

EE5609 Assignment 1

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Abstract—This assignment involves finding a vector which is perpendicular to given two vectors and non-perpendicular to a third vector.

The python solution code for this problem can be downloaded from

https://github.com/vimalkb007/EE5609/blob/master/Assignment_1/codes/assignment1_solution.py

The python verification code for this problem can be downloaded from

https://github.com/vimalkb007/EE5609/blob/master/Assignment_1/codes/assignment1_solution_verify.py

1 PROBLEM STATEMENT

Let $\mathbf{a} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 3 \\ -2 \\ 7 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$. Find a vector \mathbf{d} such that $\mathbf{d} \perp \mathbf{a}$, $\mathbf{d} \perp \mathbf{b}$ and $\mathbf{d}^T \mathbf{c} = 15$.

2 THEORY

If two vectors are perpendicular, then their dot product is 0. If we have two vectors \mathbf{x} , \mathbf{y} is given by $\mathbf{x} \cdot \mathbf{y} = |\mathbf{x}||\mathbf{y}|\cos(\theta)$.

When $\theta = \pi/2$ (90°), then $\cos \theta = 0 \implies \mathbf{x} \cdot \mathbf{y} = 0$.

If we have 3 equations and 3 unknowns, we can use Gaussian Elimination method in order to find the unknowns.

3 SOLUTION

Lets consider vector \mathbf{d} as $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

It is given that $\mathbf{d} \perp \mathbf{a}$, then their corresponding dot product will be 0.

$$\mathbf{d} \cdot \mathbf{a} = 0 \implies \begin{pmatrix} x \\ y \\ z \end{pmatrix}^T \times \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} = 0$$

$$x + 4y + 2z = 0 \quad (3.0.1)$$

Similarly, as $\mathbf{d} \perp \mathbf{b}$,

$$\mathbf{d} \cdot \mathbf{b} = 0 \implies \begin{pmatrix} x \\ y \\ z \end{pmatrix}^T \times \begin{pmatrix} 3 \\ -2 \\ 7 \end{pmatrix} = 0$$

$$3x - 2y + 7z = 0 \quad (3.0.2)$$

Since, it is given that $\mathbf{d}^T \mathbf{c} = 15$, we can write it

$$\text{as } (x \ y \ z) \times \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} = 15.$$

$$2x - y + 4z = 15 \quad (3.0.3)$$

Using equations 3.0.1, 3.0.2, 3.0.3, we can use Gaussian Elimination Method in order to find the values of x , y , z .

$$\left(\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 3 & -2 & 7 & 0 \\ 2 & -1 & 4 & 15 \end{array} \right) \quad (3.0.4)$$

$$\begin{array}{l} \xleftarrow{R_3 \leftarrow R_3 - 2R_1} \\ \xleftarrow{R_2 \leftarrow R_2 - 3R_1} \end{array} \left(\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 0 & -14 & 1 & 0 \\ 0 & -9 & 0 & 15 \end{array} \right) \quad (3.0.5)$$

$$\xleftarrow{R_3 \leftarrow R_3 - \frac{9}{14}R_2} \left(\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 0 & -14 & 1 & 0 \\ 0 & 0 & \frac{-9}{14} & 15 \end{array} \right) \quad (3.0.6)$$

We get the following equations as,

$$x + 4y + 2z = 0 \quad (3.0.7)$$

$$-14y + z = 0 \quad (3.0.8)$$

$$\frac{-9}{14}z = 15 \quad (3.0.9)$$

Equating the above equations we get, $x = 53.333$, $y = -1.667$, $z = -23.333$

The resultant vector $\mathbf{d} = \begin{pmatrix} 53.333 \\ -1.667 \\ -23.333 \end{pmatrix}$