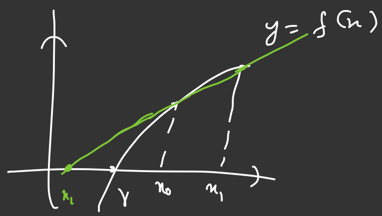
1) One point { x_0

two points - compute derivation



$x_1, f(x_1)$
$x_2, f(x_2)$ } $y = 0$ $\quad \frac{y - f(x_1)}{x - x_1} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$

$$\Rightarrow x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)}$$

- unique
- no solution
- ∞ solutions

unknown = equations

1. mul by const
2. Add 1C to 2nd equation
3. swap two eq.

Gauss elim -> upper D'lar

\# steps $\lceil \log_2 (b-a) \rceil$ how? $\quad e_k = \frac{1}{2} e_{k-1}$ $\quad$ 0 10

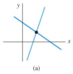
1. Bisection -o
2. Newton => $\boxed{\text{new } o - e_k = c\, e_{k-1}^2}$
3. fixed-point
4. secant -> second o $e_k = c\, e_{k-1}^{1.62}$

---

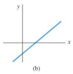
1. Gaussian elimination

Not every system has a unique solution.

There are three different possible solutions

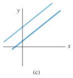


- a unique solution (exactly one solution)



- infinitely many solutions

} the system is called **consistent**



- no solution

} the system is called **inconsistent**

3 Permitted Row Operations :

(remember: every row represents an equation)

a) Multiply a row by a number

$$\left[\begin{array}{ccc|c} \boxed{3} & -9 & 6 & 15 \\ 5 & -2 & 4 & -1 \\ 2 & -4 & 2 & -2 \end{array}\right] \tfrac{1}{3}\cdot R_1 \Rightarrow \left[\begin{array}{ccc|c} 1 & -3 & 2 & 5 \\ 5 & -2 & 4 & -1 \\ 2 & -4 & 2 & -2 \end{array}\right]$$

b) Switch rows

$$\left[\begin{array}{ccc|c} 1 & 7 & 3 & 1 \\ 0 & \boxed{-2} & 4 & -1 \\ 0 & 1 & 2 & -2 \end{array}\right] R_2 \leftrightarrow R_3 \left[\begin{array}{ccc|c} 1 & 7 & 3 & 1 \\ 0 & 1 & 2 & -2 \\ 0 & -2 & 4 & -1 \end{array}\right]$$

c) Add a multiple of $\underbrace{\text{one row}}_{\text{Row that is not changing}}$ to $\underbrace{\text{another row}}_{\text{Row you want to replace}}$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ \boxed{3} & -2 & 4 & -1 \\ 2 & 1 & 3 & 1 \end{array}\right] -3R_1 + R_2 = New\ R_2 \quad \Rightarrow \begin{array}{r|cccc} -3R_1 & -3 & 6 & -3 & \vert\ 3 \\ +\ R_2 & 3 & -2 & 4 & \vert\ -1 \\ \hline New R_2 & 0 & 4 & 1 & \vert\ 2 \end{array} \quad \Rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 4 & 1 & 2 \\ 2 & 1 & 3 & 1 \end{array}\right]$$

5. Use Gaussian elimination to solve the following system of equations:

$$\begin{cases} 2x_1 + x_2 - x_3 = 4 \\ -4x_1 - 2x_2 + 2x_3 = -6 \\ 6x_1 + 3x_2 - 3x_3 = 12 \end{cases}$$

Solution: Writing this as an augmented matrix, we get

$$\left(\begin{array}{ccc|c} 2 & 1 & -1 & 4 \\ -4 & -2 & 2 & -6 \\ 6 & 3 & -3 & 12 \end{array}\right) \overset{II+2I, III-3I}{\longrightarrow} = \left(\begin{array}{ccc|c} 2 & 1 & -1 & 4 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

So this system of equations has no solutions.

2

$$\begin{pmatrix} 1 & a \\ 4 & 8 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ b \end{pmatrix} \Rightarrow \left(\begin{array}{cc|c} 1 & a & 2 \\ 4 & 8 & b \end{array}\right)$$

$\Rightarrow$

$R_2 \to R_2 - 4R_1$

$$\left(\begin{array}{cc|c} 1 & a & 2 \\ 0 & 8-4a & b-8 \end{array}\right)$$

i) $a \neq 2 \to$ unique solution

ii) $a = 2$, $b \neq 8 \Rightarrow$ no solution

iii) $a = 2$, $b = 8 \to$ infinite.

10. Use Gaussian elimination to solve the following system of equations:

$$\begin{cases} z - 3y = -6 \\ x - 2y - 2z = -14 \\ 4y - x - 3z = 5 \end{cases}$$

Solution: First write it in an augmented matrix.

$$\left(\begin{array}{ccc|c} 0 & -3 & 1 & -6 \\ 1 & -2 & -2 & -14 \\ -1 & 4 & -3 & 5 \end{array}\right) \overset{I \leftrightarrow II}{\longrightarrow} \left(\begin{array}{ccc|c} 1 & -2 & -2 & -14 \\ 0 & -3 & 1 & -6 \\ -1 & 4 & -3 & 5 \end{array}\right) \overset{III+I}{\longrightarrow} \left(\begin{array}{ccc|c} 1 & -2 & -2 & -14 \\ 0 & -3 & 1 & -6 \\ 0 & 2 & -5 & -9 \end{array}\right)$$

$$\overset{I-2/3II, III+2/3II}{\longrightarrow} \left(\begin{array}{ccc|c} 1 & 0 & -8/3 & -10 \\ 0 & -3 & 1 & -6 \\ 0 & 0 & -13/3 & -13 \end{array}\right) \overset{III*3/-13}{\longrightarrow} \left(\begin{array}{ccc|c} 1 & 0 & -8/3 & -10 \\ 0 & -3 & 1 & -6 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

$$\overset{I+8/3III, II-III}{\longrightarrow} \left(\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & -3 & 0 & -9 \\ 0 & 0 & 1 & 3 \end{array}\right) \overset{II/-3}{\longrightarrow} \left(\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

Thus the solution is $(-2, 3, 3)$.

$\Rightarrow$ use cramers rule.

$$\det A_x = \begin{vmatrix} -6 & -3 & 1 \\ -14 & -2 & -2 \\ 5 & 4 & -3 \end{vmatrix}$$

$$\det A = \begin{vmatrix} 0 & -3 & 1 \\ 1 & -2 & -2 \\ -1 & 4 & -3 \end{vmatrix}$$

③ Horners & Newton Participation exercise

$x^4 - 2x^3 + 3x^2 + 4x - 10$

I.

1	1	-2	3	4	-10
	0	1	-1	2	6
1	1	-1	2	6	$\boxed{-4}$
	0	1	0	2	
	1	0	2	$\boxed{8}$	

$x_1 = 1 - \frac{-4}{8}$

$= 3/2$

II.

1.5	1	-2	3	4	-10
	0	1.5	-0.75	3.375	11.0625
1	1	-0.5	2.25	7.375	$\boxed{1.0625}$
	0	1.5	1.5	5.625	
	1	1	3.75	$\boxed{13}$	

$x_2 = 1.5 - \frac{1.0625}{13}$

$= 1.418269$

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3.

r0	a4	a3	a2	a1	a0		
1.418269231	1	-2	3	4	-10	r0 =	1.418269231
	0	1.418269231	-0.8250508506	3.084663457	10.04796019	r1 =	1.414222645
	1	-0.5817307692	2.174949149	7.084663457	0.04796019155	error =	0.004046585914
	0	1.418269231	1.18643676	4.767350209			
	1	0.8365384615	3.36138591	11.85201367			

4.

r0	a4	a3	a2	a1	a0		
1.414222645	1	-2	3	4	-10	r0 =	1.414222645
	0	1.414222645	-0.8284196005	3.071098176	10.00010716	r1 =	1.414213562
	1	-0.5857773551	2.1715804	7.071098176	0.0001071646533	error =	0.000009082436683
	0	1.414222645	1.171606089	4.728010038			
	1	0.8284452897	3.343186488	11.79910821			

First Root (x1) = 1.414222645 with error = 0.000009082436683

Participation exercise 2/2
Due: Multiple Due Dates - 01:198:323:03 NUMER ANAL COMPUTING
42/51 Graded 8.67 / 10 (8 Average
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2.

r0	a4	a3	a2	a1		
-1.4142098	1	-0.5857773551	2.1715804	7.071098176	r0 =	-1.4142098
	0	-1.4142098	2.828401435	-7.071023311	r1 =	-1.414217417
	1	-1.999987155	4.999981835	0.0000746468121	error =	0.000007617200125
	0	-1.4142098	4.828390794			
	1	-3.414196955	9.828372629			

Second Root (x2) = -1.414217417 with error = 0.000007617200125

New deflated function:

$x^2 - 1.999987155x + 4.999981835 = 0$

Solving using Quadratic formula for remaining two roots:

Graded
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	1	0.8284452897	3.343186488	11.79910821

First Root (x1) = 1.414222645 with error = 0.000009082436683

New deflated function:

$x^3 - 0.5857773551x^2 + 2.1715804x + 7.071098176 = 0$

1.

r0	a4	a3	a2	a1		
1	1	-0.5857773551	2.1715804	7.071098176	r0 =	1
	0	1	0.4142226449	2.585803045	r1 =	-1.4142098
	1	0.4142226449	2.585803045	9.656901221	error =	2.4142098
	0	1	1.414222645			
	1	1.414222645	4.00002569			

① Secant method

$x = \cos x = 0$ find roots

$P_0 = \frac{\pi}{4} \; (1/\sqrt{2})$.

$\ddot{0}$

secant update $= 0$

$n \geq 2$

$$P_n = P_{n-1} - \frac{(P_{n-1} - P_{n-2})(\cos P_{n-1} - P_{n-1})}{(\cos P_{n-1} - P_{n-1}) - (\cos P_{n-2} - P_{n-2})}$$

$P_0 = 0.5$

$P_1 = \pi/4$

0.5
0.785397
0.736384.
0.73905

Fancy questions (Not be asked in this exam)

① loan → A dollars

repaid = "n" equal monthly install of M dollars
starting 1 month after loan is taken

"r" → interest rate

$$Ar = M\left(1 - \frac{1}{(1+r)^n}\right)$$

\$10000 → 60 months of 250\$
what is interest rate.

$\Rightarrow \quad 10000\,r = 250\left(1 - \frac{1}{(1+r)^{60}}\right)$

$\Rightarrow$

$f(r) = 40r + \frac{1}{(1+r)^{60}} - 1$

$f'(r) = 40 - 60/(1+r)^{61}$

$\Rightarrow \quad r_{n+1} = r_n - \frac{40r_n + 1/(1+r_n)^{60} - 1}{40 - 60/(1+r)^{61}}$

1.439%

Starting point

$r_0 = 0.015$

SOLUTIONS: ASSIGNMENT 9

6.2.6 *Use Gaussian elimination to find the determinant of the matrix.*

$$\det\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 4 & 4 \\ 1 & -1 & 2 & -2 \\ 1 & -1 & 8 & -8 \end{bmatrix} = \det\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & -2 & 1 & -3 \\ 0 & -2 & 7 & -9 \end{bmatrix} = \det\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & -2 & 1 & -3 \\ 0 & 0 & 6 & 12 \end{bmatrix}$$

$$= \det\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & -2 & 1 & -3 \\ 0 & 0 & 0 & 6 \end{bmatrix} = \det -\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -2 & 1 & -3 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

$$= -1 \cdot (-2) \cdot 3 \cdot 6 = 36$$

6.2.30 *Consider two distinct numbers a and b. We define the function*

$$f(t) = \det\begin{bmatrix} 1 & 1 & 1 \\ a & b & t \\ a^2 & b^2 & t^2 \end{bmatrix}.$$

(a) *Show that $f(t)$ is a quadratic function. What is the coefficient of t^2?*

Expanding down the third column, we have $f(t) = D_1 - D_2 t + D_3 t^2$, where the D_i are determinants of 2×2 matrices that contain no factor of t. Since $D_3 = b - a$, $f(t)$ is a quadratic function with leading coefficient $b - a$.

(b) *Explain why $f(a) = f(b) = 0$. Conclude that $f(t) = k(t-a)(t-b)$, for some constant k. Find k, using your work in part (a).*

If $t = a$, the first and third columns of the matrix are the same, so it has determinant 0. Likewise, if $t = b$, the second and third columns of the matrix are the same. This shows that $f(a) = f(b) = 0$. It follows that $f(t)$ has the factors $t - a$ and $t - b$; since $f(t)$ is quadratic, it can have no other non-constant factors, so $f(t) = k(t-a)(t-b)$ for some constant k. This constant is equal to the leading coefficient of $f(t)$ which is $b - a$ by part (a).

(c) *For which values of t is the matrix invertible?*

7. Find $\begin{pmatrix} 1 & 3 & 1 \\ 0 & 1 & 1 \\ 2 & 5 & 2 \end{pmatrix}^{-1}$.

Solution: We need to use Gaussian elimination to reduce

$$\left(\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 2 & 5 & 2 & 0 & 0 & 1 \end{array}\right) \xrightarrow{III-2I} \left(\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & -2 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{I-3II,III+II} \left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & -3 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -2 & 1 & 1 \end{array}\right) \xrightarrow{I+2III,II-III} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & -1 & 2 \\ 0 & 1 & 0 & 2 & 0 & -1 \\ 0 & 0 & 1 & -2 & 1 & 1 \end{array}\right)$$

Thus the inverse is $\begin{pmatrix} -3 & -1 & 2 \\ 2 & 0 & -1 \\ -2 & 1 & 1 \end{pmatrix}$.