

# EE5609 Assignment 1

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**Abstract**—This assignment involves finding a vector which is perpendicular to given two vectors and non-perpendicular to a third vector.

The python solution code for this problem can be downloaded from

[https://github.com/vimalkb007/EE5609/blob/master/Assignment\\_1/codes/assignment1\\_solution.py](https://github.com/vimalkb007/EE5609/blob/master/Assignment_1/codes/assignment1_solution.py)

The python verification code for this problem can be downloaded from

[https://github.com/vimalkb007/EE5609/blob/master/Assignment\\_1/codes/assignment1\\_solution\\_verify.py](https://github.com/vimalkb007/EE5609/blob/master/Assignment_1/codes/assignment1_solution_verify.py)

## 1 PROBLEM STATEMENT

Let  $\mathbf{a} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$ ,  $\mathbf{b} = \begin{pmatrix} 3 \\ -2 \\ 7 \end{pmatrix}$  and  $\mathbf{c} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$ . Find a vector  $\mathbf{d}$  such that  $\mathbf{d} \perp \mathbf{a}$ ,  $\mathbf{d} \perp \mathbf{b}$  and  $\mathbf{d}^T \mathbf{c} = 15$ .

## 2 THEORY

If two vectors are perpendicular, then their dot product is 0. If we have two vectors  $\mathbf{x}$ ,  $\mathbf{y}$  is given by  $\mathbf{x} \cdot \mathbf{y} = |\mathbf{x}||\mathbf{y}|\cos(\theta)$ .

When  $\theta = \pi/2$  ( $90^\circ$ ), then  $\cos \theta = 0 \implies \mathbf{x} \cdot \mathbf{y} = 0$ .

If we have 3 equations and 3 unknowns, we can use Gaussian Elimination method in order to find the unknowns.

## 3 SOLUTION

Lets consider vector  $\mathbf{d}$  as  $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ .

It is given that  $\mathbf{d} \perp \mathbf{a}$ , then their corresponding dot product will be 0.

$$\mathbf{d} \cdot \mathbf{a} = 0 \implies \begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} = 0$$

$$x + 4y + 2z = 0 \quad (3.0.1)$$

Similarly, as  $\mathbf{d} \perp \mathbf{b}$ ,

$$\mathbf{d} \cdot \mathbf{b} = 0 \implies \begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -2 \\ 7 \end{pmatrix} = 0$$

$$3x - 2y + 7z = 0 \quad (3.0.2)$$

Since, it is given that  $\mathbf{d}^T \mathbf{c} = 15$ , we can write it

$$\text{as } (x \ y \ z) \times \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} = 15.$$

$$2x - y + 4z = 15 \quad (3.0.3)$$

Using equations 3.0.1, 3.0.2, 3.0.3, we can use Gaussian Elimination Method in order to find the values of  $x$ ,  $y$ ,  $z$ .

$$\left[ \begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 3 & -2 & 7 & 0 \\ 2 & -1 & 4 & 15 \end{array} \right]$$

Applying  $R_2 \leftarrow R_2 - 3R_1$ ,  $R_3 \leftarrow R_3 - 2R_1$

$$\left[ \begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 0 & -14 & 1 & 0 \\ 0 & -9 & 0 & 15 \end{array} \right]$$

Applying  $R_3 \leftarrow R_3 - \frac{9}{14}R_2$ ,

$$\left[ \begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 0 & -14 & 1 & 0 \\ 0 & 0 & \frac{-9}{14} & 15 \end{array} \right]$$

We get the following equations as,

$$x + 4y + 2z = 0 \quad (3.0.4)$$

$$-14y + z = 0 \quad (3.0.5)$$

$$\frac{-9}{14}z = 15 \quad (3.0.6)$$

Equating the above equations we get,  $x = 53.333$ ,  
 $y = -1.667$ ,  $z = -23.333$

The resultant vector  $\mathbf{d} = \begin{pmatrix} 53.333 \\ -1.667 \\ -23.333 \end{pmatrix}$