We have $E = \sigma L_0 + \frac{\sigma}{2}\int_0^{L_0} dx \left(\frac{dh}{dx}\right)^2$ where L_0 is large.

$$\Rightarrow \quad E = \sigma L_0 + \frac{\sigma}{2}\int_0^{L_0} dx \, \frac{1}{L_0} \sum_k \sum_{k'} \eta_k \eta_{k'} e^{ikx} e^{ik'x} (ik)(ik')$$

$$= \sigma L_0 - \frac{\sigma}{2L_0} \sum_k \sum_{k'} \eta_k \eta_{k'} kk' \int_0^{L_0} dx \, e^{i(k+k')x}$$

If L_0 is large $\int_0^{L_0} dx \, e^{i(k+k')x} \simeq L_0 \, \delta_{k,-k'}$

$$\Rightarrow \quad E = \sigma L_0 + \frac{\sigma}{2} \sum_k \eta_k \eta_{-k} k^2$$

$$= \sigma L_0 + \frac{\sigma}{2} \sum_k k_2 |\eta_k|^2$$

The correlation function is now:

$$\langle (h(x) - h(x'))^2 \rangle = \langle h(x)^2 - 2 h(x) h(x') + h(x')^2 \rangle =$$

$$= 2 \langle h(x)^2 \rangle - 2 \langle h(x) h(x') \rangle$$

~~Recalling that~~ $\cancel{\langle h(x) h(x') \rangle = \int \frac{dk}{2\pi} \langle |\hat{h}_k|^2 \rangle e^{ikx}}$

we have:

$$\cancel{\langle (h(x) - h(x'))^2 \rangle = 2 \int \frac{dk}{2\pi} \langle |\hat{h}_k|^2 \rangle - 2 \int \frac{dk}{2\pi} \langle |\hat{h}_k|^2 \rangle e^{ikx}}$$

Recalling that $\langle h(0) h(x) \rangle = \frac{1}{L_0} \sum_k |\hat{h}_k|^2 e^{ikx}$

$$\langle (h(x) - h(x'))^2 \rangle = \frac{2}{L_0} \sum_k \langle |\hat{h}_k|^2 \rangle - \frac{2}{L_0} \sum_k \langle |\hat{h}_k|^2 \rangle e^{ikx}$$

By using the equipartition result $\langle |\tilde{h}_k|^2 \rangle = \frac{k_B T}{\sigma k^2}$

$$\langle (h(0) - h(x))^2 \rangle = \frac{2}{L_0} \sum_k \frac{k_B T}{\sigma k^2} \left(1 - e^{ikx}\right)$$

The argument of the sum is even in k. Hence $e^{ikx} = \cos kx + i \sin kx$ will only be nonzero for the $\cos(kx)$ part.

Finally: $$\boxed{\langle (h(0) - h(x))^2 \rangle = \frac{2}{L_0} \sum_k \frac{k_B T}{\sigma k^2} \left(1 - \cos kx\right)}$$