Maharashtra State Board
Class X Mathematics - Algebra
Board Paper – 2016

Time: 2 hours **Total Marks: 40**

Note: - (1) All questions are compulsory.
(2) Use of calculator is not allowed.

1. Attempt any five question from the following: 5

i. Write the first two terms of the sequence whose nth term is $t_n = 3n - 4$.

ii. Find the value of a, b, c in the following quadratic equation :
$$2x^2 - x - 3 = 0$$

iii. Write the quadratic equation whose roots are −2 and −3.

iv. Find the value of determinant:
$$\begin{vmatrix} 4 & -2 \\ 3 & 1 \end{vmatrix}$$

v. Write the sample space for selecting a day randomly of the week.

vi. Find the class mark of the classes 20-30 and 30-40.

2. Attempt any four sub-questions from the following: 8

i. Write the first three terms of the A.P. whose common difference is −3 and first term is 4.

ii. Solve the following quadratic equation by Factorisation method:
$$x^2 + 7x + 10 = 0$$

iii. If the value of determinant $\begin{vmatrix} m & 2 \\ -5 & 7 \end{vmatrix}$ is 31, find the value of m.

iv. A die is thrown, then find the probability of the following events:
A is an Event: getting an odd number on the top upper surface of the die.
B is an Event: getting a perfect square on the upper surface of the die.

v. Below is the given frequency distribution of words in an essay:

Number of Words	**Number of Candidates**
600 – 800	8
800 – 1000	22
1000 – 1200	40
1200 – 1400	18
1400 - 1600	12

Find the mean number of words written.

vi. Subjectwise marks obtained by a student in an examination are given below:

Subject	**Marks**
Marathi	85
Hindi	85
Science	90
Mathematics	100
Total	360

Draw pie diagram.

3. Attempt any three of the following sub questions: 9

i. Solve the following quadratic equation by using formula method:

$$5m^2 + 5m - 1 = 0$$

ii. There are three boys and two girls. A committee of two is to be formed. Find the probability of the following events:
Event A: The committee contains at least one girl
Event B: The committee contains one boy and one girl

iii. The measurements (in mm) of the diameters of the head of the screws are given below:

Diameter (in mm)	No. of Screws
33 — 35	10
36 — 38	19
39 — 41	23
42 — 44	21
45 — 47	27

Calculate mean diameter of head of a screw by ‘Assumed Mean Method’.

iv. The marks scored by students in Mathematics in a certain Examination are given below:

Marks Scored	Number of Students
0 — 20	3
20 — 40	8
40 — 60	19
60 — 80	18
80 — 100	6

Draw histogram for the above data.

v. Draw the frequency polygon for the following frequency distribution:

Rainfall (in cm)	No. of Years
20 — 25	2
25 — 30	5
30 — 35	8
35 — 40	12
40 — 45	10
45 — 50	7

4. Attempt any two of the following sub questions: 8

i. The 11^{th} term and the 21^{st} term of an A.P. are 16 and 29 respectively, then find:
(a) The first term and common difference
(b) The 34^{th} term
(c) 'n' such that $t_n = 55$

ii. Solve the following simultaneous equations:
$$\frac{7}{2x+1}+\frac{13}{y+2}=27,\quad \frac{13}{2x+1}+\frac{7}{y+2}=33.$$

iii. In a certain race, there are three boys A, B, C. The winning probability of A is twice than B and the winning probability of B is twice than C. If $P(A) + P(B) + P(C) = 1$, then find the probability of win for each boy.

5. Attempt any two of the following sub questions: 10

i. The divisor and quotient of the number 6123 are same and the remainder is half the divisor. Find the divisor.

ii. Find the sum of all numbers from 50 to 350 which are divisible by 6. Hence find the 15^{th} term of that A.P.

iii. A three digit number is equal to 17 times the sum of its digits. If 198 is added to the number, the digits are interchanged. The addition of first and third digit is 1 less than middle digit. Find the number.

Maharashtra State Board

Class X Mathematics - Algebra

Board Paper – 2016 Solution

1.

i. $t_n = 3n - 4$

For n = 1, $t_1 = 3 \times 1 - 4 = 3 - 4 = -1$

For n = 2, $t_2 = 3 \times 2 - 4 = 6 - 4 = 2$

Hence, the first two terms of the sequence are -1 and 2.

ii. Given equation is $2x^2 - x - 3 = 0$.

Comparing the given equation with general form of quadratic equation $ax^2 + bx + c$, we have

$a = 2$, $b = -1$ and $c = -3$

iii. Let the roots be $\alpha = -2$ and $\beta = -3$.

$\therefore \alpha + \beta = (-2) + (-3) = -5$ and $\alpha\beta = (-2)(-3) = 6$

Hence, the required quadratic equation is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

i.e. $x^2 - (-5)x + 6 = 0$

i.e. $x^2 + 5x + 6 = 0$

iv. $\begin{vmatrix} 4 & -2 \\ 3 & 1 \end{vmatrix} = (4 \times 1) - (-2 \times 3) = 4 + 6 = 10$

v. The sample space ‘S’ for selecting a day randomly of the week is given by

S = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}

vi. $\text{Class Mark} = \dfrac{\text{Upper Limit} + \text{Lower Limit}}{2}$

$\therefore \text{Class Mark of the class } 20 - 30 = \dfrac{30 + 20}{2} = \dfrac{50}{2} = 25$

$\text{Class Mark of the class } 30 - 40 = \dfrac{40 + 30}{2} = \dfrac{70}{2} = 35$

2.

i. $a = 4$, $d = -3$

Hence,

$t_1 = 4$

$t_2 = t_1 + d = 4 + (-3) = 4 - 3 = 1$

$t_3 = t_2 + d = 1 + (-3) = 1 - 3 = -2$

Thus, the first three terms of the A.P. are 4, 1 and -2.

ii. $x^2 + 7x + 10 = 0$

Splitting the middle term 7x as 2x + 5x, we get

$x^2 + 2x + 5x + 10 = 0$

$\therefore x(x + 2) + 5(x + 2) = 0$

$\therefore (x + 2)(x + 5) = 0$

$\therefore x + 2 = 0$ or $x + 5 = 0$

$\therefore x = -2$ or $x = -5$

iii. $\begin{vmatrix} m & 2 \\ -5 & 7 \end{vmatrix} = 31$

$\therefore (m \times 7) - (-5 \times 2) = 31$

$\therefore 7m - (-10) = 31$

$\therefore 7m + 10 = 31$

$\therefore 7m = 21$

$\therefore m = 3$

iv. When a die is thrown, the sample space (S) is given by

$S = \{1, 2, 3, 4, 5, 6\}$

$\therefore n(S) = 6$

Let A be the event of getting an odd number on the upper surface of the die.

Then $A = \{1, 3, 5\}$

$\therefore n(A) = 3$

$\therefore P(A) = \frac{n(A)}{n(S)}$

$= \frac{3}{6}$

$= \frac{1}{2}$

Let B be the event of getting a perfect square on the upper surface of the die.

Then $B = \{1, 4\}$

$\therefore n(B) = 2$

$\therefore P(B) = \frac{n(B)}{n(S)}$

$= \frac{2}{6}$

$= \frac{1}{3}$

v.

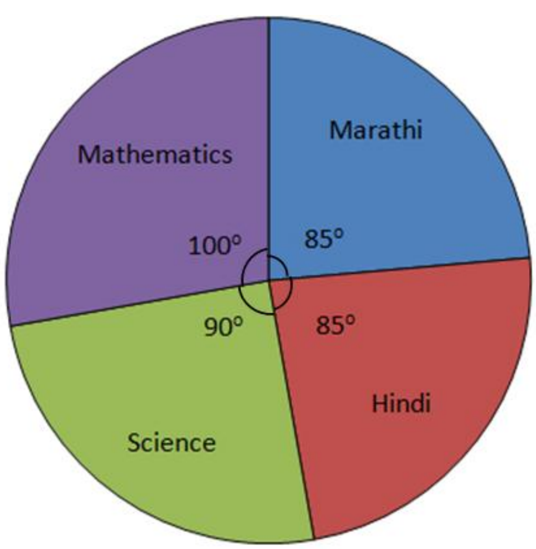
(Number of words) Class intervals	Class Mark x_i	(Number of candidates) Frequency f_i	f_ix_i
600 – 800	700	8	5600
800 – 1000	900	22	19800
1000 – 1200	1100	40	44000
1200 – 1400	1300	18	23400
1400 – 1600	1500	12	18000
Total		$\sum f_i = 100$	$\sum f_ix_i = 110800$

$$\text{Mean} = \bar{x} = \frac{\sum f_ix_i}{\sum f_i} = \frac{110800}{100} = 1108$$

$\therefore$ Mean number of words written in an essay is 1108.

vi. Central angle for each subject is computed in the following table:

Subject	Marks	Measure of central angle
Marathi	85	$\frac{85}{360} \times 360° = 85°$
Hindi	85	$\frac{85}{360} \times 360° = 85°$
Science	90	$\frac{90}{360} \times 360° = 90°$
Mathematics	100	$\frac{100}{360} \times 360° = 100°$
Total	360	$360°$



3.

i. Given quadratic equation is $5m^2 + 5m - 1 = 0$

Comparing with general form $ax^2 + bx + c = 0$, we have

$a = 5$, $b = 5$ and $c = -1$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{5^2 - 4(5)(-1)}}{2 \times 5}$$

$$= \frac{-5 \pm \sqrt{25 + 20}}{10}$$

$$= \frac{-5 \pm \sqrt{45}}{10}$$

$$= \frac{-5 \pm 3\sqrt{5}}{10}$$

$\therefore \dfrac{-5 + 3\sqrt{5}}{10}$ and $\dfrac{-5 - 3\sqrt{5}}{10}$ are the roots of the given equation.

ii. Here, there are three boys B_1, B_2, B_3 and two girls G_1, G_2.

A committee of two is to be formed.

Thus, the sample space (S) is given by

$S = \{B_1B_2, B_1B_3, B_2B_3, B_1G_1, B_1G_2, B_2G_1, B_2G_2, B_3G_1, B_3G_2, G_1G_2\}$

$\therefore n(S) = 10$

A is the event that the committee contains at least one girl.

Then, $A = \{B_1G_1, B_1G_2, B_2G_1, B_2G_2, B_3G_1, B_3G_2, G_1G_2\}$

$\therefore n(A) = 7$

$$\therefore P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{7}{10}$$

B is the event that the committee contains one boy and one girl.

Then, $B = \{B_1G_1, B_1G_2, B_2G_1, B_2G_2, B_3G_1, B_3G_2\}$

$\therefore n(B) = 6$

$$\therefore P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{6}{10}$$

$$= \frac{3}{5}$$

iii. Let A be the assumed mean.

A is taken as the class mark of the middle class.

Hence, let us take 40 as the assumed mean.

Then A = 40 and deviation $d_i = x_i - A = x_i - 40$

Diameter (in mm)	Class marks x_i	Deviations $d_i = x_i - A$ $d_i = x_i - 40$	Number of Screws f_i	$f_i d_i$
33-35	34	−6	10	−60
36-38	37	−3	19	−57
39-41	40 = A	0	23	0
42-44	43	3	21	63
45-47	46	6	27	162
Total	...	...	$\sum f_i = 100$	$\sum f_i d_i = 108$

Here, $\sum f_i d_i = 108$, $\sum f_i = 100$

$$\bar{d} = \frac{\sum f_i d_i}{\sum f_i} = \frac{108}{100} = 1.08$$

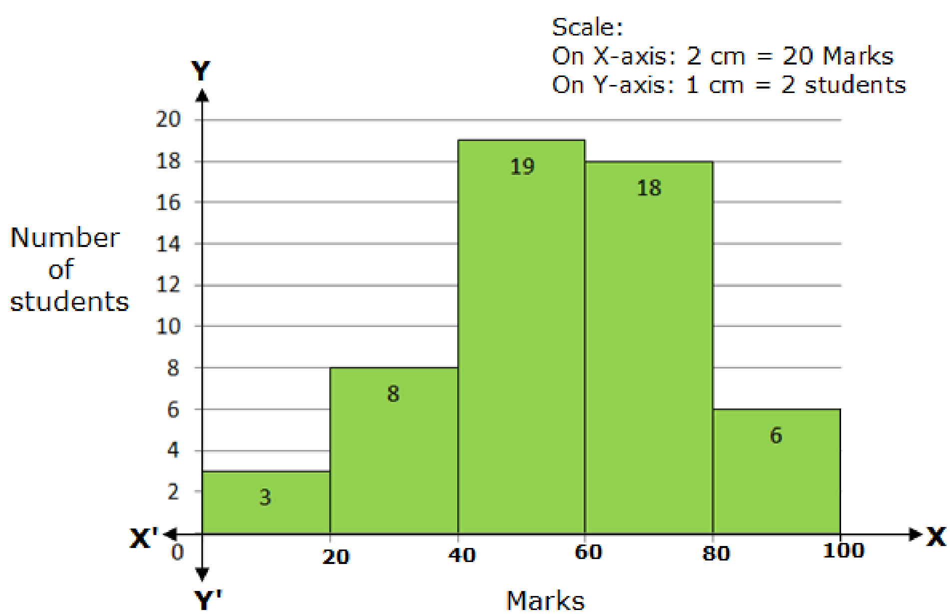
$$\bar{x} = A + \bar{d}$$

$$= 40 + 1.08$$

$$= 41.08$$

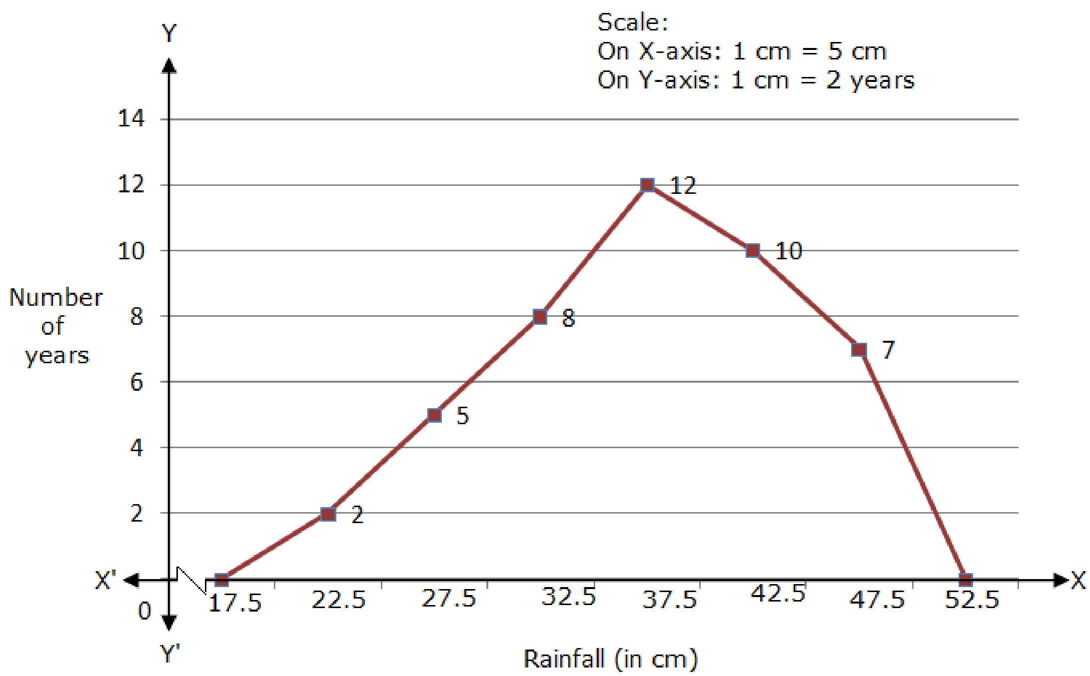
Thus, the mean diameter of the screw heads is 41.08 mm.

iv.



v.

Class mark	No. of Years
17.5	0
22.5	2
27.5	5
32.5	8
37.5	12
42.5	10
47.5	7
52.5	0



4.

i.

(a) Let 'a' be the first term and 'd' the common difference of the given A.P.

For $t_{11} = 16,\ n = 11$, we have

$t_{11} = a + (11 - 1)d$

$\therefore 16 = a + 10d \qquad \ldots (1)$

For $t_{21} = 29,\ n = 21$, we have

$t_{21} = a + (21 - 1)d$

$\therefore 29 = a + 20d \qquad \ldots (2)$

Subtracting (1) from (2), we get

$13 = 10d \Rightarrow d = 1.3$

Substituting $d = 1.3$ in (1), we get

$16 = a + 10(1.3)$

$\therefore 16 = a + 13 \Rightarrow a = 3$

Thus, the first term is 3 and the common difference is 1.3.

(b) For 34^{th} term, $n = 34,\ a = 3,\ d = 1.3$

$t_n = a + (n-1)d$

$\therefore t_{34} = 3 + (34-1)(1.3)$

$= 3 + 33 \times 1.3$

$= 3 + 42.9$

$= 45.9$

Thus, the 34^{th} term is 45.9.

(c) $t_n = 55,\ a = 3,\ d = 1.3$

$\therefore t_n = a + (n-1)d$

$\therefore 55 = 3 + (n-1)(1.3)$

$\therefore 55 - 3 = (n-1)(1.3)$

$\therefore (n-1)(1.3) = 52$

$\therefore n - 1 = \dfrac{52}{1.3}$

$\therefore n - 1 = 40$

$\therefore n = 40 + 1$

$\therefore n = 41$

ii.

$\dfrac{7}{2x+1} + \dfrac{13}{y+2} = 27$ ....(1)

$\dfrac{13}{2x+1} + \dfrac{7}{y+2} = 33$ ....(2)

Substituting $\dfrac{1}{2x+1} = m$ and $\dfrac{1}{y+2} = n$ in equations (1) and (2), we get

$7m + 13n = 27$ ....(3)

and $13m + 7n = 33$ ....(4)

Adding equations (3) and (4), we get

$20m + 20n = 60$

$\therefore m + n = 3$ ....(5)

Subtracting equation (3) from equation (4), we get

$6m - 6n = 6$

$\therefore m - n = 1$ ....(6)

Adding equations (5) and (6), we get

$2m = 4$

$\therefore m = 2$

Substituting $m = 2$ in equation (5), we get

$2 + n = 3$

$\therefore n = 1$

Resubstituting the values of m and n, we get

$\frac{1}{2x+1} = 2$ and $\frac{1}{y+2} = 1$

$\therefore 4x + 2 = 1$ and $y + 2 = 1$

$\therefore 4x = -1$ and $y = 1 - 2$

$\therefore x = -\frac{1}{4}$ and $y = -1$

iii. Given: P(A) = 2P(B) and P(B) = 2P(C)

$\therefore$ P(A) = 2[2P(C)] = 4P(C)

Now, P(A) + P(B) + P(C) = 1

$\therefore$ 4P(C) + 2P(C) + P(C) = 1

$\therefore$ 7P(C) = 1

$\therefore P(C) = \frac{1}{7}$

$\therefore P(A) = 4P(C) = 4 \times \frac{1}{7} = \frac{4}{7}$ and $P(B) = 2P(C) = 2 \times \frac{1}{7} = \frac{2}{7}$

5.

i. Dividend = 6123

Now, divisor and quotient are same

Let divisor = quotient = d

Now, remainder $= \frac{\text{divisor}}{2} = \frac{d}{2}$

Since dividend = divisor × quotient + remainder, we have

$6123 = d^2 + \frac{d}{2}$

$\therefore 12246 = 2d^2 + d$

$\therefore 2d^2 + d - 12246 = 0$

Comparing with $ax^2 + bx + c$, we get

$a = 2, b = 1, c = -12246$

$$\therefore d = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1^2 - 4(2)(-12246)}}{4}$$

$$= \frac{-1 \pm \sqrt{97969}}{4}$$

$$= \frac{-1 \pm 313}{4}$$

$\therefore d = \frac{-1+313}{4}$ or $d = \frac{-1-313}{4}$

$\therefore d = 78$ or $d = -78.5$

Ignoring the negative value, the divisor is 78.

ii. The numbers from 50 to 150 which are divisible by 6 are 54, 60, 66, ……, 348.

$\therefore$ First term = a = t_1 = 54, d = 6 and t_n = 348

$t_n = a + (n-1)d$

$\therefore 348 = 54 + (n-1)6$

$\therefore 294 = (n-1)6$

$\therefore 49 = n - 1$

$\therefore n = 50$

$S_n = \frac{n}{2}(t_1 + t_n)$

$\therefore S_{50} = \frac{50}{2}(54 + 348)$

$= 25 \times 402$

$= 10050$

$t_{15} = 54 + 14(6) = 54 + 84 = 138$

Thus, the sum of all numbers from 50 to 350, which are divisible by 6, is 10050 and the 15^{th} term of this A.P. is 138.

iii. Let the three-digit number be xyz.

Its numerical value = 100x + 10y + z

According to first information provided in the question,

100x + 10y + z = 17(x + y + z)

$\therefore$ 100x + 10y + z = 17x + 17y + 17z

$\therefore$ 83x − 7y − 16z = 0 ....(1)

Number obtained by reversing the digits: zyx

Its numerical value = 100z + 10y + x

According to second information provided in the question,

(100x + 10y + z) + 198 = 100z + 10y + x

$\therefore$ 99z − 99x = 198

$\therefore$ z − x = 2

$\therefore$ z = x + 2 ....(2)

According to third information provided in the question,

x + z = y − 1

$\therefore$ x + x + 2 = y − 1 ....[from (2)]

$\therefore$ y = 2x + 3

Substituting the values of z and y in equation (1),

83x − 7(2x + 3) − 16(x + 2) = 0

$\therefore$ 83x − 14x − 21 − 16x − 32 = 0

$\therefore$ 53x − 53 = 0

$\therefore$ 53x = 53

$\therefore$ x = 1

$\therefore$ y = 2x + 3 = 2(1) + 3 = 2 + 3 = 5

$\therefore$ z = x + 2 = 1 + 2 = 3

Thus, the three-digit number is 153.