2.6. GRAPHICAL SOLUTION OF EQUATIONS

Let the equation be $f(x) = 0$.

(*i*) *Find the interval* (a, b) *in which a root of* $f(x) = 0$ *lies.*

(*ii*) *Write the equation* $f(x) = 0$ *as* $\phi(x) = \psi(x)$ *where* $\psi(x)$ *contains only terms in* x *and the constants.*

(*iii*) *Draw the graphs of* $y = \phi(x)$ *and* $y = \psi(x)$ *on the same scale and with respect to the same axes.*

(*iv*) *Read the abscissae of the points of intersection of the curves* $y = \phi(x)$ *and* $y = \psi(x)$. *These are the initial approximations to the roots of* $f(x) = 0$.

Sometimes it may not be convenient to write the given equation $f(x) = 0$ in the form $\phi(x) = \psi(x)$. In such cases, we proceed as follows :

(*i*) Form a table for the value of x and $y = f(x)$ directly.

(*ii*) Plot these points and pass a smooth curve through them.

(*iii*) Read the abscissae of the points where this curve cuts the x-axis.

These are rough approximations to the roots of $f(x) = 0$.

Example 2.12. *Find graphically an approximate value of the root of the equation* $3 - x = e^{x-1}$.

Sol. Let
$$f(x) = e^{x-1} + x - 3 = 0 \qquad \ldots(i)$$
$$f(1) = 1 + 1 - 3 = -\text{ ve} \quad \text{and} \quad f(2) = e + 2 - 3 = 2.718 - 1 = +\text{ ve}$$

$\therefore$ A root of (*i*) lies between $x = 1$ and $x = 2$.

Let us write (*i*) as $e^{x-1} = 3 - x$.

The abscissa of the point of intersection of the curves
$$y = e^{x-1} \quad \ldots(ii) \qquad \text{and} \qquad y = 3 - x \quad \ldots(iii)$$
will give the required root.

To plot (*ii*), we form the following table of values :

x	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
$y = e^{x-1}$	1.11	1.22	1.35	1.49	1.65	1.82	2.01	2.23	2.46	2.72

Taking the origin at (1, 1) and 1 small unit along either axis = 0.02, we plot these points and pass a smooth curve through them as shown in Fig. 2.2.

To draw the line (*iii*), we join the points (1, 2) and (2, 1) on the same scale and with the same axes.

From the figure, we get the required root to be $x = 1.44$ nearly.

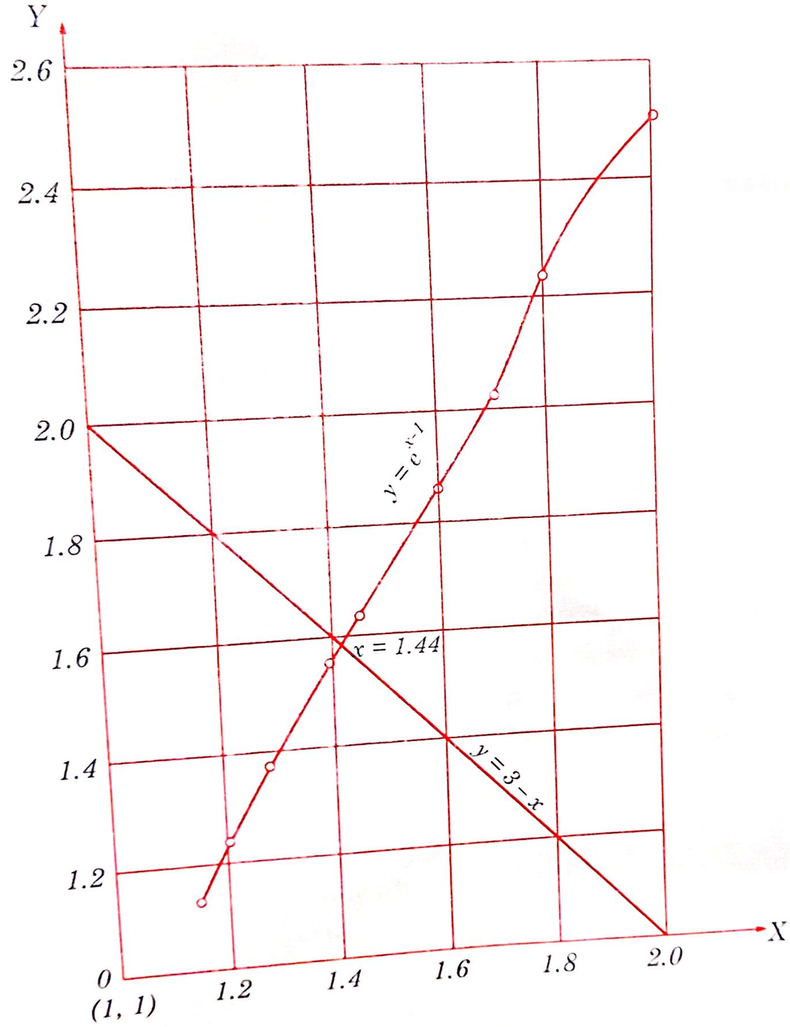

Fig. 2.2

2.13. *Obtain graphically an approximate value of the root of $x = \sin x + \pi/2$.*

Fig. 2.2

Example 2.13. *Obtain graphically an approximate value of the root of $x = \sin x + \pi/2$.*

Sol. Let us write the given equation as $\sin x = x - \pi/2$.

The abscissa of the point of intersection of the curve $y = \sin x$ and the line $y = x - \pi/2$ will give a rough estimate of the root.

To draw the curve $y = \sin x$, we form the following table :

x	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
y	0	0.71	1	0.71	0

Taking 1 unit along either axis = $\pi/4$ = 0.8 nearly, we plot the curve as shown in Fig. 2.3.

Also we draw the line $y = x - \pi/2$ to the same scale and with the same axes.
From the graph, we get $x = 2.3$ radians approximately.

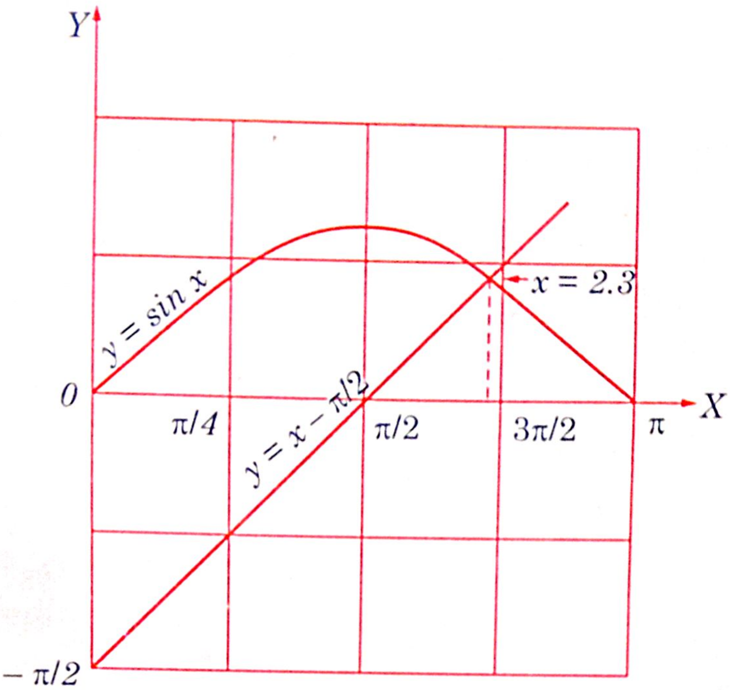

Fig. 2.3

Example 2.14. *Obtain graphically an approximate value of the lowest root of* $\cos x \cosh x = -1$.

Sol. Let $f(x) = \cos x \cosh x + 1 = 0$ ...(*i*)

$\because$ $f(0) = +$ ve, $f(\pi/2) = +$ ve and $\pi = -$ ve.

$\therefore$ The lowest root of (*i*) lies between $x = \pi/2$ and $x = \pi$.

Let us write (*i*) as $\cos x = -\text{sech } x$.

The abscissa of the point of intersection of the curves

$y = \cos x$ ...(*ii*) and $y = -\text{sech } x$ ...(*iii*)

will give the required root.

To draw (ii), we form the following table :

$x =$	$\pi/2 = 1.57$	$3\pi/4 = 2.36$	$\pi = 3.14$
$y = \cos x$	0	− 0.71	− 1

Taking the origin at (1.57, 0) and 1 unit along either axis $= \pi/8$, $= 0.4$ nearly, we plot the cosine curve as shown in Fig. 2.4.

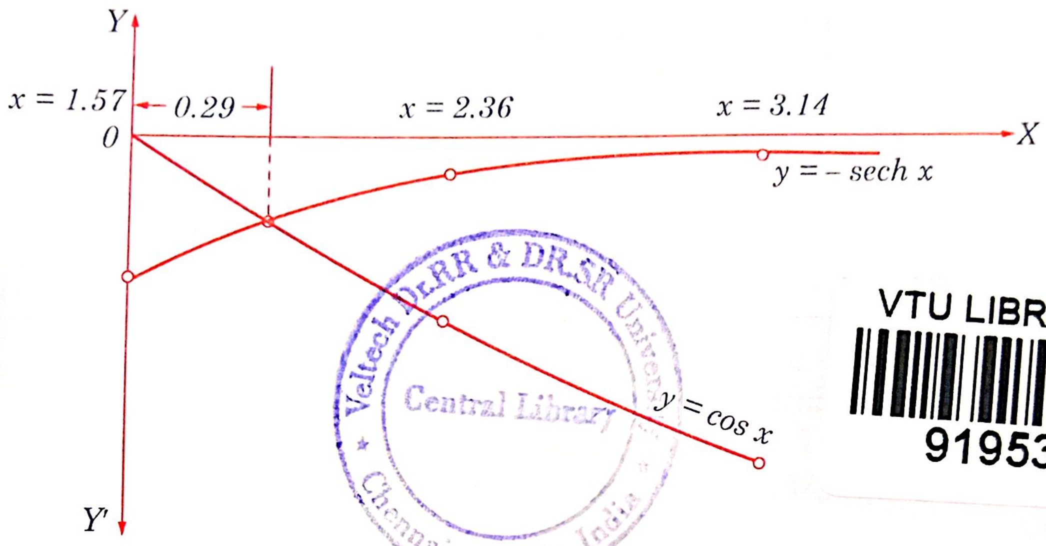

Fig. 2.4

To draw (iii), we form the following table :

x	1.57	2.36	3.14
$\cosh x$	2.51	5.34	11.57
$y = -\operatorname{sech} x$	– 0.4	– 0.19	– 0.09

Then we plot the curve (*iii*) to the same scale with the same axes.

From the above figure, we get the lowest root to be approximately $x = 1.57 + 0.29 = 1.86$.

PROBLEMS 2.3

Find the approximate value of the root of the following equations graphically (1—4) :

1. $x^3 - x - 1 = 0$ (*Madras B.E., 2000 S*)

2. $x^3 - 6x^2 + 9x - 3 = 0$

3. $\tan x = 1.2\,x$

4. $x = 3\cos(x - \pi/4)$.