CHAPTER

15

RATIO AND PROPORTION

Ratio

The ratio of two quantities in the same units is the fractions that one quantity is of the other.

Or

It is a tool to compare two or more numbers of same quantities. Thus, the ratio a to b is the fraction $\frac{a}{b}$ written as $a : b$.

Note In the ratio $a : b$, the first term a is antecedent and second term b is consequent.

Properties of Ratio

(i) The value of a ratio remains unchanged, if each one of its term is multiplied or divided by a same non-zero number.

(ii) $a^2 : b^2$ is the duplicate ratio of $a : b$.

(iii) $a^3 : b^3$ is the triplicate ratio of $a : b$.

Example **1.** If $p : q = 3 : 4$ and $q : r = 8 : 9$. Find the ratio of $p : q : r$ is

(1) 2:4:3 (2) 9:6:8 (3) 6:8:9 (4) 9:8:6

Sol. (3) $\frac{p}{q} = \frac{3}{4}$ and $\frac{q}{r} = \frac{8}{9}$

$\Rightarrow \quad \frac{p}{q} = \frac{3\times2}{4\times2} = \frac{6}{8}$ and $\frac{q}{r} = \frac{8}{9}$

$\therefore\ p : q : r = 6 : 8 : 9$

Example **2.** If $A : B = 3 : 4, B : C = 5 : 6$ and $C : D = 11 : 9$, then find the ratio of $A : D$ is

(1) 55:72 (2) 73:55 (3) 11:9 (4) 55:73

Sol. (1) $\frac{A}{D} = \left(\frac{A}{B}\times\frac{B}{C}\times\frac{C}{D}\right) = \left(\frac{3}{4}\times\frac{5}{6}\times\frac{11}{9}\right) = \frac{55}{72}$

$\Rightarrow \quad A : D = 55 : 72$

Proportion

The equality of two ratios is called proportion.

Let a, b, c and d are four quantities, then the proportional are $a : b :: c : d$ or $\frac{a}{b} = \frac{c}{a}$.

Properties of Proportion

(i) Third proportional of a and b; $\frac{b^2}{a}$

(ii) Mean proportional between a and b is $\sqrt{ab}$.

(iii) Fourth proportional of a, b and $c = \frac{bc}{a}$.

Example **3.** $x : 75 :: 15 : 45$. Find the value of x.

(1) 52 (2) 25
(3) 16 (4) 9

Sol. (2) $\frac{x}{75} = \frac{15}{45} \Rightarrow x = \frac{15\times75}{45} \Rightarrow x = 25$

Example **4**. Find the ratio in between 20 paise and ₹ 3.

(1) 2:3 (2) 15:1
(3) 1:15 (4) 3:2

Sol. (3) ₹ 3 = 300 paise

$\therefore$ Required ratio $= \frac{20}{300} = \frac{1}{15} = 1 : 15$

Example **5**. Salaries of Vivek and Vimal are ₹ 1400 and ₹ 1600. Find out the ratio of their salaries.

(1) 7:8 (2) 15:16
(3) 4:5 (4) 3:4

Sol. (1) Ratio $= \frac{\text{Vivek's salary}}{\text{Vimal's salary}} = \frac{1400}{1600} = \frac{14}{16} = \frac{7}{8}$

$\therefore$ Ratio = 7 : 8

Entrance Corner

1. Two numbers are in the ratio 2 : 3. If 9 is added to each, they will be in the ratio 3 : 4 the numbers are **[JNV 2017, 2009]**
(1) 12, 28 (2) 18, 27 (3) 8, 12 (4) 10, 15

2. A, B and C divide an amount of ₹ 9861 amongst themselves in the ratio of 3 : 11 : 5, respectively. What is the B's share in the amount?
(1) ₹ 4671 (2) ₹ 5709 (3) ₹ 6228 (4) ₹ 7266

3. If $a : b = 5 : 14$ and $b : c = 7 : 3$, then find $a : b : c$. **[JNV 2012]**
(1) 7 : 3 : 6 (2) 5 : 14 : 6
(3) 8 : 3 : 5 (4) 5 : 6 : 9

4. What must be added to each term of the ratio 49 : 68, so that it becomes 3 : 4? **[JNV 2000]**
(1) 8 (2) 9 (3) 10 (4) 11

5. 2 : 5 :: 8 : x. Find the value of x. **[JNV 2000]**
(1) 15 (2) 17 (3) 20 (4) 11

6. If $a : b = 2 : 3$ and $b : c = 4 : 5$, the ratio $a : b : c$ is equal to **[JNV 1999]**
(1) 8 : 13 : 17 (2) 8 : 14 : 16
(3) 8 : 13 : 18 (4) 8 : 12 : 15

7. Divide ₹ 4000 among A, B and C, so that their shares may be in the ratio of 5 : 7 : 8
(1) ₹ 1000, ₹ 1400, ₹ 1600 **[JNV 1999]**
(2) ₹ 3000, ₹ 6000, ₹ 9000
(3) ₹ 2000, ₹ 4000, ₹ 6000
(4) ₹ 1000, ₹ 2000, ₹ 3000

8. If $A : B = 6 : 7$ and $B : C = 8 : 9$, then find $A : C$. **[JNV 1998]**
(1) 13 : 15 (2) 16 : 21 (3) 13 : 14 (4) 18 : 21

9. 1 : 9 :: 9 : x. Find the value of x. **[JNV 1998]**
(1) 80 (2) 81 (3) 84 (4) 85

10. 3 : 5 :: 60 : x, find the value of x. **[JNV 1997]**
(1) 100 (2) 120 (3) 140 (4) 160

11. If 0.75 : x :: 5 : 8, then find x. **[JNV 1997]**
(1) 1.5 (2) 1.8 (3) 1.2 (4) 1.4

12. The ratio of number of boys and girls in a school is 4 : 3. If there are 480 boys in the school, find the number of girls in the school. **[JNV 1996]**
(1) 300 (2) 320 (3) 340 (4) 360

13. What is the mean proportional of 9 and 16?
(1) 12 (2) 16 (3) 25 (4) 7

14. In a ratio which is equal to 3 : 7, if the antecedent is 33, what is the consequent? **[JNV 1995]**
(1) 75 (2) 76 (3) 80 (4) 77

15. Speed of one bus is 80 km/h and other is 60 km/h. What is the ratio of the speeds of two buses? **[JNV 1995]**
(1) 3 : 4 (2) 4 : 3 (3) 5 : 6 (4) 6 : 5

16. The prices of a cycle and a scooter are the ratio of 9 : 5. If a cycle costs ₹ 4200 more than a scooter, what is the price of scooter? **[JNV 1995]**
(1) ₹ 5160 (2) ₹ 5250 (3) ₹ 6000 (4) ₹ 6230

17. The ratio of two numbers is 3 : 8 and their difference is 116. What is the largest number? **[JNV 1994]**
(1) 181 (2) 182 (3) 183 (4) 184

18. What is ratio in between 7 months and 7 yr? **[JNV 1994]**
(1) 1 : 12 (2) 1 : 13 (3) 1 : 14 (4) 1 : 15

19. What must be subtracted from each term of the ratio 3 : 2. So, that the ratio becomes 2 : 5 ? **[JNV 1993]**
(1) $\frac{1}{2}$ (2) $\frac{1}{3}$ (3) $\frac{1}{4}$ (4) $\frac{1}{5}$

20. If $A : B = 3 : 4$, $B : C = 5 : 6$ and $C : D = 11 : 9$, then $A : D$ is **[JNV 1993]**
(1) 55 : 72 (2) 55 : 74 (3) 55 : 84 (4) 55 : 71

21. If $p : q = 3 : 4$ and $q : r = 8 : 9$. Find $p : q : r$. **[JNV 1993]**
(1) 6 : 7 : 8 (2) 6 : 8 : 9
(3) 6 : 9 : 10 (4) 7 : 9 : 11

Answers

1. (2)	**2.** (2)	**3.** (2)	**4.** (1)	**5.** (3)	**6.** (4)	**7.** (1)	**8.** (2)	**9.** (2)	**10.** (1)
11. (3)	**12.** (4)	**13.** (1)	**14.** (4)	**15.** (2)	**16.** (2)	**17.** (4)	**18.** (1)	**19.** (2)	**20.** (1)
21. (2)									

Hints and Solutions

1. Let numbers be $2x$ and $3x$.

Then, $\frac{2x+9}{3x+9}=\frac{3}{4}$

$\Rightarrow \quad 4(2x+9)=3(3x+9)$

$\Rightarrow \quad 8x+36=9x+27$

$\Rightarrow \quad 9x-8x=36-27$

$\Rightarrow \quad x=9$

$\therefore$ Numbers $=2x=2\times9=18$

and $\quad 3x=3\times9=27$

2. B's share in the amount

$$=\frac{\text{Ratio term for } B}{\text{Total sum of ratios}}\times\text{Total amount}$$

$$=\frac{9861\times11}{(3+11+5)}$$

$$=\frac{9861\times11}{19}=₹\,5709$$

3. $a:b=5:14$

$b:c=7:3$ or $b:c=7\times2:3\times2$

$\therefore\ a:b:c=5:14:3\times2$

$=5:14:6$

4. Let x is to added. Then

$$\frac{49+x}{68+x}=\frac{3}{4}$$

$\Rightarrow 196+4x=204+3x$

$\Rightarrow \quad 4x-3x=204-196$

$\Rightarrow \quad x=8$

$\therefore$ The number is 8.

5. $\frac{2}{5}=\frac{8}{x}$

$\therefore\ x=\frac{8\times5}{2}=20$

6. $\frac{a}{b}=\frac{2}{3}$ and $\frac{b}{c}=\frac{4}{5}$

or $\frac{a}{b}=\frac{2\times4}{3\times4}=\frac{8}{12}$ and $\frac{b}{c}=\frac{4\times3}{5\times3}=\frac{12}{15}$

$\therefore a:b:c=8:12:15$

7. Total money $=₹\,4000$

Ratio of A, B and $C=5:7:8$

Total $=5+7+8=20$

$\therefore$ Share of $A=\frac{5}{20}\times\frac{4000}{1}=₹\,1000$

$\therefore$ Share of $B=\frac{7}{20}\times4000=₹\,1400$

$\therefore$ Share of $C=\frac{8}{20}\times4000=₹\,1600$

8. $\frac{A}{C}=\left(\frac{A}{B}\times\frac{B}{C}\right)=\frac{6}{7}\times\frac{8}{9}=\frac{16}{21}=16:21$

9. $\frac{1}{9}=\frac{9}{x}\ \Rightarrow\ x=\frac{9\times9}{1}=81$

10. $3:5::60:x$

$\Rightarrow \quad \frac{3}{5}=\frac{60}{x}$

$\Rightarrow \quad x=\frac{60\times5}{3}=100$

11. $0.75:x::5:8$

$\Rightarrow \quad \frac{0.75}{x}=\frac{5}{8}$

$\Rightarrow \quad x=\frac{0.75\times8}{5}=1.2$

12. Let the number of girls is x. Then

$4:3=480:x$

$\Rightarrow \quad \frac{4}{3}=\frac{480}{x}$

$\Rightarrow \quad x=\frac{480\times3}{4}=360$ girls

13. Here, $a=9,\ b=16$

we know that, the mean proportional of a and b

$=\sqrt{ab}$

$=\sqrt{9\times16}=3\times4=12$

14. $\frac{3}{7}=\frac{33}{x}$, where x is the consequent.

$\therefore \quad x=\frac{7\times33}{3}=77$

15. Ratio $=\frac{80}{60}=\frac{4}{3}=4:3$

16. The cost of cycle $=₹\,9x$

The cost of scooter $=₹\,5x$

According to the question,

$\Rightarrow \quad 9x-5x=4200$

$\Rightarrow \quad 4x=4200$

$\Rightarrow \quad x=\frac{4200}{4}=1050$

$\therefore$ Cost of scooter $=5\times1050=₹\,5250$

17. Let numbers be $3x$ and $8x$.

According to the question,

$8x-3x=115$

$\Rightarrow \quad 5x=115$

$\Rightarrow \quad x=\frac{115}{5}=23$

$\therefore$ Largest number $=8\times23=184$

18. 7 yr = 7 × 12 months = 84 months

$\therefore$ Ratio $= \frac{7}{84} = \frac{1}{12} = 1:12$

19. Let x is to be subtracted. Then

$$\frac{3-x}{7-x} = \frac{2}{5}$$

$\Rightarrow 15 - 5x = 14 - 2x \Rightarrow 15 - 14 = 5x - 2x$

$\Rightarrow 1 = 3x \Rightarrow x = \frac{1}{3}$

$\therefore$ The number is $\frac{1}{3}$.

20. $\frac{A}{D} = \left(\frac{A}{B} \times \frac{B}{C} \times \frac{C}{D}\right) = \left(\frac{3}{4} \times \frac{5}{6} \times \frac{11}{9}\right) = \frac{55}{72}$

21. $\frac{p}{q} = \frac{3}{4}$ and $\frac{q}{r} = \frac{8}{9}$

$$\frac{p}{q} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8} \text{ and } \frac{q}{r} = \frac{8}{9}$$

$\therefore$ $p : q : r = 6 : 8 : 9$

Practice Exercise

1. Write the ratio in the simplest form 25 : 35.
(1) 5 : 7 (2) 7 : 5
(3) 25 : 35 (4) None of these

2. The ratio between the 1h to 1 day
(1) 1 : 6 (2) 1 : 1
(3) 1 : 24 (4) 24 : 1

3. If A, B, C and D are four numbers such that $A : B = 2 : 3$, $B : C = 4 : 5$, $C : D = 5 : 8$. Then, $A : D$ is equal to
(1) 1 : 3 (2) 3 : 1
(3) 2 : 3 (4) 3 : 2

4. The sum of the squares of three numbers is 116 and their ratio is 2 : 3 : 4. The numbers are
(1) 2, 3, 4 (2) 4, 9, 16
(3) 4, 6, 8 (4) 8, 12, 6

5. If $\frac{a}{b} = \frac{7}{9}$ and $\frac{b}{c} = \frac{3}{5}$. Then, the value of $a : b : c$ is
(1) 7 : 9 : 15 (2) 7 : 9 : 5
(3) 21 : 35 : 45 (4) 7 : 3 : 15

6. What must be added to each term of the ratio 7 : 13. So, that the ratio becomes 2 : 3?
(1) 5 (2) 1
(3) 2 (4) 3

7. A sum of money is to be distributed between Ajay and Sanjay in the proportion of 7 : 11, respectively. Sanjay gets ₹ 6000 more than Ajay. How much did Ajay get?
(1) ₹ 3818.18
(2) ₹ 8400
(3) ₹ 10500
(4) Cannot be determined

8. The ratio between boys and girls in a school is 4 : 6, respectively. If the number of boys is increased by 200 the ratio becomes 5 : 6, respectively. How many girls are there in the school?
(1) 1200 (2) 800
(3) 1000 (4) Cannot be determined

9. The total number of students in a school is 1224. If the number of girls in the school is 600, then what is the respective ratio of the total number of boys to the total number of girls in the school?
(1) 26 : 25 (2) 21 : 17
(3) 18 : 13 (4) 5 : 4

10. A bag contains ₹ 102 in the form of rupee, 50 paise and 10 paise coins in the ratio 3 : 4 : 10. The number of 10 paise coins is
(1) 340 (2) 60
(3) 80 (4) 170

11. I have ₹ 1 coins, 50 paise coins and 25 paise coins. The number of coins are in the ratio of 2.5 : 3 : 4. If the total amount is ₹ 210. The number of ₹1 coin is
(1) 90 (2) 85 (3) 100 (4) 105

12. What is the fourth proportional of 3, 4 and 6?
(1) 8 (2) 9 (3) 12 (4) 2

13. If $x : 3 :: 12 : 4$, then value of x is
(1) 9 (2) 16
(3) 12 (4) 18

14. $A : B = 5 : 7$ and $B : C = 6 : 1$. So, $A : B : C$ is
(1) 5 : 7 : 6 (2) 5 : 7 : 1
(3) 30 : 7 : 42 (4) 30 : 42 : 7

15. An amount of ₹ 450 is shared by A and B in the ratio 4 : 5. The shares of A and B will be
(1) ₹ 400, ₹ 50 (2) ₹ 50, ₹ 400
(3) ₹ 250, ₹ 200 (4) ₹ 200, ₹ 250

16. The two numbers are in ratio 11 : 9. If sum of these two numbers is 40, then product of these two numbers is
(1) 396 (2) 432
(3) 440 (4) 384

17. The ratio of copper and zinc is 11: 6. How much zinc is there in 850 kg of brass?
(1) 510 kg
(2) 200 kg
(3) 300 kg
(4) 550 kg

18. Which of the following is smallest?
(1) 1 : 3 (2) 3 : 5
(3) 7 : 9 (4) 10 : 12

Answers

1. (1)	**2.** (3)	**3.** (1)	**4.** (3)	**5.** (1)	**6.** (1)	**7.** (3)	**8.** (1)	**9.** (1)	**10.** (4)
11. (4)	**12.** (1)	**13.** (1)	**14.** (4)	**15.** (4)	**16.** (1)	**17.** (3)	**18.** (1)		

Hints and Solutions

1. $\because 25:35 = \frac{25}{35} = \frac{5}{7}$ or 5 : 7

2. $\because$ One day = 24 h

$\therefore$ 1 h : 24 h = 1 : 24

3. Given, $\frac{A}{B} = \frac{2}{3}, \frac{B}{C} = \frac{4}{5}, \frac{C}{D} = \frac{5}{8}$

$\therefore \frac{A}{D} = \frac{A}{B} \times \frac{B}{C} \times \frac{C}{D} = \frac{2 \times 4 \times 5}{3 \times 5 \times 8} = \frac{1}{3}$

4. Let the numbers be $2x$, $3x$ and $4x$.

$\therefore (2x)^2 + (3x)^2 + (4x)^2 = 116$

$\Rightarrow 4x^2 + 9x^2 + 16x^2 = 116$

$\Rightarrow 29x^2 = 116 \Rightarrow x^2 = 4$

$\Rightarrow x = 2$

($\because x$ cannot be negative)

Hence, required numbers are 4, 6 and 8.

5. $a : b = 7 : 9$

$b : c = 3 : 5 = 9 : 15$

$\therefore \quad a : b : c = 7 : 9 : 15$

6. Let the number to be added to each term be x. Then

$$\frac{7 + x}{13 + x} = \frac{2}{3}$$

$\Rightarrow 3(7 + x) = 2(13 + x)$

$\Rightarrow 21 + 3x = 26 + 2x$

$\Rightarrow 3x - 2x = 26 - 21$

$\therefore \quad x = 5$

7. Let Ajay and Sanjay get ₹ $7x$ and ₹ $11x$, respectively. Then

$11x - 7x = 6000$

$\Rightarrow 4x = 6000$

$\therefore \quad x = 1500$

$\therefore$ Ajay's share = $7x = 1500 \times 7 =$ ₹ 10500

8. Let the number of boys and girls be $4x$ and $6x$, respectively.

According to the question, $\frac{4x + 200}{6x} = \frac{5}{6}$

$\Rightarrow 5x = 4x + 200$

$\Rightarrow x = 200$

Therefore, number of girls

$= 6x = 6 \times 200 = 1200$

9. Total number of students in the school = 1224

Number of girls = 600

$\therefore$ Number of boys = $1224 - 600 = 624$

$\therefore$ Required ratio = 624 : 600 = 26 : 25

10. Ratio of the number of coins = ₹ 1 : 50 paise : 10 paise = 3 : 4 : 10

$\therefore$ Ratio of total values of coins of ₹ 1 : 50 paise : 10 paise = $(100 \times 3) : (50 \times 4) : (10 \times 10)$

$= 300 : 200 : 100 = 3 : 2 : 1$

Total value of 10 paise coins in ₹ 102

$= \frac{1}{3 + 2 + 1} \times 102 = \frac{102}{6} =$ ₹ 17 = 1700 paise

$\therefore$ Number of 10 paise coins = $\frac{1700}{10} = 170$

11. Let number of ₹ 1, 50 paise and 25 paise coins be $2.5x$, $3x$ and $4x$, respectively.

Value of ₹ 1 coins $= 1 \times 2.5x = 2.5x$

Value of 50 paise coins $= 0.50 \times 3x = 1.5x$

Value of 25 paise coins $= 0.25 \times 4x = 1x$

Total value = ₹210

$\therefore \quad 2.5x + 1.5x + 1x = 210$

$5x = 210 \Rightarrow x = 42$

Thus, number of ₹1 coins $= 2.5x$

$= 2.5 \times 42 = 105$

12. Here, $a = 3,\ b = 4,\ c = 6$

$\because$ Fourth proportional of a, b and c

$$= \frac{bc}{a} = \frac{4 \times 6}{3} = 8$$

13. $\because$ First $\times$ Fourth = Second $\times$ Third

$$\Rightarrow \qquad x = \frac{3 \times 12}{4} = 9$$

14. $A : B = 5 : 7$ or $\frac{A}{B} = \frac{5}{7}$

and $\quad B : C = 6 : 1$ or $\frac{B}{C} = \frac{6}{1}$

Now, $\quad \frac{A}{B} = \frac{5 \times 6}{7 \times 6} = \frac{30}{42}$

and $\quad \frac{B}{C} = \frac{6 \times 7}{1 \times 7} = \frac{42}{7}$

So, $\quad A : B : C = 30 : 42 : 7$

[$\because B = 42$ in both ratios]

15. A's share $= \frac{\text{Ratios terms of } A}{\text{Total sum of Ratios}} \times \text{Total amount}$

$$= \frac{4}{5+4} \times 450 = \frac{4}{9} \times 450 = ₹\,200$$

Similarly, B's share

$$= \frac{5}{5+4} \times 450 = \frac{5}{9} \times 450 = ₹\,250$$

16. Let the two numbers are $11x$ and $9x$.

According to the question,

$$11x + 9x = 40$$

$$\Rightarrow \qquad 20x = 40$$

$$\therefore \qquad x = \frac{40}{20} = 2$$

Product of numbers $= 11x \times 9x = 99x^2$

$$= 99 \times (2)^2 = 99 \times 4 = 396$$

17. Zinc $= \frac{6}{17} \times 850 = 300$ kg

18. Given ratios can be written as

$1 : 3 = \frac{1}{3}$, $3 : 5 = \frac{3}{5}$, $7 : 9 = \frac{7}{9}$

and $\quad 10 : 12 = \frac{10}{12}$

Now, $\frac{1}{3} = 0.33\ldots.$; $\frac{3}{5} = 0.6$

$\frac{7}{9} = 0.77\ldots.$; $\frac{10}{12} = 0.83\ldots$

So, $\frac{1}{3}$ or 1 : 3 is smallest ratio.

Self Practice

1. The ratio of the length and breadth of a rectangle is 5 : 4. If the length is 25 m, the breadth is
(1) 15 m (2) 20 m (3) 10 m (4) 12 m

2. If 10, 20, x and 40 are in proportion, the value of x will be
(1) 5 (2) 20 (3) 30 (4) 80

3. Distribute ₹ 800 between Mohan and Sohan in the ratio of 5 : 3. How much amount will Mohan get?
(1) ₹ 400 (2) ₹ 500 (3) ₹ 350 (4) ₹ 250

4. The ratio between men and women working in a garden is 3 : 2. If total number of men and women is 165, the number of men working in the garden is
(1) 105 (2) 100 (3) 99 (4) 150

5. The ratio between water and alcohol in a mixture is 2 : 3. If water content is 4 L, how much is the alcohol in the mixture?
(1) 10 L (2) 6 L (3) 8 L (4) 2 L

6. Which of the following is greatest?
(1) 3 : 4 (2) 4 : 5 (3) 5 : 6 (4) 6 : 7

7. The third proportional to 12 and 30 is
(1) 40 (2) 45 (3) 50 (4) 75

8. What is the ratio of 15 and 25?
(1) 2 : 5 (2) 3 : 5 (3) 4 : 5 (4) 15 : 50

9. Two numbes are in the ratio of 2:3. If 15 added to both the number, then the ratio between two numbers becomes the numbers becomes $\frac{11}{14}$. Find the greater number.
(1) 29 (2) 27 (3) 29 (4) 30

10. If $x : y = 7 : 5$, then the value of $(5x - 2y) : (3x + 2y)$ is
(1) $\frac{24}{13}$ (2) $\frac{25}{31}$ (3) $\frac{23}{25}$ (4) $\frac{26}{31}$

Answers

1. (2)	**2.** (2)	**3.** (2)	**4.** (3)	**5.** (2)	**6.** (4)	**7.** (4)	**8.** (2)	**9.** (2)	**10.** (2)