#### CHAPTER

# 13

# SURFACE AREA AND VOLUME

Any figure bounded by one or more surfaces is called a **solid figure**. Hence, a solid figure must have length, breadth (width) and thickness (depth or height).

**Surface Area** The surface area is the sum of all the areas of all the shapes that cover the surface of the object.

**Volume** The amount of space occupied by a solid is called its volume.

## Surface Area and Volume of Plane Figures

**Cuboid** (Rectangular Solid)



Let length, breadth and height are respectively l, b and h. Total number of faces = 6

**Surface Area** (SA) = 2(bh + lh)

**Base Area** (B) = lb

**Total** (C + 2B) = 2(bh + lh + lb)

**Volume**  $(V) = l \times b \times h = \sqrt{A_1 A_2 A_3}$ 

where,  $A_1$ ,  $A_2$  and  $A_3$  are areas of base, side and end face respectively.

This chapter is very important for entrance. In this chapter, we study the surface area and volume of various solid figures such as cuboid, cube, cylinder, cone, sphere etc.

#### Cube



It has six equal faces of each side (edge) *a*.

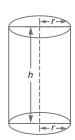
**Surface Area** 
$$(SA) = 4a^2$$

**Base Area** 
$$(B) = a^2$$

**Total** 
$$(C + 2B) = 6a^2$$

**Volume** 
$$(V) = a^3$$

#### Cylinder



Let the radius of base and height be respectively r and h.

#### **Surface Area** (SA)

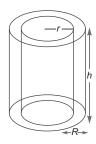
= (base perimeter) × (height) = 
$$2\pi rh$$

**Base Area** (*B*) =  $\pi r^2$ 

**Total** 
$$(C + 2B) = 2\pi r(h + r)$$

**Volume**  $(V) = \pi r^2 h$ 

#### Hollow cylinder



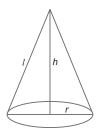
**Surface Area** (*SA*) =  $2\pi Rh + 2\pi rh$ 

**Base Area** (*B*) = 
$$\pi (R^2 - r^2)$$

**Total** 
$$(C + 2B) = 2\pi h(R + r) + 2\pi (R^2 - r^2)$$

**Volume** 
$$(V) = \pi (R^2 - r^2)h$$

#### Cone (Right Circular)



Let the radius of base, altitude and slant height be respectively r, h and l.

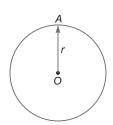
**Surface Area** (*SA*) = 
$$\pi rl$$
, where  $l = \sqrt{h^2 + r^2}$ 

**Base Area** (B) = 
$$\pi r^2$$

**Total** 
$$(C + 2B) = \pi r(l + r)$$

**Volume** (V) = 
$$\frac{1}{3}$$
 (Base area) × Altitude =  $\frac{1}{3}\pi r^2 \times h$ 

#### **Sphere**



Let radius of sphere be r.

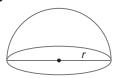
**Surface Area** (*SA*) =  $4\pi r^2$ 

**Base Area** (B) = No Base

**Total**  $(C + 2B) = 4\pi r^2$ 

**Volume**  $(V) = \frac{4}{3}\pi r^3$ 

#### Hemisphere



**Surface Area** (*SA*) =  $2\pi r^2$ 

**Base Area**  $(B) = \pi r^2$ 

**Total**  $(C + 2B) = 3\pi r^2$ 

**Volume**  $(V) = \frac{2}{3}\pi r^3$ 

#### SURFACE AREA AND VOLUME

**Example 1** The volume and surface area of a cube sides measures 4 cm are respectively

(a) 
$$64 \text{ cm}^3$$
,  $96 \text{ cm}^2$ 

(b) 
$$64 \text{ cm}^3$$
,  $80 \text{ cm}^2$ 

(c) 
$$64 \text{ cm}^3$$
,  $90 \text{ cm}^2$ 

(d) 
$$60 \text{ cm}^3$$
,  $96 \text{ cm}^2$ 

**Sol.** (a) Volume = 
$$a^3 = (4)^3 = 64 \text{ cm}^3$$

and surface area =  $6a^2$ 

$$= 6 \times 4 \times 4 = 96 \text{ cm}^2$$

**Example 2** How many bricks each measuring  $25 \text{ cm} \times 11.5 \text{ cm} \times 6 \text{ cm}$  will be needed to construct a wall 8 m long, 6 m high and 22.5 cm thick?

- (a) 6262
- (b) 6260
- (c) 6624
- (d) 6520

**Sol.** (b) Number of bricks required

$$= \frac{\text{Volume of wall in cm}^3}{\text{Volume of 1 brick in cm}^3}$$
$$= \frac{800 \times 600 \times 22.5}{25 \times 11.5 \times 6} = 6260$$

**Example 3** If the radius of a cylinder is increased from 6 cm to 14 cm and the surface area of it kept same. If its height is 5 cm, what will be its new height?

(a) 
$$\frac{15}{7}$$
 cm

(b) 
$$\frac{15}{8}$$
 cm

(c) 
$$\frac{17}{7}$$
 cm

(d) None of these

**Sol.** (a) When r = 6 cm and b = 5 cm, then surface area of cylinder  $S_1 = 2\pi rh$ 

$$=2\pi \times 6 \times 5 = 60\pi \text{ cm}^2$$

When  $r_1 = 14$  cm and height  $b_1$  cm, then the surface area of cylinder

$$S_2 = 2\pi r_1 h_1 = 2\pi \times 14 h_1$$

According to the given condition,

$$S_1 = S_2$$

$$\Rightarrow 60\pi = 28\pi h_1$$

$$\Rightarrow h_1 = \frac{60}{28} = \frac{15}{7} \text{ cm}$$

**Example 4** The volume and curved surface area of a cylinder of length 60 cm with diameter of the base 7 cm are respectively

(a) 
$$2310 \text{ cm}^3$$
,  $1320 \text{ cm}^2$  (b)  $2410 \text{ cm}^3$ ,  $1320 \text{ cm}^2$ 

(c) 
$$2310 \text{ cm}^3$$
,  $1350 \text{ cm}^2$  (d)  $2410 \text{ cm}^3$ ,  $1350 \text{ cm}^2$ 

**Sol.** (a) Volume of cylinder =  $\pi r^2 h$ 

$$= \frac{22}{7} \times 3.5 \times 3.5 \times 60 = 2310 \text{ cm}^3$$

and curved surface area =  $2\pi rh$ 

$$= 2 \times \frac{22}{7} \times 3.5 \times 60 = 1320 \text{ cm}^2$$

**Example 5** The slant height, volume and curved surface area of a cone of base radius 21 cm and height 28 cm are respectively

- (a) 35 cm 12936 cm<sup>3</sup>, 2310 cm<sup>2</sup>
- (b) 35 cm 12930 cm<sup>3</sup>, 2320 cm<sup>2</sup>
- (c) 36 cm 12940 cm<sup>3</sup>, 2325 cm<sup>2</sup>
- (d) None of the above

**Sol.** (a) Slant height, 
$$l = \sqrt{r^2 + b^2} = \sqrt{(21)^2 + (28)^2}$$
  
=  $\sqrt{1225} = 35$  cm

Volume of cone = 
$$\frac{1}{3}\pi r^2 h$$

$$=\frac{1}{3} \times \frac{22}{7} \times 21 \times 21 \times 28 = 12936 \text{ cm}^3$$

and curved surface area of cone =  $\pi rl$ 

$$=\frac{22}{7}\times21\times35=2310$$
 cm<sup>2</sup>

**Example 6** A circus tent is cylindrical to a height of 4 m and conical above it. If its diameter is 105 m and its slant height is 40 m. The total area of the canvas required (in m<sup>2</sup>) is

- (a) 7928 m<sup>2</sup> (c) 7923 m<sup>2</sup>
- (b)  $7920 \text{ m}^2$
- (d) None of these

**Sol.** (b) Total area of canvas =  $(2\pi rh + \pi rl)$ 

$$= \left(2 \times \frac{22}{7} \times \frac{105}{2} \times 4 + \frac{22}{7} \times \frac{105}{2} \times 40\right)$$
$$= 1320 + 6600 = 7920 \text{ m}^2$$

**Example 7** The volume and total surface area of a hemisphere of diameter 21 cm are respectively

- (a)  $2420 \text{ cm}^3$ ,  $1038 \text{ cm}^2$
- (b) 2422 cm<sup>3</sup>, 1039 cm<sup>2</sup>
- (c)  $2425.5 \text{ cm}^3$ ,  $1039.5 \text{ cm}^2$
- (d) None of the above

**Sol.** (c) Volume of hemisphere =  $\frac{2}{3}\pi r^3$ 

$$= \frac{2}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2}$$
$$= 2425.5 \text{ cm}^3$$

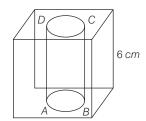
and total surface area of hemisphere

$$= 3\pi r^2 = 3 \times \frac{22}{7} \times 10.5 \times 10.5$$
$$= 1039.5 \text{ cm}^2$$

**Example 8** Find the volume of cylinder which is exactly fit into a cube of side 6 cm.

- (a) 171 cm<sup>3</sup>
- (b) 169.71 cm<sup>3</sup>
- (c)  $173 \text{ cm}^3$
- (d) None of the above

**Sol.** (*b*)



Here, height of cylinder =6 cm

Radius of cylinder =  $\frac{6}{2}$  = 3 cm

Now, volume of cylinder =  $\pi r^2 h$ 

$$= \frac{22}{7} \times (3)^2 \times 6$$
$$= \frac{1188}{7} = 169.71 \text{ cm}^3$$

# PRACTICE EXERCISE

- 1. The volume of a cuboid is 440 cm<sup>3</sup>, the area of its base is 88 cm<sup>2</sup>, then its height is (a) 5 cm (b) 10 cm (c) 11 cm (d) 6 cm
- **2.** The surface area of a cube is 486 sq m, then its volume is
  - (a)  $729 \text{ m}^3$  (b)  $781 \text{ m}^3$  (c)  $625 \text{ m}^3$  (d)  $879 \text{ m}^3$
- **3.** Rectangular sand box is 5 m wide and 2 m long. How many cubic metres of sand are needed to fill the box upto a depth of 10 cm?
  - (a)  $1 \text{ m}^3$
- (b) 10 m<sup>3</sup>
- (c) 100 m<sup>3</sup>
- (d) 1000 m<sup>3</sup>
- **4.** If the height of a cylinder becomes 1/4 of the original height and the radius is doubled, then which of the following will be true?
  - (a) Volume of the cylinder will be doubled
  - (b) Volume of the cylinder will remain unchanged
  - (c) Volume of the cylinder will be halved
  - (d) Volume of the cylinder will be  $\frac{1}{4}$  of the original volume

- **5.** A cube whose side is 5 cm will have surface area is equal to
  - (a)  $125 \text{ cm}^2$
- (b)  $50 \text{ cm}^2$
- (c)  $100 \text{ cm}^2$
- (d) None of these
- **6.** The maximum length of a pencil that can be kept in a rectangular box of dimensions  $8 \text{ cm} \times 6 \text{ cm} \times 2 \text{ cm}$  is
  - (a)  $2\sqrt{54}$  cm
- (b)  $2\sqrt{26}$  cm
- (c)  $2\sqrt{14}$  cm
- (d)  $2\sqrt{13}$  cm
- 7. The sum of the length, breadth and depth of a cuboid is 20 cm and its diagonal is  $4\sqrt{5}$  cm, then its surface area is
  - (a)  $400 \text{ cm}^2$
- (b)  $420 \text{ cm}^2$
- (c)  $300 \text{ cm}^2$
- (d) 320 cm<sup>2</sup>
- **8.** How many 6 m cubes can be cut from a cuboid measuring  $18 \text{ m} \times 15 \text{ m} \times 8 \text{ m}$ ?
  - (a) 8 (1
- (b) 9
- (c) 10 (d) 7
- **9.** The ratio of radii of two cylinders is 1 : 2 and heights are in the ratio 2 : 3. The ratio of their volumes is
  - (a) 1:6
- (b) 1:9
- (c) 1:3
- (d) 2:9

#### SURFACE AREA AND VOLUME

10.	Two cubes have volumes in the ratio 1:64.
	The ratio of the areas of a face of first cube
	to that of the other is

- (a) 1:4
- (b) 1:8
- (c) 1:16 (d) 1:32
- **11.** If the volumes of two cubes are in the ratio 8:1, then ratio of their edges is
  - (a) 2:1
- (b) 4:1
- (c)  $2\sqrt{2}:1$  (d) 8:1
- **12**. The total surface area of a right circular cylinder whose height is 15 cm and the radius of the base is 7 cm. is
  - (a) 968 cm<sup>2</sup>
- (b)  $2310 \text{ cm}^2$
- (c)  $488 \text{ cm}^2$
- (d)  $1860 \text{ cm}^2$
- **13**. The outer dimensions of a closed wooden box are  $10 \text{ cm} \times 8 \text{ cm} \times 7 \text{ cm}$ . Thickness of the wood is 1 cm. The total cost of wood required to make box, if 1 cm<sup>3</sup> of wood cost ₹ 2 is
  - (a) ₹ 540 (b) ₹ 640
- (c) ₹ 740
- (d) ₹ 780
- **14**. The diameter of a right circular cone is 12 m and the slant height is 10 m. The total surface area of cone is
  - (a)  $\frac{2412}{7}$  m<sup>2</sup>
- (b)  $\frac{2312}{7}$  m<sup>2</sup>
- (c)  $\frac{2112}{7}$  m<sup>2</sup> (d)  $\frac{2012}{7}$  m<sup>2</sup>
- **15**. The dimensions of a field are  $12 \text{ m} \times 10 \text{ m}$ . A pit 5 m long, 4 m wide and 2 m deep is dug in one corner of the field and the earth removed has been evenly spread over the remaining area of the field. The level of the field is raised by

- (a) 30 cm (b) 35 cm (c) 38 cm (d) 40 cm
- **16**. A plate of metal 1 cm thick, 9 cm broad, 81 cm long is melted into a cube. The difference in the surface area of two solids is
  - (a) 1152 cm<sup>2</sup>
- (b) 1150 cm<sup>2</sup>
- (c)  $1052 \text{ cm}^2$
- (d) 1050 cm<sup>2</sup>
- **17.** The sum of the radius of the base and the height of a cylinder is 37 m. If the total surface area of the solid cylinder is 1628 m<sup>2</sup>. The circumference of base of cylinder
  - (a) 11 m
- (b) 22 m
- (c) 33 m
- (d) 44 m

- **18**. The volume of a metallic cylindrical pipe is 748 cm<sup>3</sup>. Its length is 14 cm and its external radius is 9 cm. Then, its thickness
  - (a) 1 cm
- (b) 1.5 cm
- (c) 2 cm
- (d) 2.5 cm
- **19.** A 20 m deep well with diameter 14 m is dug up and the earth from digging is spread evenly to form a platform  $22m \times 14$  m. The height of platform is
  - (a) 10 m
- (b) 15 m
- (c) 20 m
- (d) 25 m
- **20**. The circumference of the base of a 9 m high wooden solid cone is 44 m. The slant height of the cone is
  - (a)  $\sqrt{120}$  m
- (b)  $\sqrt{130}$  m
- (c)  $\sqrt{150}$  m
- (d)  $7\sqrt{5}$  m
- **21.** How many metres of cloth 50 m wide will be required to make a conical tent, the radius of whose base is 7 m and whose height is 24 m?
  - (a) 9 m
- (b) 11 m
- (c) 12 m
- (d) 13 m
- **22.** It is required to make a hollow cone 24 cm high whose base radius is 7 cm. The area of sheet required including the base is
  - (a)  $700 \text{ cm}^2$
- (b) 704 cm<sup>2</sup>
- (c)  $708 \text{ cm}^2$
- (d) 710 cm<sup>2</sup>
- **23.** The radius of a sphere whose surface area is  $154 \, \mathrm{cm}^2$ , is
  - (a) 3.5 cm
- (b) 3.6 cm
- (c) 3.7 cm
- (d) None of these
- **24.** A hemispherical bowl made of brass has inner diameter 10.5 cm. The cost of tin plating it on the inside at the rate of ₹ 16 per 100 cm<sup>2</sup> is
  - (a) ₹ 28
- (b) ₹ 27.72
- (c) ₹29.27
- (d) ₹28.52
- **25.** If the volume of a sphere is double that of the other sphere, then the ratio of their radii is
  - (a)  $2\sqrt{2}:1$
- (b)  $\sqrt[3]{2}:1$
- (c) 1:  $\sqrt[3]{2}$
- (d) 2:1

- **26.** The internal and external diameters of a hollow hemispherical vessel are 24 cm and 25 cm respectively. The total area to be painted is
  - (a)  $\frac{13211}{7}$  cm<sup>2</sup>
- (b)  $\frac{26961}{14}$  cm<sup>2</sup>
- (c)  $\frac{6961}{14}$  cm<sup>2</sup>
- (d)  $\frac{16951}{14}$  cm<sup>2</sup>
- **27.** Three cubes each of side 10 cm are joined end to end. The surface area of the resultant figure is
  - (a) 1400 cm<sup>2</sup>
- (b) 1500 cm<sup>2</sup>
- (c) 1450 cm<sup>2</sup>
- (d) 1550 cm<sup>2</sup>
- **28.** Height of a solid cylinder is 10 cm and diameter 8 cm. Two equal conical holes have been made from its both ends. If the diameter of the hole is 6 cm and height 4 cm. The volume of remaining portion is
  - (a)  $24 \pi \text{ cm}^3$
- (b)  $36\pi \text{ cm}^3$
- (c)  $72\pi \text{ cm}^3$
- (d)  $136\pi \text{ cm}^3$
- **29.** The length, breadth and height of a room are in the ratio of 3:2:1. If its volume be  $1296 \text{ m}^3$ , its breadth is
  - (a) 12 m
- (b) 18 m
- (c) 16 m
- (d) 24 m
- **30.** The diameters of two cones are equal. If their slant height be in the ratio 5 : 7, the ratio of their curved surface areas is
  - (a) 25:7
- (b) 25:49
- (c) 5:49
- (d) 5:7
- **31.** A rectangular paper 11 cm by 8 cm can be exactly wrapped to cover the curved surface of a cylinder of height 8 cm. The volume of the cylinder is

- (a) 66 cm<sup>3</sup>
- (b)  $77 \text{ cm}^3$
- (c)  $88 \text{ cm}^3$
- (d)  $12 \text{ cm}^3$
- **32.** A cylindrical tube open at both ends is made of metal. The internal diameter of the tube is 11.2 cm and its length is 21 cm. The metal everywhere is 0.8 cm thick. The volume of the metal is
  - (a)  $316 \text{ cm}^3$
- (b)  $310 \text{ cm}^3$
- (c)  $306.24 \text{ cm}^3$
- (d) 280.52 cm<sup>3</sup>
- **33.** A solid right circular cylinder of radius 8 cm and height 2 cm is melted into a right circular cone with radius of the base 8 cm. Its height is
  - (a) 5 cm
- (b) 6 cm
- (c) 5.75 cm
- (d) 6.25 cm
- **34.** A hemispherical bowl is made from a metal sheet having thickness 0.3 cm. The inner radius of the bowl is 24.7 cm. The cost of polishing its outer surface at the rate of ₹ 4 per 100cm<sup>2</sup> is
  - (take  $\pi = 3.14$ )
  - (a) ₹ 159
- (b) ₹ 157
- (c) ₹ 160
- (d) ₹ 165
- **35.** If the radius of a cylinder is increased from 7 m to 10 m and the surface area of it kept same. If its height is 4 m, then new height will be
  - (a) 2.8 m
- (b) 3.1 m
- (c) 3.6 m
- (d) 3.3 m
- **36.** Find the volume of the cone which is exactly fit in the cube of side 8 cm.
  - (a) 133 cm<sup>3</sup>
- (b) 134 cm<sup>3</sup>
- (c)  $135 \text{ cm}^3$
- (d) None of these

### Answers

1	(a)	2	(a)	3	(a)	4	(b)	5	(c)	6	(b)	7	(d)	8	(c)	9	(a)	10	(c)
11	(a)	12	(a)	13	(b)	14	(c)	15	(d)	16	(a)	17	(d)	18	(a)	19	(a)	20	(b)
21	(b)	22	(b)	23	(a)	24	(b)	25	(b)	26	(b)	27	(a)	28	(d)	29	(a)	30	(d)
31	(b)	32	(c)	33	(b)	34	(b)	35	(a)	36	(c)								

### **Hints and Solutions**

**1.** Height =  $\frac{\text{Volume of the cuboid}}{\text{Area of its base}}$ 

$$=\frac{440}{88}$$
 = 5 cm

**2.** Let edge of cube be *a*.

Surface area of the cube =  $6a^2$ 

$$\therefore \qquad 6a^2 = 486$$

$$\Rightarrow$$
  $a^2 = 81$ 

$$\therefore \text{ Volume of the cube} = (\text{edge})^3$$
$$= (9)^3 = 729 \text{ m}^3$$

**3.** Sand needed to fill the tank

$$= \left(5 \times 2 \times \frac{10}{100}\right) = 1 \text{ m}^3$$

**4.** We know that, the volume of a cylinder having base radius r and height h is  $V = \pi r^2 h$ 

Now, if new height is  $\frac{1}{4}$ th of the original height and the radius is doubled, i.e.

$$h' = \frac{1}{4}h$$
 and  $r' = 2r$ , then

New volume, 
$$V' = \pi (2r)^2 \times \frac{1}{4}h = 4\pi r^2 \times \frac{1}{4}h$$
  
=  $\pi r^2 h = V$ 

Hence, the new volume of cylinder is same as the original volume.

**5.** Now, surface area of cube  $=4(\text{side})^2$ 

$$= 4 \times (5)^2 = 100 \text{ cm}^2$$

**6.** Length of longest pencil

$$= \sqrt{8^2 + 6^2 + 2^2} = \sqrt{104} = 2\sqrt{26} \text{ cm}$$

7. Given, l + b + h = 20 cmand  $\sqrt{l^2 + h^2 + h^2} = 4\sqrt{5}$ 

.. Surface area = 
$$2 (lb + bh + hl)$$
  
=  $(l + b + h)^2 - (l^2 + b^2 + h^2)$   
=  $(20)^2 - (4\sqrt{5})^2$   
=  $400 - 80 = 320 \text{ cm}^2$ 

**8.** Volume of cube =  $6 \times 6 \times 6 \text{ m}^3$ 

Volume of cuboid =  $18 \times 15 \times 8 \text{ m}^3$ 

:. Required number of cube

$$= \frac{\text{Volume of cuboid}}{\text{Volume of cube}}$$
$$= \frac{18 \times 15 \times 8}{6 \times 6 \times 6} = 10$$

**9.** Let  $r_1$ ,  $r_2$  be radii of two cylinders and  $h_1$ ,  $h_2$  be their heights.

Then, 
$$\frac{r_1}{r_2} = \frac{1}{2}$$
 and  $\frac{h_1}{h_2} = \frac{2}{3}$ 

$$\therefore \frac{V_1}{V_2} = \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2} = \left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2} = \left(\frac{1}{2}\right)^2 \times \frac{2}{3}$$
$$= \frac{1}{4} \times \frac{2}{3} = \frac{1}{6} = 1:6 = \frac{1}{4} \times \frac{2}{3} = \frac{1}{6} = 1:6$$

**10.** Let *a* and *b* be the edges of the two cubes, respectively.

Then, according to the question,

$$a^3: b^3 = 1: 64$$
 [: volume of cube =  $(edge)^3$ ]

$$\Rightarrow \frac{a^3}{h^3} = \frac{1}{64}$$

$$\Rightarrow \qquad \left(\frac{a}{b}\right)^3 = \left(\frac{1}{4}\right)^3$$

$$\Rightarrow \frac{a}{b} = \frac{1}{4}$$
 [taking cube roots on both sides]

Now, ratio of areas, 
$$\left(\frac{a}{b}\right)^2 = \left(\frac{1}{4}\right)^2$$

[: surface area of cube =  $6 \times (edge)^2$ ]

$$\Rightarrow \frac{a^2}{b^2} = \frac{1}{16}$$

$$a^2:b^2=1:16$$

**11.** Let the edges of cubes be x and y, then volumes are  $x^3$  and  $y^3$  respectively.

$$\frac{x^3}{y^3} = \frac{8}{1}$$

$$\Rightarrow \frac{x}{y} = \frac{2}{1}$$

12. Total surface area of right circular cylinder

$$= 2\pi r (h + r) = 2 \times \frac{22}{7} \times 7 (15 + 7)$$
$$= 2 \times 22 \times 22 = 968 \text{ cm}^2$$

**13.** External volume of the box

$$=10 \times 8 \times 7 = 560 \text{ cm}^3$$

Thickness of wood

$$=1 \, \mathrm{cm}$$

Internal length = 10 - 2 = 8 cm,

breadth = 8 - 2 = 6 cm, height = 7 - 2 = 5 cm

- $\therefore$  Internal volume =  $8 \times 6 \times 5 = 240 \text{ cm}^2$
- ⇒ Volume of wood

= External volume – Internal volume

$$= 560 - 240 = 320 \text{ cm}^3$$

:. Total cost of wood required to make the box

**14.** Total surface area =  $\pi r (l + r)$ 

$$= \frac{22}{7} \times 6 \times (10 + 6) = \frac{2112}{7} \text{ m}^2$$

**15.** Area of the field = Length  $\times$  Breadth

$$=12 \times 10 = 120 \text{ m}^2$$

Area of the pit's surface =  $5 \times 4 = 20 \text{ m}^2$ 

Area on which the earth is to be spread

$$=120-20=100 \text{ m}^2$$

Volume of earth dug out =  $5 \times 4 \times 2 = 40 \text{ m}^3$ 

- $\therefore \text{ Level of field raised} = \frac{40}{100} = \frac{2}{5} \text{ m}$  $= \frac{2}{5} \times 100 = 40 \text{ cm}$
- **16.** Let the edge of the cube be 'x'.

Then, volume of the cube is

$$x^3 = 9 \times 81 \times 1 \text{ cm}^3 = 729 \text{ cm}^3$$

$$x = \sqrt[3]{729} = 9 \text{ cm}$$

Surface area of metal plate

$$= 2 (81 \times 9 + 9 \times 1 + 1 \times 81) = 2 \times (819)$$
$$= 1638 \text{ cm}^2$$

Total surface area of the cube

$$= 6 \text{ (edge)}^2 = 6 \text{ (9)}^2 = 486 \text{ cm}^2$$

:. Difference of surface area of two solids

$$=1638 - 486 = 1152 \text{ cm}^2$$

**17.** Given, r + h = 37 m

and total surface area = 1628m<sup>2</sup>

$$= 2\pi r (h + r) = 1628 \text{ m}^2 \implies 2\pi r (37) = 1628$$

$$\Rightarrow \qquad r = \frac{1628 \times 7}{2 \times 22 \times 37} = 7$$

 $\therefore$  Circumference of its base =  $2\pi r$ 

$$= 2 \times \frac{22}{7} \times 7 = 44 \text{ m}$$

**18.** External radius,  $R = 9 \, \text{cm}$ 

Internal radius be r cm.

Length of pipe = 14 cm

Since, volume of pipe =  $770 \text{ cm}^3$ 

⇒ Volume of hollow cylinder = 748

$$\pi (R^2 - r^2) h = 748$$

$$\Rightarrow 81 - r^2 = \frac{748 \times 7}{22 \times 14} = 17$$

$$\Rightarrow$$
  $r^2 = 64 \Rightarrow r = 8 \text{ cm}$ 

$$\therefore$$
 Thickness =  $R - r = 9 - 8 = 1$  cm

**19.** Volume of earth dug out from the well

$$\pi r^2 h = \frac{22}{7} \times \left(\frac{14}{2}\right)^2 \times 20$$
$$= 22 \times 7 \times 20 \text{ m}^3$$

Let height of platform be *h* m.

$$\therefore$$
 Volume of platform =  $22 \times 14 \times h$ 

$$\Rightarrow 22 \times 14 \times h = 22 \times 7 \times 20$$

$$\Rightarrow h = \frac{22 \times 7 \times 20}{22 \times 14} = 10 \text{ m}$$

**20.** Since, circumference of cone = 44 m

$$\Rightarrow 2\pi r = 44 \Rightarrow r = \frac{44}{2\pi} = 7 \text{m}$$

∴ Slant height = 
$$\sqrt{r^2 + h^2}$$
 =  $\sqrt{49 + 81}$   
=  $\sqrt{130}$  m<sup>3</sup>

**21.** Slant height = 
$$\sqrt{r^2 + h^2} = \sqrt{24^2 + 7^2}$$
  
=  $\sqrt{576 + 49}$   
=  $\sqrt{625} = 25$ 

#### **SURFACE AREA AND VOLUME**



Curved surface area = 
$$\pi rl = \frac{22}{7} \times 7 \times 25 = 550 \text{ m}^2$$

Since, width of cloth = 
$$50 \, \text{m}$$

∴ Length of required cloth = 
$$\frac{550}{50}$$
 = 11 m

**22.** Given, 
$$h = 24$$
 cm,  $r = 7$  cm

Now, slant height

$$l = \sqrt{r^2 + h^2}$$

$$= \sqrt{24^2 + 7^2} = \sqrt{576 + 49}$$

$$= \sqrt{625} = 25 \text{ cm}$$

:. Area of metal sheet required

= Total surface area of cone  
= 
$$\pi r (l + r) = \frac{22}{7} \times 7 (7 + 25)$$
  
= 22 (32) = 704 cm<sup>2</sup>

**23.** Let the radius of the sphere be 
$$r$$
 cm.

Surface area of the sphere = 
$$154 \text{ cm}^2$$

$$\therefore 4\pi r^2 = 154$$

[: surface area of a sphere = 
$$4\pi r^2$$
]

$$\Rightarrow$$
  $4 \times \frac{22}{7} \times r^2 = 154$ 

$$\Rightarrow$$
  $r^2 = \frac{154 \times 7}{22 \times 4} = 12.25$ 

$$\Rightarrow$$
  $r = \sqrt{12.25} = 3.5 \text{ cm}$ 

Hence, the radius of the sphere is 3.5 cm.

**24.** We have, inner diameter = 
$$10.5 \text{ cm}$$

$$\therefore \text{ Inner radius } (r) = \frac{10.5}{2} = 5.25 \text{ cm}$$

Curved surface area of hemispherical bowl of

inner side = 
$$2\pi r^2 = 2 \times \frac{22}{7} \times (5.25)^2$$

$$=2\times\frac{22}{7}\times5.25\times5.25$$

$$=173.25 \, \text{cm}^2$$

∴ Cost of tin plating on inside for 
$$100 \text{ cm}^2$$
  
= ₹16

$$=\frac{16 \times 173.25}{100} = ₹27.72$$

# **25.** Let $r_1$ and $r_2$ be the radii and $V_1$ , $V_2$ be its volume of spheres respectively.

Given, 
$$V_1 = 2V_2 \Rightarrow \frac{V_1}{V_2} = 2$$

$$\therefore \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{2}{1} \Rightarrow \frac{r_1^3}{r_2^3} = \frac{2}{1}$$

$$\Rightarrow \qquad \frac{r_1}{r_2} = \frac{\sqrt[3]{2}}{1}$$

#### **26.** Internal radius (r) = 12 cm

and external radius 
$$(R) = \frac{25}{2}$$
 cm

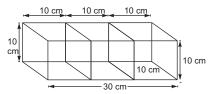
= Internal area + External area + Area of edge 
$$2\pi r^2 + 2\pi R^2 + \pi (R^2 - r^2)$$

$$= 2 \times \frac{22}{7} \times 12 \times 12 + 2 \times \frac{22}{7} \times \frac{25}{2} \times \frac{25}{2} + \frac{22}{7} \left(\frac{25}{2} \times \frac{25}{2} - 12 \times 12\right)$$

$$= \frac{6336}{7} + \frac{6875}{7} + \frac{539}{14}$$
$$= \frac{26422}{14} + \frac{539}{14} = \frac{26961}{14} \text{ cm}^2$$

## **27.** If three cubes each of side 10 cm are joined, then a cuboid will be formed of dimensions

$$30 \,\mathrm{cm} \times 10 \,\mathrm{cm} \times 10 \,\mathrm{cm}$$



$$\therefore$$
 Surface area of the cuboid =  $2[lb + bh + hl]$ 

$$= 2[30 \times 10 + 10 \times 10 + 30 \times 10]$$

$$= 2[300 + 100 + 300] = 2[700] = 1400 \text{ cm}^2$$

**28.** Volume of cylinder =  $\pi(4)^2 \times 10 = 160\pi \text{ cm}^3$ 

Volume of one cone = 
$$\frac{1}{3} \times \pi \times 3^2 \times 4$$

$$=12\pi \text{ cm}^{3}$$

- $\therefore$  Volume of both cones =  $24 \pi \text{ cm}^3$
- :. Volume of remaining portion =  $160\pi - 24\pi = 136\pi \text{ cm}^3$
- **29.** Let the sides be 3x, 2x and 1x.

$$\therefore$$
 Volume =  $l \times b \times h$ 

$$\Rightarrow$$
 1296 = 3 $x \times 2x \times x$ 

$$\Rightarrow$$
  $6x^3 = 1296$ 

$$\Rightarrow$$
  $x^3 = 216 \Rightarrow x = 6$ 

- $\therefore$  Breadth = 2 × 6 = 12 m
- **30.** Ratios of two curved surface area

= 
$$C_1$$
:  $C_2 = \pi r l_1 : \pi r l_2$   
=  $l_1 : l_2 = 5 : 7$ 

**31.** : Surface area of cylinder = Area of paper

$$\Rightarrow 2\pi rh = l \times b$$

$$\Rightarrow 2\pi r \times 8 = 11 \times 8$$

$$\Rightarrow 2\pi r = 11 \Rightarrow r = \frac{11 \times 7}{2 \times 22} = \frac{7}{4}$$

$$\therefore$$
 Volume =  $\pi \left(\frac{7}{4}\right)^2 \times 8 = 77 \text{cm}^3$ 

**32.** Volume of metal =  $\pi [R^2 - r^2] h$ 

$$=\frac{22}{7} [6^2 - (5.6)^2] \times 21$$

$$= 66 \times 4.64$$

$$= 306.24 \text{ cm}^3$$

**33.** Volume of circular cylinder =  $\pi(8)^2(2) = 128\pi$  cm

Volume of right circular cone  $=\frac{1}{3}\pi r^2 h$ 

$$\therefore \qquad \frac{1}{3}\pi r^2 h = 128\pi$$

$$\Rightarrow \frac{1}{3} \times \pi \times (8)^2 \times h = 128\pi$$

$$\Rightarrow$$
  $h = 6 \text{ cm}$ 

**34.** Given, inner radius of the hemispherical bowl  $= 24.7 \, \text{cm}$ 

Thickness of metal sheet =  $0.3 \, \text{cm}$ 

Now, outer radius of the hemispherical bowl

$$= 24.7 + 0.3 = 25 \text{ cm}$$

 $\therefore$  Outer surface area of the hemispherical bowl

$$=2\pi r^2$$

$$= 2 \times 3.14 \times (25)^2 = 157 \times 25 = 3925 \text{ cm}^2$$

Now, cost of polishing  $100 \text{ cm}^2 = ₹ 4$ 

:. Cost of polishing 3925 cm<sup>2</sup>

$$=\frac{4\times3925}{100}$$
=₹157

**35.** When radius r = 7 m and height h = 4 m,

then surface area of cylinder,  $S_1 = 2\pi rh$ 

$$=2\pi\times7\times4=56\pi m^2$$

When radius  $r_1 = 10 \text{ m}$  and height  $= h_1 \text{ m}$ , then surface area of new cylinder,

$$S_2 = 2\pi r_1 h_1 = 2\pi \times 10 \times h_1$$
  
=  $20\pi h_1$ 

According to the given condition,

$$S_1 = S_2$$

$$\Rightarrow$$
 56 $\pi$  = 20 $\pi \times h_1$ 

$$\Rightarrow h_1 = \frac{56}{20} = 2.8 \,\mathrm{m}$$

**36.** Here, height of cone h=8 cm



and radius of cone  $r = \frac{8}{2} = 4$  cm

Now, volume of cone =  $\frac{1}{3}\pi r^2 h$ 

$$=\frac{1}{3}\times\frac{22}{7}\times(4)^2\times8$$

$$=\frac{2816}{21}$$
=135 cm<sup>3</sup>