Langrange polynomial (explicit) | Implicit (Newton)

(x_0, y_0), (x_1, y_1), (x_2, y_2)

$$P_n = \quad x_0 \to y_0 \to \frac{(x-x_1)(x-x_2)\, y_0}{(x_0-x_1)(x_0-x_2)} +$$

$P_n(x) = a_0 + a_1 x \dots a_n x^n$

$x_0 \; y_0$

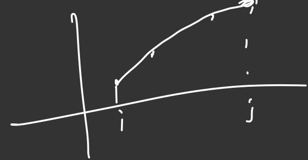
$\vdots$

$x_n \; y_n$

$$\begin{pmatrix} 1 & x_0 & \dots & x_0^n \\ \vdots & & & \\ 1 & x_n & \dots & \end{pmatrix} \to A$$

$\to \operatorname{cond}(A) = \|A\| \, \|A^{-1}\|$

error $\to$ large



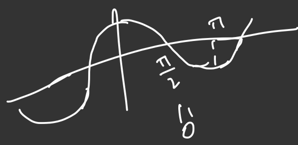
$$P_{ij} = P_{i+1\,j}(x)\frac{(x-x_i)}{x_j-x_i} + P_{i\,j-1}\frac{(x-x_j)}{(x_i-x_j)} = \frac{P_{i+1,j}(x)(x-x_i) - P_{i\,j-1}(x)(x-x_j)}{(x_j-x_i)}$$

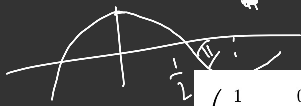
$$P_{ij}(x) = \begin{cases} y_i & x = x_i \\ y_j & x = x_j \\ y_k & \text{if } x = x_k,\; i < k < j \end{cases}$$

j

i \ j	2	0.5	4
2	0.5		
0.5		0.2	
4			0.25

$\cos \pi x$





$\cos \pi$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & \dots & 0 \\ h_0 & 2(h_0+h_1) & h_1 & 0 & 0 & \dots & 0 \\ 0 & h_1 & 2(h_1+h_2) & h_2 & 0 & \dots & 0 \\ 0 & 0 & h_2 & 2(h_2+h_3) & h_3 & \dots & 0 \\ 0 & 0 & 0 & h_3 & 2(h_3+h_4) & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & 1 \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ \vdots \\ \vdots \\ c_n \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ \frac{3}{h_1}(a_2-a_1) - \frac{3}{h_0}(a_1-a_0) \\ \frac{3}{h_2}(a_3-a_2) - \frac{3}{h_1}(a_2-a_1) \\ \frac{3}{h_3}(a_4-a_3) - \frac{3}{h_2}(a_3-a_2) \\ \vdots \\ \vdots \\ 0 \end{pmatrix}$$

0.025	0.95
0.5	0
0.75	-0.7
1.0	-1

0

$a_0 + b_0(x - 0.025) + c_0$

$a_0 = 0.95$

$a_1 = 0$

$a_2 = -0.7$

$a_3 = -1$

$h_0 = 0.475$

$h_1 = 0.25$

$h_2 = 0.25$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ h_0 & & & \\ 0 & & & \\ 0 & 0 & & \end{pmatrix}$$

Norm

- $\|A\| \geq 0$
- $\|A\bar{x}\| \leq \|A\|\|\bar{x}\|$
- $\|A+B\| \leq \|A\| + \|B\|$
- $\|AB\| \leq \|A\|\|B\|$

→ max abs row sum, col sum, eigen value } easy

$$A\bar{x} = \bar{b}$$
$$A\hat{x} = \hat{b}$$
$$A(\bar{x}-\hat{x}) = \bar{b}-\hat{b}$$
$$\|x-\hat{x}\| = \|A^{-1}(b-\hat{b})\| \leq \|A^{-1}\|\|b-\hat{b}\| \quad \frac{\|A\|}{\|A\|}$$
$$\leq \frac{\|A^{-1}\| \|b-\hat{b}\|}{\|\bar{x}\|}$$
$$\to \frac{\|x-\hat{x}\|}{\|\bar{x}\|} \leq \ldots \leq \frac{\|A^{-1}\| \|A\| \|b-\hat{b}\|}{\|b\|}$$

$$A(\underbrace{\bar{x}-\hat{x}}) = \underbrace{\bar{b}-\hat{b}} \quad \text{Ex Gaussian elimination}$$
$$A\bar{e}_x = \bar{r} = 0$$
$$\bar{e}_x = \bar{x}-\hat{x} \qquad \bar{x} = e_x + \hat{x}$$

$$e_x \leq \|A^{-1}\|\|A\| e_b$$

Jacobi

$$A\bar{x} = \bar{b}$$

0

$$x_1^{(1)} = \frac{b_1 - (\ )}{a_{11}}$$
$$x_2^{(1)} = \frac{b_2 - (\ )}{a_{22}}$$

$$|a_{ii}| > \sum |a_{ij}|$$

$$\begin{pmatrix} & U \\ L & \end{pmatrix}$$

$$(L+D+U)\bar{x} = \bar{b}$$
$$\bar{x}^1 = D^{-1}\bar{b} - D^{-1}(L+U)\bar{x}^0$$
$$\bar{e}_{k+1} = \underbrace{-D^{-1}(L+U)}_{M}\bar{e}_k$$

$$5x_1 + 2x_2 - x_3 = b_1$$

$$x_1 - 7x_2 + 2x_3 = b_2$$

$$2x_1 + 3x_2 - 4x_3 = b_3$$

b

$$x_1 = -\frac{2x_2}{5} + \frac{x_3}{5} + \frac{b_1}{5}$$

$$x_2 =$$

Cubic splines

Linear regression (problems

(demonstration/code) LR.

solve (our progs practice set

(>1 8)