

Confidence Interval, Margin of error

Population proportions & Sample proportions

Population = 1 million people

proportion of interest = % of people favoring a party

can't reach out to 1 million.

lets draw a sample of $n=100$
(stratified such that it represents the population).

∴ sample proportion $\hat{p} = 0.54$ (say)

objective: what is the 95% confidence interval that overall proportion is closer to sample proportion?



we don't have population proportion (obviously) to arrive at sampling distribution.

Sample distribution

As we have sample proportion $\hat{p} = 0.54$ with $n=100$
we can arrive at standard error of sample

$$SE_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.54 \times 0.46}{100}} = 0.05$$

Initial question was $\hat{p} = 0.54$ is it close to population parameters. 95% confidence \Rightarrow within 2.5

In other words is population parameter within 2 standard error from sample mean.

$$\begin{aligned} \therefore \text{confidence interval} &= \text{sample proportion} \pm 2 SE_{\hat{p}} \\ &= 0.54 \pm 2(0.05) \\ &= \text{within } 0.44 \text{ to } 0.64 \end{aligned}$$

so we are 95% confident that population proportion is within 0.44 & 0.64. Margin of error = $2.5 = 0.1$
Increase sample size ($n > 100$) to reduce margin of error.