CS 323 - Spring 2022 **Exam 1**

Name: _____

PROBLEMS (16 points each).

Answer the following problems, scan (take picture of) your solutions and upload them to Canvas. This exam is INDIVIDUAL you are NOT to work in groups. The exam is open book/open notes. There are several (wrong) solutions floating around on the internet. It is better to work on your own solution even if it is only partially right rather than risk using a solution found online and get a 0.

ALL OF YOUR SOLUTIONS MUST BE HANDWRITTEN (tablet OK). You can use excel to help you, but you have to copy the results by hand. You can print the exam and answer directly on it, or you can use any paper (you can use also your ipad and stylus) to answer the questions as long as it is clear which question you are answering.

THE EXAM HAS 7 PAGES

1. Do the **first 3 iterations** (means you compute 3 times Newton's formula) of Newton's Method with Horner to solve $x^3 - 2x^2 + 5x - 3 = 0$ with an initial approximation $x_0 = 0.5$

   You must:

   - Show the error after each iteration?
   - Show all your steps including all of Horner's Synthetic divisions

2. Given the following linear system

$$\begin{aligned} 2x_1 + 4x_2 - 6x_3 &= 10 \\ 5x_1 + 10x_2 &= 55 \\ 3x_1 + 9x_2 - 8x_3 &= 29 \end{aligned}$$

(a) Rewrite it in augmented matrix form

(b) Show each one of the steps used to solve it using Gaussian Elimination with maximum pivoting and backward substitution

3. Given the linear system (same as in previous question)

$$\begin{aligned} 2x_1 + 4x_2 - 6x_3 &= 10 \\ 5x_1 + 10x_2 &= 55 \\ 3x_1 + 9x_2 - 8x_3 &= 29 \end{aligned}$$

Solve it using Cramer's Rule. You must show how each one of the determinants needed, are computed using **minors**.

4. The "bi-secant method" is a combination of the secant method and the bisection method.

   As in the case of the secant method, it uses two points $(a, f(a))$, $(b, f(b))$ to find a secant line that goes through those points and computes a point c where the secant intersects the x axis.

   As in the case of the bisection method, we require that $f(a)f(b) < 0$, and assuming that $a \leq c \leq b$ the method uses sign change to determine in which side of c the solution is.

   (a) Find the equation of the secant line line between the points $(a, f(a))$ and $(b, f(b))$
   Hint: It helps to visualize this method graphically (draw the points and the line)

   (b) Find a point c where the secant line between the points $(a, f(a))$ and $(b, f(b))$ intersects the x-axis

(c) Notice that if there is a sign change in the function from a to c then the solution is in $[a, c]$ otherwise, the solution is in $[c, b]$

Use the description above, and your formula for c to write the *bi-secant algorithm* such that that given $f(x), a, b, \epsilon, N$ returns an approximation to the solution of $f(x)$ with error tolerance ϵ, or an error, if the number of iterations exceeds N

5. Given the following algorithm, and assuming *Fl(10,2,-10,10)*

```
x=350
y=5
for i=1 to 10
   x=x+y
print x
```

(a) What will the algorithm print if truncation is used?

(b) What will the algorithm print if rounding is used?