CHAPTER

08

ALGEBRAIC EXPRESSION AND IDENTITIES

In this chapter, we study the various terms of Algebraic expression, operations on polynomials, various identities, factorisation by various method.

Constant A symbol having a fixed numerical value is called a constant.

Variable A symbol which takes various numerical values is called a variable.

e.g., $P = 2(l + b)$

Here, 2 is a constant and l and b are variables.

Algebraic Expression

An algebraic expression is a combination of constants and variables connected by fundamental operations $(+, -, \times, \div)$. e.g. $2x + 3$, $8a^2b + a^3b^3 - 5a$ etc .

Terms

The separated parts of an algebraic expression are called its term.

e.g. $2x$ and 3 are the terms of expression $2x + 3$.

Like and Unlike Terms

The terms having same variable and the same exponents are like terms, otherwise it has, unlike terms.

e.g. In $x^2 - xy + 2x^2 + y$, x^2 and $2x^2$ are like terms and $-xy$ and y are unlike terms.

Factors

Each term in an algebraic expression is a product of one or more numbers and variables. These numbers and variables are known as the factors of that term.

e.g. $7x$ is the product of number 7 and variable x. So, 7 and x are factors of $7x$.

Coefficients

In a term of an algebraic expression, the numerical value of a term is called coefficient and any of the factors with the sign of the term is called the coefficient of the product of the other factors.

e.g. In $3xy$, the coefficient of y is $3x$, the coefficient of x is $3y$ and the coefficient of xy is 3.

Polynomial

An algebraic expression that contains one or more terms with non-zero coefficients, is called a polynomial.

e.g. $a + b + c + d$, $3xy$ and $7xy - 10$, etc. are all polynomials.

Note (i) An expression may contain a term involving rational power of a variable but in a polynomial, the power of each variable must be a non-negative integer.

(ii) The highest power of the variable (or in more than one variable, the sum of the highest power of the variable) is the degree of the polynomial.

Monomial

An algebraic expression that contains only one term, is called a monomial.

e.g. 3, $-6abc$ and $5x^2y$, etc., are all monomials.

Binomial

An algebraic expression that contains two terms, is called a binomial.
e.g. $x + 5$, $x^3 + 7$ and $a^2 - 2abc$, etc. are all binomials.

Trinomial

An algebraic expression that contains three terms, is called a trinomial.

e.g. $2x - y + 3$ and $3 + xyz + x^3$, etc. are all trinomials.

Quadrinomial

An algebraic expression that contains four terms, is called a quadrinomial. e.g. $a^3 + b^3 + c^3 + 3abc$ and $ab + bc + ca + abc$, etc., are all quadrinomials.

Example 1 The example of binomial is

(a) $5+2x$ (b) $2x^2$
(c) $-2x-5y-1$ (d) $3+xy+2y^2$

Sol. *(a)* We have, $5+2x$, only this algebraic contains two terms. Hence, $5+2x$ is the example of binomial.

Fundamental Operations on Polynomials

Some operations based on polynomials are discussed below

I. **Addition of Polynomials** Polynomials can be added by arranging their like terms and combining them.

II. **Subtraction of Polynomials** Polynomials can be subtracted by arranging their like terms and by changing sign of each term of the polynomial to be subtracted and then added.

III. **Multiplication of Polynomials** We know that,

(a) The product of two factors with like signs is positive and product of unlike signs is negative.

(b) If x is any variable and m, n are positive integers, then $x^m \times x^n = x^{m+n}$

Thus, $x^3 \times x^6 = x^{(3+6)} = x^9$.

IV. (a) **Division of a Monomial by a Monomial**
We have, quotient of two monomials
=(quotient of their coefficients)
$\times$ (quotients of two monomials)

(b) **Division of a Polynomial by a Monomial**

To dividing a polynomial by a monomial, we can divide each term of the polynomial by the monomial.

e.g. $\dfrac{2x^2+6x+8}{2}=\dfrac{2x^2}{2}+\dfrac{6x}{2}+\dfrac{8}{2}$

$=x^2+3x+4$

(c) **Division of a Polynomial by a Polynomial**

The following steps are given below

(i) Firstly, arrange the terms of the dividend and divisor in descending order of their degrees.

(ii) Divide the first term of the dividend by the first term of the divisor to obtain the first term of the quotient.

(iii) Multiply all the terms of the divisor by the first term of the quotient and subtract the result from the dividend.

(iv) Consider the remainder (if any) as a new dividend and proceed as before.

(v) Repeat this process till we obtain a remainder which is either 0 or a polynomial of degree less than the degree of the divisor.

Note In polynomials, we have dividend = (divisor × quotient) + remainder

Example 2 The sum of $3x^2-4x+5$ and $9x-10$ is

(a) $3x^2+5x$ (b) $3x^2+5x-5$
(c) $5x-5$ (d) $3x^2-5x+5$

Sol. *(b)* The sum of $3x^2-4x+5$ and $9x-10$, is

$$\begin{array}{r} 3x^2-4x+5 \\ +9x-10 \\ \hline 3x^2+5x-5 \\ \hline \end{array}$$

Example 3 The product of $(3x+7y)$ and $(3x-7y)$ is

(a) $9x^2-49y^2$ (b) $39x^2-49y^2$
(c) $9x^2+49y^2$ (d) $9x^2-49y$

Sol. *(a)* $(3x+7y)\times(3x-7y)$

$=3x(3x-7y)+7y(3x-7y)$

$=9x^2-21xy+21xy-49y^2=9x^2-49y^2$

Example 4 Find the quotient and the remainder when x^4+1 is divided by $x-1$.

(a) x^3+x^2+x+1, 2 (b) x^3+x^2-x+1, 2
(c) x^3+x^2+x+1, 3 (d) None of these

Sol. *(a)* Using long division method,

$$\begin{array}{r|l} & x^3+x^2+x+1 \\ \hline x-1 & x^4+1 \\ & \underline{\overset{-}{}x^4\overset{+}{-}x^3} \\ & x^3+1 \\ & \underline{\overset{-}{}x^3\overset{+}{-}x^2} \\ & x^2+1 \\ & \underline{\overset{-}{}x^2\overset{+}{-}x} \\ & x+1 \\ & \underline{\overset{-}{}x\overset{+}{-}1} \\ & 2 \end{array}$$

Hence, quotient $=x^3+x^2+x+1$ and remainder = 2.

Identity

An identity is an equality which is true for all values of the variables.

e.g. $(a+b)^2=(a)^2+2ab+(b)^2$

Some Important Identities

- $(a+b)^2=a^2+2ab+b^2$
- $(a-b)^2=a^2-2ab+b^2$
- $(a+b)(a-b)=a^2-b^2$
- $(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$
  $=a^2+b^2+c^2+2(ab+bc+ca)$
- $(a+b-c)^2=a^2+b^2+c^2+2ab-2bc-2ca$
- $(a-b+c)^2=a^2+b^2+c^2-2ab-2bc+2ca$
- $(-a+b+c)^2=a^2+b^2+c^2-2ab+2bc-2ca$
- $(a-b-c)^2=a^2+b^2+c^2-2ab+2bc-2ca$
- $(a+b)^3=a^3+b^3+3ab(a+b)$
- $(a-b)^3=a^3-b^3-3ab(a-b)$
- $a^3+b^3=(a+b)^3-3ab(a+b)$
- $a^3+b^3=(a+b)(a^2+b^2-ab)$

- $a^3 - b^3 = (a-b)^3 + 3ab(a-b)$
- $a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$
- $(x+a)(x+b) = x^2 + (a+b)x + ab$

Example 5 If $p+q=12$ and $pq=22$, then the value of p^2+q^2 is

(a) 95 (b) 105 (c) 100 (d) 110

Sol. (*c*) We know that,

$$(p+q)^2 = p^2 + 2pq + q^2$$

$$\Rightarrow \quad (12)^2 = p^2 + 2\times 22 + q^2 \quad [\because p+q=12 \text{ and } pq=22]$$

$$\Rightarrow \quad 144 = p^2 + 44 + q^2$$

$$\therefore \quad p^2+q^2 = 144 - 44 = 100$$

Example 6 If $x+\frac{1}{x}=3$, then the value of $x-\frac{1}{x}$ is

(a) $+\sqrt{5}$ (b) $-\sqrt{5}$ (c) $\pm\sqrt{5}$ (d) ± 5

Sol. (*c*) Given, $x+\frac{1}{x}=3$

$$\Rightarrow \quad \left(x+\frac{1}{x}\right)^2 = 3^2 \quad \text{[on squaring]}$$

$$\Rightarrow \quad \left(x^2+\frac{1}{x^2}+2\right)=9$$

$$\Rightarrow \quad x^2+\frac{1}{x^2}=7$$

$$\Rightarrow \quad x^2+\frac{1}{x^2}-2=7-2$$

[subtracting 2 on both sides]

$$\Rightarrow \quad x^2+\frac{1}{x^2}-2=5 \quad \Rightarrow \quad \left(x-\frac{1}{x}\right)^2=5$$

$$\Rightarrow \quad \left(x-\frac{1}{x}\right)=\pm\sqrt{5}$$

Example 7 If $4x-5z=16$ and $zx=12$, then the value of $64x^3-125z^3$ is

(a) 15600 (b) 1561
(c) 15616 (d) 15618

Sol. (*c*) $64x^2-125z^3=(4x)^3-(5z)^3$

$$= (4x-5z)[(4x)^2+4x\cdot 5z+(5z)^2]$$

$$=16[(4x-5z)^2+40zx+20zx]$$

$$=16[16^2+60\times 12]$$

$$=16\times 976 = 15616$$

Factor and Factorisation

A polynomial $g(x)$ is called **a factor** of polynomial $p(x)$, if $g(x)$ divides $p(x)$ exactly. To express polynomial as the product of polynomial of degree less than that of the given polynomial is called as **factorisation**.

Factorisation by Common Factors

A factor which occurs in each term, is called the common factor. e.g. Factorise $16x^2y+4xy$

We have, $16x^2y=2\times2\times2\times2\times x\times x\times y$

and $4xy=2\times2\times x\times y$

Here, $2\times2\times x\times y$ is common in these two terms.

Factorisation by Splitting Middle Term

Let factors of the quadratic polynomial ax^2+bx+c be $(px+q)$ and $(rx+s)$. Then,

$$ax^2+bx+c=(px+q)(rx+s)$$

$$= prx^2 + (ps+qr)x + qs$$

On comparing the coefficients of x^2, x and constant terms from both sides, we get

$$a=pr,\ b=ps+qr \text{ and } c=qs$$

Here, b is the sum of two numbers ps and qr, whose product is $(ps)(qr)=(pr)(qs)=ac$.

Thus, to factorise ax^2+bx+c, write b as the sum of two numbers, whose product is ac.

Note To factorise ax^2+bx-c and ax^2-bx-c, write b as the difference of two numbers whose product is $(-ac)$.

Example 8 Factors of $2x^2+7x+3$ are

(a) $(x+2)(x+1)$ (b) $(2x+1)(x+3)$
(c) $(x+3)(2x-1)$ (d) $(2x-2)(x-3)$

Sol. (*b*) Given polynomial is $2x^2+7x+3$

On comparing with ax^2+bx+c, we get

$$a=2,\ b=7 \text{ and } c=3$$

Now, $ac=2\times3=6$

So, all possible pairs of factors of 6 are 1 and 6, 2 and 3.

Clearly, pair 1 and 6 gives

$$1+6=7=b$$

$$\therefore \quad 2x^2+7x+3=2x^2+(1+6)x+3$$
$$=2x^2+x+6x+3$$
$$=x(2x+1)+3(2x+1)$$
$$=(2x+1)(x+3)$$

Factorisation by Algebraic Identities

To solve these types of question, we have to use some algebraic identities.

e.g. $x^2-(2y)^2=(x+2y)(x-2y)$

Here, we use $a^2-b^2=(a+b)(a-b)$ identity.

Example 9 Factors of $x^3+27y^3+8z^3-18xyz$ are

(a) $(x+3y+2z)(x^2+9y^2+4z^2-3xy-6yz-2xz)$
(b) $(x-3y-2z)(x^2-9y^2+4z^2+3xy+6yz+2xz)$
(c) $(x+3y-3z)(x^2-9y^2+4z^2-3xy-6yz-2xz)$
(d) None of the above

Sol. (*a*) We have, $x^3+27y^3+8z^3-18xyz$

$=x^3+(3y)^3+(2z)^3-3(x)(3y)(2z)$
$=(x+3y+2z)[x^2+9y^2+4z^2-x(3y)-3y(2z)-x(2z)]$
$=(x+3y+2z)(x^2+9y^2+4z^2-3xy-6yz-2xz)$

Example 10 The value of $x^4-3x^3+2x^2+x-1$ at $x=2$ is

(a) 0 (b) 1 (c) 2 (d) 3

Sol. (*b*) Let $p(x)=x^4-3x^3+2x^2+x-1$ ...(i)

On putting $x=2$ in Eq. (i), we get

$$p(2)=2^4-3\times(2)^3+2\times(2)^2+2-1$$
$$=16-24+8+1=1$$

Example 11 The value of p, if $(2x-1)$ is a factor of $2x^3+px^2+11x+p+3$ is

(a) -7 (b) 7 (c) -6 (d) 5

Sol. (*a*) Let $q(x)=2x^3+px^2+11x+p+3$

If $q(x)$ is divisible by $2x-1$, then $(2x-1)$ is a factor of $q(x)$.

$$\therefore \quad 2x-1=0$$
$$\Rightarrow \quad x=\frac{1}{2}$$

On putting $x=\frac{1}{2}$ in $q(x)$, we have

$$q\left(\frac{1}{2}\right)=2\times\left(\frac{1}{2}\right)^3+p\left(\frac{1}{2}\right)^2+11\left(\frac{1}{2}\right)+p+3=0$$
$$\Rightarrow \quad 2\times\frac{1}{8}+p\times\frac{1}{4}+\frac{11}{2}+p+3=0$$
$$\Rightarrow \quad \frac{1}{4}+\frac{p}{4}+\frac{11}{2}+p+3=0$$
$$\Rightarrow \quad \frac{1+p+22+4p+12}{4}=0$$
$$\Rightarrow \quad 5p+35=0$$
$$\Rightarrow \quad 5p=-35$$
$$\therefore \quad p=-7$$

PRACTICE EXERCISE

1. In the expression $3a^2-4ab+5b^2+7ba$ the like terms are

(a) $-4ab$, $7ba$ (b) $3a^2$, $5b^2$
(c) $4ab$, $7ba$ (d) $-4ab$, $-7ba$

2. The value of the expression

$$8y^2+\frac{1}{3}x^2+\frac{1}{5}z^2-3xy+4yz$$

at $x=1$, $y=-2$ and $z=-3$ is

(a) $64\frac{2}{15}$ (b) $64\frac{1}{15}$
(c) $64\frac{3}{15}$ (d) None of these

3. $-6x^2y^2+4x^2y^2-3x^2y^2$ is equal to

(a) $6x^2y^2$ (b) $-5x^2y^2$
(c) $-6x^2y^2$ (d) $5x^2y^2$

4. The difference of $x^3 - x^2 + 2x - 19$ and $2x^3 - x^2 + 4x - 6$ is

(a) $x^3 - 2x + 13$ (b) $-x^3 + 6x^2 - 8x + 12$
(c) $-x^3 - 2x - 13$ (d) None of the above

5. What must be added to $x^2 + 4x - 6$ to get $x^3 - x^2 + 2x - 2$?

(a) $x^3 + 6x - 8$ (b) $x^3 - 2x^2 + 2x - 4$
(c) $x^3 + 2x^2 + 2x - 4$ (d) $x^3 - 2x^2 - 2x + 4$

6. What must be subtracted from $x^4 + 2x^2 - 3x + 7$ to get $x^3 + x^2 + x - 1$?

(a) $x^4 - x^3 + x^2 - 4x + 8$
(b) $x^3 + x^4 + x^2 + 4x + 8$
(c) $x^4 + x^3 - x^2 + 4x - 8$
(d) None of the above

7. The product of $2x^2 + x - 5$ and $x^2 - 2x + 3$ is

(a) $2x^4 + 3x^3 + x^2 + 13x + 15$
(b) $2x^4 - 3x^3 + x^2 - 13x - 15$
(c) $2x^3 - 3x^2 - x - 15$
(d) $2x^4 - 3x^3 - x^2 + 13x - 15$

8. The value of $(29x - 6x^2 - 28) \div (3x - 4)$ is

(a) $(2x - 7)$ (b) $(-2x + 7)$
(c) $(2x + 7)$ (d) $(7 + 2x)$

9. What should be subtracted from $p^2 - 6p + 7$ so that it may exactly be divisible by $(p - 1)$?

(a) 4 (b) −4 (c) 2 (d) −2

10. Divide $3y^4 - 3y^3 - 4y^2 - 4y$ by $y^2 - 2y$, then remainder is

(a) $4y^2$ (b) $4y$
(c) $2y$ (d) 0

11. If $P = a^4 + a^3 + a^2 - 6$,
$Q = a^2 - 2a^3 - 2 + 3a$
and $R = 8 - 3a - 2a^2 + a^3$,
then the value of $P + Q + R$ is

(a) a^4 (b) a^3
(c) $2a^4$ (d) $3a^4$

12. If the expression $px^3 + 3x^2 - 3$ and $2x^3 - 5x + p$ when divided by $x - 4$ leave the same remainder, then the value of p is

(a) 0 (b) 1
(c) 2 (d) 3

13. The value of p and q when $px^3 + x^2 - 2x - q$ is exactly divisible by $(x - 1)$ and $(x + 1)$, is

(a) 2, 1 (b) 2, 2
(c) 1, 1 (d) 1, 0

14. The expansion of $\left(\frac{x}{5} - \frac{y}{6}\right)^2$ is

(a) $\frac{x^2}{25} - \frac{y^2}{36} - \frac{xy}{15}$ (b) $\frac{x^2}{25} + \frac{y^2}{36} + \frac{xy}{15}$
(c) $\frac{x^2}{10} + \frac{y^2}{12} - \frac{xy}{15}$ (d) $\frac{x^2}{25} + \frac{y^2}{36} - \frac{xy}{15}$

15. The irreducible factorisation $(25x^2 - 9y^2)$ is

(a) $(5x - 3y)(5x + 3y)$
(b) $(5x^2 - 3y^2)$
(c) $(5x - 3y)^2$
(d) None of the above

16. If $(a + b) = 4$ and $a^2 + b^2 = 7$, then the value of ab is

(a) $-\frac{9}{2}$ (b) $\frac{9}{2}$ (c) $\frac{9}{4}$ (d) $\frac{7}{4}$

17. If $(2x - 3)(? + 6x + 9) = 8x^3 - 27$, then ? will be replaced by

(a) $-4x^2$ (b) $4x^2$
(c) $5x^2$ (d) None of these

18. If $a^2 + b^2 + c^2 = ab + bc + ca$, then the value of $a^3 + b^3 + c^3$ is

(a) $3abc$ (b) $3a^2b^2c^2$
(c) $3(abc)^3$ (d) None of these

19. If $(x + y + z) = 10$ and $x^2 + y^2 + z^2 = 40$, then $(xy + yz + zx)$ is equal to

(a) 60 (b) 30
(c) 40 (d) 20

20. If $(x - y) = 6$ and $xy = 1$, then the value of $x^3 - y^3$ is

(a) 324 (b) 432 (c) 234 (d) 322

21. Common factors between $3x^2y^3$, $10x^3y^2$ and $-6x^2y^2z$ terms is

(a) $3x^2y$ (b) $3xz$
(c) x^2y^2 (d) $-x^2y^2$

22. Factors of $x^2yz + xy^2z + xyz^2$ are

(a) $xyz(x + y + z)$ (b) $xyz(x - y - z)$
(c) $xy(x + y + z)$ (d) $xz(x^2 + y + z)$

23. The factor form of $5x^2-20xy$ is
(a) $5x(x-4y)$
(b) $10x(x-2y)$
(c) $5(x^2-2y)$
(d) None of the above

24. The factor of form $8-4x-2x^3+x^4$ is
(a) $(2-x)(4-x^3)$ (b) $(2+x)(4-x^3)$
(c) $(2+x)(4+x^3)$ (d) $(2-x)(4+x^3)$

25. Factorise $3x^2+7x-6$
(a) $(x+3)(3x-2)$ (b) $(x-3)(3x+2)$
(c) $(x+3)(3x-3)$ (d) $(x+3)(3x-4)$

26. Factorise $8x^2-9x-14$
(a) $(x+2)(8x-7)$ (b) $(x-2)(8x-7)$
(c) $(x-2)(8x+7)$ (d) $(x+2)(8x+7)$

27. Factors of $1-8x^3$ are
(a) $(1-2x)(1+2x-4x^2)$
(b) $(1-2x)(1+2x+4x^2)$
(c) $(1-2x)(1-4x+4x^2)$
(d) $(1-2x)(1+4x-4x^2)$

28. Factors of x^6-y^6 are
(a) $(x^2-y^2)(x^4+y^4)$
(b) $(x^2+y^2)(x^4-x^2y^2+x^4)$
(c) $(x+y)(x-y)(x^2+xy+y^2)$
(d) $(x+y)(x-y)(x^2-xy+y^2)(x^2+xy+y^2)$

29. The factor form of $z^2+\frac{1}{z^2}+2-2z-\frac{2}{z}$ is
(a) $\left(z+\frac{1}{z}+2\right)\left(z-\frac{1}{z}\right)$ (b) $\left(z+\frac{1}{z}\right)\left(z+\frac{1}{z}-2\right)$
(c) $\left(z-\frac{1}{z}+2\right)\left(z+\frac{1}{z}\right)$ (d) $\left(z-\frac{1}{z}\right)\left(z-\frac{1}{z}-2\right)$

30. The factor form of $(a^4b^4-16c^4)$ is
(a) $4(a^2b^2+c^2)(ab-2c)(ab+2c)$
(b) $(a^2b^2-4c^2)(ab+2c)^2$
(c) $(a^2b^2+4c^2)(ab+2c)(ab-2c)$
(d) $(a^2b^2-4c^2)^2(ab+2c)(ab+4c)$

31. The factor form of $x^2-2\sqrt{3x}+3$ is
(a) $(x+\sqrt{3})^2$ (b) $(x-\sqrt{3})^2$
(c) $(x+\sqrt{3})(x-\sqrt{3})$ (d) $(x+2)(x+\sqrt{3})$

32. If $p(x)=x^3-5x^2+x-5$, then the remainder, when $p(x)$ is divided by $(x+1)$, is
(a) -11 (b) -12
(c) 11 (d) 12

33. Polynomial $x^{11}+1$ has factor
(a) $(x+1)$ (b) $(x-1)$
(c) $(x-2)$ (d) $(x-3)$

34. The factor of the polynomial $4x^3+3x^2-4x-3$ is
(a) $(x-2)$ (b) $(x-1)$
(c) $(x+1)$ (d) $(x-3)$

Answers

1	(a)	2	(a)	3	(b)	4	(c)	5	(d)	6	(a)	7	(d)	8	(b)	9	(c)	10	(d)
11	(a)	12	(b)	13	(a)	14	(d)	15	(a)	16	(b)	17	(b)	18	(a)	19	(b)	20	(c)
21	(c)	22	(a)	23	(a)	24	(a)	25	(a)	26	(c)	27	(b)	28	(d)	29	(b)	30	(c)
31	(b)	32	(b)	33	(a)	34	(b)												

Hints and Solutions

1. $-4ab$ and $7ba$ are like terms because variable parts are same in both side.

2. Let $E = 8y^2 + \frac{1}{3}x^2 + \frac{1}{5}z^2 - 3xy + 4yz.$

At $x = 1$, $y = -2$ and $z = -3$ is

$$E = 8(-2)^2 + \frac{1}{3}(1)^2 + \frac{1}{5}(-3)^2 - 3\times 1(-2) + 4\times(-2)\times(-3)$$

$$= 8\times 4 + \frac{1}{3} + \frac{1}{5}\times 9 + 6 + 24$$

$$= 32 + \frac{1}{3} + \frac{9}{5} + 30$$

$$= \frac{32\times 15 + 5 + 9\times 3 + 30\times 15}{15}$$

$$= \frac{480 + 5 + 27 + 450}{15} = \frac{962}{15} = 64\frac{2}{15}$$

3. $-6x^2y^2 + 4x^2y^2 - 3x^2y^2 = -5x^2y^2.$

4. Rearranging the terms of the given polynomials, changing the sign of each term of the polynomial which is to be subtracted and adding the two polynomials, we get

$$\begin{array}{rrrr}
x^3 & -\,x^2 & +\,2x & -\,19 \\
2x^3 & -\,x^2 & +\,4x & -\,6 \\
- & + & - & + \\
\hline
-\,x^3 & & -\,2x & -\,13 \\
\hline
\end{array}$$

5.

$$\begin{array}{rrrr}
x^3 & -\,x^2 & +\,2x & -\,2 \\
 & x^2 & +\,4x & -\,6 \\
 & - & - & + \\
\hline
x^3 & -\,2x^2 & -\,2x & +\,4 \\
\hline
\end{array}$$

6.

$$\begin{array}{rrrrr}
x^4 & +\,0x^3 & +\,2x^2 & -\,3x & +\,7 \\
 & x^3 & +\,x^2 & +\,x & -\,1 \\
 & - & - & - & + \\
\hline
x^4 & -\,x^3 & +\,x^2 & -\,4x & +\,8 \\
\hline
\end{array}$$

7. Now, $(2x^2 + x - 5)\times(x^2 - 2x + 3)$

$= 2x^2(x^2 - 2x + 3) + x(x^2 - 2x + 3) - 5(x^2 - 2x + 3)$

$= 2x^4 - 4x^3 + 6x^2 + x^3 - 2x^2 + 3x - 5x^2 + 10x - 15$

$= 2x^4 - x^3(4 - 1) + x^2(6 - 5 - 2) + x(3 + 10) - 15$

[arranging like terms]

$= 2x^4 - 3x^3 - x^2 + 13x - 15$

8. Arranging the terms of the dividend and the divisor in descending power and then dividing, we get

$$\begin{array}{r|l}
 & -2x + 7 \\
\cline{2-2}
3x - 4 & -6x^2 + 29x - 28 \\
 & -6x^2 + 8x \\
 & +\quad - \\
\cline{2-2}
 & \quad 21x - 28 \\
 & \quad 21x - 28 \\
 & \quad -\quad + \\
\cline{2-2}
 & \qquad \times \\
\cline{2-2}
\end{array}$$

$\therefore (29x - 6x^2 - 28) \div (3x - 4) = (-2x + 7)$

9.

$$\begin{array}{r|l}
 & p - 5 \\
\cline{2-2}
p - 1 & p^2 - 6p + 7 \\
 & p^2 - p \\
 & -\quad + \\
\cline{2-2}
 & \quad -5p + 7 \\
 & \quad -5p + 5 \\
 & \quad +\quad - \\
\cline{2-2}
 & \qquad 2 \\
\end{array}$$

Remainder = 2

Hence, 2 is subtracted from it.

10.

$$\begin{array}{r|l}
 & 3y^2 + 3y + 2 \\
\cline{2-2}
y^2 - 2y & 3y^4 - 3y^3 - 4y^2 - 4y \\
 & 3y^4 - 6y^3 \\
 & -\quad + \\
\cline{2-2}
 & \quad 3y^3 - 4y^2 - 4y \\
 & \quad 3y^3 - 6y^2 \\
 & \quad -\quad + \\
\cline{2-2}
 & \qquad 2y^2 - 4y \\
 & \qquad 2y^2 - 4y \\
 & \qquad -\quad + \\
\cline{2-2}
 & \qquad\quad 0 \\
\cline{2-2}
\end{array}$$

$\therefore$ Remainder is 0.

11. $P + Q + R = (a^4 + a^3 + a^2 - 6) + (a^2 - 2a^3 - 2 + 3a) + (8 - 3a - 2a^2 + a^3)$

$= a^4 + 0 + 0 + 0 + 0$

$= a^4$

12. Let $f(x)=px^3+3x^2-3$ and $g(x)=2x^3-5x+p$

$\therefore \quad f(4)=p(4)^3+3(4)^2-3$

and $\quad g(4)=2(4)^3-5(4)+p$

$\Rightarrow \quad f(4)=64p+45$ and $g(4)=p+108$

Since, $f(4)=g(4) \Rightarrow 64p+45=p+108$

$\Rightarrow \quad p=1$

13. Let $f(x)=px^3+x^2-2x-q=0$

$f(1)=p+1-2-q=0$

$\Rightarrow \quad p-q=1 \quad \ldots(i)$

$f(-1)=p(-1)^3+(-1)^2-2(-1)-q=0$

$\Rightarrow \quad p+q=3 \quad \ldots(ii)$

On solving Eqs. (i) and (ii), we get

$$p=2, q=1$$

14. $\left(\frac{x}{5}-\frac{y}{6}\right)^2=\left(\frac{x}{5}\right)^2-2\times\frac{x}{5}\times\frac{y}{6}+\left(\frac{y}{6}\right)^2$

[by using identity $(a-b)^2=a^2-2ab+b^2$]

$$=\frac{x^2}{25}-\frac{xy}{15}+\frac{y^2}{36}$$

$$=\frac{x^2}{25}+\frac{y^2}{36}-\frac{xy}{15}$$

15. $(25x^2-9y^2)=[(5x)^2-(3y)^2]$

$=(5x-3y)(5x+3y) \quad [\because (a^2-b^2)=(a-b)(a+b)]$

16. We have, $a+b=4$ and $a^2+b^2=7$

$\therefore \quad (a+b)^2=(4)^2$

$\Rightarrow \quad a^2+b^2+2ab=16$

$\Rightarrow \quad 7+2ab=16$

$\Rightarrow \quad 2ab=16-7=9$

$\therefore \quad ab=\frac{9}{2}$

17. Now, $\quad 8x^3-27=(2x)^3-(3)^3$

$=(2x-3)[(2x)^2+2x\times3+3^2]$

$[\because a^3-b^3=(a-b)(a^2+ab+b^2)]$

$=(2x-3)(4x^2+6x+9)$

So, ? will be replaced by $4x^2$.

18. $\because \ a^3+b^3+c^3-3abc$

$=(a^2+b^2+c^2-ab-bc-ca)(a+b+c)$

$=[(a^2+b^2+c^2)-(ab+bc+ca)](a+b+c)$

But, $a^2+b^2+c^2=ab+bc+ca$ [given]

$\therefore \ a^3+b^3+c^3-3abc=(a+b+c)\times 0=0$

$\therefore \quad a^3+b^3+c^3=3abc$

19. $\because \ (x+y+z)=10$

On squaring both sides, we get

$$(x+y+z)^2=(10)^2$$

$\Rightarrow \ x^2+y^2+z^2+2xy+2yz+2zx=100$

$\Rightarrow \quad 40+2(xy+yz+zx)=100$

$\Rightarrow \quad 2(xy+yz+zx)=100-40$

$\Rightarrow \quad 2(xy+yz+zx)=60$

$\therefore \quad (xy+yz+zx)=\frac{60}{2}=30$

20. $\because (x-y)=6$

On squaring both sides, we get

$$(x-y)^2=(6)^2$$

$\Rightarrow \quad x^2+y^2-2xy=36$

$\Rightarrow \quad x^2+y^2-2\times1=36$

$\Rightarrow \quad x^2+y^2=36+2=38$

$\because \quad x^3-y^3=(x-y)(x^2+y^2+xy)$

$\therefore \quad x^3-y^3=(6)(38+1)=6\times39=234$

21. We have,

$$3x^2y^3=3\times x\times x\times y\times y\times y,$$

$$10x^3y^2=2\times5\times x\times x\times x\times y\times y$$

and $\ -6x^2y^2z=-2\times3\times x\times x\times y\times y\times z$

Here, x, x, y, y are common factors.

Hence, x^2y^2 is common factor.

22. We have, $x^2yz+xy^2z+xyz^2$

$=x\times x\times y\times z+x\times y\times y\times z+x\times y\times z\times z$

$=xyz(x+y+z)$

23. We have, $5x^2-20xy=5x(x-4y)$

24. $8-4x-2x^3+x^4$

$=4(2-x)-x^3(2-x)=(2-x)(4-x^3)$

25. We have, $3x^2+7x-6$

$=3x^2+9x-2x-6$

$=3x(x+3)-2(x+3)$

$=(3x-2)(x+3)$

26. We have, $8x^2-9x-14$

$=8x^2-16x+7x-14$

$=8x(x-2)+7(x-2)$

$=(8x+7)(x-2)$

27. We have, $1-8x^3=1^3-(2x)^3$

$=(1-2x)(1+2x+4x^2)$

$[\because a^3-y^3=(a-b)(a^2+ab+b^2)]$

28. $x^6 - y^6 = (x^3)^2 - (y^3)^2 = (x^3 + y^3)(x^3 - y^3)$

$[\because a^2 - b^2 = (a+b)(a-b)]$

$= (x+y)(x^2 - xy + y^2)(x-y)(x^2 + xy + y^2)$

$\begin{bmatrix} \because & a^3 + b^3 = (a+b)(a^2 - ab + b^2) \\ \text{and} & a^3 - b^3 = (a-b)(a^2 + ab + b^2) \end{bmatrix}$

$= (x+y)(x-y)(x^2 - xy + y^2)(x^2 + xy + y^2)$

29. We have , $z^2 + \frac{1}{z^2} + 2 - 2z - \frac{2}{z}$

$= \left(z + \frac{1}{z}\right)^2 - 2\left(z + \frac{1}{z}\right) \quad \left[\because z^2 + \frac{1}{z^2} + 2 = \left(z + \frac{1}{z}\right)^2\right]$

$= \left(z + \frac{1}{z}\right)\left(z + \frac{1}{z} - 2\right)$

30. $(a^4 b^4 - 16c^4) = [(a^2b^2) - (4c^2)^2]$

$= (a^2b^2 + 4c^2)(a^2b^2 - 4c^2)$

$= (a^2b^2 + 4c^2)[(ab)^2 - (ac)^2]$

$= (a^2b^2 + 4c^2)(ab + 2c)(ab - 2c)$

31. $x^2 - 2\sqrt{3x} + 3$

$= x^2 - \sqrt{3}x - \sqrt{3}x + 3$

$= x(x - \sqrt{3}) - \sqrt{3}(x - \sqrt{3})$

$= (x - \sqrt{3})(x - \sqrt{3}) = (x - \sqrt{3})^2$

32. Given, that

$p(x) = x^3 - 5x^2 + x - 5$

$\therefore \quad p(-1) = (-1)^3 - 5(-1)^2 + (-1) - 5$

$= -1 - 5 - 1 - 5 = -12$

33. Let $p(x) = x^{11} + 1$

$\therefore \quad p(-1) = (-1)^{11} + 1 = -1 + 1 = 0$

Here, $\quad x = -1$

i.e. $\quad x + 1 = 0$

So, $(x+1)$ is factor of given polynomial.

34. We have, $p(x) = 4x^3 + 3x^2 - 4x - 3$

By options,

(a) $p(2) = 4 \times 2^3 + 3 \times 2^2 - 4 \times 2 - 3$

$= 32 + 12 - 8 - 3 = 33$

(b) $p(1) = 4 \times 1^3 + 3 \times 1^2 - 4 \times 1 - 3$

$= 4 + 3 - 4 - 3 = 0$

Here, by $p(1)$, i.e. $x = 1 \Rightarrow (x-1) = 0$

So, $(x-1)$ is a factor of given polynomial.