EE5609: Matrix Theory
Assignment 8

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***Abstract*—This document explains the relationship between the basis and the dimension of a vector space.**

Download all solutions from

https://github.com/vimalkb007/EE5609/tree/master/Assignment_8

1 Problem

Let $\mathbf{V}$ be the vector space of all 2×2 matrices over the field $\mathbf{F}$. Prove that $\mathbf{V}$ has dimension 4 by exhibiting a basis for $\mathbf{V}$ which has four elements.

2 Solution

Let

$$v_{11} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad v_{12} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \tag{2.0.1}$$

$$v_{21} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad v_{22} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \tag{2.0.2}$$

Suppose $av_{11} + bv_{12} + cv_{21} + dv_{22} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$. Then

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{2.0.3}$$

The only values of a, b, c, d which makes the equation 2.0.3 satisfied is, when $a = b = c = d = 0$. Thus v_1, v_2, v_3, v_4 are linearly independent.

Now, let $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be any 2×2 matrix. Then $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = av_{11} + bv_{12} + cv_{21} + dv_{22}$. Thus $v_{11}, v_{12}, v_{21}, v_{22}$ span the space of 2×2 matrix.

Thus $v_{11}, v_{12}, v_{21}, v_{22}$ are both linearly independent and they span the span of all 2×2 matrices. So, $v_{11}, v_{12}, v_{21}, v_{22}$ constitute a basis for the space of all 2×2 matrices.

We know that, the dimension of a vector space $\mathbf{V}$, denoted by $dim(\mathbf{V})$, is the number of basis for $\mathbf{V}$. Therefore, $dim(\mathbf{V}) = 4$.