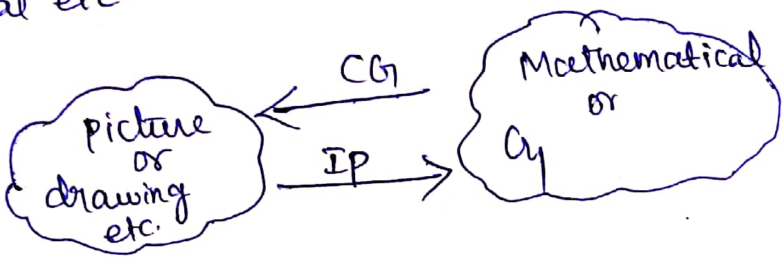
CGIP

Computer graphics:

Conversion of mathematical or Geometrical form to picture or drawing etc. using computer or any digital media.

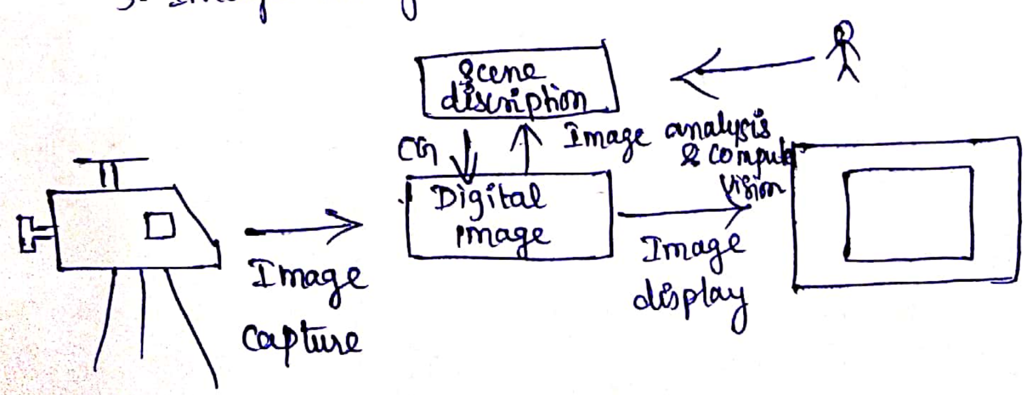
Image processing:

Conversion of Picture, drawing into mathematical or Geometrical model. Acquired from photograph, scanner, medical etc.



Purpose:

1. Visualization
2. Image Sharpening & restoration
3. Image retrival
4. Measurement of pattern
5. Image Recognization.



DDA ALGORITHM [Digital differential analyzer]

Basic line drawing formula, $y = mx + c \rightarrow ①$

where
m - slope
c - y intercept

Give two end points of line is (x_1, y_1) & (x_2, y_2)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} \rightarrow ②$$

Replace $y_2 \rightarrow y_{K+1}$ & $y_1 \rightarrow y_K$

If two points are x_K, y_K Then next points are x_{K+1}, y_{K+1}

$$m = \frac{y_{K+1} - y_K}{x_{K+1} - x_K}$$

Case 1: $m < 1$

x change in unit interval

$x_{K+1} = x_K + 1$

$y_{K+1} = ?$

$$m = \frac{y_{K+1} - y_K}{x_{K+1} - x_K}$$

$$= \frac{y_{K+1} - y_K}{\cancel{x_K} + 1 - \cancel{x_K}}$$

$$m = y_{K+1} - y_K$$

$$\boxed{y_{K+1} = y_K + m}$$

Case 2: $m > 1$

y change in unit interval

$y_{K+1} = y_K + 1$

$x_{K+1} = ?$

$$m = \frac{y_{K+1} - y_K}{x_{K+1} - x_K}$$

$$= \frac{\cancel{y_K} + 1 - \cancel{y_K}}{x_{K+1} - x_K}$$

$$m = x_{K+1} - x_K$$

$$\boxed{x_{K+1} = x_K + 1/m}$$

Case 3: $m = 1$

Both x, y change in unit interval

$$\boxed{\begin{aligned} x_{K+1} &= x_K + 1 \\ y_{K+1} &= y_K + 1 \end{aligned}}$$

Calculate Slope m,

If $m<1$, x chganges in Unit interval

y moves with deviation

$(x_{k+1}, y_{k+1}) = (x_k+1, y_k+m)$

If $m>1$, y changes in unit interval

x moves with deviation

$(x_{k+1}, y_{k+1}) = (x_k+1/m, y_k+1)$

If $m=1$,

x, y changes in Unit interval

$(x_{k+1}, y_{k+1}) = (x_k+1, y_k+1)$

Example:

Calculate the intermediate point betwen $(0,0)$ & $(4,5)$ using DDA algorithm.

$(0,0)$ $(4,5)$
x_1 y_1 x_2 y_2

$$m = \frac{5-0}{4-0} = 5/4 > 1$$

y moves in unit interval.

$x \rightarrow x_k + 1/m$

$(x_{k+1}, y_{k+1}) = (x_k+1/m, y_k+1)$

$x_{k+1} = x_k + 1/m$ $\quad$ $y_{k+1} = y_k+1$

$m = 5/4 \Rightarrow 1/m = 4/5 = 0.8$

≥ 0.5 high
< 0.5 floor

previous value + $1/m$

x	y	x-plot	y-plot	x, y
0	0	0	0	(0,0)
0.8	1	1	1	(1,1)
1.6	2	2	2	(2,2)
2.4	3	2	3	(2,3)
3.2	4	3	4	(3,4)
4	5	4	5	(4,5)

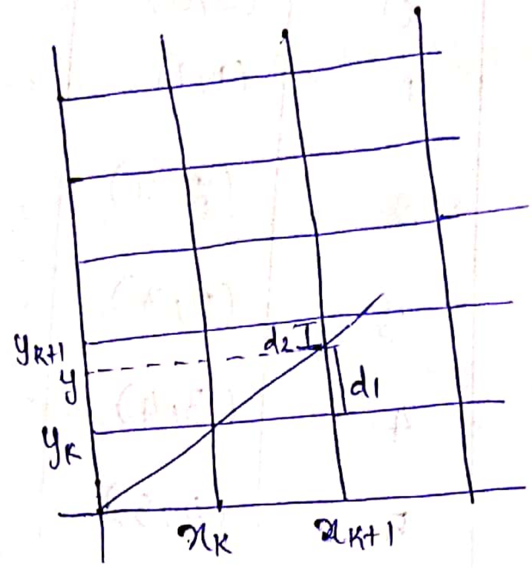
The intermediate points/pixels between (0,0) and (4,5) are (1,1), (2,2), (2,3), (3,4).

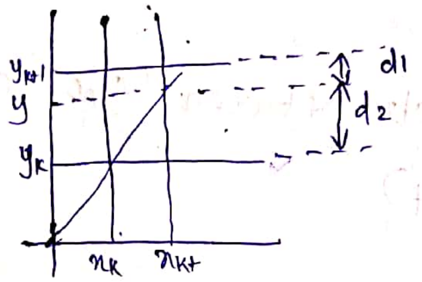
Disadvantage:

1. Rounding function [ceiling function]

BRESENHAM'S ALGORITHM

Bresenham's algorithm is used to overcome the disadvantage of DDA algorithm





Case (i) m<1

x change in unit interval

$y = mx + c$

y coordinate at pixel position at x_{k+1} is,

$$\boxed{y = m(x_k + 1) + c}$$

Decision parameter [whether to select d_1 or d_2]

dp ⇒ P_K $\boxed{P_K = \Delta x(d_1 - d_2)}$

$d_1 = y - y_K$ $\qquad$ $d_2 = y_{K+1} - y$

$\boxed{d_1 = m(x_K+1) + c - y_K}$ $\qquad$ $\boxed{d_2 = y_K + 1 - m(x_K+1) - c}$

$$d_1 - d_2 = m(x_K+1) + c - y_K - y_K - 1 + m(x_K+1) + c$$

$$\boxed{d_1 - d_2 = 2m(x_K+1) - 2y_K + 2c - 1}$$

$$\Delta x\,(d_1 - d_2) = \Delta x\left[2\cdot\frac{\Delta y}{\Delta x}(x_K+1) - 2y_K + 2c - 1\right]$$

$$= 2\Delta y\, x_K + 2\Delta y - 2\Delta x\, y_K + 2\Delta x\, c - \Delta x$$

$$\boxed{P_K = 2\Delta y\, x_K + 2\Delta y - 2\Delta x\, y_K + \Delta x(2c-1)}$$

$P_{K+1} = ?$ replace $K \to K+1$ in P_K

$$P_{K+1} = 2\Delta y\, x_{K+1} + 2\Delta y - 2\Delta x\, y_{K+1} + \Delta x(2c-1)$$

$$P_{K+1} - P_K = 2\Delta y\, x_{K+1} + \cancel{2\Delta y} - 2\Delta x\, y_{K+1} + \Delta x\cancel{(2c-1)}$$
$$- 2\Delta y\, x_K - \cancel{2\Delta y} + 2\Delta x\, y_K - \Delta x\cancel{(2c-1)}$$

$$= 2\Delta y\,(x_{K+1} - x_K) - 2\Delta x\,(y_{K+1} - y_K)$$

$$P_{K+1} - P_K = 2\Delta y\,(x_K + 1 - \cancel{x_K}) - 2\Delta x\,(y_{K+1} - y_K)$$

$$\boxed{P_{K+1} = P_K + 2\Delta y - 2\Delta x\,(y_{K+1} - y_K)}$$

Initial point will be (x_K, y_K).

Simplify P_K. $\qquad \because y = mx + c$, $c = y - mx$

$$P_K = 2\Delta y\, x_K + 2\Delta y - 2\Delta x\, y_K + \Delta x\,(2(y - m(x)) - 1)$$

$$= 2\Delta y\, x_K + 2\Delta y - 2\Delta x\, y_K + \Delta x\left(2y - 2\cdot\frac{\Delta y}{\Delta x}(x)\right) - 1$$

$$= 2\Delta y\, x_K + 2\Delta y - 2\Delta x\, y_K + 2\Delta x\, y - 2\Delta y\, x - \Delta x$$

Replace $x \to x_K$ & $y \to y_K$

$$P_K = 2\cancel{\Delta y}\, x_K + 2\Delta y - 2\cancel{\Delta x}\, y_K + 2\cancel{\Delta x}\, y_K - 2\cancel{\Delta y}\, x_K - \Delta x$$

$$\boxed{P_K = 2\Delta y - \Delta x}$$

If $P_K \geq 0$

$x_{K+1} = x_K + 1$

$y_{K+1} = y_K + 1$

$\therefore (x_{K+1}, y_{K+1}) = (x_K + 1, y_K + 1)$

If $P_K < 0$

$x_{K+1} = x_K + 1$

$y_{K+1} = y_K$

$\therefore (x_{K+1}, y_{K+1}) = (x_K + 1, y_K)$

<u>Case 2 : m > 1</u> $P_K = 2\Delta x - \Delta y$

$P_{K+1} = P_K + 2\Delta x - 2\Delta y\,(x_{K+1} - x_K)$

<u>if $P_K > 0$</u> $x_{K+1} = x_K + 1$

$y_{K+1} = y_K + 1$

$\therefore (x_{K+1}, y_{K+1}) = (x_K + 1, y_K + 1)$

If $P_K < 0$

$x_{K+1} = x_K$

$\therefore (x_{K+1}, y_{K+1}) = (x_K, y_K + 1)$

Points to remember:

If $m<1$

$P_K = 2\Delta y - \Delta x$

$P_{K+1} = P_K + 2\Delta y - 2\Delta x\,(y_{K+1} - y_K)$

If $P_K \geq 0 \Rightarrow y_{K+1} = y_K + 1$
Next coordinate (x_K+1, y_K+1)

If $P_K < 0 \Rightarrow y_{K+1} = y_K$
Next Coordinate (x_K+1, y_K)

If $m>1$

$P_K = 2\Delta x - \Delta y$

$P_{K+1} = P_K + 2\Delta x - 2\Delta y\,(x_{K+1} - x_K)$

$P_K \geq 0 \Rightarrow x_{K+1} = x_K + 1$ Next Coordinate (x_K+1, y_K+1)

$P_K < 0 \Rightarrow x_{K+1} = x_K$ Next coordinate (x_K, y_K+1)

Example:

Find the intermediate point between (35,40) & (43,45) by using Bresenham's algorithm.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{45-40}{43-35} = 5/8 = 0.6 < 1$

$m<1$

$P_K \geq 0 \Rightarrow (x_K+1, y_K+1)$

$P_K < 0 \Rightarrow (x_K+1, y_K)$

$\Delta x = (43-35)$

$\Delta x = 8$	$2\Delta x = 16$

$\Delta y = 45-40 = 5$

$\Delta y = 5$	$2\Delta y = 10$

Initial decision parameter

$P_K = 2\Delta y - \Delta x$

$= 10 - 8$

$\boxed{P_K = 2}$

K	(x_K, y_K)	P_K	(x_{K+1}, y_{K+1})	
0	(35, 40)	2	(36, 41)	$P_{K+1} = 2+10-16(41-40) = 2+10-16(1) = -4$
1	(36, 41)	-4	(37, 41)	$P_{K+1} = -4+10-16(0) = 6$
2	(37, 41)	6	(38, 42)	$P_{K+1} = 6+10-16(42-41) = 0$
3	(38, 42)	0	(39, 43)	$P_{K+1} = 0+10-16(1) = -6$
4	(39, 43)	-6	(40, 43)	$P_{K+1} = -6+10-16(0) = 4$
5	(40, 43)	+4	(41, 44)	$P_{K+1} = 4+10-16(44-43) = 0$
6	(41, 44)	-2	(42, 44)	
7	(42, 44)	8	(43, 45)	$P_{K+1} = -2+10-16(44-44) = 8$

The intermediate points between (35, 40) & (43, 45) are (36, 41), (37, 41), (38, 42), (39, 43), (40, 43), (41, 44), (42, 44).

Example 2:

Find the intermediate points between (1,1) (6,7) by using bresenham's algorithm.

Sol:

(1,1) (6,7)

$$m = \frac{7-1}{6-1} = 6/5 = 1.2 > 1$$

m>1

$$P_K = 2\Delta x - \Delta y$$

$$P_{K+1} = P_K + 2\Delta x - 2\Delta y(x_{K+1} - x_K)$$

$$P_K \geq 0 \Rightarrow (x_K+1, y_K+1)$$

$$P_K < 0 \; (x_K, y_K+1)$$

$\Delta x = 5$ | $\Delta y = 6$

$2\Delta x = 10$ | $2\Delta y = 12$

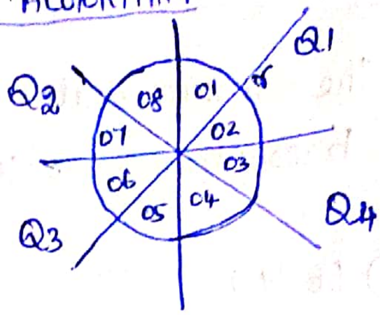
$$P_K = 2\Delta x - \Delta y = 10 - 6$$

$$\boxed{P_K = 4}$$

K	x_K, y_K	P_K	(x_{K+1}, y_{K+1})	
0	(1,1)	4	(2,2)	
1	(2,2)	2	(3,3)	$P_{K+1} = 4+10-12(2-1) = 2$
2	(3,3)	0	(4,4)	$P_{K+1} = 2+10-12(3-2) = 0$
3	(4,4)	-2	(4,5)	$P_{K+1} = 0+10-12(4-3) = -2$
4	(4,5)	8	(5,6)	$P_{K+1} = -2+10-12(0) = 8$
5	(5,6)	6	(6,7)	$P_{K+1} = 8+10-12(5-4) = 6$

The intermediate points between (1,1) and (6,7) are (2,2), (3,3), (4,4), (4,5), (5,6).

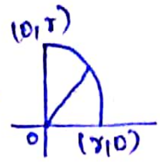
MID POINT CIRCLE DRAWING ALGORITHM



x moves in Unit interval

$y = ?$

$y_K = y_K - 1$ ←



Next coordinates may be (x_K+1, y_K) (x_K+1, y_K-1)

Mid point $= \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$

$$= \frac{x_K+1+x_K+1}{2}, \frac{y_K+y_K-1}{2}$$

$$MP = \left(x_K+1, y_K - 1/2\right)$$

From circle formula, $x^2+y^2=r^2$

$$P_K = (x_K+1)^2 + (y_K - 1/2)^2 - r^2$$

$$P_{K+1} = (x_{K+1}+1)^2 + (y_{K+1} - 1/2)^2 - r^2$$

$$P_{K+1} - P_K = (x_{K+1}+1)^2 + (y_{K+1} - 1/2)^2 - \cancel{r^2} - (x_K+1)^2 - (y_K - 1/2)^2 + \cancel{r^2}$$

$$= ((x_K+1)+1)^2 + (y_{K+1} - 1/2)^2 - (x_K+1)^2 - (y_K - 1/2)^2$$

$$= \cancel{(x_K+1)^2} + 1 + 2(x_K+1) + y_{K+1}^2 + \cancel{1/4} - y_{K+1} - \cancel{(x_K+1)^2}$$
$$- y_K^2 - 1/4 + y_K$$

$$P_{K+1} - P_K = 2(x_K+1) + (y_{K+1}^2 - y_K^2) - (y_{K+1} - y_K) + 1$$

$$P_{K+1} = P_K + 2(x_K+1) + (y_{K+1}^2 - y_K^2) - (y_{K+1} - y_K) + 1$$

Need to calculate mid point between two possible points (x_k+1, y_k) & (x_k+1, y_k-1)

$P_k = 0, r$

Sub $(0, r)$ in P_K

$$P_K = (0+1)^2 + (r - 1/2)^2 - r^2$$

$$= 1^2 + \cancel{r^2} + 1/4 - r - \cancel{r^2}$$

$$= 5/4 - r$$

$$\boxed{P_K = 1 - r}$$

If $P_K \geq 0$ $\quad y_{K+1} = y_K - 1$

Next coordinates (x_K+1, y_K-1)

If $P_K < 0$ $\quad y_{K+1} = y_K$

Next coordinates (x_K+1, y_K)

When $x \geq y$ stop doing the algorithm.

Example:

Draw a circle with $r=8$ by midpoint circle drawing algorithm.

Sol: $r = 8$

$$\boxed{P_k = 1 - r} \qquad P_{K+1} = P_K + 2(x_K+1) + (y_{K+1}^2 - y_K^2) - (y_{K+1} - y_K) + 1$$

$$= 1 - 8$$

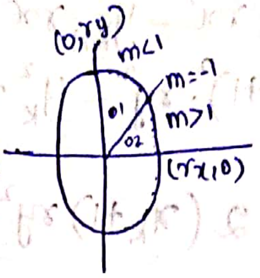
$$\boxed{P_K = -7}$$

K	x_K, y_K	P_K	x_{K+1}, y_{K+1}	
0	(0,8)	-7	(1,8)	
1	(1,8)	-4	(2,8)	$P_{K+1} = -7+2(1)+(0)-(0)+1$ $= -7+2+1 = -4$
2	(2,8)	1	(3,7)	$P_{K+1} = -4+2(2)+(0)-(0)+1$ $= -4+4+1 = 1$
3	(3,7)	-6	(4,7)	$P_{K+1} = 1+2(3)+(49-64)-(7-8)+1$ $= 1+6-15+1+1 = -6$
4	(4,7)	3	(5,6)	$P_{K+1} = -6+2(3+1)+(0)-(0)+1$ $= -6+8+1 = 3$
5	(5,6)	2	(6,5)	$P_{K+1} = 3+2(4+1)+(36-49)-(6-7)+1$ $= 3+10-13+1+1 = 2$

The points are (0,8) (1,8) (2,8) (3,7) (4,7) (5,6) (6,5)

$Q_1(+,+)$	$Q_2(-x, y)$	$Q_3(-x, -y)$	$Q_4(x, -y)$
(0,8)	(0,8)	(0,-8)	(0,-8)
(1,8)	(-1,8)	(-1,-8)	(1,-8)
(2,8)	(-2,8)	(-2,-8)	(2,-8)
(3,7)	(-3,7)	(-3,-7)	(3,-7)
(4,7)	(-4,7)	(-4,-7)	(4,-7)
(5,6)	(-5,6)	(-5,-6)	(5,-6)
(6,5)	(-6,5)	(-6,-5)	(6,-5)
(7,4)	(-7,4)	(-7,-4)	(7,-4)
(7,3)	(-8,2)	(-8,-2)	(8,-2)
(8,2)	(-8,1)	(-8,-1)	(8,-1)
(8,1)	(-8,0)	(-8,0)	(8,0)
(8,0)			

MID POINT ELLIPSE DRAWING ALGORITHM



m<1

x chagesin unit interval

y = ?

$y_{k+1} = y_k / y_k - 1$

Next coordinates (x_k+1, y_k) (x_k+1, y_k-1)

m>1

y changes in Unit interval

x = ?

$x_{k+1} = x_k / x_k+1$ [left to right, so x will be increased]

Next coordinates [(x_k, y_k-1) (x_k+1, y_k-1)]

From Ellipse formula,

$$\frac{x^2}{rx^2} + \frac{y^2}{ry^2} = 1$$

$$x^2 ry^2 + y^2 rx^2 - rx^2 ry^2 = 0$$

$m = \frac{dy}{dx}$

$\boxed{2ry^2 x \geq 2 rx^2 y}$ Condition to stop octect 1 & Start octect 2.

Octect 1

$$x^2 ry^2 + y^2 rx^2 - rx^2 ry^2 = 0 \longrightarrow (1)$$

m<1 ⇒ x chg in UI, $y \rightarrow y_k / y_k - 1$

(x_k+1, y_k) (x_k+1, y_k-1)

Mid point = $(x_k+1, y_k - \frac{1}{2})$

Apply mid point in equation (1)

$$P1_k = (x_k+1)^2 ry^2 + (y_k - \tfrac{1}{2})^2 rx^2 - rx^2 ry^2$$

$$P1_{k+1} = (x_{k+1}+1)^2 ry^2 + (y_{k+1} - \tfrac{1}{2})^2 rx^2 - rx^2 ry^2$$

$$= ((x_k+1)+1)^2 ry^2 + (y_{k+1} - \tfrac{1}{2})^2 rx^2 - rx^2 ry^2$$

$$P_{1K+1} - P_{1K} = (\cancel{x_K+1})^2 ry^2 + ry^2 + 2(x_K+1)ry^2 + y_{K+1}^2 \cdot rx^2$$
$$+ \cancel{1/4}\, rx^2 - y_{K+1}\, rx^2 - \cancel{rx^2}\, ry^2$$
$$- (\cancel{x_K}+1)^2 \cdot ry^2 - y_K^2\, rx^2 - 1/4 \cancel{rx^2} + y_K rx^2 + \cancel{rx^2 ry^2}$$

$$P_{1K+1} = P_{1K} + ry^2 + 2(x_K+1)ry^2 + rx^2(y_{K+1}^2 - y_K^2) - rx^2(y_{K+1} - y_K)$$

Substitute $(0, ry)$ in P_{1K},

$$P_{1K} = 1^2 \cdot ry^2 + (ry - 1/2)^2\, rx^2 - rx^2 \cdot ry^2$$
$$= ry^2 + \cancel{ry^2\, rx^2} + \frac{rx^2}{4} - ry\, rx^2 - \cancel{rx^2\, ry^2}$$

$$\boxed{P_{1K} = ry^2 + \frac{rx^2}{4} - ry\, rx^2}$$

If $P_{1K} \geq 0 \Rightarrow x_K+1, y_K-1$

If $P_{1K} < 0 \Rightarrow x_K+1, y_K$

<u>P_{2K} for second octant</u>

<u>$m > 1$</u> y-xchgs in U.I

$x \rightarrow x_K+1 / x_K$

Next coordinate (x_K, y_K-1) (x_K+1, y_K-1)

Mid point $= (x_K + 1/2, y_K - 1)$

Apply mid point in equ ①

$$P_{2K} = (x_K + 1/2)^2 ry^2 + (y_K-1)^2 rx^2 - rx^2 \cdot ry^2$$

$$P_{2K+1} = (x_{K+1} + 1/2)^2 ry^2 + (y_{K+1} - 1)^2 rx^2 - rx^2 \cdot ry^2$$
$$= (x_{K+1} + 1/2)^2 ry^2 + ((y_K - 1) - 1)^2 \cdot rx^2 - rx^2 \cdot ry^2$$

$$P_{2K+1} - P_{2K} = x_{K+1}^2\, ry^2 + \cancel{\frac{ry^2}{4}} + x_{K+1} \cdot ry^2 + \cancel{(y_K-1)^2} \cdot rx^2$$
$$+ rx^2 - 2(y_K - 1)rx^2 - \cancel{rx^2}\, ry^2$$
$$- x_K^2 \cdot ry^2 - \cancel{\frac{ry^2}{4}} - x_K ry^2 - \cancel{(y_K-1)^2 rx^2} + \cancel{rx^2 ry^2}$$

$$P_{2k+1} = P_{2k} + rx^2 - 2rx^2(y_k - 1) + ry^2(x_{k+1}^2 - x_k^2) + ry^2(x_{k+1} - x_k)$$

If $P_{2k} \geq 0 \Rightarrow (x_k, y_k - 1)$

If $P_{2k} < 0 \Rightarrow (x_k + 1, y_k - 1)$

Points to remember

Region - 1 $(m<1)$

$P_{1k} \geq 0$ $NC(x_k+1, y_k-1)$

$P_{1k} < 0$ $NC(x_k+1, y_k)$

$$\boxed{2ry^2 x_{k+1} \geq 2rx^2 y_{k+1}}$$

Region - 2 $(m>1)$

$P_{2k} \geq 0$ $NC(x_k, y_k-1)$

$P_{2k} < 0$ $NC(x_k+1, y_k-1)$

2D TRANSFORMATION

Moving an object from one position to another is called as Transformation.

(i) Translation (tx, ty)

(ii) Scaling (sx, sy)

(iii) Rotation (clock wise, Anti clock wise)

(iv) Shearing (X-Shear, Y-Shear)

(v) Reflection (X-Axis, Y-Axis, origin, $y=x$)

Translation

→ Changing the position of an object

→ 2 translation parameter (tx, ty)

$P(x, y)$ new point before translation

$P'(x', y')$ point after translation.

$$\boxed{\begin{aligned} x' &= x + tx \\ y' &= y + tx \end{aligned}}$$

$x' = x + tx$

$y' = y + tx$

$p' = p + T \quad [x' \ y'] = [x \ y] + [tx \ ty]$

(or)

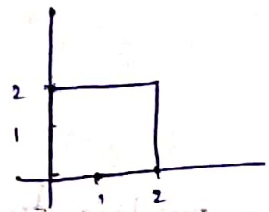
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} tx \\ ty \end{bmatrix}$$

Example:

Consider a square with (0,0) (2,0) (0,2) (2,2) and perform translation with respect to x as 2 units, with respect to y as 3 units.

Sol: Square (0,0) (2,0) (0,2) (2,2)

$tx = 2 \quad ty = 3$



Before translation.

(0,0)

$x' = tx + x$

$= 0 + 2 = 2$

$y' = 0 + 3 = 3$

$(0,0) \rightarrow (2,3)$

(0,2)

$x' = (0+2) = 2$

$y' = (2+3) = 5$

$(0,2) \rightarrow (2,5)$

(2,0)

$x' = 2 + 2 = 4$

$y' = 0 + 3 = 3$

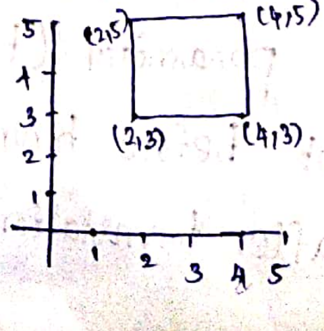
$(2,0) \rightarrow (4,3)$

(2,2)

$x' = 2 + 2 = 4$

$y' = 2 + 3 = 5$

$(2,2) \rightarrow (4,5)$



After translation.

Scaling

→ Resizing the object
→ 2 Scaling parameter S_x, S_y
→ If S_x, S_y are in between 0 & 1 point is closer to origin. so it will decrease.
→ If S_x, S_y are greater than 1 point is away from origin. so it will increase in size.
→ If S_x, S_y are equal, scaling will be done uniform.
→ $P = (x, y)$ before scaling
$P' = (x', y')$ After Scaling

$$\boxed{\begin{aligned} x' &= x \cdot S_x \\ y' &= y \cdot S_y \end{aligned}} \qquad [x' \; y'] = [x \; y] \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$$

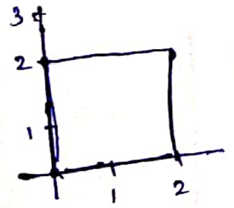
Points to remember

$S_x \neq S_y$ - Change in shape
$S_x = S_y$ - No change in shape
S_x & $S_y < 1$:- Size decrease
S_x & $S_y > 1$ - Size increase.
$x' = x \cdot S_x$
$y' = y \cdot S_y$

Example:

Consider a square with (0,0), (2,0), (0,2) (2,2) and perform ~~translation~~ ~~opar~~ scaling. where $S_x = 2$ & $S_y = 3$

Sol: (0,0) (2,0) (0,2) (2,2)
$S_x = 2$, $S_y = 3$ ⟶ $S_x, S_y > 1$ - size increase.



Before Scaling

(0,0)
$x' = 0 \times 2 = 0$
$y' = 0 \times 3 = 0$
$(0,0) \rightarrow (0,0)$

(0,2)

$x' = 0 \times 2 = 0$

$y' = 2 \times 3 = 6$

$(0,2) \rightarrow (0,6)$

(2,0)

$x' = 2 \times 2 = 4$

$y' = 0 \times 3 = 0$

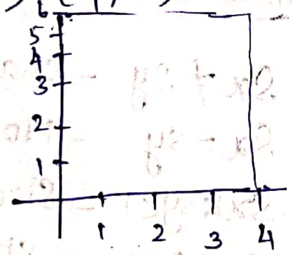
$(2,0) \rightarrow (4,0)$

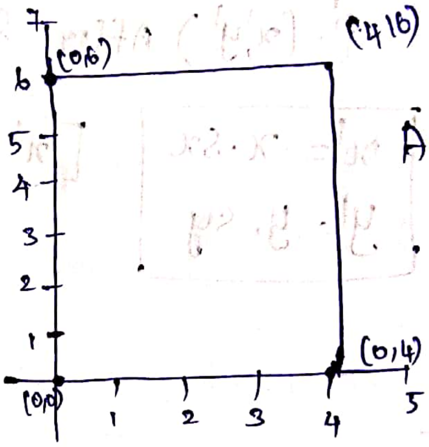
(2,2)

$x' = 2 \times 2 = 4$

$y' = 2 \times 3 = 6$

$(2,2) \rightarrow (4,6)$





After Scaling

Example 2:

square (0,0) (2,0) (0,2) (2,2)

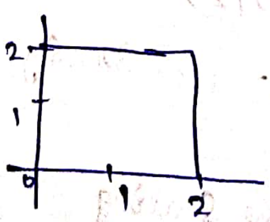
$S_x = 0.5$ $S_y = 0.5$ ← $S_x, S_y < 1$ size decrease

$(0,0) \rightarrow (0,0)$

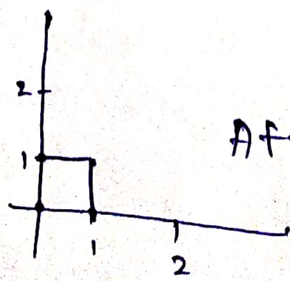
$(2,0) \rightarrow (1,0)$

$(0,2) \rightarrow (0,1)$

$(2,2) \rightarrow (1,1)$

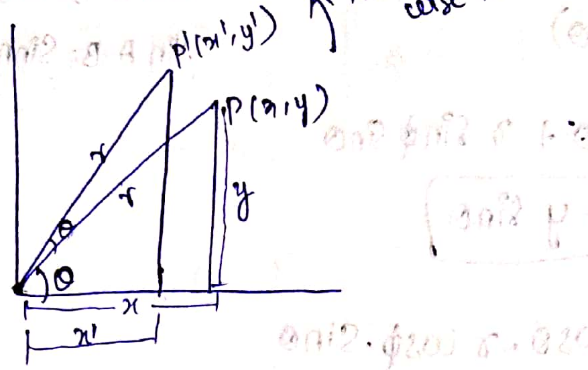


before scaling



After Scaling

Rotation:



$\because \cos\phi = \frac{\text{Adj.Side}}{\text{Hyp}} = x/r$

$x = r\cos\phi$

$\sin\phi = \frac{\text{opp side}}{\text{Hyp}} = \frac{y}{r}$

$y = r\sin\phi$

→ New angle after rotation

P to $P' = (\phi + \theta)$

$\cos(\phi+\theta) = \frac{x'}{r}$

$x' = r\cos(\phi+\theta)$

$\sin(\phi+\theta) = \frac{y'}{r}$

$y' = r.\sin(\phi+\theta)$

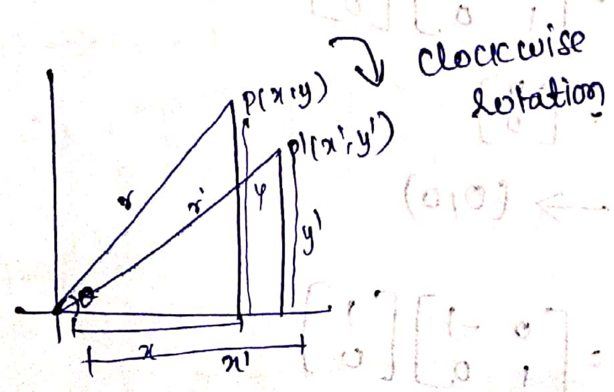
$x' = \underbrace{r.\cos\phi}_{x} \cos\theta - \underbrace{r.\sin\phi}_{y} \sin\theta$

$\boxed{x' = x.\cos\theta - y\sin\theta}$

$y' = \underbrace{r.\sin\phi}_{y} \cos\theta + \underbrace{r.\cos\phi}_{x} \sin\theta$

$\boxed{y' = x\sin\theta + y\cos\theta}$

$$[x' \ y'] = [x \ \ y] \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$



New angle after rotation P to P' $\Big\} = (\phi - \theta)$

$x' = r.\cos(\phi - \theta)$

$y' = r.\sin(\phi - \theta)$

$\cos A-B = \cos A \cos B + \sin A \sin B$

$\sin A-B = \sin A \cos B - \cos A \sin B$

$x' = r.\cos\phi \cos\theta + r.\sin\phi \sin\theta$

$\boxed{x' = x\cos\theta + y\sin\theta}$

$y' = r.\sin\phi\cos\theta - r.\cos\phi.\sin\theta$

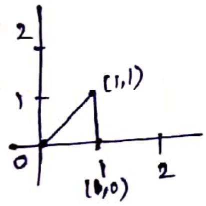
$\boxed{y' = y\cos\theta - x\sin\theta}$

$$[x' \ y'] = [x \ y] \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

<u>Example:</u>

Consider a triangle with (0,0) (1,0) (1,1) and rotate it with $\theta = 90°$ in anti clock wise.

<u>Sol:</u>



Before rotation.

$\sin 90° = 1$

$\cos 90° = 0$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

<u>(0,0)</u>

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$(0,0) \rightarrow (0,0)$

<u>(1,0)</u>

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

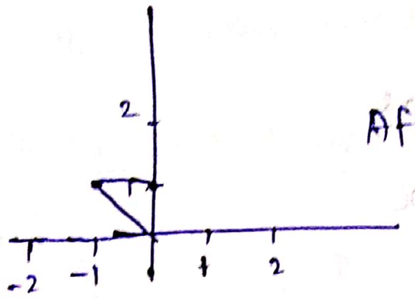
$(1,0) \rightarrow (0,1)$

$\underline{(1,1)}$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$(1,1) \rightarrow (-1,1)$

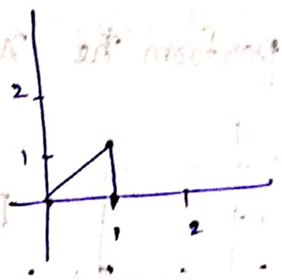


After rotation

Example:

$(0,0)$ $(1,0)$ $(1,1)$: $\theta = 90^\circ$ in clock wise

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



$\underline{(0,0)}$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

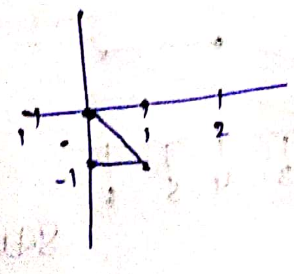
$(0,0) \rightarrow (0,0)$

$\underline{(1,0)}$ $$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$(1,0) \rightarrow (0,-1)$

$\underline{(1,1)}$ $$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$(1,1) \rightarrow (1,-1)$



After rotation.

Shearing

→ Tilting the image

→ one coordinate fixed and I will make changes in the other co-ordinate.

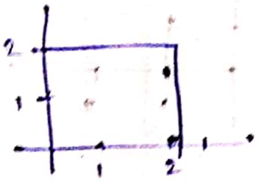
(i) x - Shear
(ii) y - Shear } Skewing.

x - Shear	y-shear
$x' = x + Sh_x \cdot y$	$y' = y + Sh_y \cdot x$
$y' = y$	$x' = x$

Example:

Consider a square with (0,0), (0,2), (2,0), (2,2) and perform the x-Shear with $Sh_x = 2$ units.

Sol:



Before Shearing.

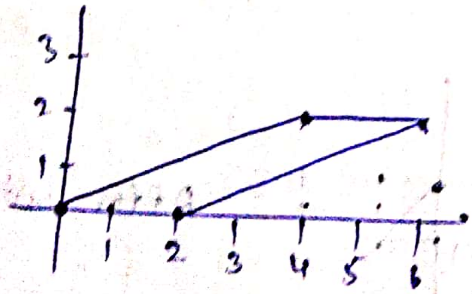
(0,0) (0,2) (2,0) (2,2) $Sh_x = 2$ units.

(0,0): $y' = 0$ $x' = 0 + 2 \cdot 0 \rightarrow x' = 0 \Rightarrow (0,0)$

(0,2): $y' = 2$ $x' = 0 + 2 \cdot 2 \rightarrow x' = 4 \Rightarrow (4,2)$

(2,0): $y' = 0$ $x' = 2 + 2 \cdot 0 \rightarrow x' = 2 \Rightarrow (2,0)$

(2,2): $y' = 2$ $x' = 2 + 2 \cdot 2 \rightarrow x' = 6 \Rightarrow (6,2)$



After Shearing

Square became parallelogram

Reflection:

→ Getting an mirror image

→ Rotating object in 180°

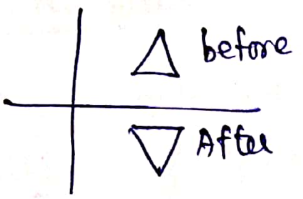
(i) Reflect with respect to X-Axis

(ii) Reflect with Respect to Y-axis

(iii) Reflect with respect to origin

(iv) Reflection towards $y=x$

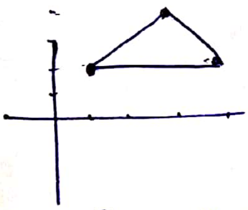
(i) Reflection WRT X-axis



$$\begin{bmatrix} x' \\ y' \\ h' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Example:

(2,2) (4,2) (3,4) perform Reflection with respect to x-axis.

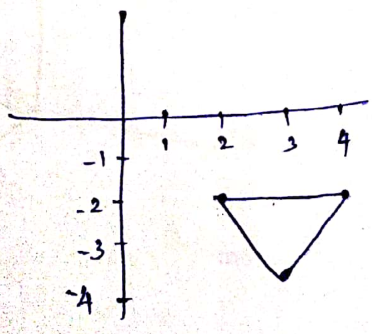


Before reflection

$$\begin{bmatrix} x' \\ y' \\ h' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \quad (2,2) \rightarrow (2,-2)$$

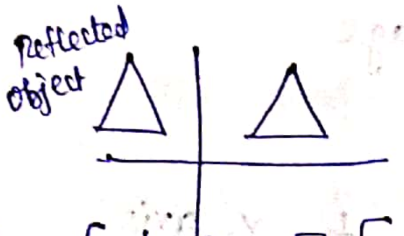
$$\begin{bmatrix} x' \\ y' \\ h' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix} \quad (4,2) \rightarrow (4,-2)$$

$$\begin{bmatrix} x' \\ y' \\ h' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix} \quad (3,4) \rightarrow (3,-4)$$



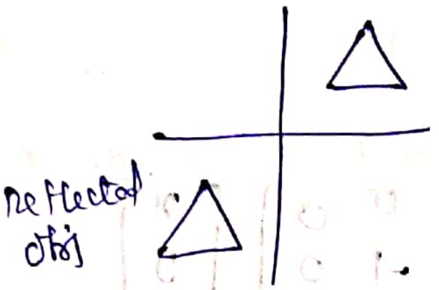
After Reflection

(ii) Reflection with respect to y-axis



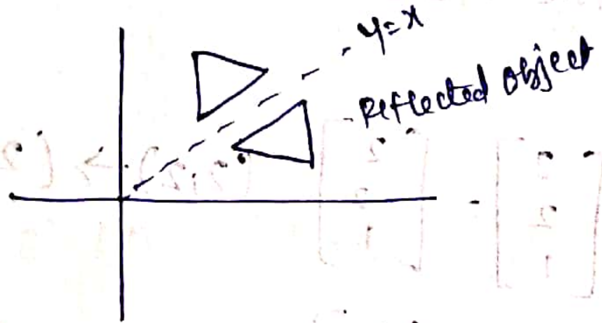
$$\begin{bmatrix} x' \\ y' \\ h' \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

(iii) Reflection with respect to origin



$$\begin{bmatrix} x' \\ y' \\ h' \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ h \end{bmatrix}$$

(iv) Reflection towards $y=x$



$$\begin{bmatrix} x' \\ y' \\ h' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

3-D Transformation

Moving an object from one position to another position is called as transformation.

(i) Sc Translation (t_x, t_y, t_z)

(ii) Scaling (s_x, s_y, s_z)

(iii) Rotation (x Roll, y Roll, z Roll)

(iv) Shearing $(x' = x + sh_x \cdot y,\ y' = y + sh_y \cdot x,\ z' = z + sh_z \cdot x)$

(v) Reflection - (x plane, y plane, z plane)

Translation:

$$x' = x + t_x$$
$$y' = y + t_y$$
$$z' = z + t_z$$

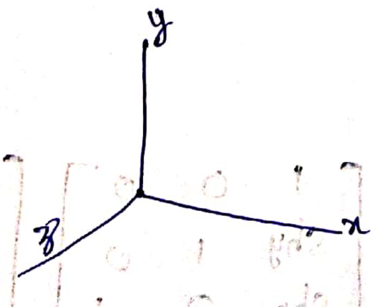
$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} + \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Scaling:

$$x' = x . s_x$$
$$y' = y . s_x$$
$$z' = z . s_z$$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & s_z \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Rotation:



→ x Roll

→ y Roll

→ z Roll

Z Roll

$z' = z$

$x' = x\cos\theta - y\sin\theta$

$y' = x\sin\theta + y\cos\theta$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

x Roll

$x' = x$

$y' = y\cos\theta - z\sin\theta$

$z' = y\sin\theta + z\cos\theta$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

y-Roll

$y' = y$

$z' = z\cos\theta - x\sin\theta$

$x' = z\sin\theta + x\cos\theta$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} -\sin\theta & 0 & \cos\theta \\ 0 & 1 & 0 \\ \cos\theta & 0 & \sin\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Shearing

z-Shear $\boxed{Shx, Shy}$

$z' = z$

$x' = x + z.Shx$

$y' = y + z.Shy$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & Shx \\ 0 & 1 & Shy \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

x-Shear Shy, Shz

$x' = x$

$y' = y + x.Shy$

$z' = z + x.Shz$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ Shy & 1 & 0 \\ Shz & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Y-shear (sh_x, sh_z)

$y' = y$

$x' = x + y \cdot sh_x$

$z' = z + y \cdot sh_z$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & sh_x & 0 \\ 0 & 1 & 0 \\ 0 & sh_z & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Reflection:

(i) Reflection with respect to "xy Plane"

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

(ii) Reflection with respect to "yz plane"

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

(iii) Reflection with respect to zx plane

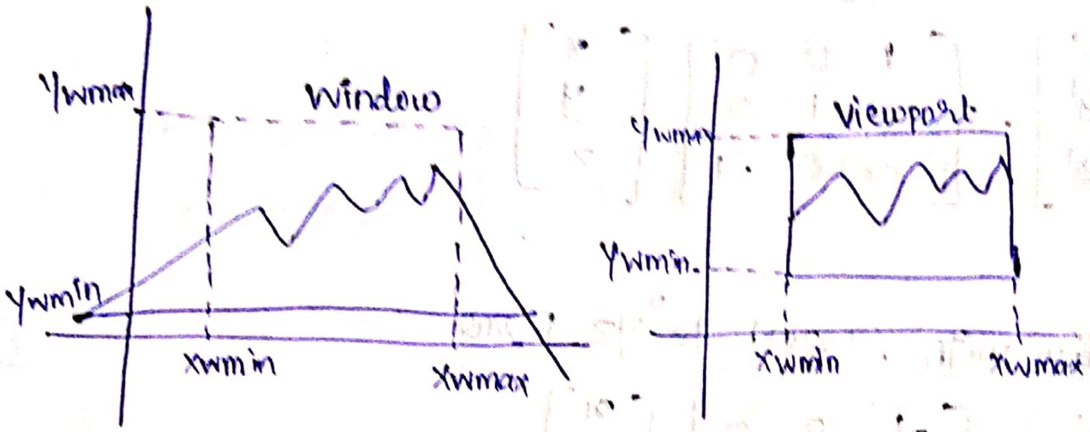
$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

CLIPPING.

WINDOW & VIEWPORT

A world co-ordinate area selected for display is called a window.

An area on a display device to which a window is mapped is called a viewport.



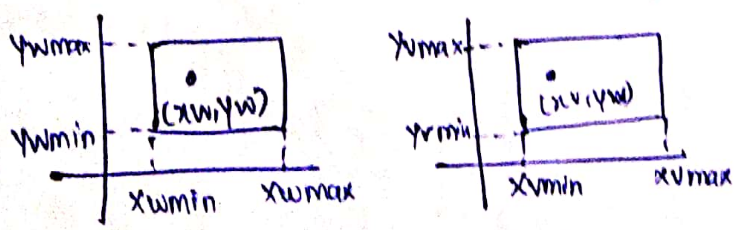
Window to viewport coordinate transformation:

A point at position (xw, yw) in the window is mapped into position (xv, yv) in the associated viewport.

To maintain the same relative placement in the viewport as in the window, we require that,

$$\frac{xv - xvmin}{xvmax - xvmin} = \frac{xw - xwmin}{xwmax - xwmin}$$

$$\frac{yv - yvmin}{yvmax - yvmin} = \frac{yw - ywmin}{ywmax \cdot ywmin}$$



$$xv - xvmin = (xvmax - xvmin)\underbrace{\left(\frac{xw - xwmin}{xwmax - xwmin}\right)}_{\text{Scaling factor } sx}$$

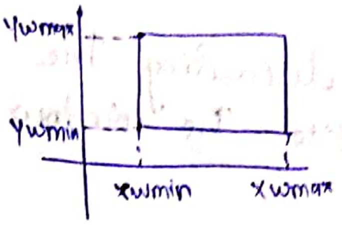
$$xv - xvmin = (xw - xwmin) \cdot sx$$

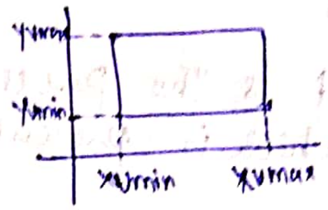
$$\boxed{xv = xvmin + (xw - xwmin) \cdot sx}$$

$$\boxed{yv = yvmin + (yw - ywmin) \cdot sy}$$

sx & sy are Scaling factors

Example:





$xwmin = 20$	$xvmin = 30$
$xwmax = 80$	$xvmax = 60$
$ywmin = 40$	$yvmin = 40$
$ywmax = 80$	$yvmax = 60.$
$(xw, yw) = (30, 80)$	$(xv, yv) = ?$

Sol:

$$\frac{xv - 30}{60 - 30} = \frac{30 - 20}{80 - 20}$$

$$xv - 30 = \left(\frac{10}{\cancel{60}_{2}}\right)\cancel{30}$$

$$xv - 30 = 5$$

$$\boxed{xv = 35}$$

$$\frac{yv - 40}{60 - 40} = \frac{80 - 40}{80 - 40}$$

$$yv - 40 = (60 - 40) \cdot 1$$

$$yv - 40 = 20$$

$$\boxed{yv = 60}$$

$$\therefore (xv, yv) = (35, 60)$$

CLIPPING:

Clipping is a process of discarding the object which is present outside the window.

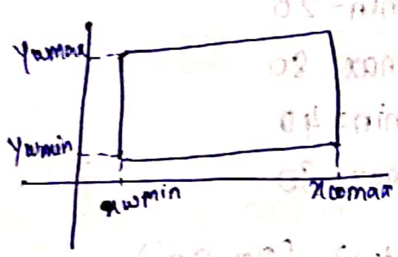
(i) Point clipping
(ii) Line clipping
(iii) Polygon clipping

Point clipping:

It is the process of discarding the point (x,y) which is present outside the window.

Condition

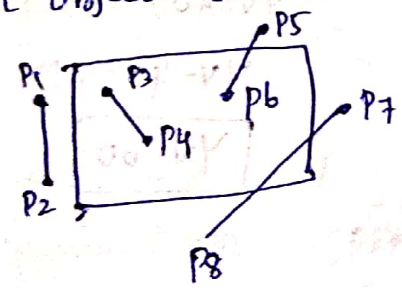
$x_{wmin} \le x \le x_{wmax}$
$y_{wmin} \le y \le y_{wmax}$



$\Rightarrow x_{wmin} \le x \quad y_{wmin} \le x$
$x_{wmax} \ge x \quad y_{wmax} \ge x.$

Line clipping:

Discarding (clipping) the excess line from the given object which is present outside the window.

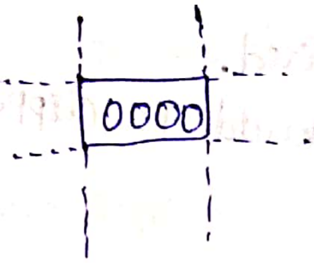


where $P_1P_2 \rightarrow$ Rejected (clipped)
$P_3P_4 \rightarrow$ Accepted
$P_5P_6 \rightarrow$ clipping required
$P_7P_8 \rightarrow$ clipping required.

The line clipping can be done using Cohen Sutherland algorithm.

In which The window size is always be 0000.

The neighbour nodes of window must be defined by The algorithm.



The neighbour nodes value can be filled by using ABRL (Above Below Right Left) property. with respect to window.

Above - ABRL - 1000 (A B R L)

Below - ABRL - 0100 (A B R L)

Right - ABRL - 0010

Left - ABRL - 0001

Above left - ABRL - 1001

Below left - ABRL - 0101

Above right - ABRL - 1010

Below right - ABRL - 0110

1001	1000	1010
0001	0000	0010
0101	0100	0110

P_1 & P_2 ⇒ $P_1 \rightarrow 0001$ - Non zero

$P_2 \rightarrow 0001$ - Non zero

0001 - Non zero

P_1 & P_2 are Rejected.

P_3 & P_4

⇒ $P_3 = 0000$ - zero

$P_4 = 0000$ - zero

0000 - zero

P_3 & P_4 are Accepted.

P_5 & P_6

$P_5 = 1000$ - Non zero

$P_6 = 0000$ - zero

So, here P_5 value is non zero & P_6 value is zero, So clipping is required.

The P_5 point Should be clipped and changed as P_5'.

$P_5' = 0000$ zero

$P_6 = 0000$ zero

0000 zero

P_5' & P_6 are accepted.

P_7 & P_8

$P_8 = 0100$ Non zero

$P_7 = 0010$ non zero

P_8, P_7 are non zero value, So here clipping is required.

$P_8' = 0000$ zero

$P_7' = 0000$ zero

0000 zero

P_8', P_7' are accepted.

polygon clipping

Discarding the polygon which is present outside the window.

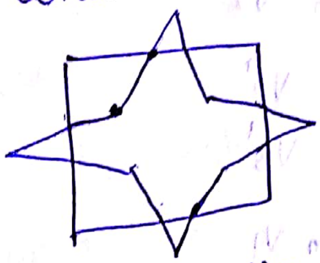
polygon clipping can be achieved using 'Sutherland hodgman algorithm.

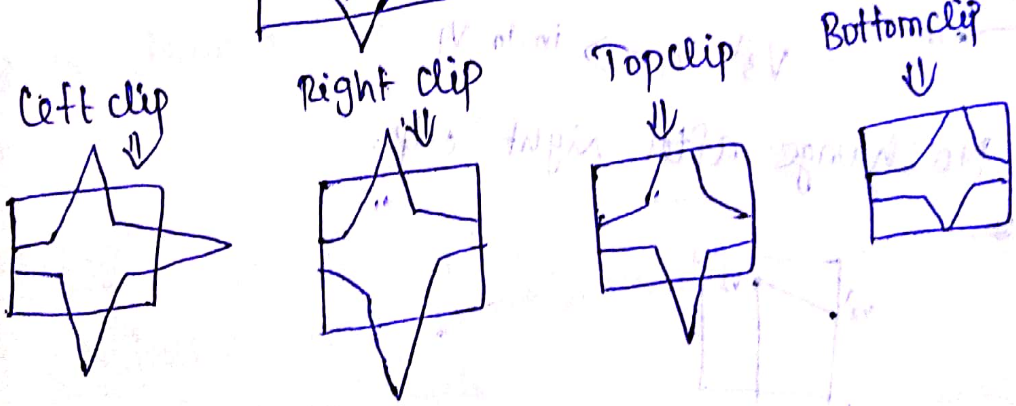
In Sutherland hodgman we need to do 4 types of clipping.

(i) Left clip (ii) Right clip (iii) Top clip (iv) Bottom clip.

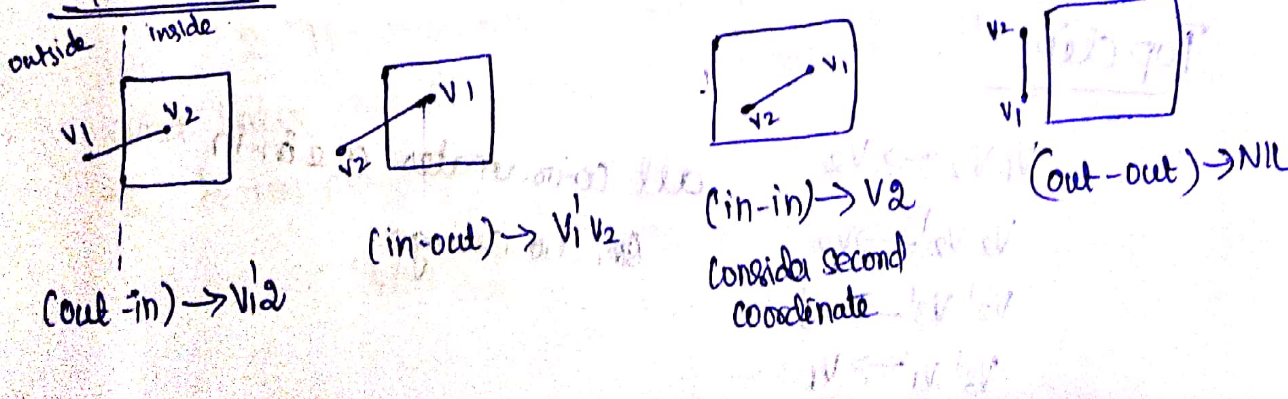
The o/p of left clip will be i/p of right clip.
The o/p of right clip will be i/p of top clip.
The o/p of Top clip will be i/p of Bottom clip.
The o/p of bottom clip will be the finalized.
The o/p of

Left clip Consider the polygon diagram,

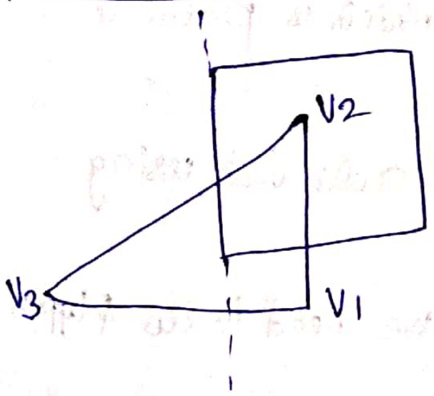




4 cases :



Example :



left clip

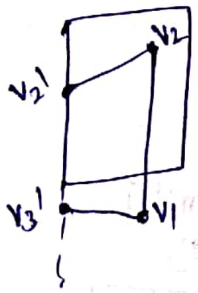
Edges

$V_1 V_2$ – in & in – V_2

$V_2 V_3$ – in & out – V_2'

$V_3 V_1$ – out & in – $(V_3' V_1)$

Right clip



$V_1 V_2 \rightarrow$ in-in V_2

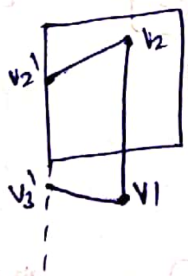
$V_2 - V_2' \rightarrow$ in-in V_2'

$V_2' - V_3' \rightarrow$ in-in V_3'

$V_3' - V_1 \rightarrow$ in-in V_1

No change after right clip.

so,



Top clip:

$V_1 V_2 \rightarrow V_2$

$V_2 V_2' \rightarrow V_2'$

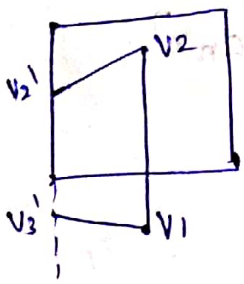
$V_2' V_3' \rightarrow V_3'$

$V_3' V_1 \rightarrow V_1$

all coordinates are in-in

so, no change.

Bottom clip:

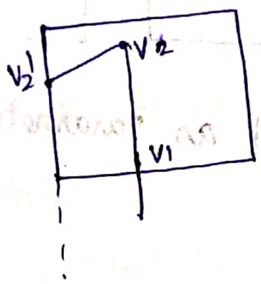


$V_1 V_2 \rightarrow$ (out-in) $\Rightarrow V_1' V_2$

$V_2 V_2' \rightarrow$ (in-in) $\Rightarrow V_2'$

$V_2' V_3' \rightarrow$ (in-out) $\rightarrow V_2''$

$V_3' V_1$ — NLL.



Bezier Curve

The parametric equation which is used to draw a smooth line is called as a Bezier curve.

Explanation:

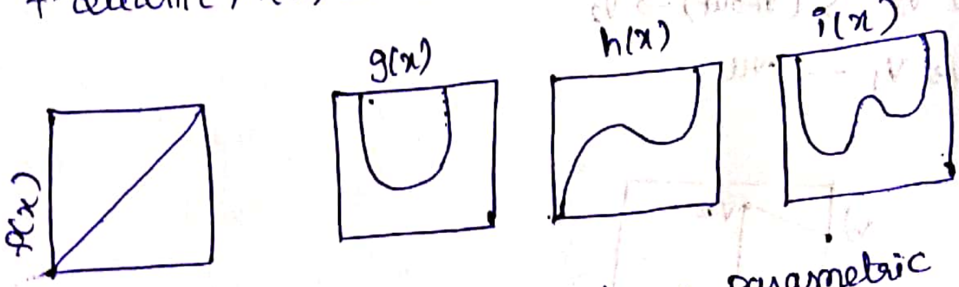
1. Bezier curve selection can be fitted to any number of control points.

2. The number of control points to be appropriated and their relative position to determine the degree of the bezier polynomial.

$\therefore$ control points $= n+1$

3. The Polynomial equation of bezier curve is $\sum_{i=0}^{n} a_i x_n = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n$

1. Linear, $f(x) = x+1$
2. Quadratic, $g(x) = x^2 + x + 1$
3. Cubic, $h(x) = x^3 + x^2 + x + 1$
4. Quartic, $i(x) = x^4 + x^3 + x^2 + x + 1$



4. Bezier Curve can be expressed as parametric equations are,

$$x = (1-t)x_0 + t x_1 \rightarrow ①$$

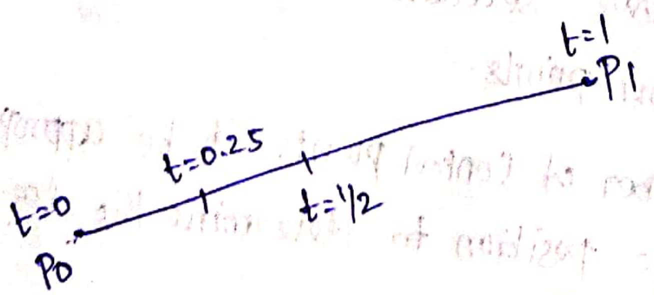
$$y = (1-t)y_0 + t y_1 \rightarrow ②$$

Solve 1 & 2

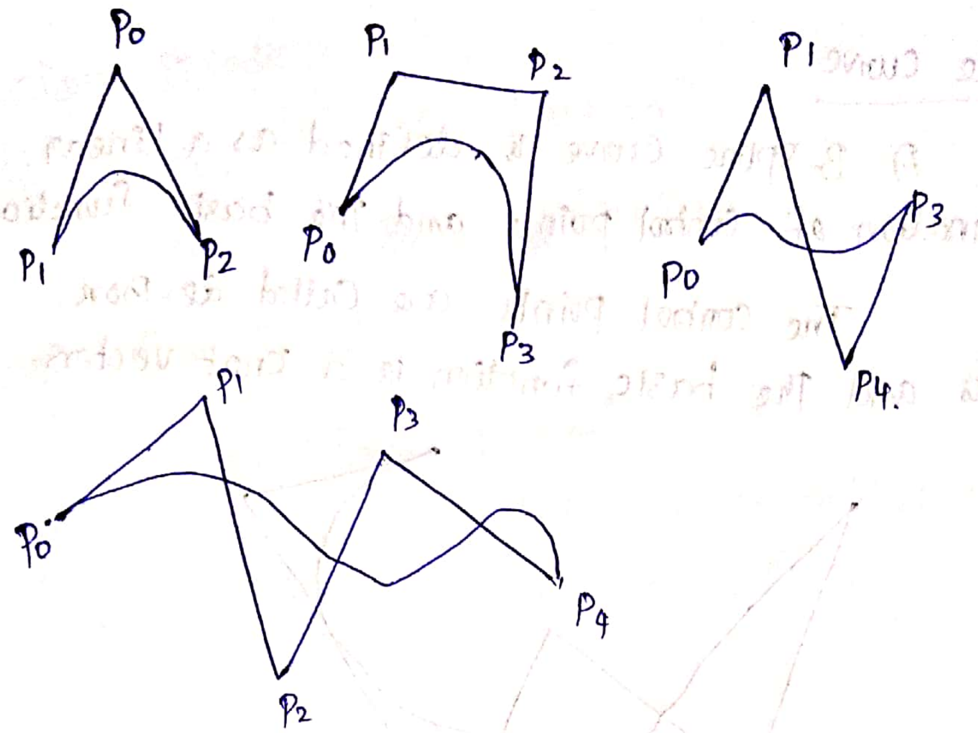
$$p = (1-t)P_0 + t P_1$$

where, $p = (x, y)$

$P_0 = (x_0, y_0)$ & $P_1 = (x_1, y_1)$



5. Bezier is a polynomial of a degree one less than The number of control points used are 3 points generates a parabola. 4 points generate a cubic curve and so forth.



Properties:

1. It is always passes through the first & last Control points.

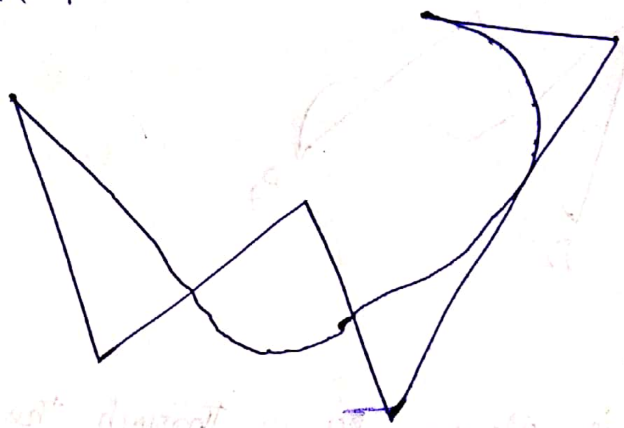
$\therefore P(0) = P_0$

$\therefore P(n) = P_n$

2. It lies with in the convex hull of the control points.

B-Spline Curve

A B-Spline curve is defined as a linear combination of control points and the basic functions. The control points are called de-Boor points and the basic function is a knot vectors.



properties:

1. The polynomial curve has a degree (d-1) and d-2 continuously over the range of u.

2. For n+1, control points the curve is described with n+1 blending function.

3. Each blending function $B_{k}d$ is defined over d subintervals of the total range of u.

4. Each section of spline curve is influenced by 'd' control points

5. Any one control point can effect the shape of the curve.

Types:

(i) Uniform periodic

(ii) Non-Uniform periodic.

Uniform periodic:

$$B_{k,d}(u) = B_{k+1}(u+\Delta u)$$

$$= B_{k+2,d}(u+2\Delta u)$$

Non Uniform Periodic

$$\therefore B_{0,3}(u) = \begin{cases} \frac{1}{2}u^2; & 0 \le u \le 1 \\ \frac{1}{2}u(2-u)+\frac{1}{2}(u-1)(3-u); & 1 \le u \le 2 \\ \frac{1}{2}(3-u)^2; & 2 \le u \le 3. \end{cases}$$