EE5609: Matrix Theory
Assignment 9

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***Abstract*—This document explains the concept of vector space over complex numbers.**

Download all solutions from

https://github.com/vimalkb007/EE5609/tree/master/Assignment_9

1 Problem

Let **V** be the vector space over the complex numbers of all functions from **R** into **C**, i.e. the space of all complex-valued functions on the real line. Let $f_1(x) = 1,\ f_2(x) = e^{ix},\ f_3(x) = e^{-ix}$.

(a) Prove that f_1, f_2, and f_3 are linearly dependent.

(b) Let $g_1(x) = 1,\ g_2(x) = \cos(x),\ g_3(x) = \sin(x)$. Find an invertible 3×3 matrix P such that $g_i = \sum_{i=1}^{3} P_{ii} f_i$

2 Solution

(a) To check for independence, lets represent the function in a polynomial format as

$$\alpha f_1 + \beta f_2 + \gamma f_3 = 0 \quad (2.0.1)$$
$$\alpha + \beta e^{ix} + \gamma e^{-ix} = 0 \quad (2.0.2)$$

Multiply the whole equation with e^{ix}

$$\beta(e^{ix})^2 + \alpha e^{ix} + \gamma = 0 \quad (2.0.3)$$

Let $y = e^{ix}$

$$\beta y^2 + \alpha y + \gamma = 0 \quad (2.0.4)$$

The above quadratic polynomial in y can be zero for atmost two values of y. But,

$$y = e^{ix} \quad x \in \mathbf{R} \quad (2.0.5)$$

So (2.0.4) cannot be zero for all $y = e^{ix}$. Which implies there is a contradiction.

Then, the only case when (2.0.4) gets satisfied is

$$\alpha = \beta = \gamma = 0 \quad (2.0.6)$$

Therefore, f_1, f_2, f_3 are linearly independent.

(b) We need to find the coordinates of vectors g_i where $i = 1, 2, 3$ in ordered basis

$$B = \begin{pmatrix} f_1 & f_2 & f_3 \end{pmatrix} \quad (2.0.7)$$

It is given that $g_1 = 1$, which can be written as

$$g_1 = f_1 \quad (2.0.8)$$
$$\implies g_1 = \begin{pmatrix} f_1 & f_2 & f_3 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad (2.0.9)$$

We can use the following identities:-

$$\cos(x) = \frac{1}{2} e^{ix} + \frac{1}{2} e^{-ix} \quad (2.0.10)$$
$$\sin(x) = \frac{1}{2i} e^{ix} - \frac{1}{2i} e^{-ix} \quad (2.0.11)$$

Comparing equations (2.0.10) and (2.0.11) with f_2, f_3, we can write g_2 and g_3 as

$$g_2 = \frac{1}{2} f_2 + \frac{1}{2} f_3 \quad (2.0.12)$$
$$\implies g_2 = \begin{pmatrix} f_1 & f_2 & f_3 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \quad (2.0.13)$$

$$g_3 = \frac{1}{2i} f_2 - \frac{1}{2i} f_3 \tag{2.0.14}$$

$$\implies g_3 = \begin{pmatrix} f_1 & f_2 & f_3 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{1}{2i} \\ \frac{-1}{2i} \end{pmatrix} \tag{2.0.15}$$

Therefore, the required matrix P is

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2i} \\ 0 & \frac{1}{2} & \frac{-1}{2i} \end{pmatrix} \tag{2.0.16}$$