proof by contradiction.

Greedy Algorithm for scheduling.

say 2 jobs i, j with priority/weights w_i, w_j and time taken l_i & l_j respectively. so based on Greedy algorithm, we define a ratio w/l for every job. Higher the ratio, higher/earlier the job will be executed.

say $w_i/l_i > w_j/l_j$ so job i would be executed first.

$$\frac{w_i}{l_i} > \frac{w_j}{l_j} \rightarrow (1)$$

(we call it σ^* - Algo soln).

now lets arbitarily say a solution with j being executed first and i later is what we now call optimal or σ^*.

If we prove σ^* is not optimal, then we can say Algo soln - σ was better, and no better solution is available.

so by switching j & i.

the benefit is for job j.

$$\text{Benefit} = w_j \times l_i$$

cost is for job i

$$\text{cost} = w_i \times l_j$$

$$(1) \Rightarrow \frac{w_i}{l_i} > \frac{w_j}{l_j} \Rightarrow w_i l_j > w_j l_i$$

$$\text{cost} > \text{Benefit}$$

so the soln σ^* has more cost than Benefit so we contradict σ^*, and conclude that σ is the better solution.

QED.