Exponents

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Any number of the form x^n, where n is a natural number and 'x' is a real number is called the exponents. Here n is called the power of the number x. Here x is the base and n is exponent (or index or power). Power may be positive or negative. For any rational number $\left(\frac{x}{y}\right)^n$, n is called the power of the rational number.

So, $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n} = \frac{x}{y} \times \frac{x}{y} \times \frac{x}{y} \times \frac{x}{y} \times \frac{x}{y} \times ------\times \frac{x}{y}$ (n times)

Uses of Exponents

The exponents can be used for various purposes such as comparing large and small numbers, expressing large and small numbers in the standard forms. It is used to express the distance between any two celestial bodies which cannot be expressed in the form of normal denotation. It is also useful in writing the numbers in scientific notation. The size of the microorganisms is very-very small and it cannot be written in normal denotation and can easily be expressed in exponential form.

Radicals Expressed with Exponents

Radicals are the fractional exponents of any number. Index of the radical becomes the denominator of the fractional power.

$n\sqrt{a} = \frac{1}{a^n}$ or, $\sqrt{9} = \sqrt[2]{9} = \frac{1}{9^2} = 3$

Let us convert the radicals to exponential expressions, and then apply laws of exponent to combine the terms. For example:

$\sqrt[3]{2}\,\sqrt[4]{2} = 2^{\frac{1}{3}}\,2^{\frac{1}{4}} = 2^{\frac{1}{3}+\frac{1}{4}} = 2^{\frac{7}{12}} = \sqrt[12]{2^7}$

- **Example:** Simplify: $\frac{\sqrt{5}}{\sqrt[3]{5}}$

(a) $5^{1/3}$ (b) $5^{1/5}$
(c) $5^{1/6}$ (d) $5^{3/8}$
(e) None of these
Answer (c)

Explanation: $\frac{5^{\frac{1}{2}}}{5^{\frac{1}{3}}} = 5^{\frac{1}{2}-\frac{1}{3}} = 5^{\frac{1}{6}}$

- **Example:** $\frac{2^4}{3}$ is equal to:

(a) $\frac{4}{9}$ (b) $\frac{16}{81}$
(c) $\frac{32}{27}$ (d) $\frac{8}{81}$
(e) None of these
Answer (b)

Explanation: $\left(\frac{2}{3}\right)^4 = \left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right) = \frac{2\times2\times2\times2}{3\times3\times3\times3} = \frac{2^4}{3^4} = \frac{16}{81}$

Squares and Square Roots

A number x is called a square number if it can be expressed in the form y^2, here y is called the square root of x. Symbol used for square root is $\sqrt{\ }$.

Properties of square Numbers

- Every square number can be expressed as the sum of odd natural numbers.
- Square Number can only end with digits 0, 1, 4, 5, 6 and 9.
- If the last digit of a number is 0, its square ends with 00 and the preceding digits must also form a square.
- If the last digit of a number is 1 or 9, its square ends with 1 and the number formed by its preceding digits must be divisible by four.
- If the last digit of a number is 2 or 8, its square ends with 4 and the preceding digits must be even.
- If the last digit of a number is 3 or 7, its square ends with 9 and the number formed by its preceding digits must be divisible by four.
- If the last digit of a number is 4 or 6, its square ends with 6 and the preceding digits must be odd.
- If the last digit of a number is 5, its square ends with 5 and the preceding digits must be 2.
- A square number cannot be a perfect: number.

Pythagorean Triplet

A Pythagorean triplet consists of three positive integers a, b, and c, such that $a^2+b^2=c^2$. Pythagorean Theorem states that, in any right triangle, the sum of squares of base and height is equal to the square of its hypotenuse. Pythagorean triplets describe a relation among three sides of a right angled triangle. For every natural number $n>1$, the Pythagorean triplet is given by $(2n, n^2-1, n^2+1)$

Let $n=3$, then the corresponding Pythagorean triplet is obtained as:

$$2n=2\times 3=6$$
$$n^2-1=3^2-1=8$$
$$n^2+1=3^2+1=10$$

Hence 6, 8, 10 are Pythagorean triplets.

Finding Square Root of a Number

Square root of a number can be found by using the following three methods.

- Repeated Subtraction Method
- Prime factorisation method
- Long division method

Repeated Subtraction Method

In this method we subtract the successive odd numbers from the given number starting from 1 till we get the result zero. The number of steps required to reduce the given number to zero will be the square root of the given number.

- **Example:**

Find the square root of 64 by repeated subtraction method.

Solution: $64-1=63 \Rightarrow 63-3=60$

$$\Rightarrow 60-5=55 \Rightarrow 55-7=48$$
$$\Rightarrow 48-9=39 \Rightarrow 39-11=28$$
$$\Rightarrow 28-13=15 \Rightarrow 15-15=0$$

There are eight steps required to reduce the number to 0.
Therefore, square root of 64 is 8.

Prime Factorisation Method

To find the square root of a number by prime factorization method, follow these steps:
Step 1: Find the prime factors of the given number.
Step 2: Make pairs of these prime factors
Step 3: Take one prime factor from each pair.
Step 4: Find the product of these factors which is the required square root of the given number.

- **Example:** Find the square root of 900 by prime factorisation method.

Solution: Prime factors of 900 are:
$900 = 2\times2\times3\times3\times5\times5$
Now make pairs of these factors i.e. $\underline{2\times2}\times\underline{3\times3}\times\underline{5\times5}$
Now take one factor from each pair i.e. $2\times3\times5 = 30$
So, square root of 900 is 30.

Long Division Method

To find the square root of a number by long division method, follow these steps:
Step1: Form pairs of digits from right to left in the given number.
Step 2: Find greatest number whose square is less than or equal to the digits in the first group.
Step 3: Taking this number as the divisor find the remainder.
Step 4: Add the divisor and the quotient and make it divisor for the second group.
Step 5: Continue this process till remainder becomes 0.

- **Example:**

Find the square root of 15876 by long division method.

	126
1	15876
22	058 44
246	1476 1476
	0

So, square root of 15876 is 126.

Cube of a Number

The word cube is used in geometry. In geometry the word cube refers to the solid having equal sides. In algebra a given number is said to be a perfect cube if it can be expressed as a product of triplets of equal factors. In other words the cube of a number n is its third power. If a number multiplied three times by itself the resultant number is called cube of that number. For example, $n^3 = n\times n\times n$. In this expression if $n\times n\times n = m$ then we can say that m is cube of n.

Cubes of Some Numbers

$1^3 = 1$	$2^3 = 8$	$3^3 = 27$	$4^3 = 64$
$5^3 = 125$	$6^3 = 216$	$7^3 = 343$	$8^3 = 512$
$9^3 = 729$	$10^3 = 1000$	$11^3 = 1331$	$12^3 = 1728$
$13^3 = 2197$	$14^3 = 2744$	$15^3 = 3375$	$16^3 = 4096$
$17^3 = 4913$	$18^3 = 5832$	$19^3 = 6859$	$20^3 = 8000$

- **Example:**

Find the unit digit in the cube of the number 3331.
(a) 1 (b) 8
(c) 4 (d) 9
(e) None of these
Answer (a)

Explanation: $(3331)^3 = 3331\times3331\times3331$
So unit digit in the product will be 1.

Cube Roots

The inverse operation of the cube of a number is called its cube root. It is normally denoted by $\sqrt[3]{n}$ or $(n)^{\frac{1}{3}}$. The cube root of a number can be found by using the prime factorization method. For example the cube root of 8 is 2 because $2^3 = 2 \times 2 \times 2 = 8$. In symbolic form, the cube root of 8 is written as $\sqrt[3]{8}$.

- **Example:**

  Find the smallest number by which we must divide 8788 so that it becomes a perfect cube.

  (a) 2 (b) 4

  (c) 7 (d) 13

  (e) None of these

  Answer (b)

  Explanation: Prime factorization of 8788 is as follows:

  $8788 = 2 \times 2 \times \underline{13 \times 13 \times 13}$

  So, 8788 must be divided by 4 to make it a perfect cube.