Central limit theorem:

combined multiple random variables which are independent identically distributed.

Summing up multiple random variables.

$X_1, X_2, \ldots X_n$ with each variable having a mean $= \mu$ and variance $= \sigma^2$, n random variables.

$$S_n = X_1 + X_2 \ldots X_n.$$

$$E(S_n) = E(X_1 + X_2 + \cdots X_n) = E(X_1) + E(X_2) \ldots E(X_n)$$

$$= \mu + \mu + \ldots \mu = n\mu$$

$$Var(S_n) = Var(X_1 + X_2 \ldots X_n).$$

$$= n\sigma^2$$ (other covariance pairs $= 0$ as the variables are independent)

$$\therefore SD(S_n) = \sqrt{n} \times \sigma$$

$\therefore$ for any given S, how many S.D away from $E(S_n)$

$$= \frac{S - S_\mu}{S_\sigma} = \frac{S - n\mu}{\sigma\sqrt{n}} = Z_n$$

mean of multiple random variables.

$$M_n = \frac{X_1 + X_2 + \ldots X_n}{n}$$

$$E(M_n) = E\left(\frac{X_1 + X_2 \ldots X_n}{n}\right) = \frac{1}{n} \times E(X_1 + X_2 \ldots X_n) = \frac{1}{n} \times \mu n$$

$$= \mu$$

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$$Var(M_n) = Var\left(\frac{X_1 + X_2 + \dots X_n}{n}\right) = \frac{1}{n^2} \times Var(X_1 + X_2 + \dots X_n)$$

$$= \frac{1}{n^2} \times n\sigma^2 = \frac{\sigma^2}{n}.$$

$$\therefore SD(M_n) = \frac{\sigma}{\sqrt{n}}$$

to understand how likely the mean will take a value M is given by

$$\frac{M - \mu_M}{\sigma_M} = \frac{M - \mu}{\sigma/\sqrt{n}} = Z_M$$

<u>what this means?</u>

probability of observing a mean upto M

$$= p(z \leq Z_m).$$

from the mean, if we travel 2σ on either sides, we would cover 95% probability.

so we can say that $\mu \pm 2\frac{\sigma}{\sqrt{n}}$ would be the 95% confidence limit.

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