RATIO AND PROPORTION

In this chapter, we study the ratio and proportion with their properties and also proportion.

Ratio

The ratio of two quantities of the same kind and in the same unit is the fraction that one quantity is of the other.

Note : In the ratio $a : b$, the first term a is antecedent and second term b is consequent.

Properties of Ratios

(i) The value of a ratio remains unchanged, if each one of its terms is multiplied or divided by a same non-zero number.

(ii) If $a : b$ and $c : d$ are two ratios, then the compounded ratio is $ac : bd$.

Example 1 If $a : b = 2 : 5$, the value of $(3a + 4b) : (4a + 5b)$ is

(a) $\dfrac{26}{33}$ (b) $\dfrac{33}{26}$ (c) $\dfrac{44}{23}$ (d) $\dfrac{33}{25}$

Sol. *(a)* We have, $\dfrac{3a + 4b}{4a + 5b} = \dfrac{3\left(\dfrac{a}{b}\right) + 4}{4\left(\dfrac{a}{b}\right) + 5} = \dfrac{3 \times \dfrac{2}{5} + 4}{4 \times \dfrac{2}{5} + 5} = \dfrac{26}{33}$ $\left[\because \dfrac{a}{b} = \dfrac{2}{5} \text{ given}\right]$

Proportion

If two ratio are equal, then we can say that both ratio are in proportion .

Direct Proportion

If the values of two quantities depend on each other in such a way that, a change in one, results in a corresponding change in the other and *vice-versa*, then the two quantities are said to be in direct proportion.

In other words, if two quantities a and b vary with each other in such a manner that the ratio $\dfrac{a}{b}$ (or $a : b$) remains constant, i.e. $\dfrac{a}{b} = k$ or $a = kb$ (where, k is any positive constant), then we say that a and b vary directly with each other or a and b are in direct proportion or a and b have a direct variation.

Example 2 The variable x varies directly as y and $x = 80$ when y is 160. What is y when x is 64?

(a) 127   (b) 128   (c) 129   (d) 130

Sol. *(b)* If x varies directly as y.

$\therefore$ $x / y = k$ (constant)   ...(i)

If $x = 80$ and $y = 160$

$\therefore$ $\dfrac{x}{y} = \dfrac{80}{160} = \dfrac{1}{2} \Rightarrow k = \dfrac{1}{2}$

When $x = 64$, then from Eq. (i),

$$\dfrac{64}{y} = \dfrac{1}{2} \quad \text{[putting the value of } k]$$

$\Rightarrow$ $y = 64 \times 2 = 128$

Inverse Proportion

The two quantities may vary in such a way that if one increases, the other decreases and *vice-versa,* then the two quantities are said to be inverse proportion. In other words if two quantities a and b vary with each other in such a manner that the product ab remains constant and is positive, then we say that a varies inversely with b and b varies inversely with a. Thus, two quantities a and b are said to vary in inverse proportion, if there exists a relation of the type $ab = k$ between them, where k is a positive constant.

Let a, b, c and d are four quantities, then the proportional are $a : b :: c : d$.

where a, b, c, d are known as first proportional, second proportional, third proportional and fourth proportional respectively.

Note (i) In the proportion $a : b :: c : d$, a and d are extreme values and b and c are mean values. i.e. Product of means $=$ Product of extreme.

(ii) If x is the third proportional to a, b, then $a : b :: b : x$.

Example 3 If x varies inversely as y and $y = 60$ when $x = 1.5$. Find x, when $y = 4.5$.

(a) 20   (b) 21   (c) 22   (d) 23

Sol. *(a)* If x varies inversely as y.

$\therefore$ $xy = k$ (constant)   ...(i)

If $x = 1.5$ and $y = 60$

$\therefore$ $xy = 1.5 \times 60 = 90$

$\Rightarrow$ $k = 90$

When $y = 4.5$, then from Eq. (i),

     $4.5 \times x = k$

$\Rightarrow$ $4.5 \times x = 90$ [putting the value of k]

$\Rightarrow$ $x = \dfrac{90}{4.5} = 20$

Example 4 In a camp, there is enough flour for 300 persons for 42 days. How long will the flour last, if 20 more persons join the camp?

(a) $\dfrac{315}{7}$ day   (b) $\dfrac{315}{8}$ day

(c) $\dfrac{126}{8}$ day   (d) None of these

Sol. *(b)* $\because$ For 300 persons flour is enough for 42 days.

$\therefore$ For 1 person flour enough

         $= 300 \times 42 = 12600$ days

Now, 20 more persons join the camp.

So, total persons $= 300 + 20 = 320$

$\therefore$ For 320 persons flour enough

     $= \dfrac{12600}{320} = \dfrac{315}{8}$ day

Example 5 The fourth proportional of 6, 11 and 12 is

(a) 29   (b) 23

(c) 72   (d) 22

Sol. *(d)* Let fourth proportional be x, then

     $6 : 11 :: 12 : x$

     [$\because$ Product of means $=$ Product of extreme]

$\Rightarrow$ $6 \times x = 11 \times 12$

$\Rightarrow$ $x = \dfrac{11 \times 12}{6} = 22$

PRACTICE EXERCISE

1. The number of teeth and the age of a person vary
   (a) directly with each other
   (b) inversely with each other
   (c) neither directly nor inversely with each other
   (d) sometimes directly and sometimes inversely with each other

2. Which of the following vary inversely with each other?
   (a) Speed and distance covered
   (b) Distance covered and taxi fare
   (c) Distance travelled and time taken
   (d) Speed and time taken

3. Both x and y are in direct proportion, then $\dfrac{1}{x}$ and $\dfrac{1}{y}$ are
   (a) in indirect proportion
   (b) in direct proportion
   (c) neither in direct nor in inverse proportion
   (d) sometimes in direct and sometimes in inverse proportion

4. If two quantities p and q vary inversely with each other, then
   (a) $\dfrac{p}{q}$ remains constant
   (b) $p + q$ remains constant
   (c) $p \times q$ remains constant
   (d) $p - q$ remains constant

5. The variable x is inversely proportional to y. If x increases by $p\%$, then by what per cent will y decrease?
   (a) $p\%$
   (b) $2p\%$
   (c) $3p\%$
   (d) None of these

6. l varies directly as m and $l = 5$, when $m = \dfrac{2}{3}$. Find l when $m = \dfrac{16}{3}$.
   (a) 38
   (b) 39
   (c) 40
   (d) 41

7. The fourth proportional to 3, 5 and 21 is
   (a) 35
   (b) $\dfrac{5}{7}$
   (c) $\dfrac{7}{5}$
   (d) None of these

8. What must be added to each term of the ratio $49 : 68$, so that it becomes $3 : 4$?
   (a) 11
   (b) 10
   (c) 7
   (d) 8

9. A bag contains ₹600 in the form of one rupee, 50 paise and 25 paise coins in the ratio $3 : 4 : 12$. The number of 25 paise coins is
   (a) 800
   (b) 900
   (c) 1205
   (d) None of these

10. The cost of making an article is divided between materials, labour and overheads in the ratio $5 : 3 : 1$. If the materials cost ₹6.90, then cost of the article is
    (a) ₹12.42
    (b) ₹13.20
    (c) ₹14.00
    (d) None of these

11. If $\dfrac{a}{3} = \dfrac{b}{4} = \dfrac{c}{7}$, then $\dfrac{a + b + c}{c}$ is equal to
    (a) $\dfrac{1}{7}$
    (b) $\dfrac{1}{2}$
    (c) 7
    (d) 2

12. Out of the ratios $7 : 20 ; 13 : 25 ; 17 : 30$ and $11 : 15$, the smallest one is
    (a) $10 : 7$
    (b) $7 : 20$
    (c) $17 : 20$
    (d) None of these

13. ₹770 have been divided among A, B, C in such a way that A receives $\dfrac{2}{9}$th of what B and C together receive. Then, A's share is
    (a) ₹140
    (b) ₹154
    (c) ₹165
    (d) ₹ 170

14. If x varies inversely as y and $x = 20$ when $y = 600$, find y when $x = 400$.
    (a) 30
    (b) 40
    (c) 32
    (d) 35

15. If $2x = 3y = 4z$, then $x : y : z$ is
    (a) $6 : 4 : 3$ (b) $5 : 4 : 2$
    (c) $2 : 3 : 4$ (d) None of these

16. The ratio of number of boys and girls in a school of 720 students is $7 : 5$. How many more girls should be admitted to make the ratio $1 : 1$?
    (a) 160   (b) 145   (c) 120   (d) 170

17. The incomes of A and B are in the ratio $3 : 2$ and their expenditures in the ratio $5 : 3$. If each saves ₹1500, then B's income is
    (a) ₹6000   (b) ₹4700
    (c) ₹3000   (d) ₹7500

18. Third proportional to 9 and 12 is
    (a) 16   (b) 10.5
    (c) $6\sqrt{3}$ (d) None of these

19. If $a : b = 3 : 4$, then $(6a + b) : (4a + 5b)$ is
    (a) $1 : 2$
    (b) $3 : 5$
    (c) $7 : 9$
    (d) None of the above

20. The ratio of zinc and copper in a brass piece is $13 : 7$. How much zinc will be there in 100 kg of such a piece?
    (a) 65 kg   (b) 40 kg
    (c) 45 kg   (d) 50 kg

Answers

1	(d)	2	(d)	3	(b)	4	(c)	5	(a)	6	(c)	7	(a)	8	(d)	9	(b)	10	(a)
11	(d)	12	(b)	13	(a)	14	(a)	15	(a)	16	(c)	17	(a)	18	(a)	19	(d)	20	(a)

Hints and Solutions

1. The number of teeth and the age of a person vary sometimes directly and sometimes inversely with each other, we cannot predict about the number of teeth with exactly the age of a person. It change with person-to-person.

Hence, option (d) is correct.

2. We know that, when we increases the speed, then the time taken by vehicle decreases.

Hence, speed and time taken vary inversely with each other.

So, option (d) is correct.

3. If both x and y are in directly proportion, then $\dfrac{1}{x}$ and $\dfrac{1}{y}$ are in direct proportion.

Hence, option (b) is correct.

4. If two quantities p and q vary inversely with each other, then $p \times q$ remains constant. Since, in inverse proportion, an increase in p cause a proportional decrease in q and vice-versa.

Hence, option (c) is correct.

5. The variable x is inversely proportionls to y.

∴ $xy = k$ (constant)

Since, we know that two quantities x and y are said to be in inverse proportion, if an increase in x cause a proportional decrease in y and *vice-versa*.

So, we can say y decrease by $p\%$.

6. If l varies directly as m.

∴ $l / m = k$ (constant)   …(i)

If $l = 5$ and $m = \dfrac{2}{3}$

∴ $\dfrac{l}{m} = \dfrac{5}{2/3} = \dfrac{5}{1} \times \dfrac{3}{2} = \dfrac{15}{2}$

⇒ $k = \dfrac{15}{2}$

When, $m = \dfrac{16}{3}$, then from Eq. (i),

      $\dfrac{l}{16/3} = \dfrac{15}{2}$ [putting the value of k]

⇒ $l = \dfrac{15}{2} \times \dfrac{16}{3} = 40$

7. Let the fourth proportional be x.

∴ $3 : 5 :: 21 : x$

⇒ $x = \dfrac{5 \times 21}{3} = 35$

8. Let the number be x added in the given ratio.

$\therefore \quad \dfrac{49 + x}{68 + x} = \dfrac{3}{4}$

$\Rightarrow \quad 4(49 + x) = 3(68 + x)$

$\Rightarrow \qquad\qquad x = 8$

9. Given ratio is $3 : 4 : 12$.

The ratio of values in coins $= \dfrac{3}{1} : \dfrac{4}{2} : \dfrac{12}{4} = 3 : 2 : 3$

Value of 25 paise coins $= ₹\left(600 \times \dfrac{3}{8}\right) = ₹\,225$

$\therefore$ Number of these coins $= 225 \times 4 = 900$

10. If material cost be ₹5, then cost of article be ₹9.

$\therefore \qquad\qquad 5 : 9 :: 6.90 : x$

$\Rightarrow \qquad\qquad x = \dfrac{9 \times 6.90}{5}$

$\Rightarrow \qquad\qquad x = ₹\,12.42$

11. Let $\dfrac{a}{3} = \dfrac{b}{4} = \dfrac{c}{7} = k$.

Then, $a = 3k$, $b = 4k$ and $c = 7k$.

$\therefore \qquad \dfrac{a + b + c}{c} = \dfrac{3k + 4k + 7k}{7k} = 2$

12. Now, $\dfrac{7}{20} = 0.35$, $\dfrac{13}{25} = 0.52$, $\dfrac{17}{30} = 0.56$ and

$\dfrac{11}{15} = 0.73$.

$\therefore$ The smallest value is 0.35 i.e. ratio $7 : 20$.

13. $\therefore \quad A : (B + C) = 2 : 9$

$\therefore \quad A\text{'s part} = ₹\left(770 \times \dfrac{2}{11}\right) = ₹\,140$

14. If x varies inversely as y.

$\therefore \qquad\qquad xy = k\ (\text{constant}) \qquad\qquad …(i)$

If $\qquad\qquad x = 20$ and $y = 600$

$\therefore \qquad\qquad xy = 20 \times 600 = 12000$

$\Rightarrow \qquad\qquad k = 12000$

When $x = 400$, then from Eq. (i),

$\qquad\qquad y \times 400 = k$

$\Rightarrow \qquad y \times 400 = 12000$ [putting the value of k]

$\Rightarrow \qquad\qquad y = \dfrac{12000}{400} = 30$

15. Let $2x = 3y = 4z = k$

Then, $x = \dfrac{k}{2}$, $y = \dfrac{k}{3}$ and $z = \dfrac{k}{4}$

$\therefore \quad x : y : z = \dfrac{k}{2} : \dfrac{k}{3} : \dfrac{k}{4} = \dfrac{1}{2} : \dfrac{1}{3} : \dfrac{1}{4} = 6 : 4 : 3$

16. Number of boys $= 720 \times \dfrac{7}{12} = 420$

Number of girls $= 720 - 420 = 300$

$\therefore \qquad$ Number of girls to be admitted

$\qquad\qquad = 420 - 300 = 120$

17. Let their incomes be $3x$ and $2x$ and their corresponding expenditure be $5y$ and $3y$.

According to the given conditions,

$\qquad 3x - 5y = 1500$ and $2x - 3y = 1500$

On solving, we get

$\qquad\qquad x = 3000$ and $y = 1500$

Hence, B's income $= 2x = ₹\,6000$

18. Let third proportion be x.

$\therefore \qquad\qquad 9 : 12 :: 12 : x$

$\Rightarrow \qquad\qquad 9x = 12 \times 12$

$\Rightarrow \qquad\qquad x = 16$

19. Now, $\dfrac{6a + b}{4a + 5b} = \dfrac{6\left(\dfrac{a}{b}\right) + 1}{4\left(\dfrac{a}{b}\right) + 5}$

$= \dfrac{6 \times \dfrac{3}{4} + 1}{4 \times \dfrac{3}{4} + 5} = \dfrac{11}{16}$

20. Amount of zinc $= \left(100 \times \dfrac{13}{20}\right)\,\text{kg}$

$\qquad\qquad = 65\,\text{kg}$