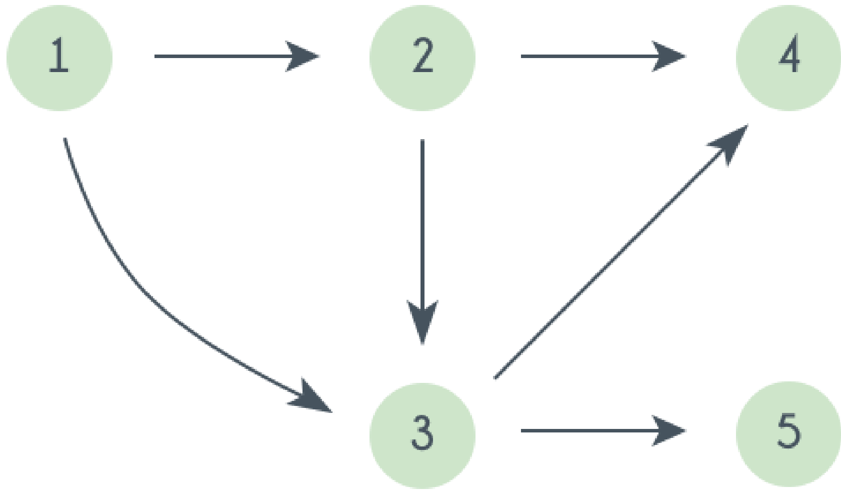
Algorithms

Topological Sort

Topological sorting of vertices of a **Directed Acyclic Graph** is an ordering of the vertices $v_1, v_2, \ldots v_n$ in such a way, that if there is an edge directed towards vertex v_j from vertex v_i, then v_i comes before v_j. For example consider the graph given below:



A topological sorting of this graph is: $1\ 2\ 3\ 4\ 5$
There are multiple topological sorting possible for a graph. For the graph given above one another topological sorting is: $1\ 2\ 3\ 5\ 4$
In order to have a topological sorting the graph must not contain any cycles. In order to prove it, let's assume there is a cycle made of the vertices $v_1, v_2, v_3 \ldots v_n$. That means there is a directed edge between v_i and v_{i+1} $(1 \leq i < n)$ and between v_n and v_1. So now, if we do topological sorting then v_n must come before v_1 because of the directed edge from v_n to v_1. Clearly, v_{i+1} will come after v_i, because of the directed from v_i to v_{i+1}, that means v_1 must come before v_n. Well, clearly we've reached a contradiction, here. So topological sorting can be achieved for only directed and acyclic graphs.

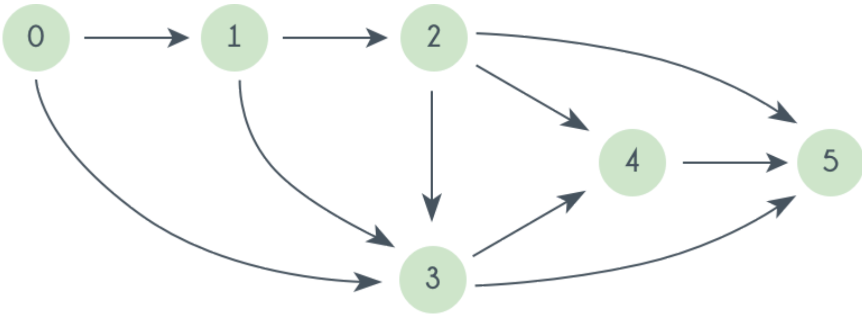
Le'ts see how we can find a topological sorting in a graph. So basically we want to find a permutation of the vertices in which for every vertex v_i, all the vertices v_j having edges coming out and directed towards v_i comes before v_i. We'll maintain an array T that will denote our topological sorting. So, let's say for a graph having N vertices, we have an array $in\_degree[]$ of size N whose i^{th} element tells the number of vertices which are not already inserted in T and there is an edge from them incident on vertex numbered i. We'll append vertices v_i to the array T, and when we do that we'll decrease the value of in_degree

by 1 for every edge from v_i to v_j. Doing this will mean that we have inserted one vertex having edge directed towards v_j. So at any point we can insert only those vertices for which the value of $in\_degree[]$ is 0. The algorithm using a BFS traversal is given below:

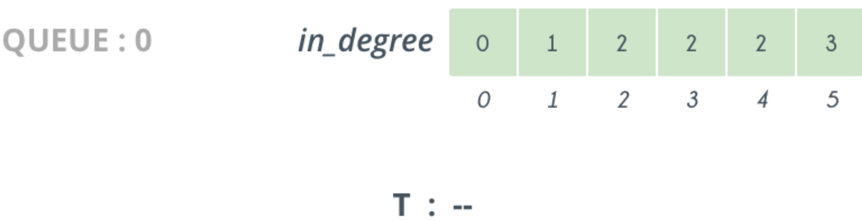

```
topological_sort(N, adj[N][N])
        T = []
        visited = []
        in_degree = []
        for i = 0 to N
                in_degree[i] = visited[i] = 0

        for i = 0 to N
                for j = 0 to N
                        if adj[i][j] is TRUE
                                in_degree[j] =
in_degree[j] + 1

        for i = 0 to N
                if in_degree[i] is 0
                        enqueue(Queue, i)
                        visited[i] = TRUE

        while Queue is not Empty
                vertex = get_front(Queue)
                dequeue(Queue)
                T.append(vertex)
                for j = 0 to N
                        if adj[vertex][j] is TRUE and
visited[j] is FALSE
                                in_degree[j] =
in_degree[j] - 1
                                if in_degree[j] is 0
                                        enqueue(Queue,
j)
                                        visited[j] =
```

```
TRUE
        return T
```

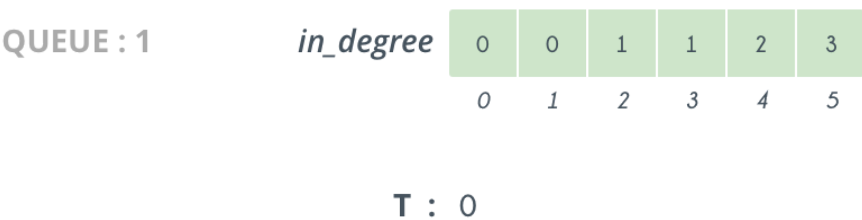
Let's take a graph and see the algorithm in action. Consider the graph given below:



Initially $in\_degree[0] = 0$ and T is empty



So, we delete 0 from $Queue$ and append it to T. The vertices directly connected to 0 are 1 and 2 so we decrease their $in\_degree[]$ by 1. So, now $in\_degree[1] = 0$ and so 1 is pushed in $Queue$.



Next we delete 1 from $Queue$ and append it to T. Doing this we decrease $in\_degree[2]$ by 1, and now it becomes 0 and 2 is pushed

Queue.

QUEUE : 2

in_degree

0	1	2	3	4	5
0	0	0	1	1	3

T : 0 1

So, we continue doing like this, and further iterations looks like as follows:

QUEUE : 3

	0	1	2	3	4	5
in_degree	0	0	0	0	1	2

T : 0 1 2

$\downarrow$

QUEUE : 4

	0	1	2	3	4	5
in_degree	0	0	0	0	0	1

T : 0 1 2 3

$\downarrow$

QUEUE : 5

	0	1	2	3	4	5
in_degree	0	0	0	0	0	0

T : 0 1 2 3 4

$\downarrow$

QUEUE : --

	0	1	2	3	4	5
in_degree	0	0	0	0	0	0

T : 0 1 2 3 4 5

So at last we get our Topological sorting in T i.e. : $0, 1, 2, 3, 4, 5$

Solution using a DFS traversal, unlike the one using BFS, does not need any special $in\_degree[]$ array. Following is the pseudo code of the DFS solution:

```
T = []
visited = []

topological_sort( cur_vert, N, adj[][] ){
    visited[cur_vert] = true
    for i = 0 to N
        if adj[cur_vert][i] is true and visited[i] is
false
        topological_sort(i)
    T.insert_in_beginning(cur_vert)
}
```

The following image of shows the state of stack and of array T in the above code for the same graph shown above.

Stack	T
0	- -
0 1	- -
0 1 2	- -
0 1 2 4	- -
0 1 2 4 5	- -
0 1 2 4	*5*
0 1 2	*4 5*
0 1 2 3	*4 5*
0 1 2	*3 4 5*
0 1	*2 3 4 5*
0	*1 2 3 4 5*
- -	*0 1 2 3 4*

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