CHAPTER
04

FACTORS AND MULTIPLES INCLUDING THEIR PROPERTIES

Factors

If a number is exactly divisible by the another number, without leaving any remainder, then the second number is said to be a factor of first number. In other words, an exact divisor of a number is called a factor of the number.

- 1 is the factor of every number.
- Every number is a factor of itself.
- Factors of a number are less than or equal to that number.
- Number of factors of that number are finite.

Example **1.** Find number of factors of 250.

    (1) 7   (2) 8   (3) 9   (4) 6

Sol. (2) $250 = 2 \times 125 = 5 \times 50 = 10 \times 25 = 250 \times 1$

So, 1, 2, 5, 10, 25, 50, 125 and 250 are all factors of 250. Hence, number of factors of 250 is 8.

Common Factors

When we find the factors of two or more numbers and then find some factors are the same ("Common") then they are the "Common Factors".

Example **2.** What are the common factors of 20 and 25?

    (1) 4   (2) 5   (3) 6   (4) 7

Sol. (2) The factors of 20 = 1, 2, 4, 5, 10, 20
The factors of 25 = 1, 5, 25
and the common factors of 20 and 25 are 1 and 5.

Multiples

A multiple of a number is the number obtained by multiplying it with other (or same) number. In other words, the product of two or more numbers is said to be a multiple of each of those numbers.

e.g. $5 \times 1 = 5, 5 \times 2 = 10, 5 \times 3 = 15, 5 \times 4 = 20$; Hence, 5, 10, 15 and 20 all are multiples of 5.

- Multiple of a number is greater than or equal to that number.
- Every number is a multiple of itself.
- Every multiple of a number is exactly divisible by the number. Number of multiples of a number are infinite.

Example **3.** Find the first five multiples of 20 between 100 and 300.

    (1) 125, 130, 145, 165, 180
    (2) 115, 130, 145, 165, 180
    (3) 125, 135, 145, 165, 180
    (4) 120, 140, 160, 180, 200

Sol. (4) Multiples of 20 between 100 and 300 are
120 (20×6), 140 (20×7), 160 (20×8), 180 (20×9), 200 (20×10).

Common Multiples

A number that can be divided exactly by two or more different numbers.

e.g. common multiple of 24 and 36 is 4, because
$$4 \times 6 = 24, 4 \times 9 = 36$$

Prime Factor

The prime factors of a quantity are all of the prime quantities that will exactly divide the given quantity.
e.g. $28 = 2 \times 2 \times 7$ etc.

Example **4.** Find the prime factors of 96.

    (1) 4   (2) 5   (3) 6   (4) 7

Sol. (3) $96 = 2 \times 48 = 2 \times 2 \times 24$
$= 2 \times 2 \times 2 \times 12 = 2 \times 2 \times 2 \times 2 \times 6 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$
Thus, the prime factors of 96 are 2, 2, 2, 2, 2 and 3.

Entrance Corner

1. Which of the following numbers is divisible by 3, 4, 5 and 6? **[JNV 2019]**
   (1) 36   (2) 60   (3) 80   (4) 90

2. A common multiple of both 9 and 7 is *A*. This number is in between 1200 and 1300. What is number *A*? **[JNV 2018]**
   (1) 1197   (2) 1260   (3) 1206   (4) 1266

3. The sum of the first four multiples of 6, is **[JNV 2016]**
   (1) 66   (2) 56   (3) 72   (4) 60

4. The sum of first five multiple of 6 is **[JNV 2015]**
   (1) 90   (2) 54   (3) 30   (4) 84

5. The difference between ten's digit and unit's digit of the sum of the first five multiple of 6 is **[JNV 2015]**
   (1) 6   (2) 7   (3) 8   (4) 9

6. Which of the following is not a factor of 316? **[JNV 2011, 2002]**
   (1) 1   (2) 8   (3) 79   (4) 158

7. What is the prime factorization of 37800? **[JNV 2005]**
   (1) $2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7 \times 7$
   (2) $2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 7$
   (3) $8 \times 27 \times 25 \times 7$
   (4) $2 \times 4 \times 25 \times 27 \times 7$

8. Factors of 30 are **[JNV 2004]**
   (1) 2, 3, 5   (2) 1, 2, 3, 5, 110
   (3) 1, 2, 3, 10, 15   (4) 1, 2, 3, 5, 6, 10, 15, 30

9. How many times does 9 come in writing the number from 1 to 100? **[JNV 2004]**
   (1) 9   (2) 100   (3) 20   (4) 21

10. The number of prime factors of 105 is **[JNV 2001]**
    (1) 2   (2) 3   (3) 4   (4) 5

11. The total number of the factors of 24 is **[JNV 2000]**
    (1) 8   (2) 7   (3) 4   (4) 9

12. The factor of each odd number is **[JNV 1999]**
    (1) 0   (2) 1
    (3) 3   (4) 5

Answers

1. (2)	**2.** (2)	**3.** (4)	**4.** (1)	**5.** (4)	**6.** (2)	**7.** (2)	**8.** (4)	**9.** (3)	**10.** (2)
11. (1)	**12.** (2)								

Hints and Solutions

1. From the options,
   Multiples of $60 = 2 \times 2 \times 3 \times 5$ or $4 \times 3 \times 5$ or 6×10
   Hence, number 60 is divisible by 3,4,5 and 6.

2. A common multiple of 9 and 7 both is *A*.
   Then number will completely divide both 9 and 7. We observed that only two numbers 1197 and 1260 is in between 1200 and 1300 is completely divide by 9 and 7. But only number 1260.
   Thus, the number *A* is 1260.

3. First four multiple of $6 = 6$, 12, 18 and 24
   Then, require sum $= 6 + 12 + 18 + 24$
   $= 60$

4. First five multiple of 6 is as follows $6 \times 1, 6 \times 2,$
   $6 \times 3, 6 \times 4, 6 \times 5.$ or 6, 12, 18, 24, 30
   $\therefore$ Required sum $= 6 + 12 + 18 + 24 + 30 = 90$

5. First five multiple of 6 is as follows $6 \times 1, 6 \times 2,$
   $6 \times 3, 6 \times 4, 6 \times 5$ or 6, 12, 18, 24, 30
   $\because$ Sum of first five multiple of 6
   $= 6 + 12 + 18 + 24 + 30 = 90$
   $\therefore$ Required difference of ten's and unit's digits
   $= 9 - 0 = 9$

6. $\because$ Factors of 316 are 1×316, 2×158 and 4×79
   (1, 2, 4, 79, 158, 316)
   $\therefore 8$ is not a factor of 316.

7.

2	37800
2	18900
2	9450
3	4725
3	1575
3	525
5	175
5	35
	7

∴Prime factorization
$$= 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 7$$

8. Factors of 30 are
$$1 \times 30, 2 \times 15, 3 \times 10 \text{ and } 5 \times 6$$
∴Factors of 30 are
1, 2, 3, 5, 6, 10, 15, 30

9. 9, 19, 29, 39, 49, 59, 69, 79, 89 $= 9$
90, 91, 92, 93, 94, 95, 96, 97, 98 $= 9$
99 $= 2$
Total $= 20$

10.

3	105
5	35
	7

Prime factors of 105 are 3, 5 and 7.
∴ Number of factors of 105 $= 3$

11. All the factors of 24 are 1×24,
$2 \times 12, 3 \times 8$ and 4×6
So, number of factors are
$(1, 2, 3, 4, 6, 8, 12, 24) = 8$

12. 1 is the factor of each odd number.

Practice Exercise

1. The total number of the factors of 81 is
(1) 6   (2) 5   (3) 4   (4) 7

2. The total number of the factors of 54 is
(1) 6   (2) 8   (3) 7   (4) 5

3. The prime factors of 120 are
(1) $2 \times 2 \times 3 \times 8$ (2) $2 \times 9 \times 5$
(3) $2 \times 2 \times 2 \times 6$ (4) $2 \times 2 \times 2 \times 3 \times 5$

4. The prime factors of 48 are
(1) $2 \times 2 \times 12$ (2) 2×24
(3) $2 \times 2 \times 2 \times 6$ (4) $2 \times 2 \times 2 \times 2 \times 3$

5. What are the numbers of multiples of 5 which are less than 45?
(1) 9   (2) 8   (3) 7   (4) 10

6. Which of the following is not a factor of 144?
(1) 2   (2) 3   (3) 5   (4) 1

7. Which of the following is not a factor of 128?
(1) 8   (2) 2   (3) 3   (4) 4

8. Total number of the factors of 210 is
(1) 16   (2) 8   (3) 10   (4) 14

9. All prime factors of 150 are
(1) 2, 3, 5   (2) 3, 5, 10
(3) 2, 3, 5, 5   (4) None of these

10. Which one of the following is true?
(1) 1 is a factor of every number
(2) The factors of a number are uncountable
(3) The multiples of a number are countable
(4) 1 is a multiple of every number

11. The sum of first five even multiples of 2 is
(1) 28   (2) 32   (3) 40   (4) 30

12. The sum of first 8 multiple of 3 is
(1) 108   (2) 110   (3) 107   (4) 105

13. The numbers $x, x + 2, x + 4$ are all prime so x is
(1) 3   (2) 2
(3) 11   (4) 17

14. Which of the following is a prime factor?
(1) $84 = 2 \times 2 \times 3 \times 7$
(2) $112 = 2 \times 2 \times 14 \times 2$
(3) $70 = 14 \times 5$
(4) $45 = 5 \times 9$

15. Which of the following is a prime factor?
(1) $48 = 2 \times 2 \times 2 \times 6$ (2) $63 = 3 \times 3 \times 7$
(3) $81 = 3 \times 3 \times 9$ (4) $54 = 2 \times 3 \times 9$

16. Common multiple number for 18 and 54 is
(1) 8   (2) 9
(3) 7   (4) 4

17. The number x, $x - 2$ and $x - 6$ are all prime numbers, so find the value of x.
(1) 15   (2) 17   (3) 19   (4) 21

18. Common multiple for the numbers 4, 8 and 10, within the first 10 multiples is
(1) 40   (2) 20   (3) 50   (4) 48

19. Which of the following is not a prime factor?
(1) $81 = 3 \times 3 \times 3 \times 3$
(2) $102 = 2 \times 3 \times 17$
(3) $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$
(4) $98 = 7 \times 14$

20. Which of the following is a prime factor of 168?
(1) $2 \times 2 \times 6 \times 7$ (2) $2 \times 4 \times 3 \times 7$
(3) $2 \times 2 \times 2 \times 21$ (4) $2 \times 2 \times 2 \times 3 \times 7$

21. Which of the following is always a factor of prime number?
(1) 1   (2) 2   (3) 4   (4) 7

22. Common multiple of numbers 6, 8 and 12, within the first 10 multiples are
(1) 24, 40   (2) 24, 48   (3) 40, 60   (4) 36, 40

23. The sum of first four multiple of 7 is
(1) 60   (2) 68   (3) 70   (4) 74

Answers

1. (2)	**2.** (2)	**3.** (4)	**4.** (4)	**5.** (2)	**6.** (3)	**7.** (3)	**8.** (1)	**9.** (3)	**10.** (1)
11. (4)	**12.** (1)	**13.** (1)	**14.** (1)	**15.** (2)	**16.** (2)	**17.** (3)	**18.** (1)	**19.** (4)	**20.** (4)
21. (1)	**22.** (2)	**23.** (3)							

Hints and Solutions

1. $\because$ Factors of 81 are 1×81, 3×27 and 9×9

$\therefore$ Number of factors = (1, 3, 9, 27, 81) = 5

2. $\because$ Factors of 54 are 1×54, 2×27

3×18 and 6×9.

$\therefore$ Number of factors = (1, 2, 3, 6, 9, 18, 27, 54)

$= 8$

3.

2	120
2	60
2	30
3	15
5	5
	1

$\therefore$ Prime factors of $120 = 2 \times 2 \times 2 \times 3 \times 5$

4.

2	48
2	24
2	12
3	6
3	3
	1

$\therefore$ Prime factors of $120 = 2 \times 2 \times 2 \times 2 \times 3$

5. Multiples of 5 less than 45

$= 5$, 10, 15, 20, 25, 30, 35, 40

Hence, required number of multiples is 8.

6. Factors of $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 1$

So, 5 is not a factor of 144.

7. Factors 128

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 1$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 4 \times 1 = 2 \times 2 \times 2 \times 2 \times 8 \times 1$

So, 3 is not the factor of 128.

8. Factors of 210 are 1×210, 2×105

3×70, 5×42, 6×35, 7×30 and 10×21, 14×15
Number of factors = 1, 2, 3, 5, 6, 7, 10, 14, 15, 21, 30, 35, 42, 70, 105, 210
Hence, number of factor is 16.

9.

2	150
3	75
5	25
5	5
	1

Prime factors of $150 = 2, 3, 5, 5$

10. 1 is a factor of every number.

11. $\because$ First 5 even multiples of $2 = 2, 4, 6, 8, 10$

Sum of these multiples $= 2 + 4 + 6 + 8 + 10$
$= 30$

12. $\because$ First 8 multiple of 3 are 3, 6, 9, 12, 15, 18, 21 and 24.

$\therefore$ Sum of these multiples
$= 3 + 6 + 9 + 12 + 15 + 18 + 21 + 24 = 108$

13. 3 ($\because x = 3$, $x + 2 = 3 + 2 = 5$ and
$x + 4 = 3 + 4 = 7$)

14. Prime factors of $84 = 2 \times 2 \times 3 \times 7$

Prime factors of $112 = 2 \times 2 \times 2 \times 2 \times 2 \times 7$

Prime factor of $70 = 2 \times 5 \times 7$

Prime factors of $45 = 3 \times 3 \times 5$

So, factors of 84 are prime factors.

15. Prime factors of $48 = 2 \times 2 \times 2 \times 2 \times 3$

Prime factors of $63 = 3 \times 3 \times 7$

Prime factors of $81 = 3 \times 3 \times 3 \times 3$

Prime factors of $54 = 2 \times 3 \times 3 \times 3$

So, factors of 63 are prime factors.

16. Factors of $18 = 2 \times 3 \times 3$

Factors of $54 = 2 \times 3 \times 3 \times 3$

$\therefore$ Common multiple $= 3 \times 3 = 9$

17. From option (3), $x = 19$,

$x - 2 = 19 - 2 = 17$

$x - 6 = 19 - 6 = 13$

18. First 10 multiples of $4 = 4, 8, 12, 16, 20, 24, 28, 32, 36, \boxed{40}$

$8 = 8, 16, 24, 32, \boxed{40}, 48, 56, 64, 72, 80$

$10 = 10, 20, 30, \boxed{40}, 50, 60, 70, 80, 90, 100$

Hence, common multiple $= 40$

19. Prime factors of $81 = 3 \times 3 \times 3 \times 3$

Prime factors of $102 = 2 \times 3 \times 17$

Prime factors of $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

Prime factors of $98 = 2 \times 7 \times 7$

So, factor of 98 are not prime factors.

20. Prime factors of $168 = 2 \times 2 \times 2 \times 3 \times 7$

21. 1 is the factor of prime number.

22. First 10 multiple of
$6 = 6, 12, 18, \boxed{24}, 30, 36, 42, \boxed{48}, 54, 60$

$8 = 8, 16, \boxed{24}, 32, 40, \boxed{48}, 56, 64, 72, 80$

$12 = 12, \boxed{24}, 36, \boxed{48}, 60, 72, 84, 96, 108, 120$

So, common multiples are 24 and 48.

23. First 4 multiples of $7 = 7, 14, 21, 28$

Sum of these multiples $= 7 + 14 + 21 + 28$
$= 70$

Self Practice

1. Sum of first five odd multiples of 3 are
   (1) 45   (2) 75   (3) 90   (4) 60

2. The number of multiples of 3 between 18 and 54
   (1) 11   (2) 12   (3) 10   (4) 14

3. Sum of first five multiples of 6 are
   (1) 90   (2) 30   (3) 60   (4) 120

4. The prime factors of 75 are
   (1) $3 \times 5 \times 3$ (2) 3×25 (3) 5×15 (4) $3 \times 5 \times 5$

5. All prime factors of 182 are
   (1) 2 and 13   (2) 2 and 7   (3) 2, 7 and 13   (4) None of these

6. The prime factors of 210 are
   (1) 5   (2) 4   (3) 3   (4) 2

7. The prime factors of 2310 are
   (1) 2, 3, 5, 7, 11   (2) 2, 3, 4, 7   (3) 2, 4, 6, 7, 11   (4) None of these

8. Prime factors of 240 are
   (1) $2 \times 2 \times 4 \times 3 \times 5$ (2) $2 \times 2 \times 2 \times 2 \times 3 \times 5$ (3) $4 \times 4 \times 3 \times 5$ (4) $4 \times 4 \times 15$

9. Multiple of first three multiple of 4
   (1) 380   (2) 384   (3) 390   (4) 400

10. The number of multiple of 7 between 21 and 77
    (1) 6   (2) 7   (3) 8   (4) 9

11. Prime factors of 68 are
    (1) 2×34 (2) $2 \times 2 \times 17$ (3) 4×17 (4) 1×68

12. Prime factors of 88 are
    (1) 3   (2) 4   (3) 6   (4) 8

13. Common multile of number 7 and 17 within 5 multiple
    (1) 14, 42   (2) 14, 30   (3) 14, 28   (4) 21, 56

14. Which of the following is true?
    (1) Every number is a factor of itself
    (2) Number of multiples of a number are infinite
    (3) A number is exactly divisible by its factors
    (4) All are true

15. Which of the following is not true?
    (1) Every number is a multiple of itself
    (2) 1 is not the factor of all numbers
    (3) Exact divisor of a number is a factor of that number
    (4) All are true

Answers

1. (2)	2. (1)	3. (1)	4. (4)	5. (3)	6. (2)	7. (1)	8. (2)	9. (2)	10. (2)
11. (2)	12. (2)	13. (3)	14. (4)	15. (2)					