EE5609 Assignment 5

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Abstract—**This assignment involves proving the equivalence of 2 lines that are formed inside an isosceles triangle because of certain given conditions via vector representation.**

Download all latex-tikz codes from

https://github.com/vimalkb007/EE5609/tree/master/Assignment_5

1 Problem Statement

In an isosceles $\triangle ABC$ with $AB = AC$, D and E are points on BC such that $BE = CD$. Show that $AD = AE$.

2 Solution

In the given $\triangle ABC$, let D and E be any arbitrary points on the side BC such that $BE = CD$.

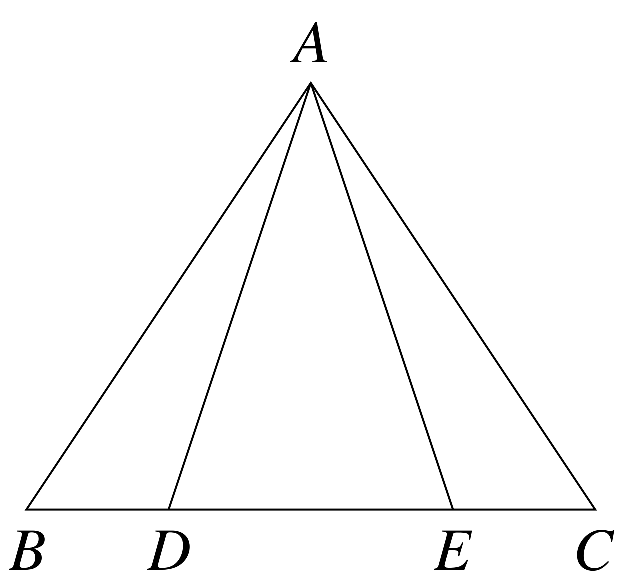

Fig. 1: Isosceles Triangle with sides AB = AC

We are given that the sides $AB = AC$, and $BE = CD$. These two can be represented as

$$\|\mathbf{B}-\mathbf{A}\| = \|\mathbf{C}-\mathbf{A}\| \tag{2.0.1}$$

$$\|\mathbf{B}-\mathbf{E}\| = \|\mathbf{D}-\mathbf{C}\| \tag{2.0.2}$$

Since the given trianle is an isoceles triangle, the angles formed by AB and AC on BC will be the same. That is

$$\angle ABC = \angle ACB = \alpha \tag{2.0.3}$$

The side AD can be represented as

$$(\mathbf{D}-\mathbf{A}) = (\mathbf{B}-\mathbf{A}) - (\mathbf{B}-\mathbf{D}) \tag{2.0.4}$$

Squaring both the sides we get

$$\|\mathbf{D}-\mathbf{A}\|^2 = \|\mathbf{B}-\mathbf{A}\|^2 + \|\mathbf{B}-\mathbf{D}\|^2 \\ -2\,\|\mathbf{B}-\mathbf{A}\|\,\|\mathbf{B}-\mathbf{D}\|\cos\alpha \tag{2.0.5}$$

The side AE can be represented as

$$(\mathbf{A}-\mathbf{E}) = (\mathbf{C}-\mathbf{A}) - (\mathbf{C}-\mathbf{E}) \tag{2.0.6}$$

Squaring both the sides we get

$$\|\mathbf{A}-\mathbf{E}\|^2 = \|\mathbf{C}-\mathbf{A}\|^2 + \|\mathbf{C}-\mathbf{E}\|^2 \\ -2\,\|\mathbf{C}-\mathbf{A}\|\,\|\mathbf{C}-\mathbf{E}\|\cos\alpha \tag{2.0.7}$$

From (2.0.2), we can further $(\mathbf{E}-\mathbf{B})$ write it as

$$(\mathbf{E}-\mathbf{B}) = (\mathbf{D}-\mathbf{B}) + (\mathbf{E}-\mathbf{D}) \tag{2.0.8}$$

Squaring both the sides we get

$$\|\mathbf{E}-\mathbf{B}\|^2 = \|\mathbf{D}-\mathbf{B}\|^2 + \|\mathbf{E}-\mathbf{D}\|^2 \\ +2\,\|\mathbf{D}-\mathbf{B}\|\,\|\mathbf{E}-\mathbf{D}\|\cos\theta \tag{2.0.9}$$

Since, both the vectors are on the same direction, the angle between them is $\theta = 0°$.

$$\|\mathbf{E}-\mathbf{B}\|^2 = \|\mathbf{D}-\mathbf{B}\|^2 + \|\mathbf{E}-\mathbf{D}\|^2 \\ +2\,\|\mathbf{D}-\mathbf{B}\|\,\|\mathbf{E}-\mathbf{D}\| \tag{2.0.10}$$

$$\|\mathbf{E}-\mathbf{B}\|^2 = (\|\mathbf{D}-\mathbf{B}\| + \|\mathbf{E}-\mathbf{D}\|)^2 \tag{2.0.11}$$

From (2.0.2), we can further $(\mathbf{C}-\mathbf{D})$ write it as

$$(\mathbf{C}-\mathbf{D}) = (\mathbf{C}-\mathbf{E}) + (\mathbf{E}-\mathbf{D}) \tag{2.0.12}$$

Squaring both the sides we get

$$\|\mathbf{C}-\mathbf{D}\|^2 = \|\mathbf{C}-\mathbf{E}\|^2 + \|\mathbf{E}-\mathbf{D}\|^2 \\ +2\,\|\mathbf{C}-\mathbf{E}\|\,\|\mathbf{E}-\mathbf{D}\|\cos\theta \tag{2.0.13}$$

Since, both the vectors are on the same direction, the angle between them is $\theta = 0°$.

$$\|\mathbf{C}-\mathbf{D}\|^2 = \|\mathbf{C}-\mathbf{E}\|^2 + \|\mathbf{E}-\mathbf{D}\|^2 \\ +2\,\|\mathbf{C}-\mathbf{E}\|\,\|\mathbf{E}-\mathbf{D}\| \tag{2.0.14}$$

$$\|\mathbf{C}-\mathbf{D}\|^2 = (\|\mathbf{C}-\mathbf{E}\| + \|\mathbf{E}-\mathbf{D}\|)^2 \tag{2.0.15}$$

From equations (2.0.2), (2.0.11), (2.0.15), we get

$$(\|\mathbf{D}-\mathbf{B}\| + \|\mathbf{E}-\mathbf{D}\|)^2 = (\|\mathbf{C}-\mathbf{E}\| + \|\mathbf{E}-\mathbf{D}\|)^2 \\ \implies \|\mathbf{D}-\mathbf{B}\| + \|\mathbf{E}-\mathbf{D}\| = \|\mathbf{C}-\mathbf{E}\| + \|\mathbf{E}-\mathbf{D}\| \\ \implies \|\mathbf{D}-\mathbf{B}\| = \|\mathbf{C}-\mathbf{E}\| \tag{2.0.16}$$

Using equations (2.0.1), (2.0.16), we apply them in the equation (2.0.5), (2.0.7). After applying it we see that the R.H.S components are getting equated to each other, then we can equate the L.H.S as well. We get

$$\|\mathbf{D}-\mathbf{A}\|^2 = \|\mathbf{A}-\mathbf{E}\|^2 \\ \|\mathbf{D}-\mathbf{A}\| = \|\mathbf{A}-\mathbf{E}\| \tag{2.0.17}$$

Therefore, we can say that $AD = AE$.