$

$\Rightarrow k^{1/z} = k^{1/x} \cdot k^{2/y}$

$\Rightarrow k^{1/z} = k^{\left(\frac{1}{x}+\frac{2}{y}\right)}$

$\therefore \dfrac{1}{z} = \dfrac{1}{x} + \dfrac{2}{y}$

$\Rightarrow z = \dfrac{xy}{(2x+y)}$

PRACTICE EXERCISE

1. The value of $\dfrac{5^0+2^1}{3^2+8^0}$ is

(a) $\frac{3}{10}$ (b) $\frac{5}{3}$

(c) $\frac{2}{5}$ (d) $\frac{3}{5}$

2. The multiplicative inverse of 10^{-100} is

(a) 10 (b) 100
(c) 10^{100} (d) 10^{-100}

3. The value of $\dfrac{5}{(121)^{-1/2}}$ is

(a) -55 (b) $\frac{1}{55}$ (c) $-\frac{1}{55}$ (d) 55

4. The value of $3 \times 9^{-3/2} \times 9^{1/2}$ is

(a) $\frac{1}{3}$ (b) 3

(c) 27 (d) $-\frac{1}{3}$

5. The simplified form of $(-4)^5 \div (-4)^8$ is

(a) $\frac{1}{4^3}$ (b) $\frac{1}{(-4)^3}$

(c) $\frac{1}{4^4}$ (d) None of these

6. The value of $\left(\frac{1}{2^3}\right)^2$ is

(a) $\frac{1}{62}$ (b) $\frac{1}{64}$

(c) $\frac{1}{32}$ (d) None of these

7. Evaluate $(-3)^4 \times \left(\frac{5}{3}\right)^4$

(a) 5^7 (b) 5^6

(c) 5^3 (d) 5^4

8. Evaluate, $\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1}$

(a) −1 (b) − 2

(c) − 3 (d) − 4

9. If $5^a = 3125$, then the value of 5^{a-3} is

(a) 625 (b) 25

(c) 5 (d) 225

10. The standard form of 0.0000078 is

(a) 78×10^{-6} (b) 78×10^{6}

(c) 78×10^{-5} (d) None of these

11. If $3^x = \frac{1}{9}$, the value of x is

(a) 2 (b) −2 (c) 1/2 (d) 1

12. The value of $(12^2 + 5^2)^{1/2}$ is

(a) 11 (b) 13 (c) 12 (d) 15

13. The value of $(0.000064)^{5/6}$ is

(a) $\frac{32}{100000}$ (b) $\frac{16}{10000}$

(c) $\frac{16}{100000}$ (d) None of these

14. The value of $\left[\left(\frac{25}{9}\right)^{5/2}\right]^{3/5}$ is

(a) $\frac{25}{27}$ (b) $\frac{125}{27}$

(c) $\frac{25}{9}$ (d) None of these

15. The value of $\left(-\frac{1}{125}\right)^{-2/3}$ is

(a) 5 (b) 25

(c) − 25 (d) None of these

16. The value of $\frac{(81)^{1/3} \times (576)^{1/3}}{(64)^{2/3} \times (27)^{2/3}}$ is

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$

(c) $\frac{5}{8}$ (d) None of these

17. The value of $\frac{2^{10+n} \times 4^{3n-5}}{2^{4n+1} \times 2^{3n-1}}$ is

(a) 0 (b) 1

(c) 3 (d) 4

18. If $9\sqrt{x} = \sqrt{12} + \sqrt{147}$, then the value of x is

(a) 1 (b) 2

(c) 3 (d) 4

Answers

1	*(a)*	2	*(c)*	3	*(d)*	4	*(a)*	5	*(b)*	6	*(b)*	7	*(d)*	8	*(a)*	9	*(b)*	10	*(a)*
11	*(b)*	12	*(b)*	13	*(a)*	14	*(b)*	15	*(b)*	16	*(a)*	17	*(b)*	18	*(c)*				

Hints and Solutions

1. $\dfrac{5^0 + 2^1}{3^2 + 8^0} = \dfrac{1 + 2}{9 + 1} = \dfrac{3}{10}$

2. For multiplicative inverse, let a be the multiplicative inverse of 10^{-100}.

[$\because$ If a is multiplicative inverse of b then $a \times b = 1$]

$\therefore\ a \times 10^{-100} = 1$

$$\Rightarrow \quad a = \frac{1}{10^{-100}} \frac{1}{\frac{1}{10^{100}}} = 10^{100} \quad \left[\because a^{-m} = \frac{1}{a^m}\right]$$

3. $\dfrac{5}{(121)^{-1/2}} = 5 \times 121^{1/2}$

$= 5 \times (11^2)^{1/2} = 5 \times 11 = 55$

4. $3 \times 9^{-3/2} \times 9^{1/2} = 3 \times \left(3^{2 \times -\frac{3}{2}}\right) \times \left(3^{2 \times \frac{1}{2}}\right)$

$= 3 \times (3)^{-3} \times 3 = 3 \times \left(\dfrac{1}{3}\right)^3 \times 3$

$= 3 \times \dfrac{1}{27} \times 3 = \dfrac{1}{3}$

5. We have, $(-4)^5 \div (-4)^8$

$= \dfrac{(-4)^5}{(-4)^8} = \dfrac{1}{(-4)^8 \times (-4)^{-5}} \quad \left[\because a^m = \dfrac{1}{a^{-m}}\right]$

$= \dfrac{1}{(-4)^{8-5}} = \dfrac{1}{(-4)^3} \quad [\because a^m \times a^n = a^{m+n}]$

which is the required form.

6. We have, $\left(\dfrac{1}{2^3}\right)^2$

$= \dfrac{(1)^2}{(2^3)^2} \quad \left[\because \left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}\right]$

$= \dfrac{1}{2^6} = \dfrac{1}{64} \quad [\because (a^m)^n = a^{m \times n}]$

7. We have, $(-3)^4 \times \left(\dfrac{5}{3}\right)^4$

$= (-1 \times 3)^4 \times \left(\dfrac{5}{3}\right)^4 \quad [\because -a = -1 \times a]$

$= (-1)^4 \times 3^4 \times \dfrac{5^4}{3^4}$

$\left[\because (a \times b)^m = a^m \times b^m, \left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}\right]$

$= 1 \times 5^4 = (5)^4 \quad [\because (-1)^4 = 1]$

which is the required form.

8. We have, $\left\{\left(\dfrac{1}{3}\right)^{-1} - \left(\dfrac{1}{4}\right)^{-1}\right\}^{-1}$

$= \left\{\dfrac{(1)^{-1}}{(3)^{-1}} - \dfrac{(1)^{-1}}{(4)^{-1}}\right\}^{-1} \quad \left[\because \left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}\right]$

$= \left\{\dfrac{3}{1} - \dfrac{4}{1}\right\}^{-1} \quad \left[\because a^{-m} = \dfrac{1}{a^m}\right]$

$= (3 - 4)^{-1} = (-1)^{-1} \quad \left[\because a^{-m} = \dfrac{1}{a^m}\right]$

$= \dfrac{1}{(-1)^1}$

$= \dfrac{1}{-1} = -1$

9. Given, $5^a = 3125$

$\Rightarrow \quad 5^a = 5^5$

On comparing, we get

$a = 5$

$\therefore \quad 5^{a-3} = 5^{5-3} = 25$

10. According to question,

$0.000078 = \dfrac{78}{1000000} = 78 \times 10^{-6}$

11. $\because \quad 3^x = \dfrac{1}{9}$

$\therefore \quad 3^x = \left(\dfrac{1}{3}\right)^2$

or $\quad 3^x = 3^{-2}$

On comparing both sides, we get $x = -2$

12. $(12^2 + 5^2)^{1/2} = (144 + 25)^{1/2}$

$= (169)^{1/2}$

$= (13^2)^{1/2} = 13$

13. $(0.000064)^{5/6} = \left(\frac{64}{1000000}\right)^{5/6}$

$= \left[\left\{\left(\frac{2}{10}\right)^6\right\}^{1/6}\right]^5 = \left(\frac{2}{10}\right)^5 = \frac{32}{100000}$

14. $\left[\left(\frac{25}{9}\right)^{5/2}\right]^{3/5} = \left[\left\{\left(\frac{5}{3}\right)^2\right\}^{5/2}\right]^{3/5}$

$= \left[\left(\frac{5}{3}\right)^5\right]^{3/5} = \left(\frac{5}{3}\right)^3 = \frac{125}{27}$

15. $\left(-\frac{1}{125}\right)^{-2/3} = \left[\left(-\frac{1}{5} \times -\frac{1}{5} \times -\frac{1}{5}\right)^{-1/3}\right]^2$

$= (-5)^2 = 25$

16. $\frac{(81)^{1/3} \times (576)^{1/3}}{(64)^{2/3} \times (27)^{2/3}} = \frac{(3^4)^{1/3} \times (2^6 \times 3^2)^{1/3}}{(4^3)^{2/3} \times (3^3)^{2/3}}$

$= \frac{3^{4/3} \times 2^2 \times 3^{2/3}}{4^2 \times 3^2} = \frac{3^2 \times 2^2}{4^2 \times 3^2} = \frac{9 \times 4}{16 \times 9} = \frac{1}{4}$

17. $\frac{2^{10+n} \times 4^{3n-5}}{2^{4n+1} \times 2^{3n-1}} = \frac{2^{10+n} \times 2^{6n-10}}{2^{4n+1} \times 2^{3n-1}}$

$= \frac{2^{10+n+6n-10}}{2^{4n+1+3n-1}} = \frac{2^{7n}}{2^{7n}} = 1$

18. $9\sqrt{x} = \sqrt{12} + \sqrt{147}$

$= \sqrt{2 \times 2 \times 3} + \sqrt{3 \times 7 \times 7}$

$= 2\sqrt{3} + 7\sqrt{3}$

$\Rightarrow \quad 9\sqrt{x} = 9\sqrt{3} \quad \Rightarrow \quad x^{1/2} = 3^{1/2}$

On comparing both sides, we get $x = 3$