# EE5609 Assignment 1

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*Abstract*—This assignment involves finding a vector which is perpendicular to given two vectors and non-perpendicular to a third vector.

The python solution code for this problem can be downloaded from

The python verification code for this problem can be downloaded from

## 1 PROBLEM STATEMENT

Let $\mathbf{a} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 3 \\ -2 \\ 7 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$. Find a vector $\mathbf{d}$ such that $\mathbf{d} \perp \mathbf{a}, \mathbf{d} \perp \mathbf{b}$ and $\mathbf{d}^T\mathbf{c} = 15$.

## 2 THEORY

If two vectors are perpendicular, then their dot product is 0. If we have two vectors $\mathbf{x}$, $\mathbf{y}$ is given by $\mathbf{x} \cdot \mathbf{y} = |\mathbf{x}||\mathbf{y}|\cos(\theta)$.

When $\theta = \pi/2$ (90°),then $\cos\theta = 0 \implies \mathbf{x} \cdot \mathbf{y} = 0$.

If we have 3 equations and 3 unknowns, we can use Guassian Elimination method in order to find the unknowns.

## 3 SOLUTION

Lets consider vector $\mathbf{d}$ as $\begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$.

It is given that $\mathbf{d} \perp \mathbf{a}$, then their correponding dot product will be 0.

$$\mathbf{d}^T\mathbf{a} = 0 \implies \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}^T \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} = 0$$

$$d_1 + 4d_2 + 2d_3 = 0 \qquad (3.0.1)$$

Similarly, as $\mathbf{d} \perp \mathbf{b}$,

$$\mathbf{d}^T\mathbf{b} = 0 \implies \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}^T \begin{pmatrix} 3 \\ -2 \\ 7 \end{pmatrix} = 0$$

$$3d_1 - 2d_2 + 7d_3 = 0 \qquad (3.0.2)$$

Since, it is given that $\mathbf{d}^T\mathbf{c} = 15$, we can write it as $(d_1 \quad d_2 \quad d_3) \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} = 15$.

$$2d_1 - d_2 + 4d_3 = 15 \qquad (3.0.3)$$

Using equations 3.0.1, 3.0.2, 3.0.3, we can use Guassian Elimination Method in order to find the values of $d_1$, $d_2$, $d_3$.

$$\begin{pmatrix} 1 & 4 & 2 & \bigm| & 0 \\ 3 & -2 & 7 & \bigm| & 0 \\ 2 & -1 & 4 & \bigm| & 15 \end{pmatrix} \qquad (3.0.4)$$

$$\xrightarrow[R_2 \leftarrow R_2 - 3R_1]{R_3 \leftarrow R_3 - 2R_1} \begin{pmatrix} 1 & 4 & 2 & \bigm| & 0 \\ 0 & -14 & 1 & \bigm| & 0 \\ 0 & -9 & 0 & \bigm| & 15 \end{pmatrix} \qquad (3.0.5)$$

$$\xrightarrow{R_3 \leftarrow R_3 - \frac{9}{14}R_2} \begin{pmatrix} 1 & 4 & 2 & \bigm| & 0 \\ 0 & -14 & 1 & \bigm| & 0 \\ 0 & 0 & \frac{-9}{14} & \bigm| & 15 \end{pmatrix} \qquad (3.0.6)$$

$$\xrightarrow[R_2 \leftarrow \frac{-1}{14}R_2]{R_3 \leftarrow \frac{-14}{9}R_2} \begin{pmatrix} 1 & 4 & 2 & \bigm| & 0 \\ 0 & 1 & \frac{-1}{14} & \bigm| & 0 \\ 0 & 0 & 1 & \bigm| & \frac{-210}{9} \end{pmatrix} \qquad (3.0.7)$$

$$\xrightarrow{R_1 \leftarrow R_1 + \frac{1}{14}R_3} \begin{pmatrix} 1 & 4 & 2 & \bigm| & 0 \\ 0 & 1 & 0 & \bigm| & \frac{-210}{126} \\ 0 & 0 & 1 & \bigm| & \frac{-210}{9} \end{pmatrix} \qquad (3.0.8)$$

$$\xrightarrow{R_1 \leftarrow R_1 - 4R_3} \begin{pmatrix} 1 & 0 & 2 & \bigg| & \frac{840}{126} \\ 0 & 1 & 0 & \bigg| & \frac{-210}{126} \\ 0 & 0 & 1 & \bigg| & \frac{-210}{9} \end{pmatrix} \qquad (3.0.9)$$

$$\xrightarrow{R_1 \leftarrow R_1 - 2R_3} \begin{pmatrix} 1 & 0 & 0 & \bigg| & \frac{6720}{126} \\ 0 & 1 & 0 & \bigg| & \frac{-210}{126} \\ 0 & 0 & 1 & \bigg| & \frac{-210}{9} \end{pmatrix} \qquad (3.0.10)$$

By using Guassian Elimination Method, we were able to get the vector **d** as $\begin{pmatrix} \frac{6720}{126} \\ \frac{-210}{126} \\ \frac{-210}{9} \end{pmatrix}$

The resultant vector $\mathbf{d} = \begin{pmatrix} 53.333 \\ -1.667 \\ -23.333 \end{pmatrix}$