

Solution of non linear equations

1. Solve the system of non linear equations

$$x^2 + y = 11 ; \quad y^2 + x = 7$$

we: $y = 11 - x^2 ; \quad y^2 = 7 - x$

$$\Rightarrow y^2 = (11 - x^2)^2$$

$$\Rightarrow 7 - x = 11^2 + x^4 - 22x^2$$

$$\Rightarrow x^4 - 22x^2 + x + 121 - 7 = 0$$

$$\Rightarrow x^4 - 22x^2 + x + 114 = 0$$

$x = 3$ is one of the root of $x^4 - 22x^2 + x + 114 = 0$.

$$\begin{array}{r|rrrrr} 3 & 1 & 0 & -22 & 1 & 114 \\ & & 3 & 9 & -39 & -114 \\ \hline & 1 & 3 & -13 & -38 & 0 \end{array}$$

$$x = 3 \text{ and } x^3 + 3x^2 - 13x - 38 = 0$$

$$\text{Let } f(x) = x^3 + 3x^2 - 13x - 38$$

$$f(3) = -ve ; \quad f(4) = +ve$$

Root lies between 3 and 4

$$\text{Let } x_0 = 3.5 \quad f'(x) = 3x^2 + 6x - 13$$

Newton Raphson's formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\text{when } n=0, \quad x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$\begin{aligned}
 x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)} \\
 &= 3.5 - \frac{f(3.5)}{f'(3.5)} \\
 &= 3.5 - \left[\frac{(3.5)^2 + 3(3.5)^2 - 13(3.5) - 38}{3(3.5)^2 + 6(3.5) - 13} \right] \\
 &= 3.584
 \end{aligned}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 3.5844$$

Given $y = 11 - x^2$

$$y = 11 - (3.5844)^2$$

$$= -1.848$$

$$\boxed{x \neq 2}$$

$$\boxed{x = 3.5844}$$

$$\boxed{y = -1.848}$$

Solve the system of non linear equations

$$x^2 + y = 5 ; \quad y^2 + x = 3$$

Ans: $x = -1.683 ; y = 2.164$