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EE5609 Assignment 1

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Abstract—This assignment involves finding a vector which is perpendicular to given two vectors and nonperpendicular to a third vector.

The python solution code for this problem can be downloaded from

> https://github.com/vimalkb007/EE5609/ blob/master/Assignment 1/codes/ assignment1 solution.py

The python verification code for this problem can be downloaded from

> https://github.com/vimalkb007/EE5609/ blob/master/Assignment 1/codes/ assignment1 solution verify.py

1 Problem Statement

Let
$$\mathbf{a} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$$
, $\mathbf{b} = \begin{pmatrix} 3 \\ -2 \\ 7 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$. Find a

vector **d** such that $\mathbf{d} \perp \mathbf{a}, \mathbf{d} \perp \mathbf{b}$ and $\mathbf{d}^T \mathbf{c} = 15$.

2 Theory

If two vectors are perpendicular, then their dot product is 0. If we have two vectors x, y is given by $\mathbf{x} \cdot \mathbf{v} = |\mathbf{x}| |\mathbf{v}| \cos(\theta)$.

When $\theta = \pi/2$ (90°), then $\cos \theta = 0 \implies \mathbf{x} \cdot \mathbf{y}$

If we have 3 equations and 3 unknowns, we can use Guassian Elimination method in order to find the unknowns.

3 Solution

Lets consider vector \mathbf{d} as |y|

It is given that $\mathbf{d} \perp \mathbf{a}$, then their corresponding dot y = -1.667, z = -23.333product will be 0.

$$\mathbf{d}^T \mathbf{a} = 0 \implies \begin{pmatrix} x \\ y \\ z \end{pmatrix}^T \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} = 0$$

$$x + 4y + 2z = 0 (3.0.1)$$

Similarly, as $\mathbf{d} \perp \mathbf{b}$,

$$\mathbf{d}^{T}\mathbf{b} = 0 \implies \begin{pmatrix} x \\ y \\ z \end{pmatrix}^{T} \begin{pmatrix} 3 \\ -2 \\ 7 \end{pmatrix} = 0$$
$$3x - 2y + 7z = 0 \tag{3.0.2}$$

Since, it is given that
$$\mathbf{d}^T \mathbf{c} = 15$$
, we can write it as $(x \ y \ z) \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} = 15$.
 $2x - y + 4z = 15$ (3.0.3)

Using equations 3.0.1, 3.0.2, 3.0.3, we can use Guassian Elimination Method in order to find the values of x, y, z.

$$\begin{pmatrix}
1 & 4 & 2 & | & 0 \\
3 & -2 & 7 & | & 0 \\
2 & -1 & 4 & | & 15
\end{pmatrix}$$
(3.0.4)

$$\stackrel{R_3 \leftarrow R_3 - 2R_1}{\underset{R_2 \leftarrow R_2 - 3R_1}{\longleftrightarrow}} \begin{pmatrix}
1 & 4 & 2 & 0 \\
0 & -14 & 1 & 0 \\
0 & -9 & 0 & 15
\end{pmatrix}$$
(3.0.5)

$$\stackrel{R_3 \leftarrow R_3 - \frac{9}{14}R_2}{\longleftrightarrow} \begin{pmatrix} 1 & 4 & 2 & 0 \\ 0 & -14 & 1 & 0 \\ 0 & 0 & \frac{-9}{14} & 15 \end{pmatrix}$$
(3.0.6)

We get the following equations as,

$$x + 4y + 2z = 0 ag{3.0.7}$$

$$-14y + z = 0 (3.0.8)$$

$$\frac{-9}{14}z = 15\tag{3.0.9}$$

Equating the above equations we get, x = 53.333,

The resultant vector
$$\mathbf{d} = \begin{pmatrix} 53.333 \\ -1.667 \\ -23.333 \end{pmatrix}$$