Chapter 2 – Basic Circuit Elements and Circuit Analysis

W. D. Prasad

What is Electricity?

(a) A quantity of either positive or negative charge.

(b) A flow of charged particles.

(c) An energy field produced by the motion of charged particles.

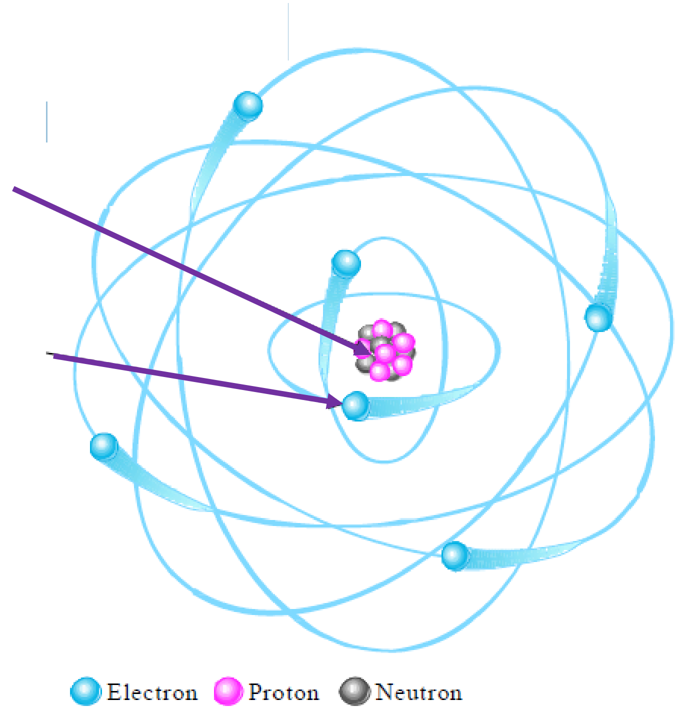
(d) The force of attraction or repulsion between the charged particles.

Charge

- The behaviour of electric/electronic components and circuits is determined by the motion of charge through the component/circuit.
- Electron is the fundamental charge carrier in electronics – the motion of electrons drives the functions and properties of all electronic components.
- The Bhor atom is useful in visualizing the atomic structure and understanding the charge.

Nucleus consists of protons and neutrons

Negatively charged electrons are in discrete shells orbiting the nucleus



Charge

- Two kinds of charge: positive charge and negative charge.
- Charge is conserved.
- Charge is quantized.

$$\boldsymbol{Q = N \times \left(1.602 \times 10^{-19}\right)}$$

Q = total charge

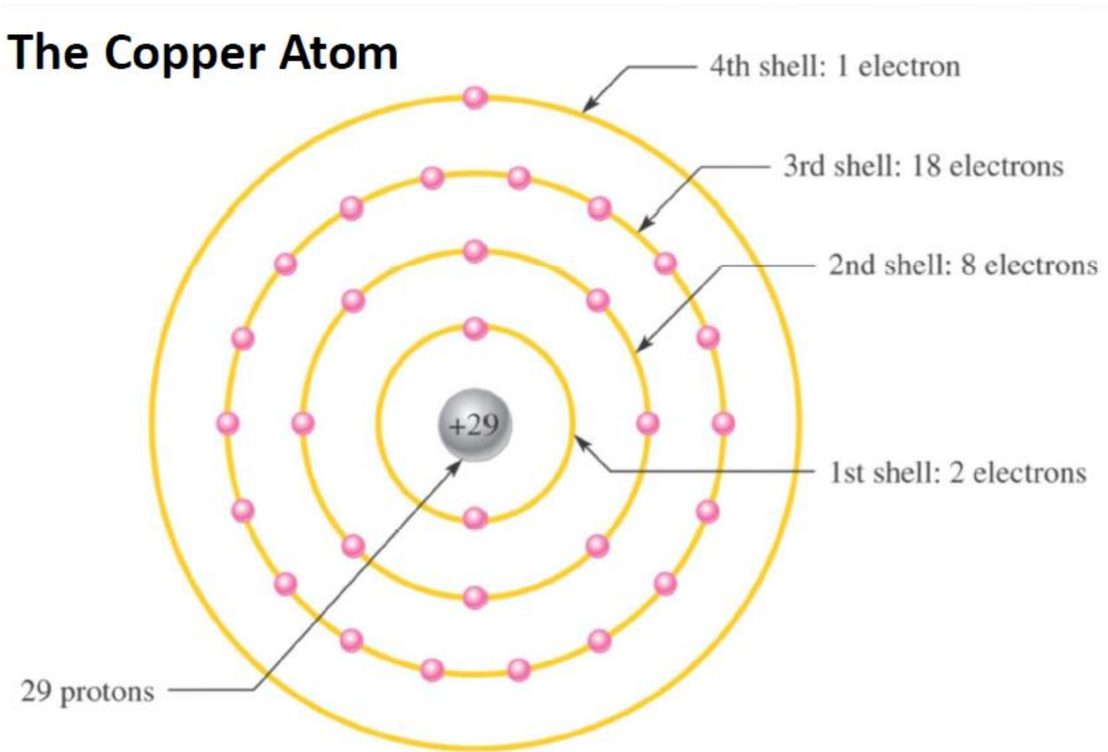
N = Number of electrons

1.602×10^{-19} is the charge per electron

- The SI unit of charge is **Coulomb [C]**.
- One Coulomb of charge corresponds to 6.25×10^{18} number of electrons.
- Time invariant charge is usually referred to using the symbol Q, and the charge that may vary with time is usually referred to using the symbol q.

Charge conduction

- The atomic structure of a material has a significant impact on the ability of electric charge to move through it.
- There are three categories of materials used in electrical and electronic circuits:
    - Conductors, insulators and semi-conductors

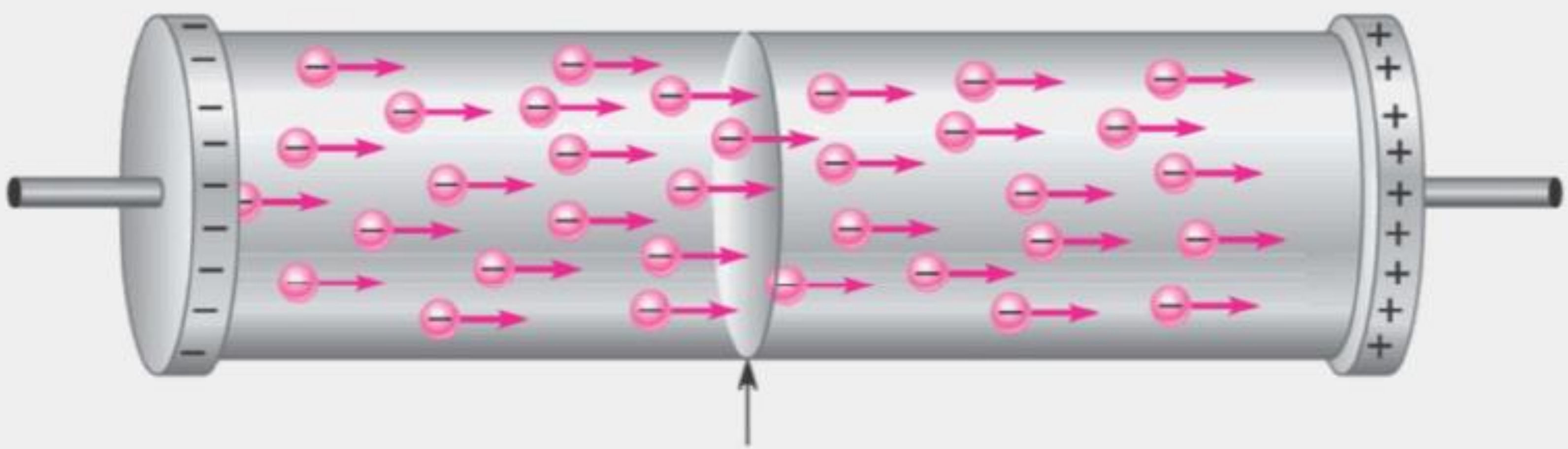


Current

- An electric current is a flow of charge in a conductor.
- Current (I) is defined as the amount of charge (Q) in Coulomb that flows past a point in a unit time (t) in second.

$$I = \frac{Q}{t}$$

- The SI unit of current is **Ampere (A)**.



When a number of electrons having a total charge of 1 C pass through a cross-sectional area in 1 s, there is 1 A of current.

Current

- By convention, current flows from more positive to more negative potentials. This is known as conventional current.
- Electron flow is in the opposite direction of the conventional current.
- Time invariant current (direct current or DC) is referred to using the symbol $\boldsymbol{I}$, current that varies with time (alternating current or AC) is usually referred to using the symbol $\boldsymbol{i}$.

- **Example:**

(1) If 300 C of charge flows through a minute, what is the current?

$$I = \frac{Q}{t} = \frac{300\ (C)}{60\ (s)} = 5\ A$$

(2) If the current was 1.5 A, how much charge through in 10 seconds?

$$Q = It = 1.5 \times 10 = 15\ C$$

Voltage

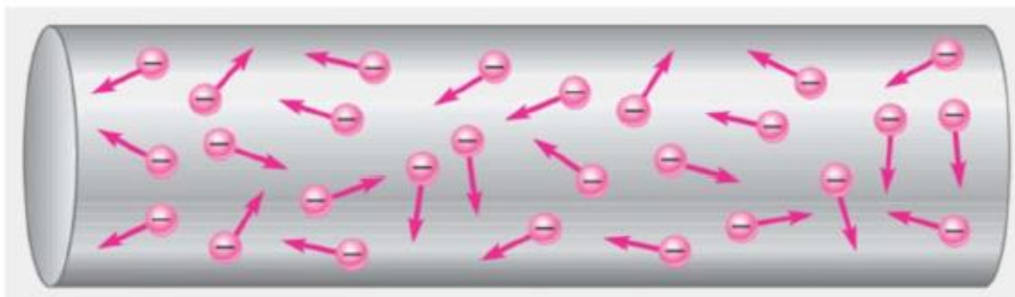
- Force is required to move a charge against the electric field (the electric field is a result of the forces of attraction or repulsion between charges).
- A difference of electrical potential, or potential difference, is defined by the work (W) which must be done to move a unit electric charge from one point to another.
- Potential difference is more commonly referred to as voltage.
- The SI unit for voltage (potential difference) is the **Volt (V)**.
- One volt is the potential difference between two points when one joule of energy is used to move one coulomb of charge from one point to the other.

$$\boldsymbol{V = \frac{E}{Q}}$$

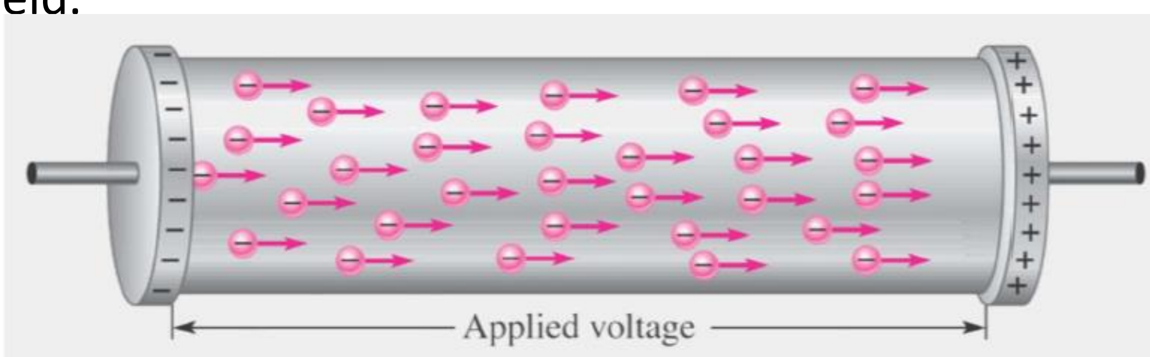
- Time invariant voltage is referred to using the symbol V, voltage that varies with time is referred to using the symbol $\boldsymbol{v}$.
- In electrical circuits, one point is always nominated as a reference point (zero potential), called a common or ground point or simply ground.
- Normally a voltage (potential difference) must be referenced between two points. $V_{AB} = V_A - V_B$

Electron motion and current flow

- Consider a conductor of length L and cross-sectional area A. Electrons are the charge carriers.
- If there is no applied potential, the electrons will exhibit a degree of random motion due to thermal energy, but the average electron velocity will be zero. The electrons will be “localized”.

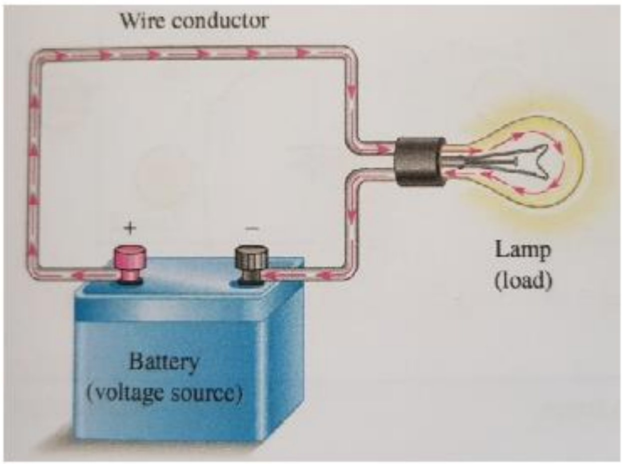


- If a potential V is applied along the length of the conductor, the potential sets up an electric field.
- Electrons placed in this electric field will experience a force opposite direction to the electric field.



Electrical circuits

- An electrical circuit essentially consists of different circuit elements connected in closed paths by wires.



Pictorial diagram

Schematic diagram

- An electrical circuit can be;
  - Closed circuit
  - Open circuit
  - Short circuit

Energy and power

- Energy $(E\ or\ W)$ is used to do the work. Power (P) is the rate at which the work is done (work done in unit time).

$$p = \frac{dw}{dt}$$

$$p = \frac{dw}{dq}\frac{dq}{dt} = vi$$

$$\boldsymbol{p = vi}$$

$$\boldsymbol{w = \int_{t_0}^{t} p dt = \int_{t_0}^{t} (vi) dt}$$

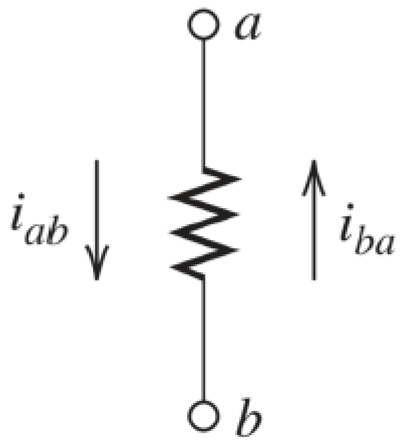
- Unit of Energy is **Joules (J)**. Unit of power is **Joules per second or Watt (W).**
- Assume a constant voltage *V* applied across a resistor *R* gives current *I* in a time period of *t* seconds.

$$Power\ (P) = VI = I^2R = \frac{V^2}{R}$$

$$Energy\ (E) = VIt = Pt$$

Double subscript notation

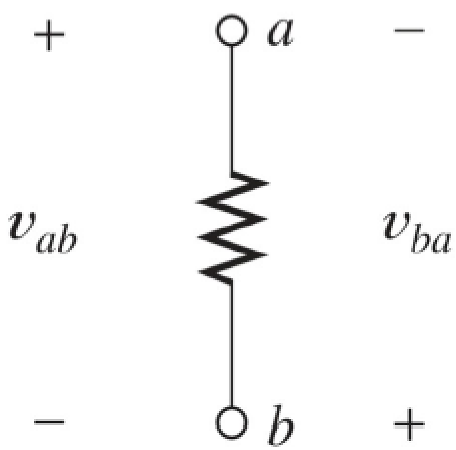
- **Current**



i_{ab} indicates current flowing from $a \rightarrow b$

$$i_{ab} = -i_{ba}$$

- **Voltage**

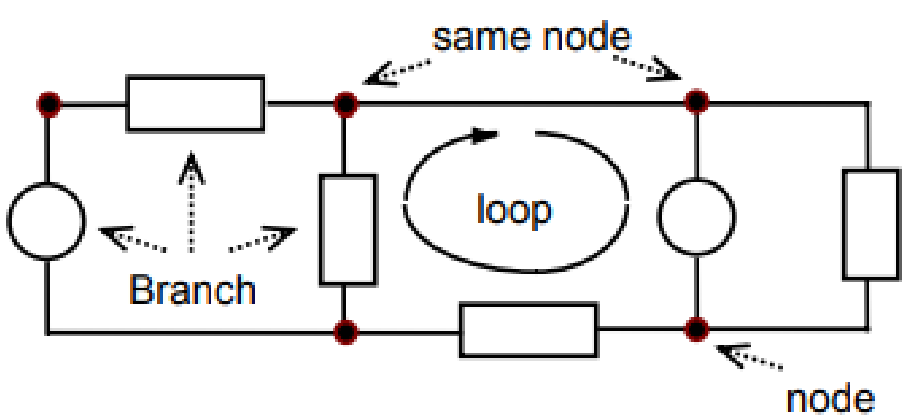


v_{ab} indicates a voltage with a more positive polarity at point a

$$v_{ab} = -v_{ba}$$

Branch, node, loop, mesh in a circuit

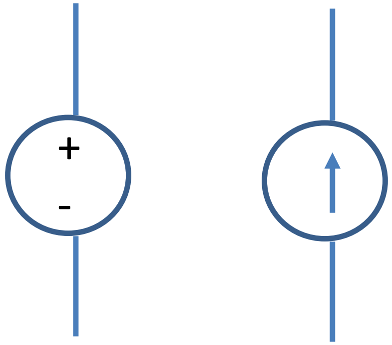
- **Branch**
  A branch represents a single element, such as a resistor or a battery. A branch is a two terminal element.
- **Node**
  A node is the point connecting two or more branches. Usually a dot is used to represent a node.
- **Loop**
  A loop is a closed path through a circuit in which no node is encountered more than once except for the same start/finish node.
- **Mesh**
  A mesh is a loop without having other loops inside it.



Circuit elements

- Electric circuits consist of two basic types of elements.
  - Active elements
  - Passive elements

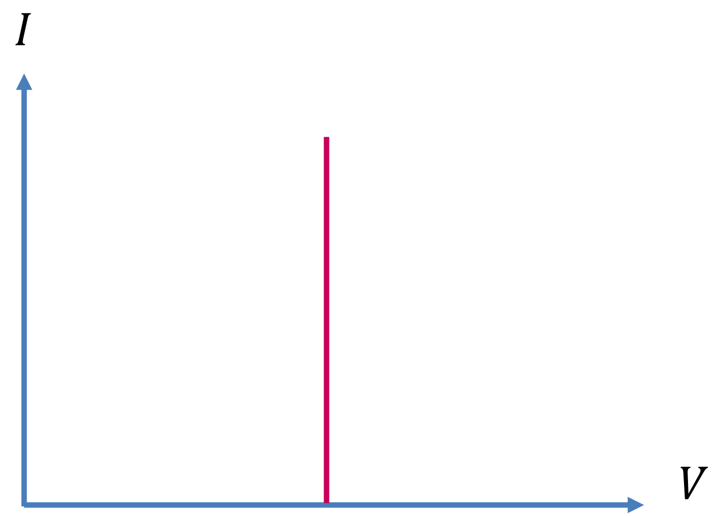
- An active circuit element is capable of generating electrical energy.
  - Voltage sources
  - Current sources



- A passive circuit element does not generate electrical energy. It either consumes electrical energy or stores it.
  - Resistors
  - Inductors
  - Capacitors

Voltage sources

- Sources of voltage:
  - Batteries (electrochemical)
  - Solar cells (photo voltaic)
  - Generators (electromagnetic)
- Ideally, a voltage source can provide a constant voltage for any current required by a circuit.
- The current vs voltage curve for an ideal voltage source has a constant voltage for all current:



- In practice, ideal sources do not exist, but they can be closely approximated by actual sources.

Resistors

- Resistance (R) of a conductor in terms of the physical dimensions is given below:

$$\boldsymbol{R = \frac{\rho l}{A}}$$

$$\rho = resistivity, l = lenght, A = cross - sectional\ area$$

- The SI unit for resistance is the **Ohm (Ω)**.

- If a voltage V is applied across a conductor, then a given current I will flow through the conductor $(V \propto I)$ – the proportionality constant is called resistance $(\boldsymbol{R})$.

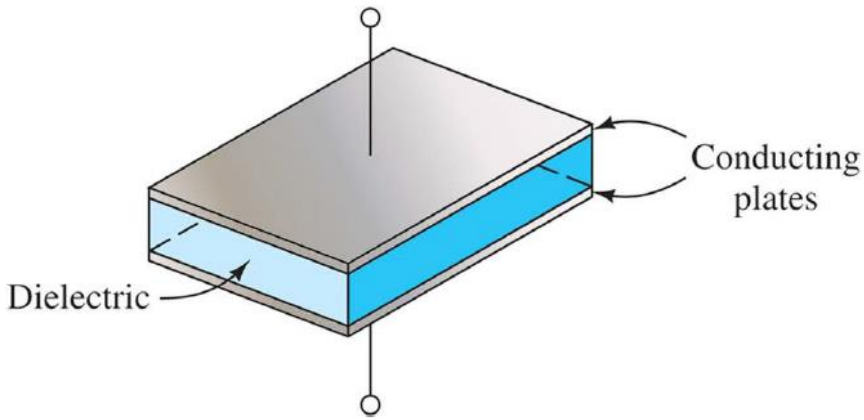
$$\boldsymbol{R = \frac{V}{I} = \frac{Voltage\ across\ the\ element}{Current\ passing\ through\ the\ element}}$$

- This resistance to current flow results from scattering of the electrons through collisions with atoms and impurities as they move through the conductor.

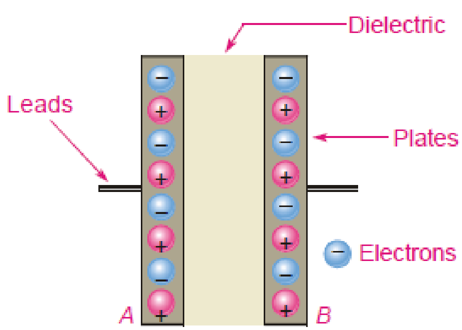
- A resistor is sized for the anticipated power.

Capacitors

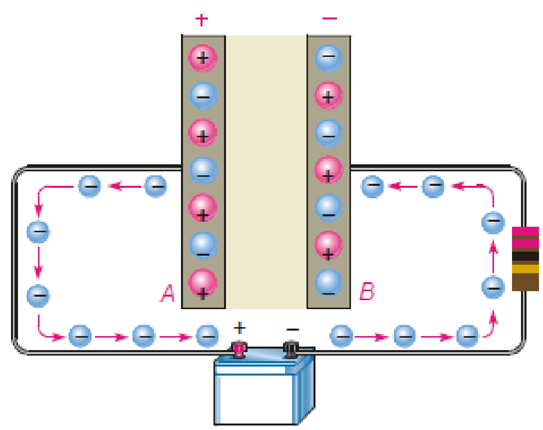
- In its most basic form, a capacitor comprises two conductive plates separated by an insulating (dielectric) layer.



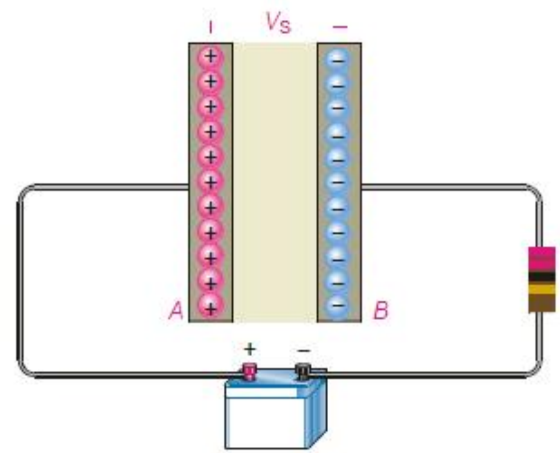
$$C = \frac{\varepsilon A}{d}$$



Initially uncharged



Charging



Fully charged

Equations describing the capacitor operation

- In an ideal capacitor, the charge imbalance Q is proportional to the voltage V across the plates.

$$\boldsymbol{Q = CV}$$

  C is the capacitance.
- Unit of capacitance is Farad (F).
- The capacitance C is a fixed constant. Therefore, the current i passing through the capacitor and voltage v across the capacitor are related as follows.

$$q = CV \rightarrow \frac{dq}{dt} = C\frac{dv}{dt}$$

$$\boldsymbol{i = C\frac{dv}{dt}}$$

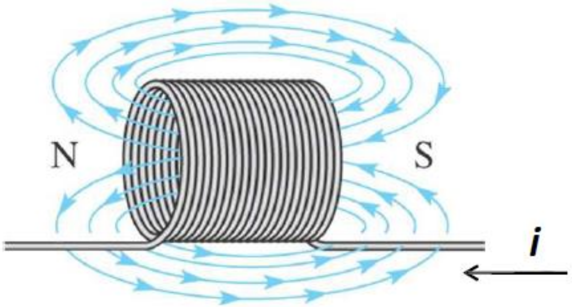
- Energy stored in the capacitor:
  Assume voltage across an Initially uncharged capacitor rises to V during a time period of t seconds.

$$e = \int_0^t p dt = \int_0^t (vi) dt = C\int_0^t v dv$$

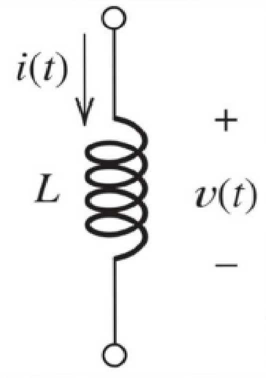
$$\boldsymbol{E = \frac{1}{2}CV^2}$$

Inductors

- When a length of wire is formed into a coil, it becomes a basic inductor.
  - When there is current in the inductor, a magnetic field is created.
- Any change in current causes the magnetic field to change, this in turn induces a voltage across the inductor that opposes the original change in current.



- For an ideal inductor, the voltage is proportional to the time rate of change of the current and the polarity is such as to oppose the change in current.
- The constant of proportionality is called inductance, L.



$$v = L\frac{di}{dt}$$

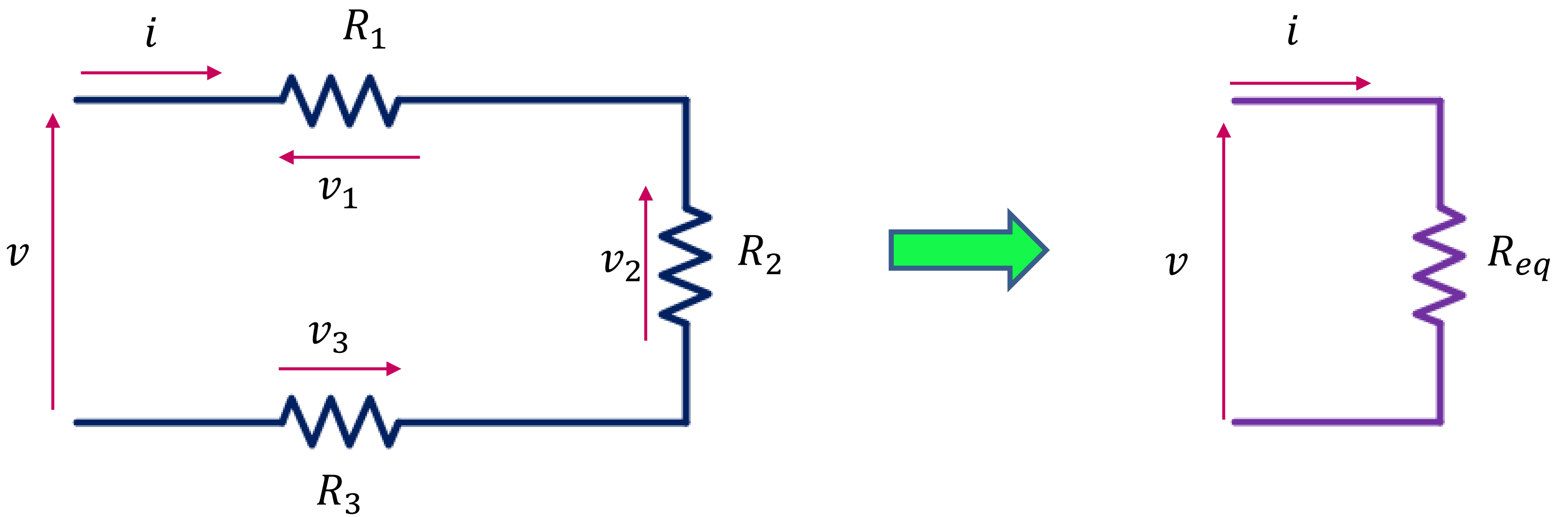
Energy stored in an inductor

- Assume current through an inductor rises to i during a time period of t seconds.

$$e = \int_0^t p dt = \int_0^t (vi) dt = L \int_0^t i\, di$$

$$\boldsymbol{E = \frac{1}{2} L i^2}$$

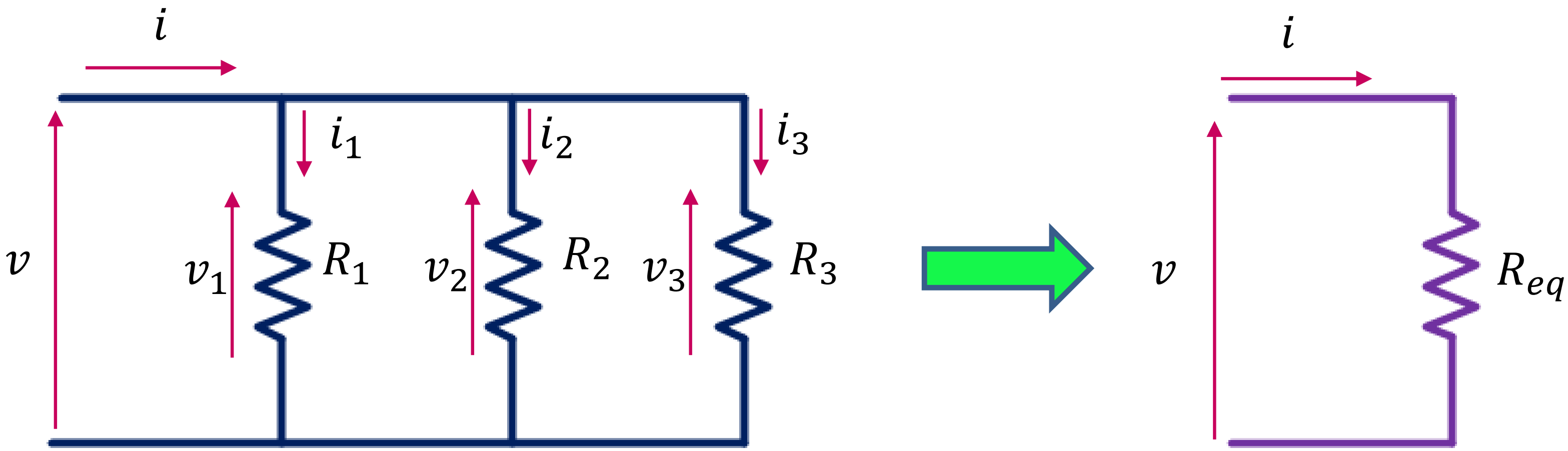
Equivalence resistance: series connection



$$v = v_1 + v_2 + v_3$$
$$v_1 = iR_1,\ v_2 = iR_2,\ v_3 = iR_3$$
$$v = iR_{eq}$$
$$R_{eq} = R_1 + R_2 + R_3$$

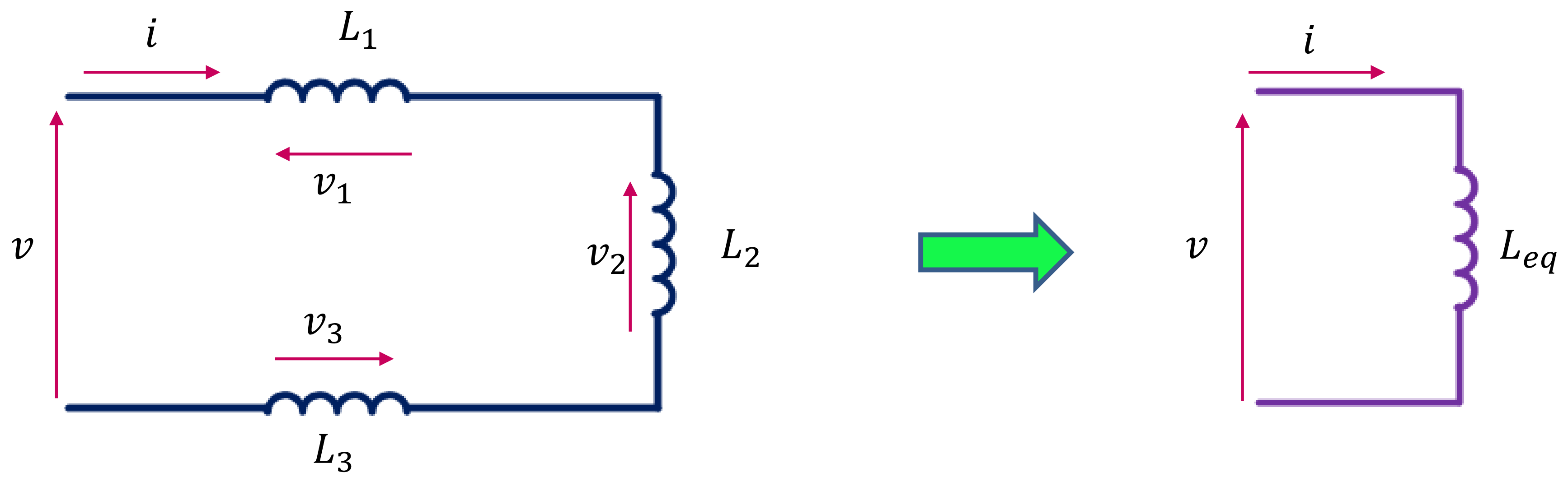
$$\boldsymbol{R_{eq} = \sum_{i=1}^{n} R_i}$$

Equivalence resistance: shunt connection



$$i = i_1 + i_2 + i_3$$
$$v_1 = v_2 = v_3 = v$$
$$v_1 = i_1 R_1,\ v_2 = i_2 R_2,\ v_3 = i_3 R_3$$
$$v = iR_{eq}$$
$$\frac{v}{R_{eq}} = \frac{v}{R_1} + \frac{v}{R_2} + \frac{v}{R_3}$$
$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$
$$\boldsymbol{R_{eq} = \frac{1}{\sum_i^n \frac{1}{R_i}}}$$

Equivalence inductance: series connection



$$v = v_1 + v_2 + v_3$$

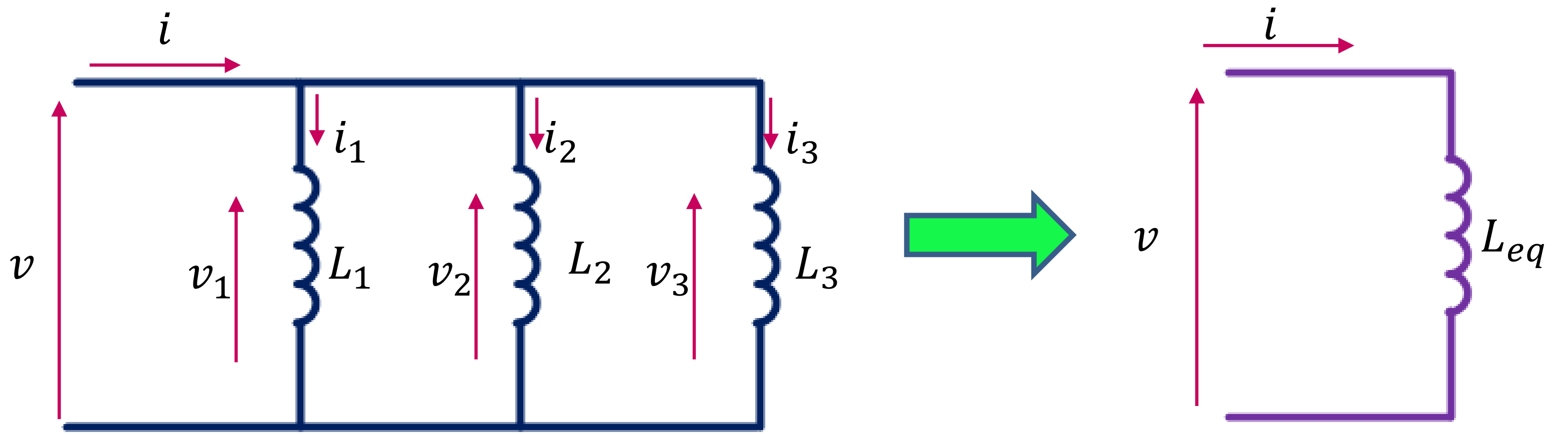
$$v_1 = L_1 \frac{di}{dt},\ v_2 = L_2 \frac{di}{dt},\ v_3 = L_3 \frac{di}{dt}$$

$$v = L_{eq} \frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 + L_3$$

$$\boldsymbol{L_{eq} = \sum_{i=1}^{n} L_i}$$

Equivalence inductance: shunt connection



$$i = i_1 + i_2 + i_3$$

$$v_1 = v_2 = v_3 = v$$

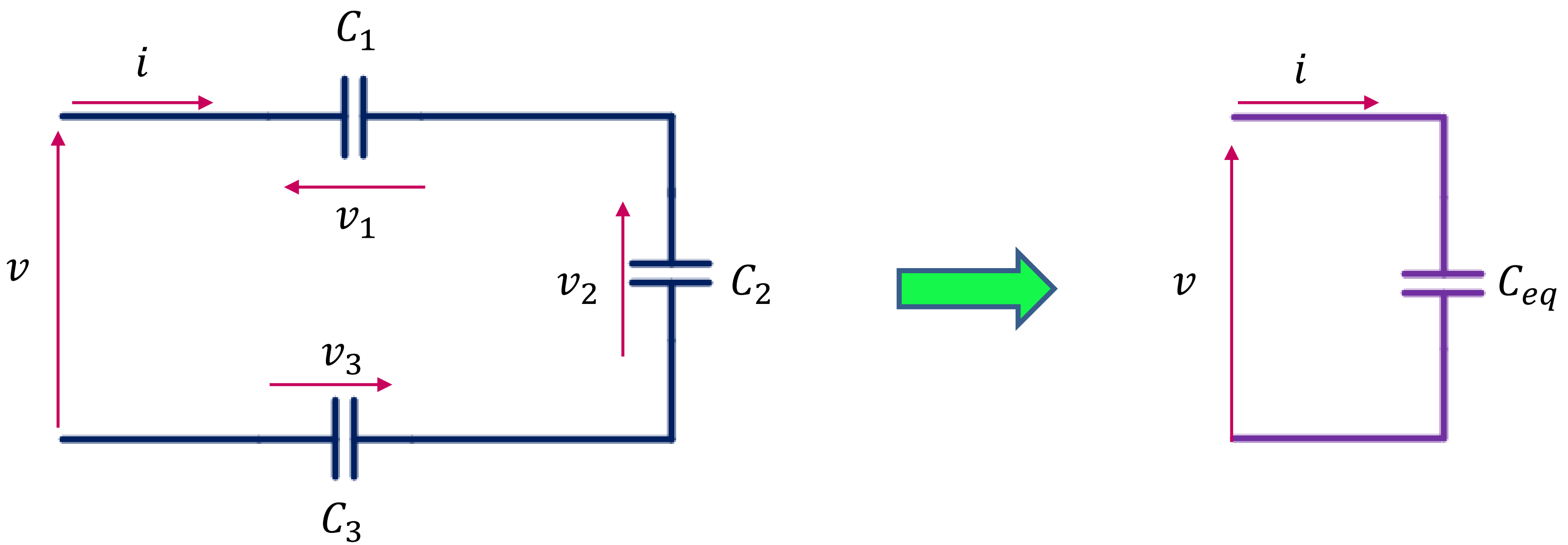
$$v_1 = L_1 \frac{di}{dt}, v_2 = L_2 \frac{di}{dt}, v_3 = L_3 \frac{di}{dt}, v = L_{eq} \frac{di}{dt}$$

$$\frac{v}{L_{eq}} = \frac{v}{L_1} + \frac{v}{L_2} + \frac{v}{L_3}$$

$$L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}}$$

$$\boldsymbol{L_{eq} = \frac{1}{\sum_i^n \frac{1}{L_i}}}$$

Equivalence capacitance: series connection



$$v = v_1 + v_2 + v_3$$

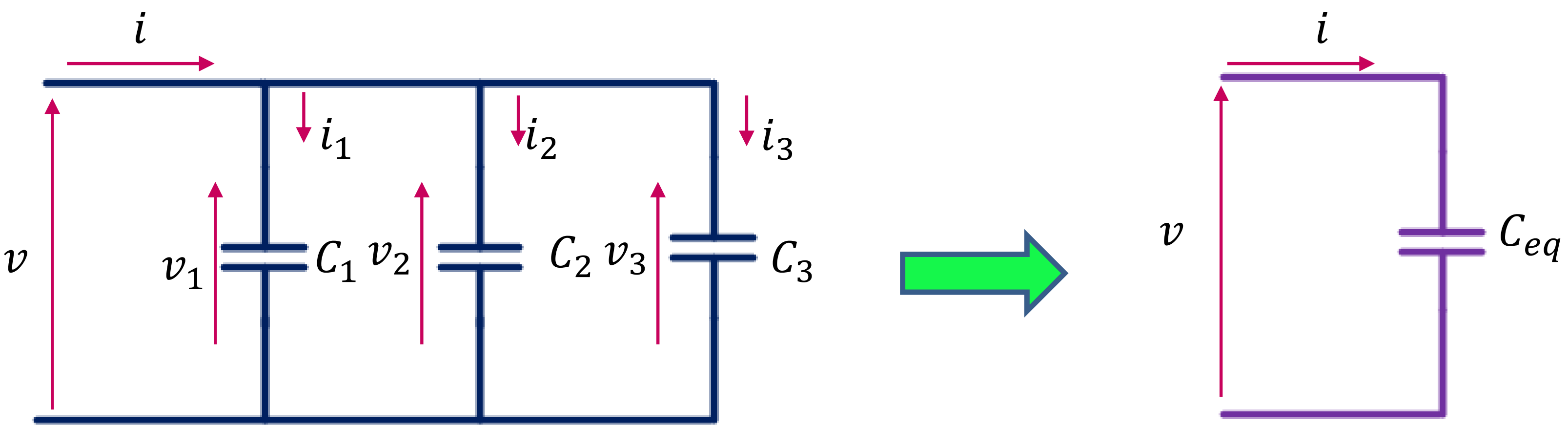
$$q = Cv \rightarrow v_1 = \frac{q}{C_1}, v_2 = \frac{q}{C_2}, v_3 = \frac{q}{C_3}, v = \frac{q}{C_{eq}}$$

$$\frac{q}{C_{eq}} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

$$\boldsymbol{C_{eq} = \frac{1}{\sum_i^n \frac{1}{C_i}}}$$

Equivalence capacitance: shunt connection



$$i = i_1 + i_2 + i_3$$
$$v_1 = v_2 = v_3 = v$$
$$q = Cv \rightarrow i = C\frac{dv}{dt}$$
$$C_{eq}\frac{dv}{dt} = C_1\frac{dv_1}{dt} + C_2\frac{dv_2}{dt} + C_3\frac{dv_3}{dt}$$
$$C_{eq} = C_1 + C_2 + C_3$$

$$\boldsymbol{C_{eq} = \sum_{i=1}^{n} C_i}$$