02

Fraction and Decimal Fractions

Fraction

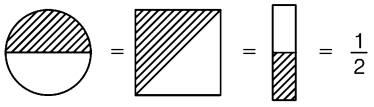
The number of the form $\frac{a}{b}$, where *a* and *b* are natural numbers are known as fraction.

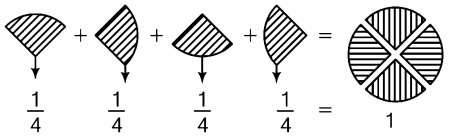
e.g. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$

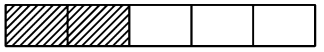
Here, a is called numerator and b is called denominator.

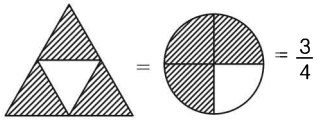
A fraction may also be defined as the quotient, obtained on dividing the numerator by denominator.

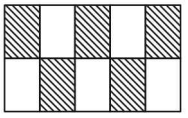
Some figures are given below to understand the fraction in better way.

1.  $= \frac{1}{2}$

2. 
$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1$

3.  $= \frac{2}{5}$

4.  $= \frac{3}{4}$

5.  $= \frac{5}{10} = \frac{1}{2}$

Types of Fraction

Proper Fraction

In a fraction when numerator is less than the denominator, it is called as proper fraction, e.g. 4/7 is a proper fraction.

Improper Fraction

In a fraction when numerator is greater than the denominator, it is called as improper fraction , e.g. 7/4 is an improper fraction.

Mixed Fraction

A fraction with a whole number part and a fractional part is called as mixed fraction, e.g. $4\frac{2}{3}$ is a mixed fraction.

Every mixed fraction can be expressed as improper fraction and *vice-versa*.

e.g. $3\frac{4}{7} = \frac{3\times7+4}{7} = \frac{25}{7}$

Decimal Fraction

This type of fractions, is a fraction whose denominator is 10 or a power of 10.

e.g. $\frac{7}{10}, \frac{9}{100}, \frac{3}{1000}$ etc.

Example 1 Express ₹ 2.25 in fraction.

(a) $\frac{5}{4}$ (b) $2\frac{1}{2}$ (c) $2\frac{1}{4}$ (d) $5\frac{1}{3}$

Sol. *(c)* $2.25 = \frac{225}{100} = \frac{9}{4} = 2\frac{1}{4}$

Operations on Fraction

Addition of Fractions

(i) If denominator of all the fractions are same, then numerators of fractions are added and their addition is divided by denominator.

e.g. $\frac{1}{2} + \frac{5}{2} + \frac{7}{2} = ?$

$\Rightarrow \quad ? = \frac{1+5+7}{2} = \frac{13}{2}$

(ii) If denominator of all the fractions are not same (i.e. unequal), take LCM of all the denominators and then divide the LCM by each denominator and then multiply the quotient to the numerator

e.g. $\frac{3}{7} + \frac{11}{3} + \frac{4}{21}$

$$= \frac{(21/7)\times 3 + (21/3)\times 11 + (21/21)\times 4}{21}$$

$$= \frac{3\times 3 + 7\times 11 + 1\times 4}{21} = \frac{9+77+4}{21}$$

$$= \frac{90}{21} = \frac{30}{7} \text{ or } 4\frac{2}{7}$$

Example 2 Find the value of fraction $\frac{2}{7} + \frac{1}{3} + \frac{4}{9} + \frac{2}{5}$ is

(a) $\frac{315}{461}$ (b) $\frac{461}{315}$ (c) $\frac{350}{443}$ (d) $\frac{72}{105}$

Sol. *(b)* $\frac{2}{7} + \frac{1}{3} + \frac{4}{9} + \frac{2}{5} = ?$

$$\Rightarrow ? = \frac{\left(\frac{315}{7}\times 2\right) + \left(\frac{315}{7}\times 1\right) + \left(\frac{315}{9}\times 4\right) + \left(\frac{315}{5}\times 2\right)}{315}$$

$$\Rightarrow ? = \frac{90+105+140+126}{315} \quad \Rightarrow ? = \frac{461}{315}$$

Subtraction of Fractions

(i) When denominator of all the fractions are same.

e.g. $\frac{8}{9} - \frac{2}{9} - \frac{4}{9} = \frac{8-2-4}{9} = \frac{8-6}{9} = \frac{2}{9}$

(ii) When denominator of all the fractions are not same (i.e. unequal)

e.g. $\frac{12}{13} - \frac{6}{8} - \frac{1}{13}$

$$= \frac{(104/13)\times 12 - (104/8)\times 6 - (104/13)\times 1}{104}$$

$$= \frac{8\times 12 - 13\times 6 - 8\times 1}{104} = \frac{96-78-8}{104}$$

$$= \frac{96-86}{104} = \frac{10}{104} = \frac{5}{52}$$

Example 3 Find the value of fraction $\frac{7}{3} - \frac{2}{5} - \frac{1}{4} - \frac{8}{15}$ is

(a) $\frac{135}{31}$ (b) $\frac{31}{120}$ (c) $\frac{69}{60}$ (d) $\frac{135}{23}$

Sol. *(c)* $\frac{7}{3} - \frac{2}{5} - \frac{1}{4} - \frac{8}{15} = ?$

$\Rightarrow \quad \frac{140-24-15-32}{60} = ? \Rightarrow \frac{69}{60} = ?$

Multiplication of Fractions

If fraction is in simple form (i.e. Proper/Improper fraction), then multiply all the numerators together and denominators separately, then reduce the fraction in its simplest form.

e.g. $\frac{1}{3}\times\frac{4}{5}\times\frac{5}{6}\times\frac{3}{2} = \frac{1\times 4\times 5\times 3}{3\times 5\times 6\times 2} = \frac{4}{12} = \frac{1}{3}$

Example 4 Multiply $5\frac{1}{2}\times\frac{2}{3}\times 2\frac{1}{11}$.

(a) $7\frac{2}{3}$ (b) $2\frac{1}{11}$

(c) $2\frac{2}{11}$ (d) $5\frac{3}{4}$

Sol. *(a)* $5\frac{1}{2}\times\frac{2}{3}\times 2\frac{1}{11} = ? \Rightarrow \frac{11}{2}\times\frac{2}{3}\times\frac{23}{11} = ?$

$\Rightarrow \quad \frac{23}{3} = ? \Rightarrow 7\frac{2}{3} = ?$

Division of Fractions

If fraction is in simple form, retain the fraction as it is, but take the inverse of all the fraction(s) after division, then multiply them.

e.g. $\frac{2}{3} \div \frac{4}{5} = \frac{2}{3}\times\frac{5}{4} = \frac{10}{12} = \frac{5}{6}$

For multiplication or division of mixed fractions convert them into improper fractions first.

Example 5 Simplify the expression $11\frac{4}{3} \div 15\frac{1}{7}$.

(a) $\frac{279}{319}$ (b) $\frac{259}{318}$ (c) $\frac{319}{279}$ (d) $\frac{333}{123}$

Sol. (*b*) $11\frac{4}{3} \div 15\frac{1}{7} = ? \Rightarrow \frac{37}{3} \div \frac{106}{7} = ?$

$\Rightarrow \quad \frac{37}{3} \times \frac{7}{106} = ? \Rightarrow \frac{259}{318} = ?$

Comparison of Fraction

There are two Methods of comparison of fraction.

1. Comparing Like Fractions

Like fraction are easy to compare, since they have the same denominators.

In a series of two or more fractions, if denominators of all the fractions are same, then

(i) that fraction will be smallest whose Numerator will be smallest.

(ii) that fraction will be highest whose Numerator will be greatest.

e.g. Arrange $\frac{9}{10}, \frac{7}{10}, \frac{3}{10}$ in descending order.

Since, denominator of all are same, then the fraction having greater numerator must be greater as $9 > 7 > 3$. And the fraction with lesser numerator (smaller numerator) must be smaller, so the order of the fraction is given below

$$\frac{9}{10} > \frac{7}{10} > \frac{3}{10}$$

2. Comparing Unlike Fractions

Two cases arise while comparing unlike fractions.

Case I Fractions with the Same Numerator

e.g. Arrange $\frac{5}{12}, \frac{5}{23}, \frac{5}{17}, \frac{5}{11}$ in ascending order.

In this case the fraction which has a bigger denominator is smaller. Hence, in the above fractions $\frac{5}{23}$ is the smallest.

Next $\frac{5}{17}$, then $\frac{5}{12}$ and $\frac{5}{11}$ is the largest.

Therefore, $\frac{5}{23} < \frac{5}{17} < \frac{5}{12} < \frac{5}{11}$

Case II Fractions with Different Numerators

Convert these unlike fractions into like fractions by making the denominator same.

e.g. Arrange $\frac{2}{3}, \frac{4}{5}$ and $\frac{5}{6}$ in ascending order.

The easiest way to do it is to find the LCM of all the denominators. LCM of 3, 5 and 6 is 30.

Now, multiply both the numerator and the denominator of all the fractions by suitable numbers to get the denominator as 30.

$$\frac{2}{3} = \frac{2 \times 10}{3 \times 10} = \frac{20}{30}, \frac{4}{5} = \frac{4 \times 6}{5 \times 6} = \frac{24}{30}$$

$$\frac{5}{6} = \frac{5 \times 5}{6 \times 5} = \frac{25}{30}$$

Thus, $\frac{20}{30} < \frac{24}{30} < \frac{25}{30}$

Therefore, $\frac{2}{3} < \frac{4}{5} < \frac{5}{6}$

Example 6 Arrange the fractions $\frac{1}{3}, \frac{5}{7}$ and $\frac{1}{2}$ in ascending order

(a) $\frac{1}{2} < \frac{5}{7} < \frac{1}{3}$ (b) $\frac{1}{3} < \frac{5}{7} < \frac{1}{2}$

(c) $\frac{5}{7} < \frac{1}{2} < \frac{1}{3}$ (d) $\frac{1}{3} < \frac{1}{2} < \frac{5}{7}$

Sol. *(d)* First take the LCM of denominators of all the fractions

i.e. LCM of 3, 7, 2 = 42

Now, $\frac{1 \times 14}{3 \times 14} = \frac{14}{42}, \frac{5 \times 6}{7 \times 6} = \frac{30}{42}$

and $\frac{1 \times 21}{2 \times 21} = \frac{21}{42}$

In ascending order, $\frac{14}{42} < \frac{21}{42} < \frac{30}{42} \Rightarrow \frac{1}{3} < \frac{1}{2} < \frac{5}{7}$

Example 7 Find the value of $2 + 0.2 + 0.02 + 0.002$ is

(a) 2.222 (b) 2.022 (c) 2.220 (d) 2.2222

Sol. *(a)* $2 + 0.2 + 0.02 + 0.002 = ?$

$\Rightarrow \quad 2.000 + 0.200 + 0.020 + 0.002 = ?$

$\Rightarrow \quad \frac{2000}{1000} + \frac{200}{1000} + \frac{20}{1000} + \frac{2}{1000} = ?$

$\Rightarrow \quad ? = \frac{2222}{1000} = 2.222$

Practice Exercise

1. Change $\frac{75}{100}$ into decimal fraction.

(a) 0.075 (b) 0.75 (c) 0.0075 (d) 7.5

2. Convert 0.4776 into the fraction.

(a) $\frac{43}{90}$ (b) $\frac{4776}{10000}$

(c) $\frac{4776}{1000}$ (d) $\frac{4776}{100}$

3. $4+4.44+44.4=?$

(a) 52.08 (b) 50.48

(c) 52.84 (d) 5.284

4. $1+0+\frac{9}{10}+\frac{3}{100}=?$

(a) 1.93 (b) 1.093

(c) 1.903 (d) 1.0093

5. Arrange the fraction $\frac{21}{42},\frac{14}{42},\frac{30}{42}$ in desending order

(a) $\frac{14}{42}<\frac{30}{42}<\frac{21}{42}$ (b) $\frac{30}{42}<\frac{21}{42}<\frac{14}{42}$

(c) $\frac{30}{42}>\frac{21}{42}>\frac{14}{42}$ (d) $\frac{21}{42}>\frac{14}{42}>\frac{30}{42}$

6. Divide the $1.2875\div25$.

(a) 0.515 (b) 0.0515

(c) 0.00515 (d) 5.15

7. Find the sum of $\frac{3}{10}+\frac{5}{100}+\frac{8}{1000}$ in decimal form.

(a) 0.853 (b) 3.58

(c) 8.35 (d) 0.358

8. Simplify the expression $\frac{5}{2}+\frac{3}{4}$

(a) $\frac{13}{4}$ (b) $\frac{15}{4}$

(c) $\frac{14}{3}$ (d) $\frac{17}{4}$

9. Simplify the decimal fraction 1088.88 + 1800.08 + 1880.80

(a) 8790.86 (b) 15185.86

(c) 5588.80 (d) 4769.76

10. Arrange 0.583, 0.666 and 0.375 in the ascending order.

(a) 0.666 > 0.583 > 0.375

(b) 0.375 > 0.583 > 0.666

(c) 0.666 > 0.375 > 0.583

(d) 0.583 > 0.375 > 0.666

11. The number which is to be added to 0.001 to get 1.111

(a) 0.111 (b) 0.112 (c) 1.1 (d) 1.11

12. If $\frac{1}{36.18}=0.0276$, then the value of $\frac{1}{0.0003618}$ will be

(a) 276 (b) 2760

(c) 27600 (d) 0.0002760

13. Select the correct answer of $4+4.44+5.04+55.5+555=?$

(a) 472.88 (b) 495.22

(c) 577.2 (d) None of these

14. $25+\frac{3}{100}+\frac{4}{1000}$ is equal to

(a) 25.34 (b) 25.304

(c) 25.034 (d) 25.0034

15. Siddhant spent $\frac{5}{8}$ of his time studying hindi. He spent $\frac{4}{5}$ as much time studying mathematics as hindi, then what fraction of siddhant's study time was spent studying mathematics?

(a) $\frac{1}{2}$ (b) $\frac{3}{5}$ (c) $\frac{11}{12}$ (d) $\frac{3}{7}$

Answers

1	(b)	2	(b)	3	(c)	4	(a)	5	(c)	6	(b)	7	(d)	8	(a)	9	(d)	10	(a)
11	(d)	12	(b)	13	(d)	14	(c)	15	(a)										

Hints & Solutions

1. $\frac{75}{100} = 0.75$

2. $0.4776 = \frac{4776}{10^4} = \frac{4776}{10000}$

3. $4 + 4.44 + 44.4 = 8.44 + 44.40 = 52.84$

4. $1 + 0 + \frac{9}{10} + \frac{3}{100} = 1 + 0 + 0.9 + 0.03 = 1.93$

5. Here, the denominator are equal then the fraction with smaller numerator must be smallest.

In descending order $\frac{30}{42} > \frac{21}{42} > \frac{14}{42}$

6. $\frac{1.2875}{25} = \frac{12875}{25} \times 10^{-4} = 515 \times 10^{-4} = 0.0515$

7. $\frac{3}{10} + \frac{5}{100} + \frac{8}{1000} = 0.3 + 0.05 + 0.008$

$= 0.358$

8. $\frac{5}{2} + \frac{3}{4} = \frac{10+3}{4} = \frac{13}{4}$

9.
$$\begin{array}{r} 1088.88 \\ 1800.08 \\ +\ 1880.80 \\ \hline 4769.76 \end{array}$$

10. In ascending order
$0.375 < 0.583 < 0.666$

11. $1.111 - 0.001 = 1.11$

12. Multiply both sides by 10^5 to get the answer.

13. $4 + 4.44 + 5.04 + 55.5 + 555 = 623.98$

14. $25 + \frac{3}{100} + \frac{4}{1000} = 25 + 0.03 + 0.004 = 25.034$

15. Let the total time of study be 'x'. So,

$\because$ Time for Hindi $= x \times \frac{5}{8}$

$\therefore$ Time for Mathematics $= x \times \frac{5}{8} \times \frac{4}{5} = x \times \frac{1}{2}$

Hence, Siddhant spent $\frac{1}{2}$ of his time studying Mathematics.

Try Yourself

1) Simplify 0.8697 + 0.249 + 0.09 + 0.9.
(a) 2.1025 (b) 2.1087
(c) 1.1081 (d) 2.5087

2) Simplify $5\frac{2}{3} + 16\frac{1}{5} - 12\frac{1}{3}$.
(a) $9\frac{8}{15}$ (b) $6\frac{7}{15}$ (c) $5\frac{1}{15}$ (d) $2\frac{1}{15}$

3) Simplify $165 \div 15 + 5 \times 10$.
(a) 78 (b) 61 (c) 65 (d) 68

4) Simplify $\frac{4}{9} \times \frac{18}{5} \div \frac{24}{5}$.
(a) $\frac{1}{2}$ (b) $\frac{1}{8}$ (c) $\frac{1}{3}$ (d) $\frac{1}{6}$

5) Simplify 3034 – (1002 – 20.04).
(a) 2052.04 (b) 1052.04
(c) 5012.04 (d) 3052.52

6) Simplify $\frac{1 \div 35}{2 \div 7}$.
(a) $\frac{1}{5}$ (b) $\frac{1}{12}$ (c) $\frac{1}{10}$ (d) $\frac{1}{9}$

7) Twenty-six and twenty-six hundredths is written as
(a) 2626 (b) 26.26 (c) 262.6 (d) 2.626

8) $(1.1 + 1.1 + 1.1 + 1.1) \times 1.1 \times 1.1 = ? \times 0.121$, find the value of question (?) mark.
(a) 50 (b) 45 (c) 56 (d) 44

9) What should come in the following at the place of blank space?
$0.001 \times 0.01 = 0.1 + \ldots$
(a) –0.09999 (b) –0.9909
(c) –9.0009 (d) –9.999

10) A man spent $\frac{1}{4}$ part of his income on food, $\frac{2}{3}$ part on rent and the remaining ₹ 630 on other expenses. His house rent is
(a) ₹ 5040 (b) ₹ 3520
(c) ₹ 4840 (d) ₹ 4458

Answers

1	(b)	2	(a)	3	(b)	4	(c)	5	(a)
6	(c)	7	(b)	8	(d)	9	(a)	10	(a)