

EE5609: Matrix Theory

Assignment 8

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Abstract—This document explains the relationship between the basis and the dimension of a vector space.

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https://github.com/vimalkb007/EE5609/tree/master/Assignment_8

Thus $v_{11}, v_{12}, v_{21}, v_{22}$ are both linearly independent and they span the span of all 2×2 matrices. So, $v_{11}, v_{12}, v_{21}, v_{22}$ constitute a basis for the space of all 2×2 matrices.

We know that, the dimension of a vector space \mathbf{V} , denoted by $\dim(\mathbf{V})$, is the number of basis for \mathbf{V} . Therefore, $\dim(\mathbf{V}) = 4$.

1 PROBLEM

Let \mathbf{V} be the vector space of all 2×2 matrices over the field \mathbf{F} . Prove that \mathbf{V} has dimension 4 by exhibiting a basis for \mathbf{V} which has four elements.

2 SOLUTION

Let

$$v_{11} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad v_{12} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad (2.0.1)$$

$$v_{21} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad v_{22} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad (2.0.2)$$

Suppose $av_{11} + bv_{12} + cv_{21} + dv_{22} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$. Then

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad (2.0.3)$$

The only values of a, b, c, d which makes the equation (2.0.3) satisfied is, when $a = b = c = d = 0$. Thus v_1, v_2, v_3, v_4 are linearly independent.

Now, let $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be any 2×2 matrix. Then $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = av_{11} + bv_{12} + cv_{21} + dv_{22}$. Thus $v_{11}, v_{12}, v_{21}, v_{22}$ span the space of 2×2 matrix.