Christmas

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1 Christmas Party

Problem: There are n children at a Christmas party, and each of them has brought a gift. The idea is that everybody will get a gift brought by someone else.

In how many ways can the gifts be distributed?

Input: The only input line has an integer n: the number of children.

Output: Print the number of ways modulo $10^9 + 7$.

Constraints:

- $1 \leq n \leq 10^6$

Example:

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Input:
4
Output:
9
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Approach: This is a classic derangement problem. The number of valid gift distributions (where no one gets their own gift) is equal to the number of *derangements* of n elements. The recurrence relation is:

$$D(n) = (n-1) \cdot (D(n-1) + D(n-2))$$

With base cases:

$$D(1) = 0, \quad D(2) = 1$$

Use dynamic programming with modulo $10^9 + 7$ to avoid overflow.

Time Complexity: $\mathcal{O}(n)$