Variance, Covariance, Correlation, Expected Values.

Expected Value of x = Average Value of x (In terms of discrete) $= \int_{-\infty}^{\infty} x f(x)\, dx$

Summation of x across all probabilities

Variance of x $= \frac{(x - \bar{x})^2}{n}$

= squared distance from the mean

= which is also equal to $= E(x^2) - [E(x)]^2$

= how off is the summation of squared value from the summation.

Variance of a random variable = Variance between itself $= E[x^2] - E(x) E(x)$

Variance between 2 random variables $= E(XY) - E(x) E(Y)$

$\Rightarrow$ also equal to $\frac{\Sigma (x - \bar{x})(y - \bar{y})}{n}$

Combining multiple random variables.

$$Var(x) = E[x - E(x)]^2$$

$$Var(x+Y) = E(x+Y - E(x+Y))^2 \rightarrow ①$$

$$E(X+Y) = E(x) + E(y) \rightarrow ②$$

$\therefore$ ① $\Rightarrow$

$$Var(X+Y) = E(x+Y - E(x) - E(Y))^2$$

$$= E((x - E(x)) + (Y - E(Y)))^2$$

$$= E[(x - E(x))^2 + (y - E(y))^2 + 2(x - E(x))(y - E(y))]$$

$$= E((x - E(x))^2) + d((y - E(y))^2) + 2E[(x - E(x))(y - E(y))]$$

$$= Var(x) + Var(y) + 2\, cov(x, y).$$

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