Taylor_s series to Newton method

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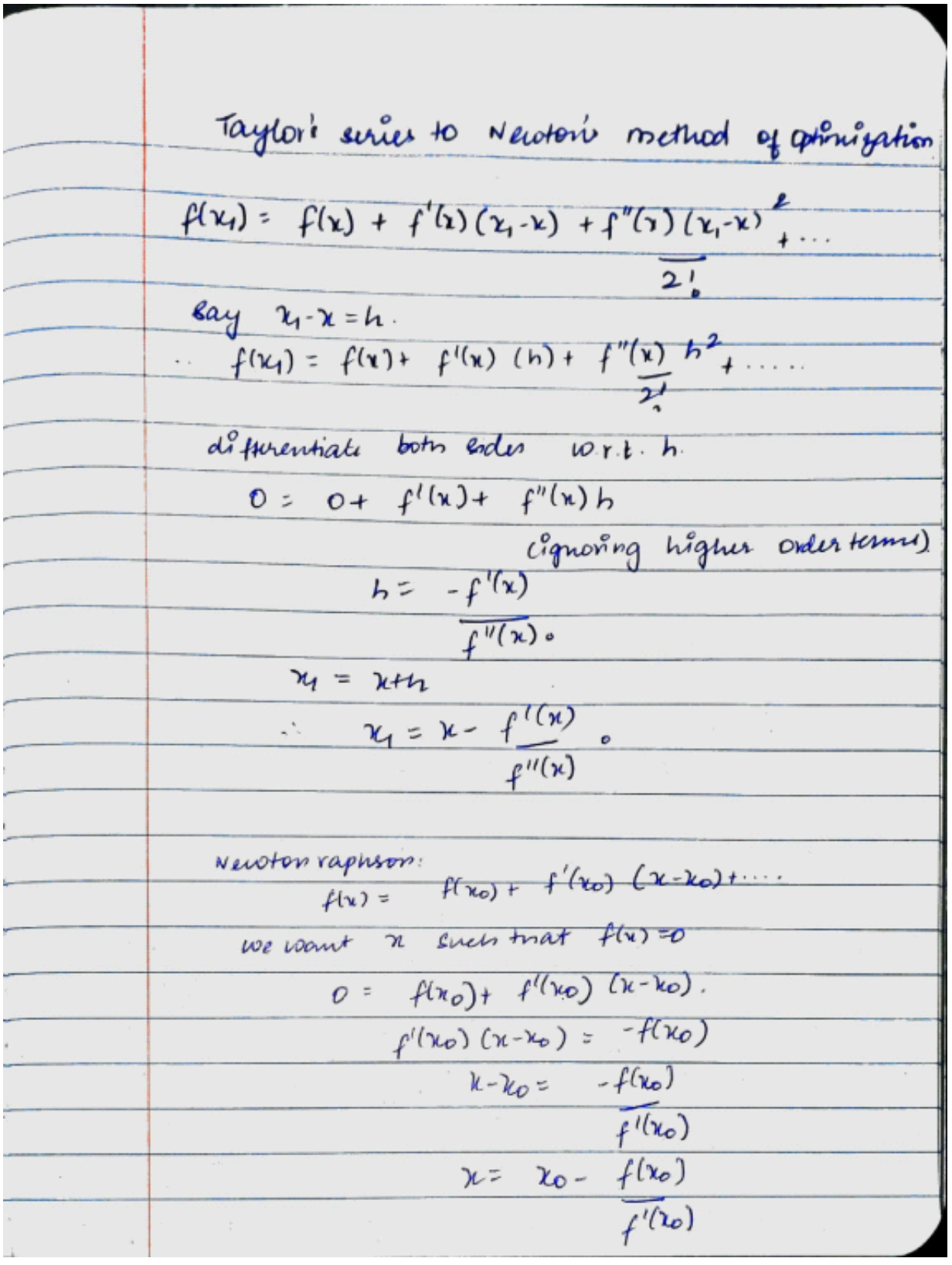

Taylor's series to Newton's method of optimization

$$f(x_1) = f(x) + f'(x)(x_1 - x) + \frac{f''(x)(x_1 - x)^2}{2!} + \dots$$

Say $x_1 - x = h$.

$$\therefore f(x_1) = f(x) + f'(x)(h) + \frac{f''(x)}{2!} h^2 + \dots$$

differentiate both sides w.r.t. h.

$$0 = 0 + f'(x) + f''(x) h$$

(ignoring higher order terms)

$$h = \frac{-f'(x)}{f''(x)}$$

$$x_1 = x + h$$

$$\therefore x_1 = x - \frac{f'(x)}{f''(x)}$$

Newton raphson:

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \dots$$

we want x such that $f(x) = 0$

$$0 = f(x_0) + f'(x_0)(x - x_0)$$

$$f'(x_0)(x - x_0) = -f(x_0)$$

$$x - x_0 = \frac{-f(x_0)}{f'(x_0)}$$

$$x = x_0 - \frac{f(x_0)}{f'(x_0)}$$