

decision boundary

$$w^T x + b = 0$$

\hat{u} is \perp to decision boundary

say x is on decision boundary

x' is on margin $(w^T x' + b = 1)$

$$\text{so } \vec{x}' - \vec{x} = \vec{r}$$

\vec{r} is in the direction of \hat{u}

$$\therefore \vec{r} = \gamma \hat{u}$$

$$\vec{r} = \gamma \frac{\vec{w}}{\|\vec{w}\|_2}$$

$$\vec{r} = \vec{x}' - \vec{x}$$

multiply \vec{w} on both sides.

$$\vec{w}^T \vec{r} = \vec{w}^T \vec{x}' - \vec{w}^T \vec{x}$$

$$\therefore \vec{w}^T \gamma \frac{\vec{w}}{\|\vec{w}\|_2} = 1 - b - b = 1$$

$$\therefore \gamma \frac{\vec{w}^T \vec{w}}{\|\vec{w}\|_2} = 1$$

$$\therefore \gamma = \frac{\|\vec{w}\|_2}{\vec{w}^T \vec{w}} = \frac{1}{\|\vec{w}\|}$$

$$\therefore \text{margin} = 2\gamma = \frac{2}{\|\vec{w}\|}$$

