EE5609 Assignment 3

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Abstract—This assignment involves finding the determinant of the given matrix.

The python solution code for this problem can be downloaded from

1 Problem Statement

Evaluate $\begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$.

2 Solution

We first multiply either the rows or the columns, and then try taking the common element out.

$$\begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$$

$$\xrightarrow[C_3\leftarrow(\sin\alpha)C_3]{C_3\leftarrow(\cos\alpha)C_3} \left(\frac{1}{\sin\alpha\cos\alpha}\right) \begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin^2\alpha\cos\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos^2\alpha\sin\alpha \end{vmatrix}$$
$$(2.0.1)$$

From 2.0.1 R_1 we take out common element $\cos\alpha$. And from row R_2 we take out common element $\sin\alpha$

$$\begin{vmatrix} \cos\beta & \sin\beta & -\sin^2\alpha \\ -\sin\beta & \cos\beta & 0 \\ \cos\beta & \sin\beta & \cos^2\alpha \end{vmatrix}$$

$$\xrightarrow{R_1\leftarrow R_1-R_3} \begin{vmatrix} 0 & 0 & -\sin^2\alpha-\cos^2\alpha \\ -\sin\beta & \cos\beta & 0 \\ \cos\beta & \sin\beta & \cos^2\alpha \end{vmatrix}$$
$$(2.0.2)$$

Now, we can expand the determinant from row 1 in 2.0.2, and we get

$$\left(-\sin^2\alpha-\cos^2\alpha\right)\left(-\sin^2\beta-\cos^2\beta\right)$$
$$\implies \left(\sin^2\alpha+\cos^2\alpha\right)\left(\sin^2\beta+\cos^2\beta\right)=1 \quad (2.0.3)$$

Therefore, the determinant of the matrix is 1.