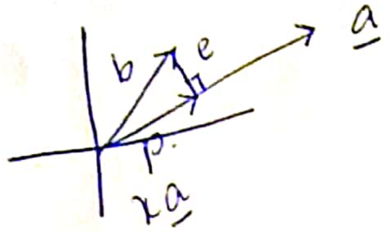

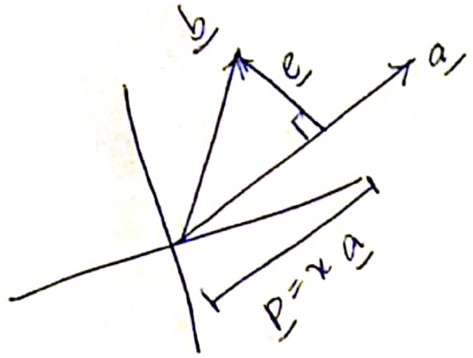

$\underline{p} + \underline{e} = \underline{b}$

$x\underline{a} + \underline{e} = \underline{b}$

$\underline{e} = \underline{b} - x\underline{a}$

$(\underline{b} - x\underline{a}) \cdot (\underline{a}) = 0$

$\underline{a} \cdot \underline{b} - x\, \underline{a} \cdot \underline{a} = 0$

$\underline{a} \cdot \underline{b} = x\, \underline{a} \cdot \underline{a}$

$x = \dfrac{\underline{a} \cdot \underline{b}}{\underline{a} \cdot \underline{a}}$

$\underline{p} = x \cdot \underline{a}$ (Scalar method)

$x = \dfrac{\underline{a} \cdot \underline{b}}{\underline{a} \cdot \underline{a}} = \dfrac{\underline{a}^T b}{\underline{a}^T \underline{a}}$

$p = \underline{a}\, \dfrac{a^T b}{a^T a} \qquad \dfrac{\underline{a}\,\underline{a}^T}{a^T a} \cdot \underline{b}$ (matrix method)

② $\underline{b} = (1,1)$ $\quad \underline{a} = (2,0)$. $\quad \underline{a} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ $b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

$$p = x . \underline{a} = \frac{\begin{bmatrix} 2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}}{\begin{bmatrix} 2 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix}} \underline{a} = \frac{2}{4} \underline{a} = \frac{\underline{a}}{2}$$

$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$p = \frac{\underline{a}\,\underline{a}^T . \underline{b}}{a^T . a} = \frac{\begin{bmatrix} 2 \\ 0 \end{bmatrix} \begin{bmatrix} 2 & 0 \end{bmatrix}}{\begin{bmatrix} 2 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix}} . \underline{b}$$

$$= \frac{\begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}}{4} \quad \frac{\begin{bmatrix} 4 \\ 0 \end{bmatrix}}{4} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Same.

$$A = \left[\frac{\underline{a}\,\underline{a}^T}{a^T a} \right] = \text{transformation matrix.}$$

A is idempotent. repeated projections onto same vector has no effect.

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(3) $I-A$ can project onto $\perp^r$ space of A.

$$I - A = I - \frac{\underline{a}\,\underline{a}^T}{\underline{a}^T a}$$

say for $a = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$$A = \frac{\begin{bmatrix} 2 \\ 0 \end{bmatrix} [2 \quad 0]}{[2 \quad 0] \begin{bmatrix} 2 \\ 0 \end{bmatrix}} = \frac{\begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix}}{4}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

projecting $\vec{b}$ on $I-A$ $\qquad \underline{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is $\perp^r$ to $\underline{a} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$I - A$ is also idempotent.

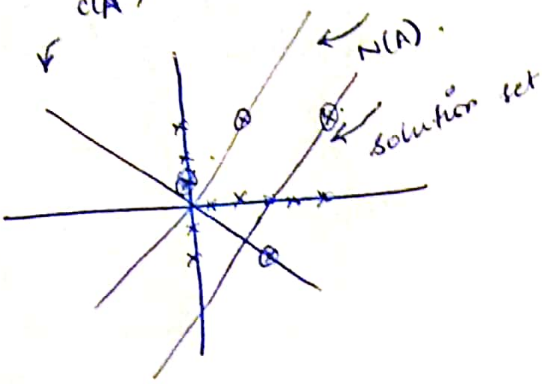
④ projection onto subspaces

consider $A = \begin{bmatrix} 3 & -2 \\ 6 & -4 \end{bmatrix}$

$$Ax = B = \begin{bmatrix} 9 \\ 18 \end{bmatrix}$$

$C(A^T) = \underset{\text{span}}{} [3 \;\; -2]$, or $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$ $\qquad N(A) = \text{span}\left(\begin{bmatrix} 2 \\ 3 \end{bmatrix}\right)$.

$\therefore$ solution set of $Ax = B$ = $\left\{ \bar{x} = \begin{bmatrix} 3 \\ 0 \end{bmatrix} + c \begin{bmatrix} 2 \\ 3 \end{bmatrix} \;\middle|\; c \in \mathbb{R} \right\}$



any point/vector in $\mathbb{R}^n$ canbe respresented as

$a\,[\text{Rowspace}] + b\,[\text{nullspace}]$

orthogonal complements//.

projection of vector $\bar{x}$ on a subspace with basis vectors $\bar{A}$ is

$$\underline{A}\,(\underline{A}^T \underline{A})^{-1} \underline{A}^T \bar{x}$$

(refer khan Academy for details).

$$\text{Proj}_A\, \bar{x} = \underline{A}\,(\underline{A}^T \underline{A})^{-1} A^T \bar{x}$$

when A is matrix of orthonormal basis.

$$A A^T \bar{x}$$

(5) Least square approximation:

$\bar{A}\bar{x} = \bar{b}$ → no solution.

$\bar{b}$ is not in column space of $\bar{A}$.

so when you search for a solution for $\bar{A}\bar{x} = \bar{b}$, if $\bar{b}$ is not in $\bar{A}$

the closest $\bar{b}$ is projection of $\bar{b}$ in $C(A)$.

say $\tilde{b}$ is projection of $\bar{b}$ on $C(A)$,

then $\bar{A}\bar{x} = \tilde{b}$ definitely has a solution.

$A\bar{x} = \bar{b}$ no solution?

no problem

solve for

$A^T A\bar{x} = A^T\bar{b}$

↳ (arrived through $A^T(A\bar{x}^* - \vec{b}) = \bar{0}$

A is transformation

$A\bar{x}^*$ ⇒ projection of b on $C(A)$.

(6) orthogonal & orthonormal Basis:

If Q is a matrix with all orthogonal vectors

then $Q Q^T = I$ $\therefore$ $Q^T = Q^{-1}$

Q- unitary matrix.

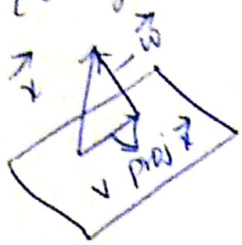
Column space of a matrix $\Rightarrow$ orthonormal vectors

Gram Schmidt.

Gram Schmidt process - khan Academy:

(7) projecting a vector $\bar{x}$ on a subspace V $\{\bar{v}_1\ \bar{v}_2\ \bar{v}_3 \ldots \bar{v}_k\}$ orthonormal basis

Actual $\bar{x}$ = projection of $\bar{x}$ on V + $\bar{w}$ {orthogonal to V}



$$= \underbrace{c_1 \bar{v}_1 + c_2 \bar{v}_2 + \ldots c_k \bar{v}_k}_{\text{projection of } \bar{x} \text{ on } V} + \bar{w}$$

$$\vec{v}_i \cdot \vec{x} = \underbrace{c_1 \vec{v}_1 \cdot \vec{v}_i + c_2 \vec{v}_2 \cdot \vec{v}_i + \ldots c_k \vec{v}_k \cdot \vec{v}_i}_{} + \underbrace{\vec{w} \cdot \vec{v}_i}_{0}$$

when $\vec{v}_i$ = any on of basis ($\vec{w}$ $\perp$ to V).

then $v_i \cdot v_i = 1$ for others $v_i \cdot v_j = 0$ (ortho)

$$\vec{v}_i \cdot \vec{x} = c_i$$

projection of $\bar{x}$ on V $= (\vec{v}_1 \cdot \vec{x})\,\vec{v}_1 + (\vec{v}_2 \cdot \vec{x})\,\vec{v}_2 \ldots (\vec{v}_k \cdot \vec{x})\,\vec{v}_k$

$= (\vec{v}_1 \cdot \vec{x})\,\vec{v}_1 + (\vec{v}_2 \cdot \vec{x})\,\vec{v}_2$