1. Iteration 1

$$
\begin{array}{r|rrrr}
x_0 = 0.5 & 1 & -2 & 5 & -3 \\
 & & 0.5 & -0.75 & 2.125 \\
\hline
 & 1 & -1.5 & 4.25 & \boxed{-0.875} \leftarrow f(0.5) \\
 & & 0.5 & -0.5 & \\
\hline
 & 1 & -1 & \boxed{3.75} & \leftarrow f'(0.5)
\end{array}
$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0.5 - \frac{-0.875}{3.75}$$

$$= 0.73333$$

$$e_1 = |0.73333 - 0.5| = 0.23333$$

Iteration 2

$$
\begin{array}{r|rrrr}
x_1 = 0.73333 & 1 & -2 & 5 & -3 \\
 & & 0.73333 & -0.92889 & 2.98548 \\
\hline
 & 1 & -1.26667 & 4.07111 & \boxed{-0.01452} \leftarrow f(0.7333) \\
 & & 0.73333 & -0.39111 & \\
\hline
 & 1 & -0.53333 & \boxed{3.68} & \leftarrow f'(0.7333)
\end{array}
$$

$$x_2 = 0.73333 - \frac{-0.01452}{3.68}$$

$$= 0.73728$$

$$e_2 = |0.73728 - 0.73333| = 3.945 \times 10^{-3}$$

Iteration 3

$$
\begin{array}{r|rrrr}
x = 0.73728 & 1 & -2 & 5 & -3 \\
 & & 0.73728 & -0.93098 & 3.00000 \\
\hline
 & 1 & -1.26272 & 4.06902 & \boxed{0.00000} \\
 & & 0.73728 & -0.38740 & \\
\hline
 & 1 & -0.52544 & \boxed{3.68163} &
\end{array}
$$

$$x_3 = 0.73728 - \frac{0}{3.68163} = 0.73728$$

$$e_3 = |0.73728 - 0.73728| = 8.62 \times 10^{-7}$$

2. a. $\left[\begin{array}{ccc|c} 2 & 4 & -6 & 10 \\ 5 & 10 & 0 & 55 \\ 3 & 9 & -8 & 29 \end{array}\right]$

b. $R_2 = R_2 - \frac{5}{2}R_1$; $\left[\begin{array}{ccc|c} 2 & 4 & -6 & 10 \\ 0 & 0 & 15 & 30 \\ 0 & 3 & 1 & 14 \end{array}\right]$ ← Column 1

$R_3 = R_3 - \frac{3}{2}R_1$;

Column 2: use maximum pivoting ; $\max\{|a_{i2}| : 2 \le r \le 3\} = \max\{0, 3\} = 3$

→ swap rows 2 and 3

$\rightarrow \left[\begin{array}{ccc|c} 2 & 4 & -6 & 10 \\ 0 & 3 & 1 & 14 \\ 0 & 0 & 15 & 30 \end{array}\right]$

We have upper triangular matrix and proceed with backward substitution:

$$X_3 = \frac{a_{3,4}}{a_{3,3}} = \frac{30}{15} = 2$$

$$X_2 = \frac{1}{a_{22}}\left(a_{2,4} - a_{3,2} \times X_3\right) = \frac{1}{3} \times (14 - 1 \cdot 2) = 4$$

$$X_1 = \frac{1}{2}\left(10 - 4 \cdot 4 - (-6) \cdot 2\right) = 3$$

3. det A ; expand by minors along row 2:

$$\det A = -5 \cdot \det\begin{bmatrix} 4 & -6 \\ 9 & -8 \end{bmatrix} + 10 \det\begin{bmatrix} 2 & -6 \\ 3 & -8 \end{bmatrix} = -5(4\cdot(-8) - (-6)\cdot 9) + 10(2\cdot(-8) - (-6)\cdot 3)$$
$$= -5 \cdot 22 + 10 \cdot 2$$
$$= -110 + 20 = -90$$

$$\det A_1 = \det\begin{bmatrix} 10 & 4 & -6 \\ 55 & 10 & 0 \\ 29 & 9 & -8 \end{bmatrix} = -55 \cdot 22 + 10 \cdot \det\begin{bmatrix} 10 & -6 \\ 29 & -8 \end{bmatrix}$$
$$= -1210 + 10 \cdot (10 \cdot (-8) - (-6) \cdot 29)$$
$$= -270$$

$$\det A_2 = \det\begin{bmatrix} 2 & 10 & -6 \\ 5 & 55 & 0 \\ 3 & 29 & -8 \end{bmatrix} = -5 \cdot \det\begin{bmatrix} 10 & -6 \\ 29 & -8 \end{bmatrix} + 55 \cdot \det\begin{bmatrix} 2 & -6 \\ 3 & 8 \end{bmatrix}$$
$$= -5 \cdot (10 \cdot -8 - (-6) \cdot 29) + 55 \cdot (2 \cdot -8 - (-6) \cdot 3)$$
$$= -470 + 110$$
$$= -360$$

$\det A_3 = \det\begin{bmatrix} 2 & 4 & 10 \\ 5 & 10 & 55 \\ 3 & 9 & 29 \end{bmatrix}$; expand on row 1 → $= 2 \det\begin{bmatrix} 10 & 55 \\ 9 & 29 \end{bmatrix} - 4 \det\begin{bmatrix} 5 & 55 \\ 3 & 29 \end{bmatrix}$

$+10 \det\begin{bmatrix} 5 & 10 \\ 3 & 9 \end{bmatrix}$

$= 2(10 \cdot 29 - 55 \cdot 9) - 4(5 \cdot 29 - 55 \cdot 3)$

$+10(5 \cdot 9 - 10 \cdot 3)$

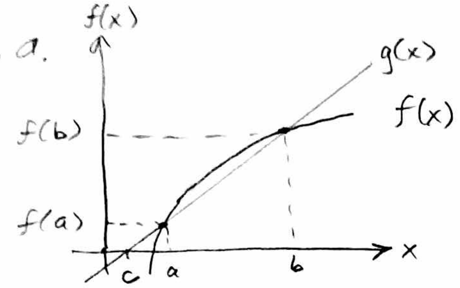
$= -410 - (-80) + 150 = -180$

Cramer's Rule:

$X_1 = \frac{\det A_1}{\det A} = \frac{-270}{-90} = 3$; $X_2 = \frac{-360}{-90} = 4$

$X_3 = \frac{-180}{-90} = 2$

4. a.



equation: $g(x) - f(b) = \dfrac{f(b) - f(a)}{b-a}(x-b)$

$$g(x) = f(b) + \frac{f(b)-f(a)}{b-a}(x-b)$$

b. We solve $g(x) = 0$ for x:

$$0 - f(b) = \frac{f(b)-f(a)}{b-a}(x-b)$$

$$\Rightarrow \frac{-f(b)\cdot(b-a)}{f(b)-f(a)} = x-b$$

$$\Rightarrow c = b - \frac{f(b)\cdot(b-a)}{f(b)-f(a)}$$

c. Bi- Secant Algorithm

Input: function $f: \mathbb{R} \to \mathbb{R}$; $a, b, \varepsilon \in \mathbb{R}$; $N \in \mathbb{N}$; initialize $i = 1$

Repeat:

Compute $c = b - \dfrac{f(b)\cdot(b-a)}{f(b)-f(a)}$

$i = i + 1$

if $(f(c) = 0)$

print "the solution is c"

stop

else if $(f(a)f(c) < 0)$

$b \leftarrow c$

else

$a \leftarrow c$

Until $|b-a| < \varepsilon$ or $i > N$

If $i \leq N$ print "the answer is c"

else print "too many iterations → error"

5. a. At iteration 1, 350 + 5 is truncated to 350

because trunc (355) = 350. This happens in all subsequent iterations as well → alg prints 350.

b. At iteration 1, 350 + 5 rounded up to 360

— " — 2, 360 + 5, rounded up to 370

Alg prints 450 at end.