EE5609 Assignment 1

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***Abstract*—This assignment involves finding a vector which is perpendicular to given two vectors and non-perpendicular to a third vector.**

The python solution code for this problem can be downloaded from

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https://github.com/vimalkb007/EE5609/
    blob/master/Assignment_1/codes/
    assignment1_solution.py
```

The python verification code for this problem can be downloaded from

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https://github.com/vimalkb007/EE5609/
    blob/master/Assignment_1/codes/
    assignment1_solution_verify.py
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1 Problem Statement

Let $\mathbf{a} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 3 \\ -2 \\ 7 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$. Find a vector $\mathbf{d}$ such that $\mathbf{d} \perp \mathbf{a}, \mathbf{d} \perp \mathbf{b}$ and $\mathbf{d}^T\mathbf{c} = 15$.

2 Theory

If two vectors are perpendicular, then their dot product is 0. If we have two vectors $\mathbf{x}$, $\mathbf{y}$ is given by $\mathbf{x}{\cdot}\mathbf{y} = |\mathbf{x}||\mathbf{y}|\cos(\theta)$.

When $\theta = \pi/2 \ \ (90°)$,then $\ \cos\theta = 0 \implies \mathbf{x}{\cdot}\mathbf{y} = 0$.

If we have 3 equations and 3 unknowns, we can use Guassian Elimination method in order to find the unknowns.

3 Solution

It is given that $\mathbf{d} \perp \mathbf{a}$, then their correponding dot product will be 0.

$$\mathbf{d}^T\mathbf{a} = 0 \tag{3.0.1}$$

which can be written as

$$\mathbf{d}^T \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} = 0 \tag{3.0.2}$$

Similarly, as $\mathbf{d} \perp \mathbf{b}$

$$\mathbf{d}^T\mathbf{b} = 0 \tag{3.0.3}$$

i.e.

$$\mathbf{d}^T \begin{pmatrix} 3 \\ -2 \\ 7 \end{pmatrix} = 0 \tag{3.0.4}$$

It is given that

$$\mathbf{d}^T\mathbf{c} = 15 \tag{3.0.5}$$

i.e.

$$\mathbf{d}^T \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} = 15 \tag{3.0.6}$$

Using equations 3.0.1, 3.0.3, 3.0.5, we can represent them in a Matrix Representation of Linear Equations $Ax{=}B$ form as:

$$\begin{bmatrix} \mathbf{a}^T \\ \mathbf{b}^T \\ \mathbf{c}^T \end{bmatrix} \mathbf{d} = \begin{bmatrix} 0 \\ 0 \\ 15 \end{bmatrix}$$

Numerically, using 3.0.2, 3.0.4, 3.0.6 the above equation can be written as,

$$\begin{bmatrix} 1 & 4 & 2 \\ 3 & -2 & 7 \\ 2 & -1 & 4 \end{bmatrix} \mathbf{d} = \begin{bmatrix} 0 \\ 0 \\ 15 \end{bmatrix}$$

we can use Guassian Elimination Method in order to find the coordinate values of $\mathbf{d}$.

$$\left(\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 3 & -2 & 7 & 0 \\ 2 & -1 & 4 & 15 \end{array}\right) \tag{3.0.7}$$

$$\xleftrightarrow[R_2 \leftarrow R_2 - 3R_1]{R_3 \leftarrow R_3 - 2R_1} \left(\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 0 & -14 & 1 & 0 \\ 0 & -9 & 0 & 15 \end{array}\right) \tag{3.0.8}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - \frac{9}{14}R_2} \left(\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 0 & -14 & 1 & 0 \\ 0 & 0 & \frac{-9}{14} & 15 \end{array}\right) \tag{3.0.9}$$

$$\xleftrightarrow[R_2 \leftarrow \frac{-1}{14}R_2]{R_3 \leftarrow \frac{-14}{9}R_2} \left(\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 0 & 1 & \frac{-1}{14} & 0 \\ 0 & 0 & 1 & \frac{-210}{9} \end{array}\right) \tag{3.0.10}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 + \frac{1}{14}R_3} \left(\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 0 & 1 & 0 & \frac{-210}{126} \\ 0 & 0 & 1 & \frac{-210}{9} \end{array}\right) \tag{3.0.11}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - 4R_3} \left(\begin{array}{ccc|c} 1 & 0 & 2 & \frac{840}{126} \\ 0 & 1 & 0 & \frac{-210}{126} \\ 0 & 0 & 1 & \frac{-210}{9} \end{array}\right) \tag{3.0.12}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - 2R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{6720}{126} \\ 0 & 1 & 0 & \frac{-210}{126} \\ 0 & 0 & 1 & \frac{-210}{9} \end{array}\right) \tag{3.0.13}$$

By using Guassian Elimination Method, we were able to get the vector $\mathbf{d}$ as $\begin{pmatrix} \frac{6720}{126} \\ \frac{-210}{126} \\ \frac{-210}{9} \end{pmatrix}$

The resultant vector $\mathbf{d} = \begin{pmatrix} 53.333 \\ -1.667 \\ -23.333 \end{pmatrix}$