A system is composed of two harmonic oscillators, each of natural frequency ω_0, and having permissible energies $(n + 1/2)\hbar\omega_0$, where n is any non-negative integer. The total energy of the system is $E' = n'\hbar\omega_0$, where n' is a positive integer.

- For a given energy, how many microstates are available to the system ? What is * the entropy of the system ?
- A second system is also composed of two harmonic oscillators, each of natural frequency $2\omega_0$. The total energy of this system is $E'' = n''\hbar\omega_0$, where n'' is an even integer.
  - How many microstate are available to this system ?
  - What is the entropy of a system composed of the two preceeding subsystems (separated and enclosed by a totally restrictive wall) ? Express the entropy as a function of E' and E''.

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We have two harmonic oscillators with energy levels:

$$E_1 = \left(n_1 + \frac{1}{2}\right)\hbar\omega_0$$

$$E_2 = \left(n_2 + \frac{1}{2}\right)\hbar\omega_0$$

Therefore, the total energy of the system is: $E' = E_1 + E_2 = n'\hbar\omega_0$ for any given n' or E',

but is also given by:

$$E' = E_1 + E_2 = \hbar\omega_0 + (n_1 + n_2)\hbar\omega_0 = n'\hbar\omega_0$$ as we wish to focus on the state of energy E'.

The levels of the two oscillators have to satisfy:

$$\boxed{n_1 + n_2 = n' - 1}$$ for any given n'.

We now have to count how many states (defined by n_1 and n_2) can the system adopt for the given n' (all are positive integers).

The answer is simple: $n_1 \in (0, 1, \ldots n'-1)$, and for any value in this set, we can find the corresponding value of n_2.

$$\Rightarrow \quad \Omega(E') = n' \quad \text{and} \quad \boxed{S(E') = k_B \ln n' = k_B T \ln \frac{E'}{\hbar\omega_0}}$$

Similarly for the two oscillators of frequency $2\omega_0$:

$$\boxed{S(E'') = k_B \ln \frac{E'}{\hbar 2\omega_0}}$$

Finally, if we treat both systems as <u>independent</u> subsystems of a larger system, then the entropy is additive:

$$\boxed{S_T = S(E') + S(E'') = k_B \ln \frac{E' \cdot E''}{2\hbar^2\omega_0^2}}$$