EE5609 Assignment 3

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***Abstract*—This assignment involves finding the determinant of the given matrix.**

The python solution code for this problem can be downloaded from

https://github.com/vimalkb007/EE5609/blob/master/Assignment_4/codes/assignment4_solution.py

1 Problem Statement

Evaluate $\begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$.

2 Solution

We first multiply either the rows or the columns, and then try taking the common element out.

$$\begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix} \xleftrightarrow[C_3\leftarrow(\sin\alpha)C_3]{C_3\leftarrow(\cos\alpha)C_3}$$

$$\left(\frac{1}{\sin\alpha\cos\alpha}\right)\begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin^2\alpha\cos\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos^2\alpha\sin\alpha \end{vmatrix} \tag{2.0.1}$$

From 2.0.1 R_1 we take out common element $\cos\alpha$. And from row R_2 we take out common element $\sin\alpha$

$$\begin{vmatrix} \cos\beta & \sin\beta & -\sin^2\alpha \\ -\sin\beta & \cos\beta & 0 \\ \cos\beta & \sin\beta & \cos^2\alpha \end{vmatrix} \xleftrightarrow{R_1\leftarrow R_1-R_3}$$

$$\begin{vmatrix} 0 & 0 & -\sin^2\alpha-\cos^2\alpha \\ -\sin\beta & \cos\beta & 0 \\ \cos\beta & \sin\beta & \cos^2\alpha \end{vmatrix} \tag{2.0.2}$$

Now, we can expand the determinant from row 1 in 2.0.2, and we get

$$\begin{aligned} &\left(-\sin^2\alpha-\cos^2\alpha\right)\left(-\sin^2\beta-\cos^2\beta\right) \\ \implies &\left(\sin^2\alpha+\cos^2\alpha\right)\left(\sin^2\beta+\cos^2\beta\right)=1 \end{aligned} \tag{2.0.3}$$

Therefore, the determinant of the matrix is 1.