

Statistical Analysis for Engineers

Worksheet on Probability

Question 1

A computer program generates a random integer number between 1 and 100, inclusive, where all numbers are equally likely to come up. Identify the experiment and the sample space in this example. Calculate the probability for each of the following events:

1. The integer is divisible by 4.
2. The integer is divisible by 6.
3. The integer has '1' as one of its digits.

Question 2

A small club lottery offers a jackpot for matching the 4 numbers on a ticket to the 4 chosen at random from 30. Identify the experiment and the sample space. Calculate the probability of the following events.

1. The four numbers on a ticket come up in the draw.
2. Three of the four numbers on a ticket come up in a draw. Use your result to explain why the prize for matching 3 rather than 4 numbers is considerably lower than that for matching all 4.
3. Calculate the probability of the club suffering three jackpots in a row.

Question 3

A small firm consists of 10 women and 4 men. Three names are chosen at random. Identify the experiment and the sample space in this example.

Identify the Universal set. Calculate the probability of the following events.

1. The three names are all men.
2. The three names are two men and one woman.
3. The three names are one man and two women.
4. The three names are all women.
5. Add up these probabilities and explain your answer.

Question 4

A standard fair dice is rolled three times in succession. Calculate the probability of the following two events:

1. The second number is larger than the first.
2. The third number is larger than the second.

Question 5

The following questions require Bayes Rule

1. Dublin's marquee forward Diarmuid Connolly may be given a 12 week ban, meaning he will not play for them until a potential All-Ireland semi-final. He is appealing the ban; the probability his appeal succeeds and he will play is estimated at $\frac{3}{4}$. Pundits have estimated the following probabilities:

- $P[\text{Dublin reach a semi-final with DC}] = 0.8$
- $P[\text{Dublin reach a semi-final without DC}] = 0.6$

After a lengthy sojourn on the moon, you return to find that Dublin did in fact reach the semi-final. Assuming Connolly's presence was the only factor affecting their chances of doing this, calculate the

probability he played for them. [This example will be overtaken by actual events; please ignore them.]

2. A blood-test on a driver determines whether they are over the legal limit of alcohol for driving. Anonymous polling has suggested that 24% of drivers will still drive after drinking enough to put them over this limit. It is proposed to apply random breathalyser-tests to drivers. It is known that:

- The probability the breathalyser test will give a positive result if the subject is over the limit is 0.95,
- The probability the breathalyser test will give a positive result if the subject is not over the limit is 0.02.

Calculate the probability a driver was not over the limit despite giving a positive result.

Question 6

A contestant on a game show has to pick one of three doors and wins the prize of whatever is behind the door; behind one door is a car and behind the other two are goats. The host knows which doors conceal the car and the goats. The contestant picks a door. The host will then open one of the two other doors showing a goat. The contestant is then asked if they wish to stick with the door they chose or switch to the remaining one. Calculate the probability they win the car if they switch doors or if they do not.

[You can approach this problem by working out conditional probabilities for every possible outcome, or by using Bayes theorem.]