Chapter 2 – Basic Circuit Elements and Circuit Analysis

Section 2: Circuit Analysis

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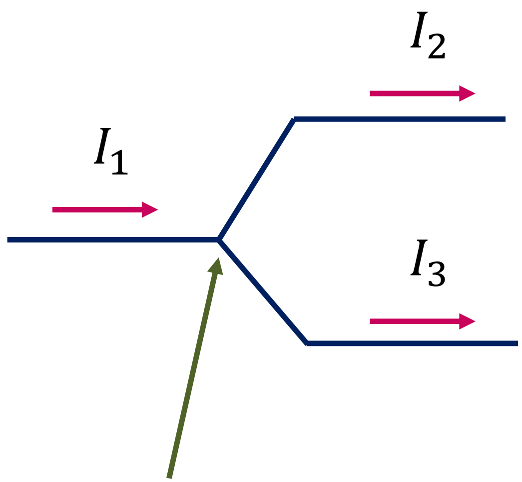
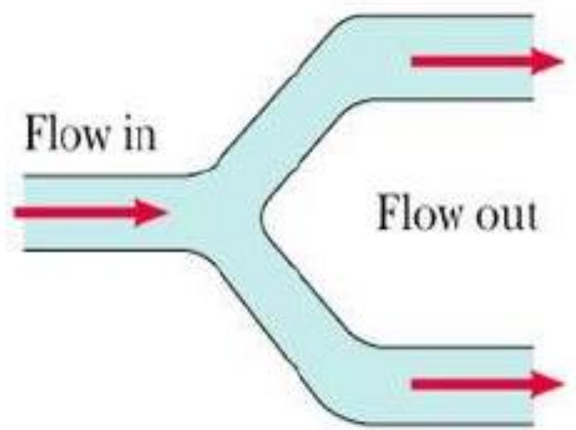
Kirchhoff Laws

- Gustav Kirchhoff developed two fundamental laws which are used as a basis for analysis of all the circuits.
- The first law is based on the "Principle of conservation of charge".



A node is a point at which two or more circuit elements are connected

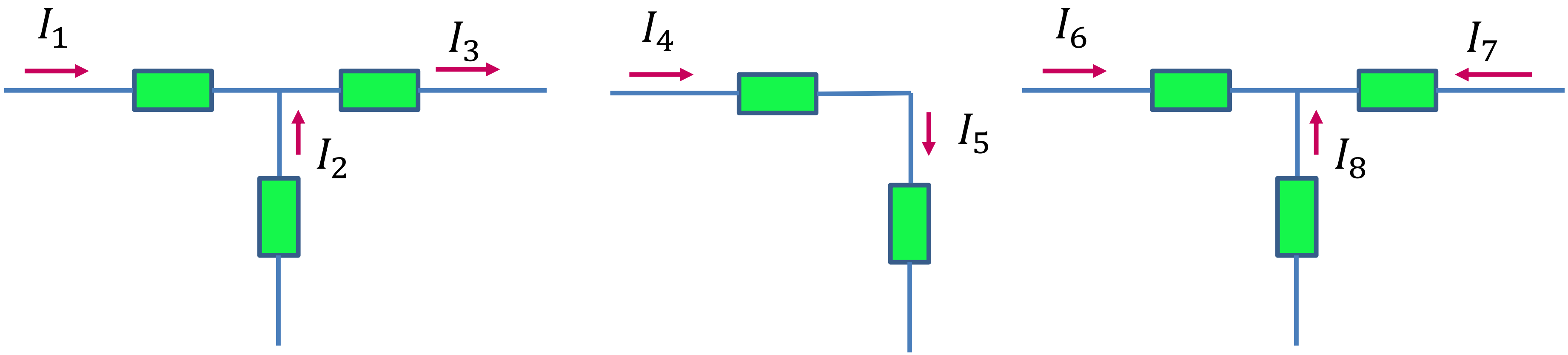


- What is the relationship between I_1, I_2, and I_3?

Kirchhoff Current Law (KCL)

- KCL states that the algebraic sum of all the currents entering and leaving a node is zero.

$$\sum_{in} I = \sum_{out} I \quad \rightarrow \quad \sum_{node} I = 0$$



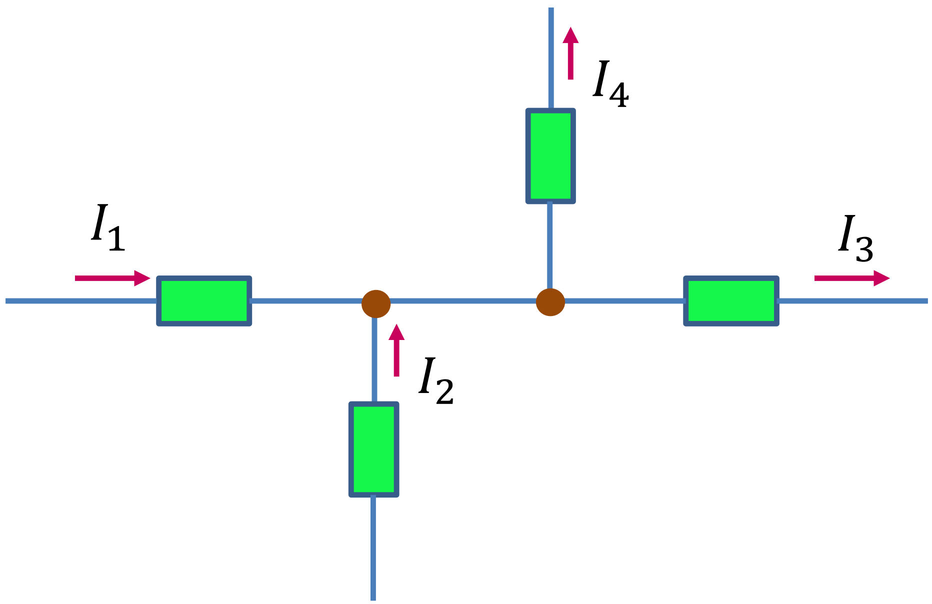
$$I_1 + I_2 + (-I_3) = 0$$
$$I_1 + I_2 = I_3$$

$$I_4 + (-I_5) = 0$$
$$I_4 = I_5$$

$$I_6 + I_7 + I_8 = 0$$

Kirchhoff Current Law (KCL)

- Example:

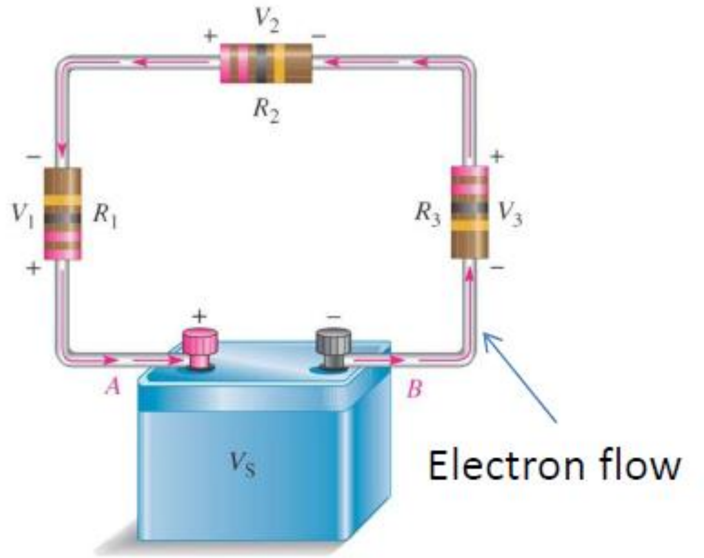


$$I_1 + I_2 + (-I_3) + (-I_4) = 0$$
$$I_1 + I_2 = I_3 + I_4$$

- **All points in a circuit that are connected directly by ideal conductors can be considered to be a single node.**

Kirchhoff Laws

- Case study: three resistors connected in series across a voltage source.



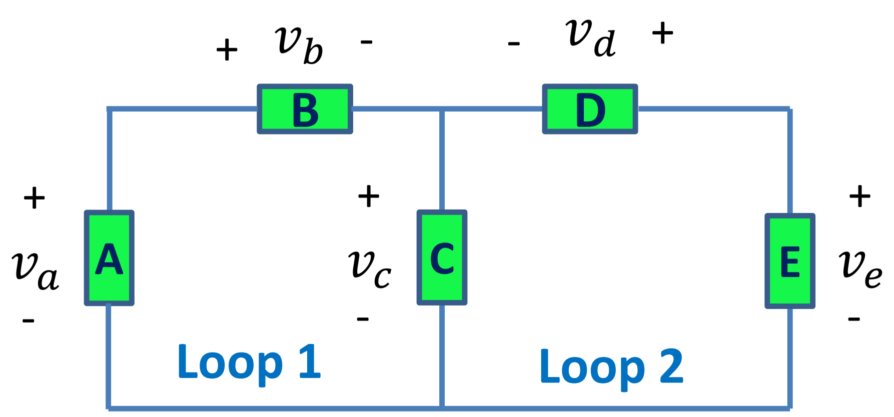
- When electrons flow through a resistor, they lose energy and are therefore at a lower energy level when they emerge.
- This drop in energy level creates a potential difference or voltage drop.
- Due to conservation of energy, the voltage from point A to point B (the source voltage) must be the same as the sum of the series resistor voltage drops.

$$V_s = V_1 + V_2 + V_3$$

Kirchhoff Voltage Law (KVL)

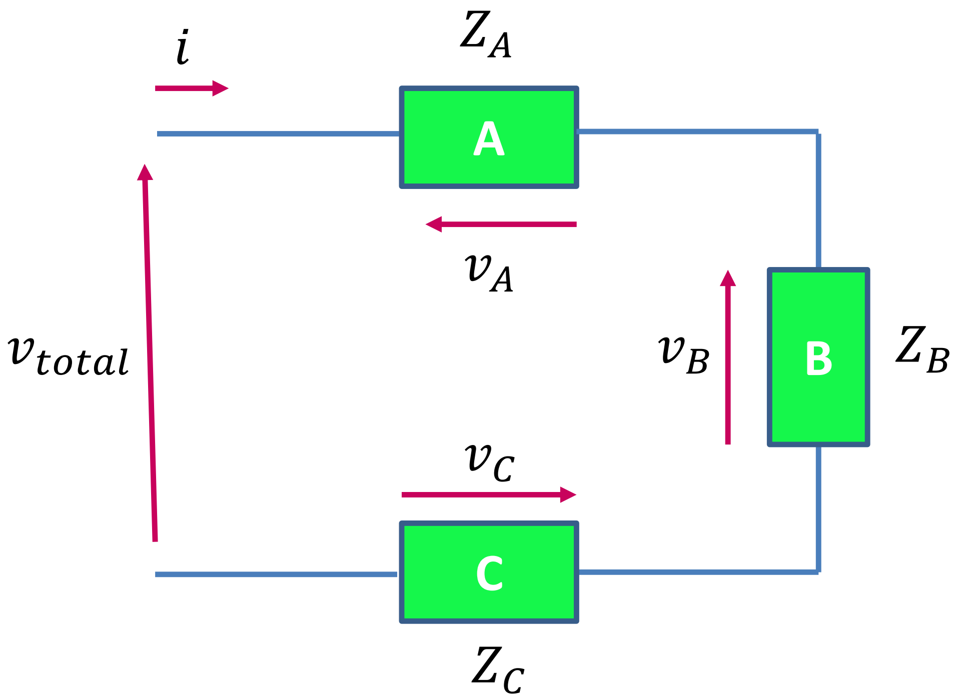
- Algebraic sum of voltages around any closed path (loop) in an electrical circuit equals zero.

$$\sum_{loop} \boldsymbol{V} = \mathbf{0}$$



$$Loop\ 1 \ \rightarrow \ \boldsymbol{v_a + (-v_b) + (-v_c) = 0} \ \rightarrow \ \boldsymbol{v_a = v_b + v_c}$$
$$Loop\ 2 \ \rightarrow \ \boldsymbol{v_c + v_d + (-v_e) = 0} \ \rightarrow \ \boldsymbol{v_e = v_c + v_d}$$

Voltage division principle



$$v_{total} = v_A + v_B + v_C$$

$$v_A = Z_A i, \qquad v_B = Z_B i, \qquad v_C = Z_C i$$

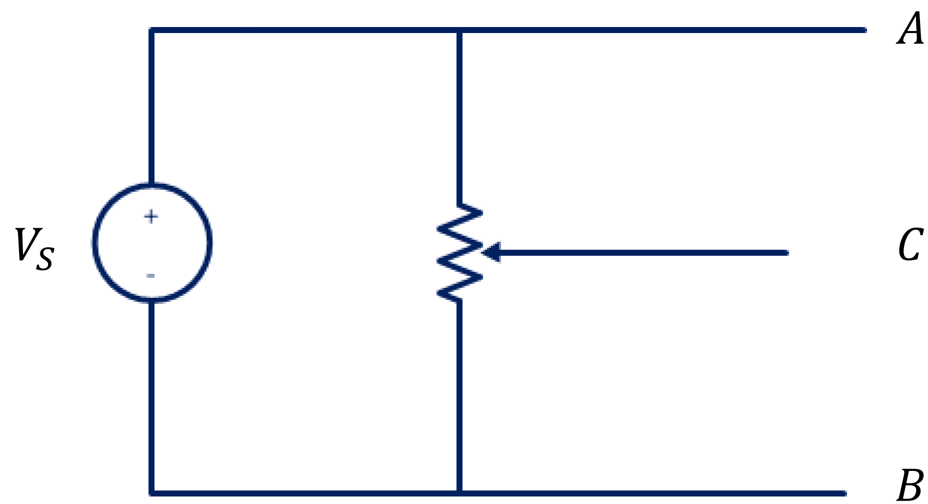
$$i = \frac{v_{total}}{Z_A + Z_B + Z_C}$$

$$\boldsymbol{v_A = \frac{Z_A}{Z_A + Z_B + Z_C} v_{total}, \qquad v_B = \frac{Z_B}{Z_A + Z_B + Z_C} v_{total},}$$

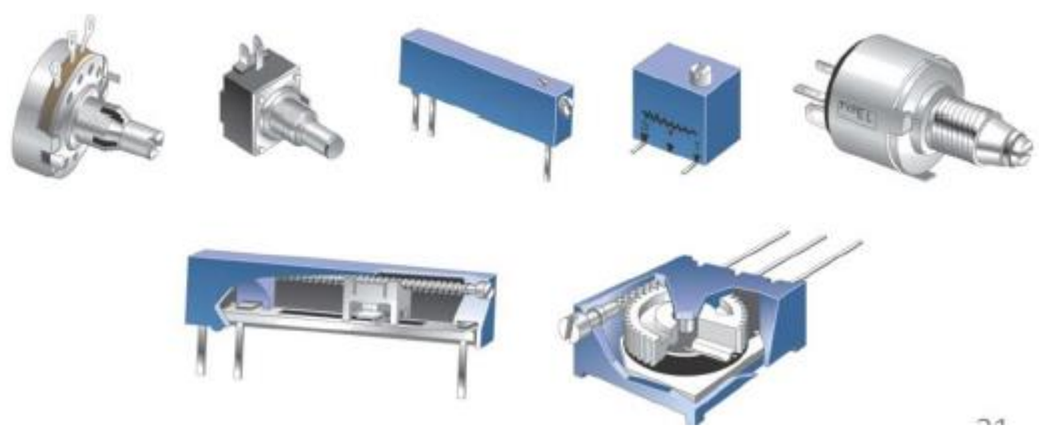
$$\boldsymbol{v_C = \frac{Z_C}{Z_A + Z_B + Z_C} v_{total}}$$

Voltage divider circuits

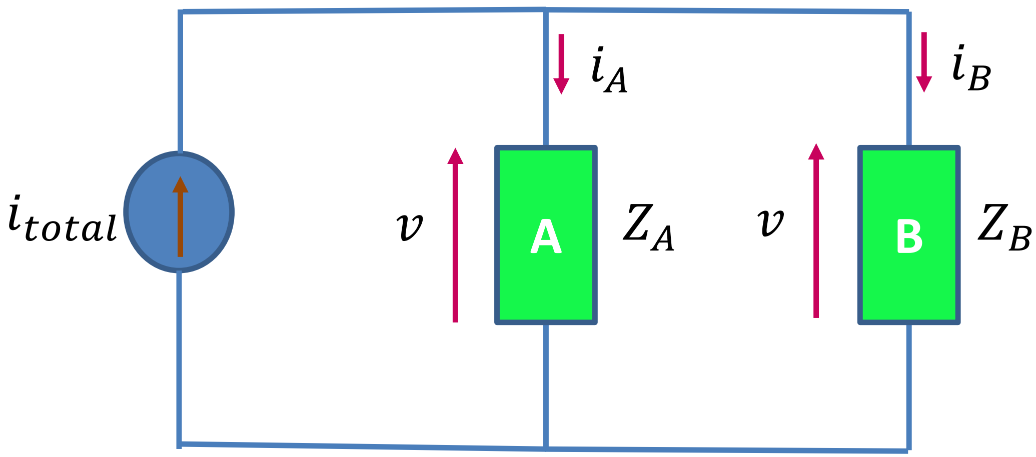
- Voltage division principle is very useful as it can be used to produce a desired (or variable) output from a fixed supply.
- Potentiometers (a form of variable resistor) are commonly used for this purpose.



$$V_{BC} = \frac{R_{var}}{R_{Total}} \times V_S$$



Current division principle



$$i_{total} = i_A + i_A$$

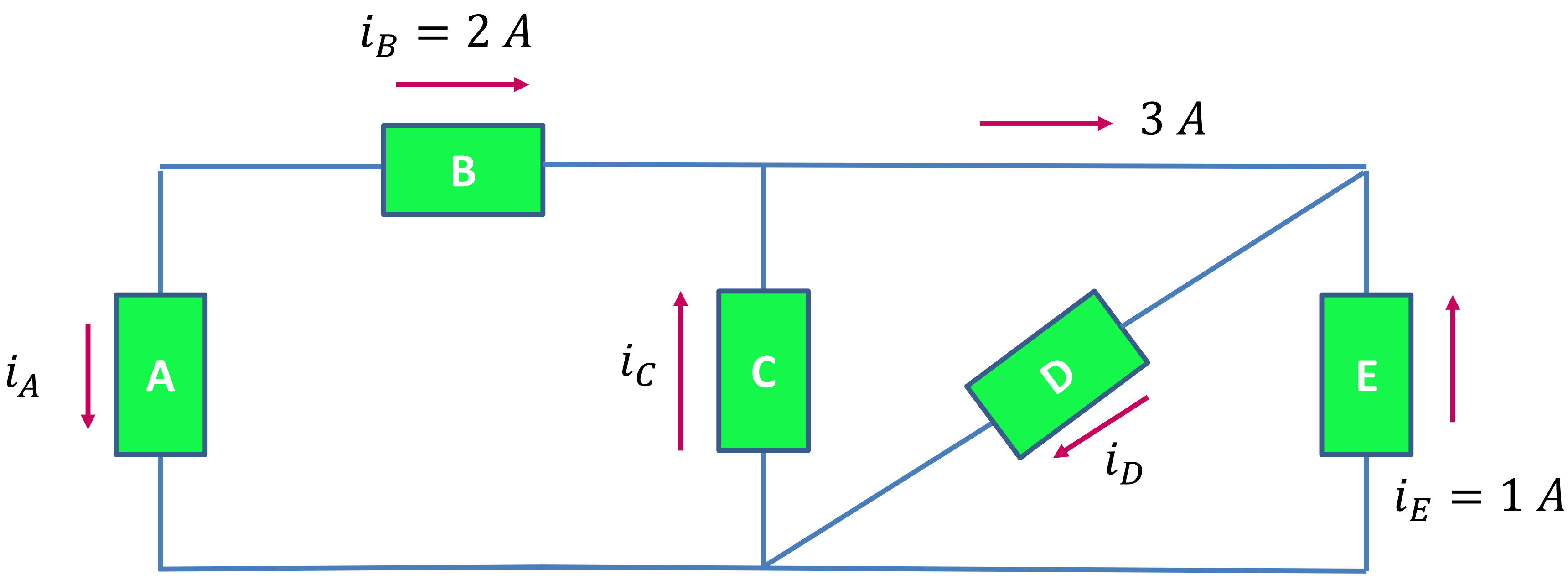
$$v = Z_{eq} i_{total} = \frac{Z_A Z_B}{Z_A + Z_B} i_{total}$$

$$\boldsymbol{i_A = \frac{v}{Z_A} = \frac{Z_B}{Z_A + Z_B} i_{total}}$$

$$\boldsymbol{i_B = \frac{v}{Z_A} = \frac{Z_A}{Z_A + Z_B} i_{total}}$$

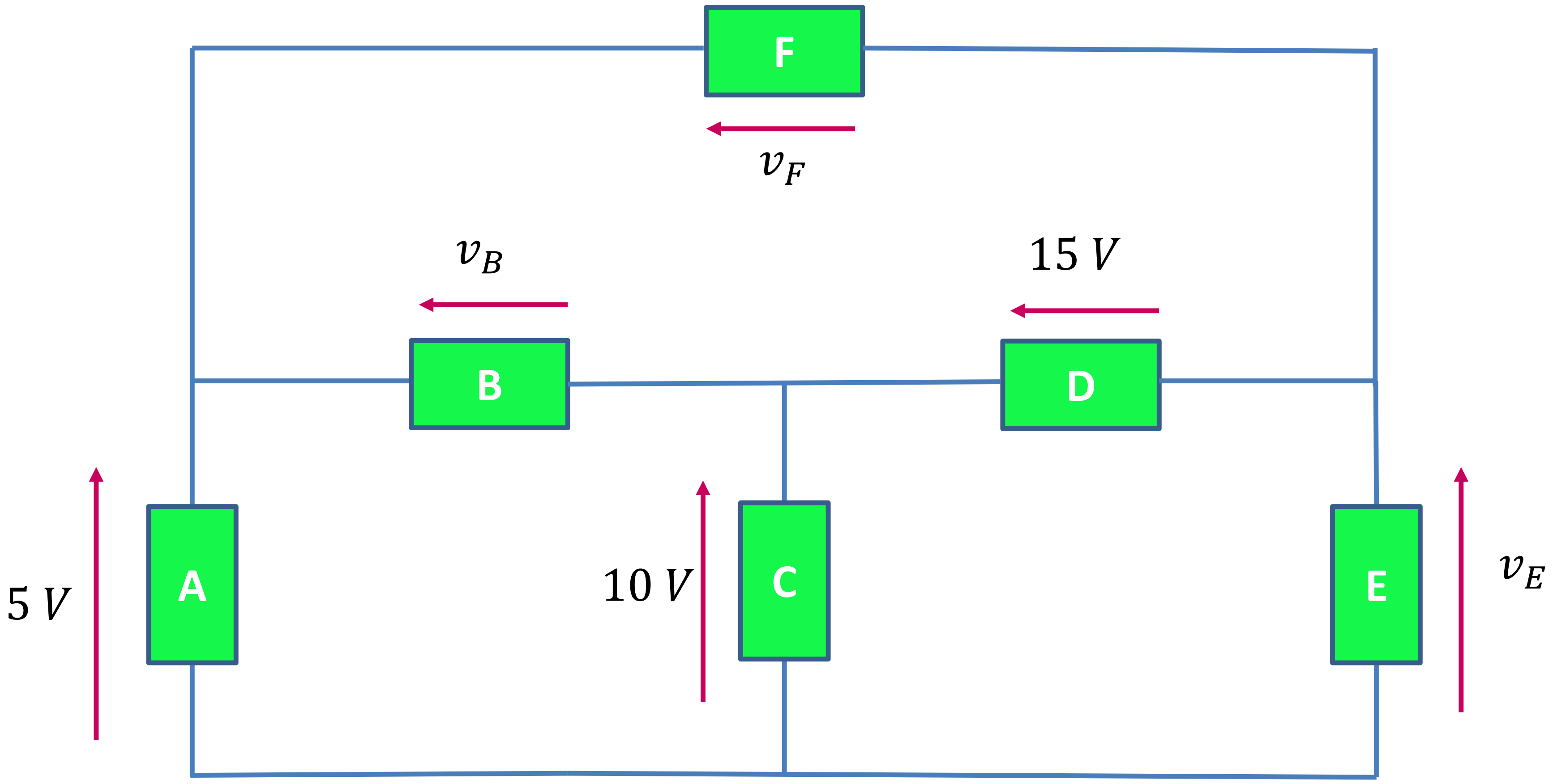
Example

1) Use KCL to find unknown currents in the circuit given below.



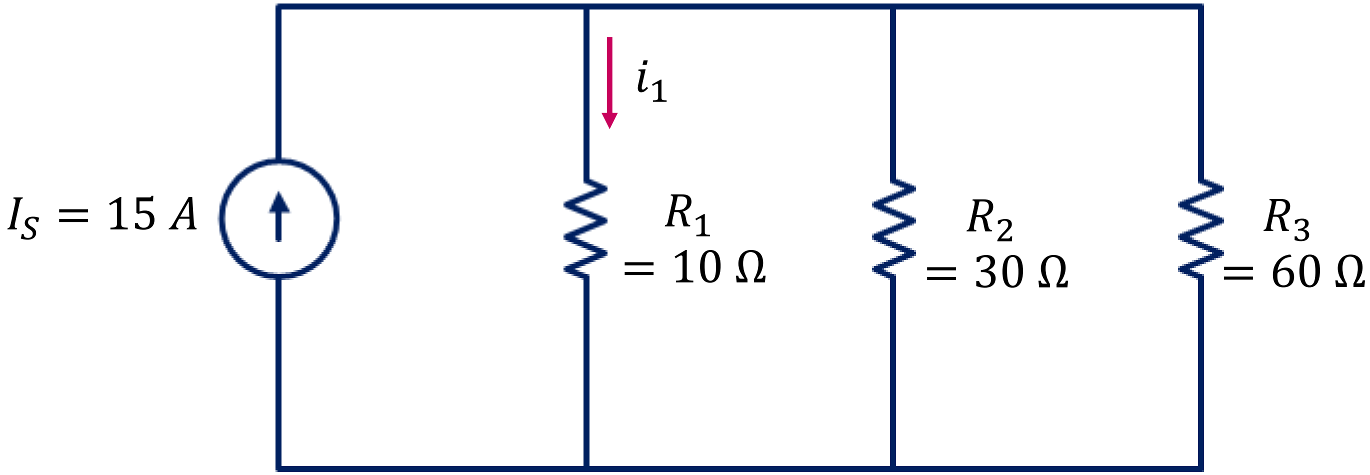
Example

2) Use KVL to find unknown voltages in the circuit given below.



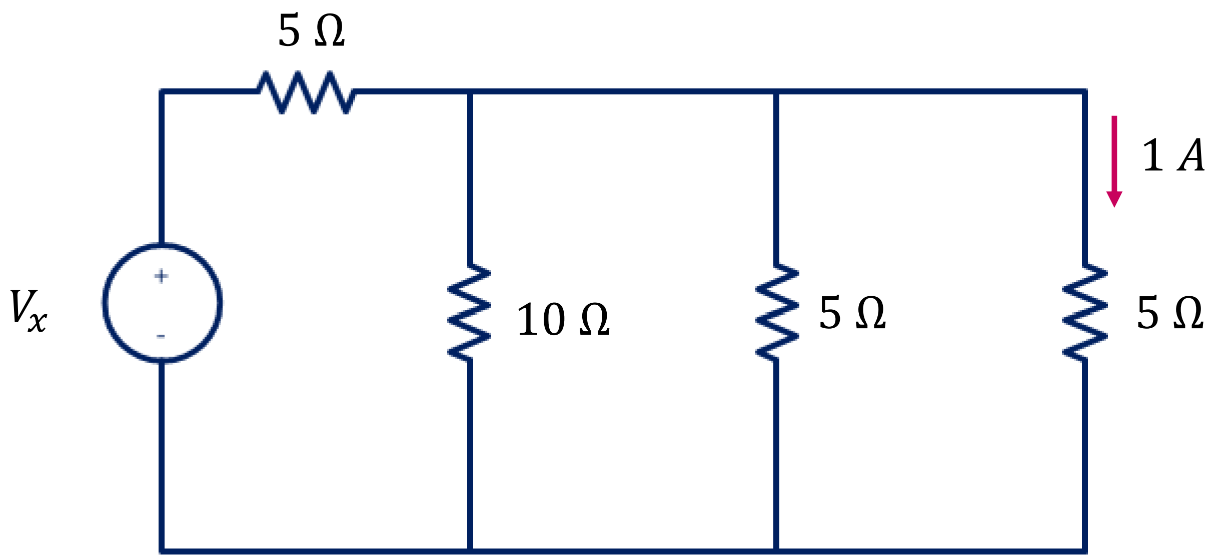
Example

3) Find the current i_1 in the circuit given below.



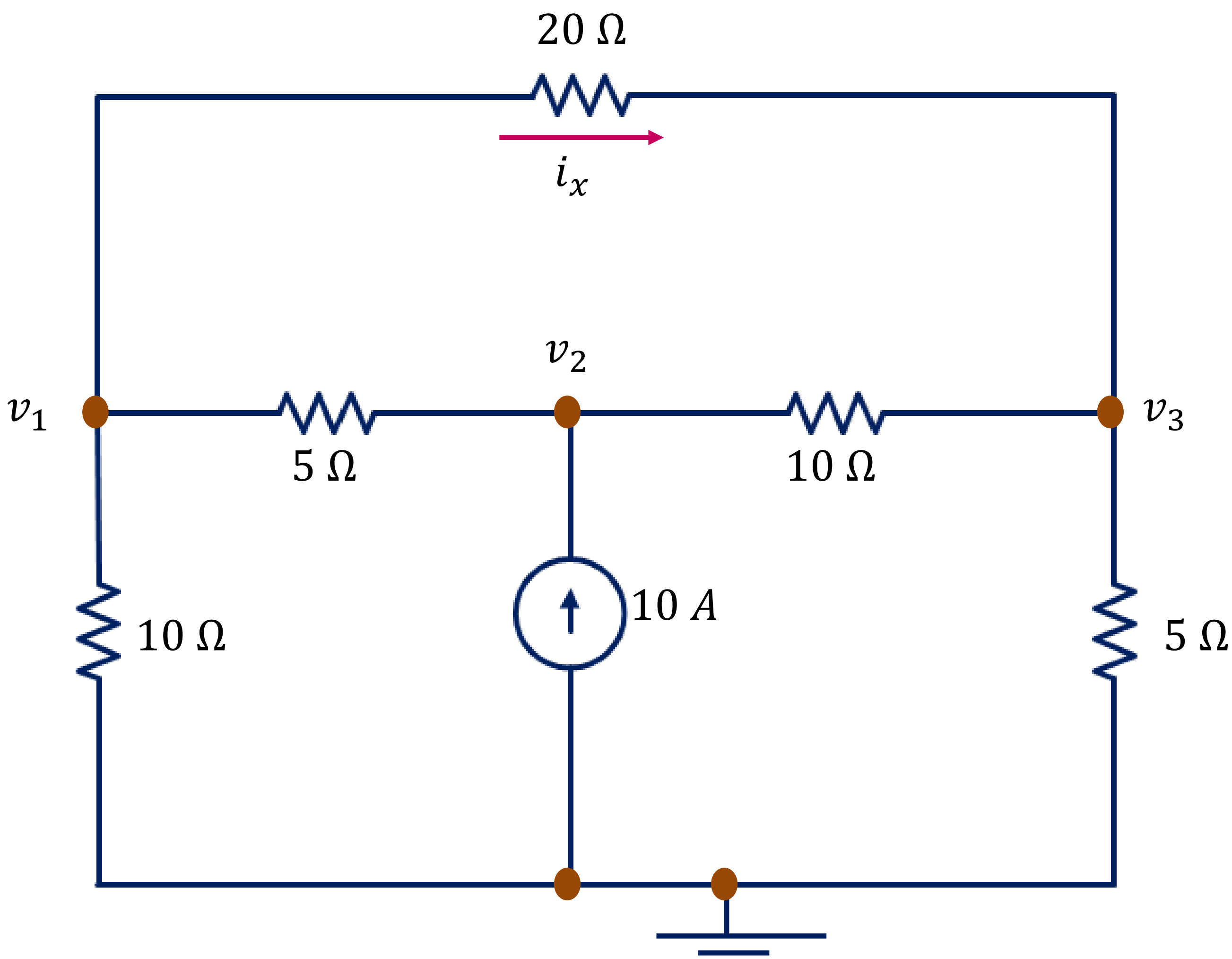
Example

4) Find the voltage V_x in the circuit given below.



Example

5) Solve for the node voltages in the circuit below to calculate the current i_x.



Example 5 solution

$$\frac{0-v_1}{10}=\frac{v_1-v_3}{20}+\frac{v_1-v_2}{5} \quad \rightarrow \mathbf{7v_1-4v_2-v_3=0}$$

$$10+\frac{v_1-v_2}{5}=\frac{v_2-v_3}{10} \quad \rightarrow \mathbf{-2v_1+3v_2-v_3=100}$$

$$\frac{v_2-v_3}{10}+\frac{0-v_3}{5}+\frac{v_1-v_3}{20}=0 \quad \rightarrow \mathbf{v_1+2v_2-7v_3=0}$$

$$\begin{bmatrix} 7 & -4 & -1 \\ -2 & 3 & -1 \\ 1 & 2 & -7 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 100 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}=\begin{bmatrix} 45.45 \\ 72.72 \\ 27.27 \end{bmatrix} V$$

$$i_x=\frac{v_1-v_3}{20}=\frac{45.45-27.27}{20}$$

$$i_x=0.91\ A$$