



# Algebraic Expression and Identities

## Constants and Variables

The symbols which have fixed numeric values are known as **constants**.

**Variables** are the symbols like  $a, b, x, y, \dots$  which take on various numerical values.

e.g. Suppose  $c = 2\pi r$  is the formula of circumference of circle, where 2 and  $\pi$  are constants, whereas  $c$  and  $r$  are variables.

## Algebraic Expression

An algebraic expression is a combination of constants and variables connected by fundamental operations ( $+, -, \times, \div$ ).

e.g.  $2x + 3$ ,  $8a^2b + a^3\sqrt{b} - 5/a$  etc.

## Terms

The separated parts of an algebraic expression are called its term.

e.g.  $2x$  and  $3$  are the terms of expression  $2x + 3$ .

## Like and Unlike Terms

The terms having same variable and the same exponents are like terms, otherwise it has, unlike terms.

e.g. In  $x^2 - xy + 2x^2 + y$ ,  $x^2$  and  $2x^2$  are like terms and  $-xy$  and  $y$  are unlike terms.

## Polynomial

An algebraic expression in which the variables involved only non-negative integral powers is called a polynomial.

e.g. Expression  $3x^2y + 3y^2 + 2x$  is a polynomial.

☑ An expression may contain a term involving rational power of a variable but in a polynomial, the power of each variable must be a non-negative integer.

## Degree of a Polynomial

Highest power of the variable in a polynomial is known as the **degree** of that polynomial.

### 1. Degree of a Polynomial in One Variable

For a polynomial in one variable, the highest power of the variable is called the **degree** of a polynomial.

e.g.  $2x^4 - 6x^3 + 4x + 1$  is a polynomial in  $x$  of degree 4. [since, the highest power of  $x$  is 4]

### 2. Degree of a Polynomial in Two or More Variables

For a polynomial in more than one variable, the sum of the powers of the variables in each term is taken up and the highest sum so obtained is called the degree of a polynomial.

e.g.  $7x^3 - 5x^2y^2 + 3xy + 6y + 8$  is a polynomial in  $x$  and  $y$  of degree 4.

[since, the highest sum of powers of  $x$  and  $y$  is  $2 + 2$ , i.e. 4]

**Example 1** Find the degree of the polynomial

$$8xy^2 - 4y + 6.$$

- (a) 3 (b) 2  
(c) 4 (d) 1

**Sol.** (a) We have,  $8xy^2 - 4y + 6$

Here, sum of the powers of variables in first term is  $1 + 2$ , i.e. 3, which is the highest, so its degree is 3.

## Classifications of Polynomials

### (i) Monomial

A polynomial containing one non-zero term, is called a **monomial** ('mono' means 'one').

e.g.  $5x$ ,  $7$ ,  $3x^3$  and  $-7x^2$  are all monomials.

### (ii) Binomial

A polynomial containing two non-zero terms, is called a **binomial** ('bi' means 'two').

e.g.  $(5 + 7x)$  and  $(7x^2y + 3y)$  are binomials.

### (iii) Trinomial

A polynomial containing three non-zero terms, is called a **trinomial** ('tri' means 'three').

e.g.  $(8 + 3x + x^2)$  and  $(7 + 5xy + 6xy^2)$  are trinomials.

**Example 2** Identify the following polynomial  $4xy + 2x$ .

- (a) Monomial (b) Binomial  
(c) Trinomial (d) None of these

**Sol.** (b) We have,  $4xy + 2x$

Here, we see that there are two terms in the polynomial. So, it is a binomial.

## Types of Polynomials

### (i) Constant Polynomial

A polynomial of degree zero, is called constant polynomial.

or

A polynomial containing only one constant term, is called a constant polynomial.

e.g.  $3$ ,  $-7$  and  $7/4$  are constant polynomials.

### (ii) Linear Polynomial

A polynomial of degree 1, is called a linear polynomial.

e.g.  $2x + 5$  is a linear polynomial in  $x$ .

### (iii) Quadratic Polynomial

A polynomial of degree 2, is called a quadratic polynomial.

e.g.  $3x^2 + 7x + 9$  is a quadratic polynomial in  $x$ .

### (iv) Cubic Polynomial

A polynomial of degree 3, is called a cubic polynomial.

e.g.  $7x^3 - 5x^2 + 3x - 9$  is a cubic polynomial in  $x$ .

### (v) Biquadratic Polynomial

A polynomial of degree 4, is called a biquadratic polynomial.

e.g.  $5x^4 - 7x^3 + 8x^2 - 12x - 10$  is a biquadratic polynomial in  $x$ .

**Example 3** Identify the type of polynomial

$$3y^2 + 4y + 5.$$

- (a) Constant polynomial  
(b) Linear polynomial  
(c) Quadratic polynomial  
(d) Cubic polynomial

**Sol.** (c) We have,  $3y^2 + 4y + 5$

Here, we see that, the highest power of variable  $y$  is 2. So, it is a quadratic polynomial.

## Fundamental Operations on Polynomials

### (i) Addition of Polynomials

Polynomials can be added by arranging their like terms and combining them.

## (ii) Subtraction of Polynomials

Polynomials can be subtracted by arranging their like terms and by changing sign of each term of the polynomial to be subtracted and then added.

## (iii) Multiplication of Polynomials

We know that

- (a) The product of two factors with like signs is positive and product of unlike signs is negative.
- (b) If  $x$  is any variable and  $m, n$  are positive integers, then  $x^m \times x^n = x^{m+n}$   
e.g.  $x^3 \times x^6 = x^{(3+6)} = x^9$ .

## (iv) Division of Polynomials

Different forms of division of one polynomial to the another polynomial is given below

### (a) Division of a Monomial by a Monomial

We have, quotient of two monomials

$$= (\text{Quotient of their coefficients}) \\ \times (\text{Quotients of two monomials})$$

### (b) Division of a Polynomial by a Monomial

To divide a polynomial by a monomial, we can divide each term of the polynomial by the monomial.

$$\text{e.g. } \frac{2x^2 + 6x + 8}{2} = \frac{2x^2}{2} + \frac{6x}{2} + \frac{8}{2} = x^2 + 3x + 4$$

### (c) Division of a Polynomial by a Polynomial

For dividing a polynomial by another polynomial, proceed according to the following steps

- (i) Arrange the terms of the dividend and divisor in descending order of their degrees.
- (ii) Divide the first term of the dividend by the first term of the divisor to obtain the first term of the quotient.
- (iii) Multiply all the terms of the divisor by the first term of the quotient and subtract the result from the dividend.
- (iv) Consider the remainder (if any) as a new dividend and proceed as before.

- (v) Repeat this process till we obtain a remainder which is either 0 or a polynomial of degree less than the degree of the divisor.

**Example 4** Subtract  $3t^4 - 4t^3 + 2t^2 - 6t + 6$  from  $-4t^4 + 8t^3 - 4t^2 - 2t + 11$ .

- (a)  $-7t^4 + 12t^3 - 6t^2 + 4t + 5$
- (b)  $7t^4 - 12t^3 + 6t^2 + 4t + 5$
- (c)  $-7t^4 + 12t^3 + 6t^2 - 4t - 5$
- (d) None of the above

**Sol.** (a) We have,  $3t^4 - 4t^3 + 2t^2 - 6t + 6$

and  $-4t^4 + 8t^3 - 4t^2 - 2t + 11$

The required difference is given by

$$\begin{aligned} & (-4t^4 + 8t^3 - 4t^2 - 2t + 11) \\ & \quad - (3t^4 - 4t^3 + 2t^2 - 6t + 6) \\ &= -4t^4 + 8t^3 - 4t^2 - 2t + 11 - 3t^4 + 4t^3 - 2t^2 + 6t - 6 \\ &= (-4t^4 - 3t^4) + (8t^3 + 4t^3) + (-4t^2 - 2t^2) \\ & \quad + (-2t + 6t) + (11 - 6) \\ & \quad \text{[grouping like terms]} \\ &= -7t^4 + 12t^3 - 6t^2 + 4t + 5 \end{aligned}$$

**Example 5** Find the quotient and the remainder, when  $(15z^3 - 20z^2 + 13z - 12)$  is divided by  $(3z - 6)$ .

- (a)  $5z^2 + \frac{10}{3}z + 11; 57$
- (b)  $5z^2 + \frac{10}{3}z + 11; 54$
- (c)  $5z^2 + 10z + 11; 54$
- (d)  $5z^2 + \frac{10}{3}z - 11; 54$

**Sol.** (b)  $5z^2 + \frac{10}{3}z + 11$

$$\begin{array}{r} 3z - 6 \overline{) 15z^3 - 20z^2 + 13z - 12} \\ \underline{15z^3 - 30z^2} \phantom{+ 13z - 12} \\ 10z^2 + 13z - 12 \\ \underline{10z^2 - 20z} \phantom{- 12} \\ 33z - 12 \\ \underline{33z - 66} \\ 54 \end{array}$$

$$\therefore \text{Quotient} = 5z^2 + \frac{10}{3}z + 11$$

and Remainder = 54

## Identity

An identity is an equality which is true for all values of the variables.

e.g.  $(a + 1)(a + 3) = a^2 + 4a + 3$  is an identity, because it is true for every value of  $a$ .

### Standard Identities

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- $(a + b)(a - b) = a^2 - b^2$
- $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$   
 $= a^2 + b^2 + c^2 + 2(ab + bc + ca)$
- $(a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$
- $(a - b + c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$
- $(-a + b + c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
- $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
- $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
- $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$
- $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$
- $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$
- $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$
- $(x + a)(x + b) = x^2 + (a + b)x + ab$
- $(x + a)(x - b) = x^2 + (a - b)x - ab$
- $(x - a)(x + b) = x^2 + (b - a)x - ab$
- $(x - a)(x - b) = x^2 - (a + b)x + ab$
- $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

If  $a + b + c = 0$  then,  $a^3 + b^3 + c^3 = 3abc$

**Example 6.** The square of the polynomial

$2xy + 5y$  is

(a)  $4x^2y^2 + 20xy^2 - 25y^2$

(b)  $4x^2y^2 + 20xy^2 + 25y^2$

(c)  $4x^2y^2 + 10xy^2 + 25y^2$

(d) None of the above

**Sol.** (b) Now,

$$(2xy + 5y)^2 = (2xy)^2 + 2(2xy) \times 5y + (5y)^2$$

$$[\because (a + b)^2 = a^2 + 2ab + b^2]$$

$$= 4x^2y^2 + 20xy^2 + 25y^2$$

**Example 7.** Using identity, while dividing

$p(4p^2 - 16)$  by  $4p(p - 2)$ , we get

(a)  $2p + 4$

(b)  $2p - 4$

(c)  $p + 2$

(d)  $p - 2$

**Sol.** (c) We have,

$$\frac{p(4p^2 - 16)}{4p(p - 2)} = \frac{p[(2p)^2 - 4^2]}{4p(p - 2)}$$

$$= \frac{(2p - 4)(2p + 4)}{4(p - 2)}$$

$$[\because a^2 - b^2 = (a + b)(a - b)]$$

$$= \frac{2(p - 2) \cdot 2(p + 2)}{4(p - 2)}$$

$$= \frac{4(p - 2)(p + 2)}{4(p - 2)} = p + 2$$

**Example 8.** Find the cube of the polynomial

$(2a - 3b)$ .

(a)  $8a^3 - 27b^3 - 36a^2b + 54ab^2$

(b)  $8a^3 - 27b^3 - 36a^2b + 54ab^2$

(c)  $8a^3 + 27b^3 - 36a^2b + 54ab^2$

(d) None of the above

**Sol.** (b)  $(2a - 3b)^3 = (2a)^3 - (3b)^3$

$$- 3 \times (2a)(3b)(2a - 3b)$$

$$[\because (x - y)^3 = x^3 - y^3 - 3xy(x - y)]$$

$$= 8a^3 - 27b^3 - 18ab(2a - 3b)$$

$$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$$

# Practice Exercise

- The product of a monomial and a binomial is a  
 (a) monomial (b) binomial  
 (c) trinomial (d) None of these
- In a polynomial, the exponents of the variables are always  
 (a) integers  
 (b) either (a) or (b)  
 (c) non-negative integers  
 (d) non-positive integers
- Constant is a polynomial of degree  
 (a) 0 (b) 1  
 (c) 3 (d) Not defined
- In the expression  $3a^2 - 4ab + 5b^2 + 7ba$ , like terms are  
 (a)  $-4ab, 7ba$  (b)  $3a^2, 5b^2$   
 (c)  $4ab, 7ba$  (d)  $-4ab, -7ba$
- The degree of the polynomial  $5x^3 - 2x^2y^2 + x^2 + 9y^2$  in  $x$  and  $y$  is  
 (a) 3 (b) 2  
 (c) 4 (d) None of these
- A polynomial of degree 2 is called  
 (a) quadratic polynomial  
 (b) linear polynomial  
 (c) cubic polynomial  
 (d) biquadratic polynomial
- Sum of  $a - b + ab, b + c - bc$  and  $c - a - ac$  is  
 (a)  $2c + ab - ac - bc$  (b)  $2c - ab - ac - bc$   
 (c)  $2c + ab + ac + bc$  (d)  $2c - ab + ac + bc$
- The difference of  $x^3 - x^2 + 2x - 19$  and  $2x^3 - x^2 + 4x - 6$  is  
 (a)  $x^3 - 2x + 13$  (b)  $-x^3 + 6x^2 - 8x + 12$   
 (c)  $-x^3 - 2x - 13$  (d) None of these
- What must be added to  $x^2 + 4x - 6$  to get  $x^3 - x^2 + 2x - 2$ ?  
 (a)  $x^3 + 6x - 8$   
 (b)  $x^3 - 2x^2 + 2x - 4$   
 (c)  $x^3 + 2x^2 + 2x - 4$   
 (d)  $x^3 - 2x^2 - 2x + 4$
- The product of  $2x^2 + x - 5$  and  $x^2 - 2x + 3$  is  
 (a)  $2x^4 + 3x^3 + x^2 + 13x + 15$   
 (b)  $2x^4 - 3x^3 + x^2 - 13x - 15$   
 (c)  $2x^3 - 3x^2 - x - 15$   
 (d)  $2x^4 - 3x^3 - x^2 + 13x - 15$
- The value of  $(29x - 6x^2 - 28) \div (3x - 4)$  is  
 (a)  $(2x - 7)$  (b)  $(-2x + 7)$   
 (c)  $(2x + 7)$  (d)  $(7 + 2x)$
- What should be subtracted from  $p^2 - 6p + 7$  so that it may exactly be divisible by  $(p - 1)$ ?  
 (a) 4 (b)  $-4$  (c) 2 (d)  $-2$
- Square of  $9x - 7xy$  is  
 (a)  $81x^2 + 49x^2y^2$   
 (b)  $81x^2 - 49x^2y^2$   
 (c)  $81x^2 + 49x^2y^2 - 126x^2y$   
 (d)  $81x^2 + 49x^2y^2 - 63x^2y$
- The expansion of  $(5x + y - 3z)^2$  is  
 (a)  $25x^2 + y^2 + 9z^2 + 10xy + 6yz + 30xz$   
 (b)  $25x^2 + y^2 - 9z^2 + 10xy - 6yz - 30xz$   
 (c)  $25x^2 + y^2 + 9z^2 + 10xy - 6yz - 30xz$   
 (d) None of the above
- The value of  $(391 \times 391 - 291 \times 291)$  is  
 (a) 68200 (b)  $-68200$   
 (c) 68280 (d) None of these
- The simplification of  $\frac{7.83 \times 7.83 - 1.17 \times 1.17}{6.66}$  gives  
 (a) 6 (b) 7 (c) 8 (d) 9
- If  $p + q = 12$  and  $pq = 22$ , then find  $p^2 + q^2$ .  
 (a) 105 (b) 100 (c) 107 (d) 109
- If  $x - y = 13$  and  $xy = 28$ , then find  $x^2 + y^2$ .  
 (a) 225 (b) 230 (c) 223 (d) 227
- The value of  $(a + b)^2 + (a - b)^2$  is  
 (a)  $2a + 2b$  (b)  $2a - 2b$   
 (c)  $2a^2 + 2b^2$  (d)  $2a^2 - 2b^2$

20. The value of  $(a + 1)(a - 1)(a^2 + 1)$  is  
 (a)  $a^4 - 1$  (b)  $a^5 - 1$  (c)  $a^3 - 1$  (d)  $a^6 - 1$

21. Simplify  $27x^3 - (3x - y)^3$ .  
 (a)  $27x^2y + y^3 - 9xy^2$  (b)  $27xy^2 + y^3 - 9xy^2$   
 (c)  $27x^2y + y^3 - 9x^2y$  (d) None of the above

22. If  $x - y = 5$  and  $xy = 84$ , then find the value of  $x^3 - y^3$ .  
 (a) 1385 (b) 1380  
 (c) 1390 (d) None of these

23. If  $a^2 + b^2 + c^2 = 15$  and  $ab + bc + ca = 5$ , then find the value of  $(a + b + c)^2$ .  
 (a) 25 (b) 26 (c) 27 (d) 28

24. If  $a^2 + b^2 + c^2 = ab + bc + ca$ , then the value of  $a^3 + b^3 + c^3$  is

(a)  $3abc$  (b)  $3a^2b^2c^2$   
 (c)  $3(ab)^3$  (d) None of these

25. The cost of a chocolate is ₹  $(x + 4)$  and Rohit bought  $(x + 4)$  chocolates. Find the total amount paid by him in terms of  $x$ . If  $x = 10$ , then find the amount paid by him.

(a)  $x^2 + 8x + 10$ ; ₹ 200  
 (b)  $x^2 + 8x + 16$ ; ₹ 196  
 (c)  $x^2 + 8x + 10$ ; ₹ 215  
 (d)  $x^2 + 10x + 12$ ; ₹ 195

## Answers

1	(b)	2	(c)	3	(a)	4	(a)	5	(c)	6	(a)	7	(a)	8	(c)	9	(d)	10	(d)
11	(b)	12	(c)	13	(c)	14	(c)	15	(a)	16	(d)	17	(b)	18	(a)	19	(c)	20	(a)
21	(a)	22	(a)	23	(a)	24	(a)	25	(b)										

## Hints & Solutions

- (b) The multiplication of a binomial by a monomial will always produce a binomial.
- (c) In a polynomial, the exponents of the variables are either positive integers or 0.
- (a) Constant is a polynomial of degree 0.
- (a)  $-4ab$  and  $7ba$  are like terms because variable parts are same in both side.
- (c) The degree of the polynomial in  $x$  and  $y$  is 4.

6. (a) A polynomial of degree 2 is called a quadratic polynomial.

7. (a) Required sum  
 $= (a - b + ab) + (b + c - bc) + (c - a - ac)$   
 $= a - b + ab + b + c - bc + c - a - ac$   
 $= 2c + ab - ac - bc$   
 [adding like terms and retaining others]

8. (c)  $x^3 - x^2 + 2x - 19$

$$\begin{array}{r} 2x^3 - x^2 + 4x - 6 \\ - \quad + \quad - \quad + \\ \hline -x^3 \quad -2x - 13 \end{array}$$

9. (d)  $x^3 - x^2 + 2x - 2$   
 $x^2 + 4x - 6$   
 $- \quad - \quad +$   
 $\hline x^3 - 2x^2 - 2x + 4$

10. (d) Now,  $(2x^2 + x - 5) \times (x^2 - 2x + 3)$

$$\begin{aligned} &= 2x^2(x^2 - 2x + 3) + x(x^2 - 2x + 3) \\ &\quad - 5(x^2 - 2x + 3) \\ &= 2x^4 - 4x^3 + 6x^2 + x^3 - 2x^2 + 3x - 5x^2 \\ &\quad + 10x - 15 \\ &= 2x^4 - x^3(4 - 1) + x^2(6 - 5 - 2) \\ &\quad + x(3 + 10) - 15 \\ &\quad \text{[arranging like terms]} \\ &= 2x^4 - 3x^3 - x^2 + 13x - 15 \end{aligned}$$

11. (b) Arranging the terms of the dividend and the divisor in descending power and then dividing, we get

$$\begin{array}{r} -2x + 7 \\ 3x - 4 \overline{) -6x^2 + 29x - 28} \\ \underline{-6x^2 + 8x} \phantom{-28} \\ 21x - 28 \\ \underline{21x - 28} \\ 0 \end{array}$$

$$\begin{aligned} \therefore (29x - 6x^2 - 28) \div (3x - 4) \\ = (-2x + 7) \end{aligned}$$

12. (c)

$$\begin{array}{r} p-5 \\ p-1 \overline{) p^2 - 6p + 7} \\ \underline{p^2 - p} \phantom{+} \\ -5p + 7 \\ \underline{-5p + 5} \phantom{+} \\ + \phantom{-} - \\ \underline{\phantom{+} - 2} \phantom{+} \\ 2 \end{array}$$

Remainder = 2

Hence, 2 is subtracted from it.

13. (c) Square of  $(9x - 7xy) = (9x - 7xy)^2$

Comparing with  $(a - b)^2$ , we get

$a = 9x$  and  $b = 7xy$

$$(9x - 7xy)^2 = (9x)^2 - 2 \cdot 9x \cdot 7xy + (7xy)^2$$

$$[\because (a - b)^2 = a^2 - 2ab + b^2]$$

$$= 81x^2 - 126x^2y + 49x^2y^2$$

$$= 81x^2 + 49x^2y^2 - 126x^2y$$

14. (c)  $(5x + y - 3z)^2 = (5x)^2 + (y)^2 + (-3z)^2$

$$+ 2(5x)(y) + 2(y)(-3z) + 2(-3z)(5x)$$

$$= 25x^2 + y^2 + 9z^2 + 10xy - 6yz - 30zx$$

15. (a)  $(391 \times 391 - 291 \times 291)$

$$= (391)^2 - (291)^2 = (391 + 291)(391 - 291)$$

$$[\because a^2 - b^2 = (a + b)(a - b)]$$

$$= 682 \times 100 = 68200$$

16. (d)  $\frac{7.83 \times 7.83 - 1.17 \times 1.17}{6.66} = \frac{(7.83)^2 - (1.17)^2}{6.66}$

$$= \frac{(7.83 + 1.17)(7.83 - 1.17)}{6.66} = \frac{9 \times 6.66}{6.66} = 9$$

17. (b) Given,  $p + q = 12$  and  $pq = 22$

$$\text{Since, } (p + q)^2 = p^2 + q^2 + 2pq$$

$$[\because (a + b)^2 = a^2 + b^2 + 2ab]$$

$$\therefore (12)^2 = p^2 + q^2 + 2 \times 22$$

$$\Rightarrow p^2 + q^2 = (12)^2 - 44$$

$$\Rightarrow p^2 + q^2 = 144 - 44 = 100$$

18. (a) Given,  $x - y = 13$  and  $xy = 28$

$$\text{Since, } (x - y)^2 = x^2 + y^2 - 2xy$$

$$[\because (a - b)^2 = a^2 + b^2 - 2ab]$$

$$\therefore (13)^2 = x^2 + y^2 - 2 \times 28$$

$$\Rightarrow x^2 + y^2 = (13)^2 + 56$$

$$\Rightarrow x^2 + y^2 = 169 + 56 \Rightarrow x^2 + y^2 = 225$$

19. (c) We have,  $(a + b)^2 + (a - b)^2 = (a^2 + b^2 + 2ab)$

$$+ (a^2 + b^2 - 2ab)$$

$$[\because (a + b)^2 = a^2 + b^2 + 2ab]$$

$$\text{and } (a - b)^2 = a^2 + b^2 - 2ab]$$

$$= (a^2 + a^2) + (b^2 + b^2) + (2ab - 2ab)$$

[combining the like terms]

$$= 2a^2 + 2b^2$$

20. (a) We have,  $(a + 1)(a - 1)(a^2 + 1) = (a^2 - 1)(a^2 + 1)$

$$[\because (a + b)(a - b) = a^2 - b^2]$$

$$= (a^2)^2 - 1^2 \text{ [again using the same identity]}$$

$$= a^4 - 1$$

21. (a)  $27x^3 - (3x - y)^3 = (3x)^3 - (3x - y)^3$

$$= [3x - (3x - y)][(3x)^2 + (3x - y)^2 + 3x \times (3x - y)]$$

$$[\because a^3 - b^3 = (a - b)(a^2 + b^2 + ab)]$$

$$= y(9x^2 + 9x^2 + y^2 - 6xy + 9x^2 - 3xy)$$

$$[\because (a - b)^2 = a^2 + b^2 - 2ab]$$

$$= y(27x^2 + y^2 - 9xy) = 27x^2y + y^3 - 9xy^2$$

22. (a)  $x^3 - y^3 = (x - y)(x^2 + y^2 + xy)$

$$= (x - y)[(x - y)^2 + 2xy + xy]$$

$$\left[ \begin{array}{l} \because (x - y)^2 = x^2 + y^2 - 2xy \\ \therefore (x - y)^2 + 2xy = x^2 + y^2 \end{array} \right]$$

$$= (x - y)[(x - y)^2 + 3xy]$$

$$= 5[(5)^2 + 3 \times 84]$$

$$[\because x - y = 5 \text{ and } xy = 84]$$

$$= 5(25 + 252) = 5 \times 277 = 1385$$

23. (a) Now,  $(a + b + c)^2 = a^2 + b^2 + c^2$

$$+ 2(ab + bc + ca)$$

$$= 15 + 2(5) = 15 + 10 = 25$$

24. (a)  $\because a^3 + b^3 + c^3 - 3abc$

$$= (a^2 + b^2 + c^2 - ab - bc - ca)(a + b + c)$$

$$= [(a^2 + b^2 + c^2) - (ab + bc + ca)](a + b + c)$$

$$\text{But, } a^2 + b^2 + c^2 = ab + bc + ca \quad [\text{given}]$$

$$\therefore a^3 + b^3 + c^3 - 3abc = (a + b + c) \times 0 = 0$$

$$\Rightarrow a^3 + b^3 + c^3 = 3abc$$

25. (b) Given, cost of a chocolate = ₹  $(x + 4)$

Rohit bought  $(x + 4)$  chocolates.

$\therefore$  The cost of  $(x + 4)$  chocolates

= Cost of one chocolate  $\times$  Number of chocolates

$$= (x + 4)(x + 4) = (x + 4)^2$$

$$= x^2 + 8x + 16 \quad [\because (a + b)^2 = a^2 + 2ab + b^2]$$

$\therefore$  The total amount paid by Rohit

$$= ₹(x^2 + 8x + 16)$$

Now, if  $x = 10$ , then amount paid by Rohit

$$= 10^2 + 8 \times 10 + 16 = 100 + 80 + 16 = ₹196$$