SVD explanation

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oct/12/20 SVD matrix A $m \times n$.

$A \times \vec{v} = \sigma \times \vec{u}$

for $A \times \vec{v} \neq 0$, $\vec{V}$ is in row space and $\vec{u}$ is in column space

say V is collection of all basis of rowspace

U for column space

then $A \times V = U \times \Sigma$

where Σ is diagonal matrix with scaling factors similar to σ for every column $\vec{u}$

since V and U are unitary

$VV^T = UU^T = I$

$\therefore A V = U \Sigma$

$A V V^T = U \Sigma V^T$

$A = U \Sigma V^T \rightarrow (1)$

$A^T A = (U \Sigma V^T)^T U \Sigma V^T$

$= V \Sigma U^T U \Sigma V^T$

$A^T A = V \Sigma^2 V^T$

$A^T A V = V \Sigma^2 V^T V = V \Sigma^2$

$A^T A V = V \Sigma^2 \rightarrow (2)$

this is of the eigen form

$\therefore$ V is eigen vectors of $A^T A$

Σ is set of eigen values of $A^T A$. //