EXERCISE 1.4 [Gaussian Elimination and Gauss - Jordan]

Solve the system by Gauss-Elimination method and Gauss-Jordan method :

1. $2x + 3y - z = 5$, $4x + 4y - 3z = 3$ and $2x - 3y + 2z = 2$

   [Ans. $x = 1, y = 2, z = 3$]

2. $3x + 4y + 5z = 18$, $2x - y + 8z = 13$, $5x - 2y + 7z = 20$

   [Ans. $x = 3, y = 1, z = 1$]

3. $4.12x - 9.68y + 2.01z = 4.93$

   $1.88x - 4.62y + 5.50z = 3.11$

   $1.10x - 0.96y + 2.72z = 4.02$

   [Ans. 4.2075, 1.3327, 0.2468]

4. $2x + 4y + 8z = 41$, $4x + 6y + 10z = 56$, $6x + 8y + 10z = 64$

   [Ans. 1.5, 2.5, 3.5]

5. $x + 0.5y + 0.33z = 1$, $0.33x + 0.25y + 0.2z = 0$, $0.5x + 0.33y + 0.25z = 0$

   [Ans. 55.56, −277.78, 255.56]

6. $2x - y + 3z + w = 9$, $3x + y - 4z + 3w = 3$, $5x - 4v + 3z - 6w = 2$, $x - 2y - z + 2w = -2$

   [Ans. 1, 2, 2, 2]

7. $10x + y + z = 18.141$, $x + 10y + z = 28.140$, $x + y + 10z = 38.139$

   [Ans. 1.234, 2.348, 3.455]

8. $6x - y + z = 13$, $x + y + z = 9$, $10x + y - z = 19$.

   [Ans. 2, 3, 4]

9. $x - 3y - z = -30$, $2x - y - 3z = 5$, $5x - y - 2z = 142$

   [Ans. 39.2, 16.7, 19]

10. $2x + 2y - z + w = 4$, $4x + 3y - z + 2w = 6$, $8x + 5y - 3z + 4w = 12$, $3x + 3y - 2z + 2w = 6$.

    [Ans. 1, 1, −1, −1]

1.5 ITERATIVE METHODS

(a) Gauss Jacobi method

(b) Gauss-Siedel method

(a) Jacobi method of iteration or Gauss-Jacobi method

We have to explain this method in the case of three equations in three unknowns.

Consider the system of equations,

$$\begin{aligned} a_1 x + b_1 y + c_1 z &= d_1 \\ a_2 x + b_2 y + c_2 z &= d_2 \\ a_3 x + b_3 y + c_3 z &= d_3 \end{aligned} \quad \text{... (1)}$$

Let us assume

$$\begin{aligned} |a_1| &> |b_1| + |c_1| \\ |b_2| &> |a_2| + |c_2| \\ |c_3| &> |a_3| + |b_3| \end{aligned}$$

Then, iterative method can be used for the system (1). Solve for x, y, z (whose coefficients are the larger values) in terms of the other variables.

i.e.,
$$\begin{aligned} x &= \frac{1}{a_1}(d_1 - b_1 y - c_1 z) \\ y &= \frac{1}{b_2}(d_2 - a_2 x - c_2 z) \\ z &= \frac{1}{c_3}(d_3 - a_3 x - b_3 y) \end{aligned} \quad \text{... (2)}$$

If $x^{(0)}, y^{(0)}, z^{(0)}$ are the initial values of x, y, z respectively, then

$$\begin{aligned} x^{(1)} &= \frac{1}{a_1}(d_1 - b_1 y^{(0)} - c_1 z^{(0)}) \\ y^{(1)} &= \frac{1}{b_2}(d_2 - a_2 x^{(0)} - c_2 z^{(0)}) \\ z^{(1)} &= \frac{1}{c_3}(d_3 - a_3 x^{(0)} - b_3 y^{(0)}) \end{aligned} \quad \text{... (3)}$$

Again using these values $x^{(1)}, y^{(1)}, z^{(1)}$ in (2), we get

$$x^{(2)} = \frac{1}{a_1}(d_1 - b_1\, y^{(1)} - c_1 z^{(1)})$$

$$y^{(2)} = \frac{1}{b_1}(d_2 - a_2\, x^{(1)} - c_2 z^{(1)}) \qquad \ldots (4)$$

$$z^{(2)} = \frac{1}{c_1}(d_3 - a_3\, x^{(1)} - b_3 y^{(1)})$$

Continuing in the same way, if the rth iterates are $x^{(r)}$, $y^{(r)}$, $z^{(r)}$, the iteration scheme reduces to

$$x^{(r+1)} = \frac{1}{a_1}(d_1 - b_1 y^{(r)} - c_1 z^{(r)})$$

$$y^{(r+1)} = \frac{1}{b_1}(d_2 - a_2 x^{(r)} - c_2 z^{(r)}) \qquad \ldots (5)$$

$$z^{(r+1)} = \frac{1}{c_3}(d_3 - a_3 x^{(r)} - b_3 y^{(r)})$$

The procedure is continued till the convergence is assured (correct to required decimals).

(b) Gauss-Siedel method of iteration

This method is only a refinement of Gauss-Jacobi method. As before,

$$x = \frac{1}{a_1}(d_1 - b_1 y - c_1 z)$$

$$y = \frac{1}{b_2}(d_2 - a_2 x - c_2 z) \qquad \ldots (1)$$

$$z = \frac{1}{c_3}(d_3 - a_3 x - b_3 y)$$

We start with the initial values $y^{(0)}$, $z^{(0)}$ for y and z and get $x^{(1)}$ from the first equation. That is,

$$x^{(1)} = \frac{1}{a_1}(d_1 - b_1 y^{(0)} - c_1 z^{(0)})$$

While using the second equation, we use $z^{(0)}$ for z and $x^{(1)}$ for x instead of $x^{(0)}$ as in the Jacobi's method, we get

$$y^{(1)} = \frac{1}{b_2}(d_2 - a_2 x^{(1)} - c_2 z^{(0)})$$

Now, having known $x^{(1)}$ and $y^{(1)}$, use $x^{(1)}$ for x and $y^{(1)}$ for y in the third equation, we get

$$z^{(1)} = \frac{1}{c_3}(d_3 - a_3 x^{(1)} - b_3 y^{(1)})$$

To find the values of the unknowns, we use the latest available values on the right hand side. If $x^{(r)}, y^{(r)}, z^{(r)}$ are the rth iterates, then the iteration scheme will be

$$x^{(r+1)} = \frac{1}{a_1}(d_1 - b_1 y^{(r)} - c_1 z^{(r)})$$

$$y^{(r+1)} = \frac{1}{b_1}(d_2 - a_2 x^{(r+1)} - c_2 z^{(r)})$$

$$z^{(r+1)} = \frac{1}{c_3}(d_3 - a_3 x^{(r+1)} - b_3 z^{(r+1)})$$

This process of iteration is continued until the convergence is confirmed. As the current values of the unknowns at each stage of iteration are used in getting the values of unknowns, the convergence in Gauss-Siedel method is very fast when compared to Gauss-Siedel and method is roughly two times than that of Gauss-Jacobi method.

Note : We emphasize, however, that without diagonal dominance, neither Jacobi nor Gauss-Seidal is sure to converge. When both methods converge, the Gauss-Seidel method converges faster. Datta (1995) discusses this and give examples.

Note : Diagonally dominant.

called diagonally dominant.

1. **Solve the following system of equations by Gauss-Jacobi method and Gauss-Seidel Method.** **[A.U. May 1999, A.U. M/J 2006]**

$$27x + 6y - z = 85$$
$$x + y + 54z = 110$$
$$6x + 15y + 2z = 72$$

Solution : As the coefficient matrix is not diagonally dominant we rewrite the equations.

$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$
$$x + y + 54z = 110$$

Since the diagonal elements are dominant in the coefficient matrix, we write x, y, z as follows :

$$x = \frac{1}{27}[85 - 6y + z]$$

$$y = \frac{1}{15}[72 - 6x - 2z]$$

$$z = \frac{1}{54}[110 - x - y]$$

(1) Gauss Jacobi method

Let the initial values be $x = 0,\ y = 0,\ z = 0$

First iteration :

$$x^{(1)} = \frac{1}{27}[85] = 3.148$$

$$y^{(1)} = \frac{1}{15}[72] = 4.8$$

$$z^{(1)} = \frac{1}{54}[110] = 2.037$$

Second iteration :

$$x^{(2)} = \frac{1}{27}[85 - 6y^{(1)} + z^{(1)}] = \frac{1}{27}[85 - 6(4.8) + (2.037)] = 2.157$$

$$y^{(2)} = \frac{1}{15}[72 - 6x^{(1)} - 2z^{(1)}] = \frac{1}{15}[72 - 6(3.148) - 2(2.037)] = 3.269$$

$$z^{(2)} = \frac{1}{54}[110 - x^{(1)} - y^{(1)}] = \frac{1}{54}[110 - 3.148 - 4.8] = 1.890$$

Third iteration :

$$x^{(3)} = \frac{1}{27}[85 - 6y^{(2)} + z^{(2)}] = \frac{1}{27}[85 - 6(3.269) + 1.890] = 2.492$$

$$y^{(3)} = \frac{1}{15}[72 - 6x^{(2)} + 2z^{(2)}] = \frac{1}{15}[72 - 6(2.157) - 2(1.890)] = 3.685$$

$$z^{(3)} = \frac{1}{54}[110 - x^{(2)} - y^{(2)}] = \frac{1}{54}[110 - 2.157 - 3.269] = 1.937$$

Fourth iteration :

$$x^{(4)} = \frac{1}{27}[85 - 6y^{(3)} + z^{(3)}] = \frac{1}{27}[85 - 6(3.685) + 1.937] = 2.401$$

$$y^{(4)} = \frac{1}{15}[72 - 6x^{(3)} - 2z^{(3)}] = \frac{1}{15}[72 - 6(2.492) - 2(1.937) = 3.545$$

$$z^{(4)} = \frac{1}{54}[110 - x^{(3)} - y^{(3)}] = \frac{1}{54}[110 - 2.492 - 3.685] = 1.923$$

Fifth iteration :

$$x^{(5)} = \frac{1}{27}[85 - 6y^{(4)} + z^{(4)}] = \frac{1}{27}[85 - 6(3.545) + 1.923] = 2.432$$

$$y^{(5)} = \frac{1}{15}[72 - 6x^{(4)} - 2z^{(4)}] = \frac{1}{15}[72 - 6(2.401) - 2(1.923) = 3.583$$

$$z^{(5)} = \frac{1}{54}[110 - x^{(4)} - y^{(4)}] = \frac{1}{54}[110 - 2.401 - 3.545] = 1.927$$

Sixth iteration :

$$x^{(6)} = \frac{1}{27}[85 - 6y^{(5)} + z^{(5)}] = \frac{1}{27}[85 - 6(3.583) + 1.927] = 2.423$$

$$y^{(6)} = \frac{1}{15}[72 - 6x^{(5)} - 2z^{(5)}] = \frac{1}{15}[72 - 6\,(2.4332) - 2\,(1.927) \quad = 3.570$$

$$z^{(6)} = \frac{1}{54}[110 - x^{(5)} - y^{(5)}] = \frac{1}{54}[110 - 2.432 - 3.583] \quad = 1.926$$

Seventh iteration :

$$x^{(7)} = \frac{1}{27}[85 - 6\,y^{(6)} + z^{(6)}] = \frac{1}{27}[85 - 6\,(3.570) + 1.926] \quad = 2.426$$

$$y^{(7)} = \frac{1}{15}[72 - 6x^{(6)} - 2z^{(6)}] = \frac{1}{15}[72 - 6\,(2.423) - 2\,(1.926)] \quad = 3.574$$

$$z^{(7)} = \frac{1}{54}[110 - x^{(6)} - y^{(6)}] = \frac{1}{54}[110 - 2.423 - 3.570] \quad = 1.926$$

Eighth iteration :

$$x^{(8)} = \frac{1}{27}[85 - 6\,y^{(7)} + z^{(7)}] = \frac{1}{27}[85 - 6\,(3.574) + 1.926] \quad = 2.425$$

$$y^{(8)} = \frac{1}{15}[72 - 6x^{(7)} - 2z^{(7)}] = \frac{1}{15}[72 - 6\,(2.426) - 2\,(1.926)] \quad = 3.573$$

$$z^{(8)} = \frac{1}{54}[110 - x^{(7)} - y^{(7)}] = \frac{1}{54}[110 - 2.426 - 3.574] \quad = 1.926$$

Nineth iteration

$$x^{(9)} = \frac{1}{27}[85 - 6\,y^{(8)} + z^{(8)}] = \frac{1}{27}[85 - 6\,(3.573) + 1.926] \quad = 2.426$$

$$y^{(9)} = \frac{1}{15}[72 - 6x^{(8)} - 2z^{(8)}] = \frac{1}{15}[72 - 6\,(2.425) - 2\,(1.926)] = 3.573$$

$$z^{(9)} = \frac{1}{54}[110 - x^{(8)} - y^{(8)}] = \frac{1}{54}[110 - 2.425 - 3.573] \quad = 1.926$$

Tenth iteration

$$x^{(10)} = \frac{1}{27}[85 - 6\,y^{(9)} + z^{(9)}] = \frac{1}{27}[85 - 6\,(3.573) + 1.926] \quad = 2.426$$

$$y^{(10)} = \frac{1}{15}[72 - 6x^{(9)} - 2z^{(9)}] = \frac{1}{15}[72 - 6\,(2.426) - 2\,(1.926)] = 3.573$$

$$z^{(10)} = \frac{1}{54}[110 - x^{(9)} - y^{(9)}] = \frac{1}{54}[110 - 2.426 - 3.573] = 1.926$$

Hence $x = 2.426$, $y = 3.573$, $z = 1.926$

Correct to three decimal places.

2. Gauss-Seidel Method

Let the initial values be $y = 0$, $z = 0$

First iteration

$$x^{(1)} = \frac{1}{27}[85 - 6y^{(0)} + z^{(0)}] = \frac{1}{27}[85 - 6(0) + 0] = 3.148$$

$$y^{(1)} = \frac{1}{15}[72 - 6x^{(1)} - 2z^{(0)}] = \frac{1}{15}[72 - 6(3.148) - 0] = 3.541$$

$$z^{(1)} = \frac{1}{54}[110 - x^{(1)} - y^{(1)}] = \frac{1}{54}[110 - 3.148 - 3.541] = 1.913$$

Second iteration

$$x^{(2)} = \frac{1}{27}[85 - 6y^{(1)} + z^{(1)}] = \frac{1}{27}[85 - 6(3.541) + 1.913] = 2.432$$

$$y^{(2)} = \frac{1}{15}[72 - 6x^{(2)} - 2z^{(1)}] = \frac{1}{15}[72 - 6(2.432) - 2(1.913)] = 3.572$$

$$z^{(2)} = \frac{1}{54}[110 - x^{(2)} - y^{(2)}] = \frac{1}{54}[110 - 2.432 - 3.572] = 1.926$$

Third iteration

$$x^{(3)} = \frac{1}{27}[85 - 6y^{(2)} + z^{(2)}] = \frac{1}{27}[85 - 6(3.572) + 1.926] = 2.426$$

$$y^{(3)} = \frac{1}{15}[72 - 6x^{(3)} - 2z^{(2)}] = \frac{1}{15}[72 - 6(2.426) - 2(1.926)] = 3.573$$

$$z^{(3)} = \frac{1}{54}[110 - x^{(3)} - y^{(3)}] = \frac{1}{54}[110 - 2.426 - 3.573] = 1.926$$

Fourth iteration

$$x^{(4)} = \frac{1}{27}[85 - 6y^{(3)} + z^{(3)}] = \frac{1}{27}[85 - 6(3.573) + 1.926] = 2.426$$

$$y^{(4)} = \frac{1}{15}[72 - 6x^{(4)} - 2z^{(3)}] = \frac{1}{15}[72 - 6(2.426) - 2(1.926)] = 3.573$$

$$z^{(4)} = \frac{1}{54}[110 - x^{(4)} - y^{(4)}] = \frac{1}{54}[110 - 2.426 - 3.573] = 1.926$$

Hence $x = 2.426,\ y = 3.573,\ z = 1.926$

This shows that the convergence is rapid in Gauss-Seidel Method when compared to Gauss-Jacobi method.

2. Solve the following equations by Gauss-Siedel method

$4x + 2y + z = 14,\ x + 5y - z = 10,\ x + y + 8z = 20$

[A.U. April/May 2005]

Solution : As the coefficient matrix is diagonally dominant solving for x, y, z we get

$$x = \frac{1}{4}[14 - 2y - z]$$

$$y = \frac{1}{5}[10 - x + z]$$

$$z = \frac{1}{8}[20 - x - y]$$

Let the initial values be $y = 0,\ z = 0$

First iteration

$$x^{(1)} = \frac{1}{4}[14 - 2(0) - (0)] = \frac{14}{4} = 3.5$$

$$y^{(1)} = \frac{1}{5}[10 - x^{(1)} + z^{(0)}] = \frac{1}{5}[10 - 3.5 + 0] = 1.3$$

$$z^{(1)} = \frac{1}{8}[20 - x^{(1)} - y^{(1)}] = \frac{1}{8}[20 - 3.5 - 1.3] = 1.9$$

Second iteration

$$x^{(2)} = \frac{1}{4}[14 - 2y^{(1)} - z^{(1)}] = \frac{1}{4}[14 - 2(1.3) - (1.9)] = 2.375$$

$$y^{(2)} = \frac{1}{5}[10 - x^{(2)} + z^{(1)}] = \frac{1}{5}[10 - 2.375 + 1.9] = 1.905$$

$$z^{(2)} = \frac{1}{8}[20 - x^{(2)} - y^{(2)}] = \frac{1}{8}[20 - 2.375 - 1.905] = 1.965$$

Third iteration

$$x^{(3)} = \frac{1}{4}[14 - 2y^{(2)} - z^{(2)}] = \frac{1}{4}[14 - 2\,(1.905) - 1.965] = 2.056$$

$$y^{(3)} = \frac{1}{5}[10 - x^{(3)} + z^{(2)}] = \frac{1}{5}[10 - 2.056 + 1.965] = 1.982$$

$$z^{(3)} = \frac{1}{8}[20 - x^{(3)} - y^{(3)}] = \frac{1}{8}[20 - 2.056 - 1.982] = 1.995$$

Fourth iteration

$$x^{(4)} = \frac{1}{4}[14 - 2y^{(3)} - z^{(3)}] = \frac{1}{4}[14 - 2\,(1.982) - 1.995] = 2.010$$

$$y^{(4)} = \frac{1}{5}[10 - x^{(4)} + z^{(3)}] = \frac{1}{5}[10 - 2.010 + 1.995] = 1.997$$

$$z^{(4)} = \frac{1}{8}[20 - x^{(4)} - y^{(4)}] = \frac{1}{8}[20 - 2.010 - 1.997] = 1.999$$

Fifth iteration

$$x^{(5)} = \frac{1}{4}[14 - 2y^{(4)} - z^{(4)}] = \frac{1}{4}[14 - 2\,(1.997) - 1.999] = 2.002$$

$$y^{(5)} = \frac{1}{5}[10 - x^{(5)} + z^{(4)}] = \frac{1}{5}[10 - 2.002 + 1.999] = 1.999$$

$$z^{(5)} = \frac{1}{8}[20 - x^{(5)} - y^{(5)}] = \frac{1}{8}[20 - 2.002 - 1.999] = 2$$

Sixth iteration

$$x^{(6)} = \frac{1}{4}[14 - 2y^{(5)} - z^{(5)}] = \frac{1}{4}[14 - 2(1.999) - 2] = 2.001$$

$$y^{(6)} = \frac{1}{5}[10 - x^{(6)} + z^{(5)}] = \frac{1}{5}[10 - 2.001 + 2] = 2$$

$$z^{(6)} = \frac{1}{8}[20 - x^{(6)} - y^{(6)}] = \frac{1}{8}[20 - 2.001 - 2] = 2$$

Seventh iteration

$$x^{(7)} = \frac{1}{4}[14 - 2y^{(6)} - z^{(6)}] = \frac{1}{4}[14 - 2(2) - 2] = 2$$

$$y^{(7)} = \frac{1}{5}[10 - x^{(7)} + z^{(6)}] = \frac{1}{5}[10 - 2 + 2] = 2$$

$$z^{(7)} = \frac{1}{8}[20 - x^{(7)} - y^{(7)}] = \frac{1}{8}[20 - 2 - 2] = 2$$

Eighth iteration

$$x^{(8)} = \frac{1}{4}[14 - 2y^{(7)} - z^{(7)}] = \frac{1}{4}[14 - 2(2) - 2] = 2$$

$$y^{(8)} = \frac{1}{5}[10 - x^{(8)} + z^{(7)}] = \frac{1}{5}[10 - 2 + 2] = 2$$

$$z^{(8)} = \frac{1}{8}[20 - x^{(8)} - y^{(8)}] = \frac{1}{8}[20 - 2 - 2] = 2$$

Hence $x = 2, \quad y = 2, \quad z = 2$

SHORT QUESTIONS AND ANSWERS

1. **Write a sufficient condition for Gauss-Seidel method to converge.**

[M.U. Oct., 1995], [M.U. Oct., 1996], [M.U. Oct., 1999] [M.U. Oct., 2000]

Solution : The process of iteration by Gauss-Seidel method will converge if in each equation of the system, the absolute value of the largest coefficient is greater than the sum of the absolute values of the remaining coefficients.

[The coefficient of matrix should be diagonally dominant]

2. **State a sufficient condition for Gauss-Jacobi method to converge.**

[M.U. April, 1996, Oct., 1997, April, 1998]

Solution : Same as Q.No. 1.

3. **Gauss elimination and Gauss Jordan are direct methods while .... and .... are iterative methods.**

Solution : (i) Gauss-Seidel method; (ii) Gauss-Jacobi method

4. **True or false 'Iteration method will always converge'.**

Solution : False

5. **Give two indirect methods to solve a system of linear equations.**

[M.U. April, 2001]

Solution : (i) Gauss-Jacobi method; (ii) Gauss-Seidel method

6. **Solve the system of equations**
$2x - 3y + 20z = 25$
$20x + y - 2z = 17$
$3x + 20y - z = -18$ by Gauss-Jacobi iteration method.
[Only two iteration] [A.U. Nov. 1996]

Solution : As the coefficient matrix is not diagonally dominant as it is we rewrite the equation.

$$20x + y - 2z = 17$$

$$3x + 20y - z = -18$$

$$2x - 3y + 20z = 25$$

Now the diagonal elements are dominant in the coefficient matrix, we write x, y, z as follows :

$$x = \frac{1}{20}[17 - y + 2z]$$

$$y = \frac{1}{20}[-18 - 3x + z]$$

$$z = \frac{1}{20}[25 - 2x + 3y]$$

Let the initial conditions be $x = 0,\ y = 0,\ z = 0$

First Iteration :

$$x^{(1)} = \frac{1}{20}[17 - 0 + 0] = 0.85$$

$$y^{(1)} = \frac{1}{20}[-18] = -0.9$$

$$z^{(1)} = \frac{1}{20}[25] = 1.25$$

Second iteration :

$$x^{(2)} = \frac{1}{20}[17 - y^{(1)} + 2z^{(1)}] = \frac{1}{20}[17 - (-0.9) + 2(1.25)] = 1.02$$

$$y^{(2)} = \frac{1}{20}[-18 - 3x^{(1)} + z^{(1)}] = \frac{1}{20}[-18 - 3(0.85) + 1.25] = -0.965$$

$$z^{(2)} = \frac{1}{20}[25 - 2x^{(1)} + 3y^{(1)}] = \frac{1}{20}[25 - 2(0.85) + 3(-0.9)] = 1.03$$

7. **Solve by Gauss-Seidel method $x - 2y = -3$, $2x + 25y = 15$ correct to four decimal places.** **[A.U. May, 2000]**

Solution : $x - 2y = -3$

$$2x + 25y = 15$$

$$x = -3 + 2y$$

$$y = \frac{1}{25}[15 - 2x]$$

Let the initial value $y = 0$

First Iteration :

$$x^{(1)} = -3 + 0 = -3$$

$$y^{(1)} = \frac{1}{25}[15 + 6] = 0.84$$

Second iteration :

$$x^{(2)} = -3 + 2y^{(1)} = -3 + 2(0.84) = -1.32$$

$$y^{(2)} = \frac{1}{25}[15 - 2x^{(2)}] = \frac{1}{25}[15 - 2(-1.32)] = 0.7056$$

Third iteration :

$$x^{(3)} = -3 + 2y^{(2)} = -3 + 2(0.7056) = -1.5888$$

$$y^{(3)} = \frac{1}{25}[15 - 2x^{(3)}] = \frac{1}{25}[15 - 2(-1.5888)] = 0.7271$$

Fourth iteration :

$$x^{(4)} = -3 + 2y^{(3)} = -3 + 2\,(0.7271) = -1.5458$$

$$y^{(4)} = \frac{1}{25}[15 - 2x^{(4)}] = \frac{1}{25}[15 - 2\,(-1.5458)] = 0.7237$$

Fifth iteration :

$$x^{(5)} = -3 + 2\,y^{(4)} = -3 + 2\,(0.7237) = -1.5526$$

$$y^{(5)} = \frac{1}{25}[15 - 2x^{(5)}] = \frac{1}{25}[15 - 2\,(-1.5526)] = 0.7242$$

Sixth iteration :

$$x^{(6)} = -3 + 2y^{(5)} = -3 + 2\,[0.7242] = [-1.5516]$$

$$y^{(6)} = \frac{1}{25}[15 - 2x^{(6)}] = \frac{1}{25}[15 - 2\,(-1.5516)] = 0.7241$$

Seventh iteration :

$$x^{(7)} = -3 + 2y^{(6)} = -3 + 2\,(0.7241) = -1.5518$$

$$y^{(7)} = \frac{1}{25}[15 - 2x^{(7)}] = \frac{1}{25}[15 - 2\,(-1.5518)] = 0.7241$$

Eighth iteration :

$$x^{(8)} = -3 + 2\,y^{(7)} = -3 + 2(0.7241) = -1.5518$$

$$y^{(8)} = \frac{1}{25}[15 - 2x^{(8)}] = \frac{1}{25}[15 - 2\,(-1.5518)] = 0.7241$$

Hence $x = -1.5518,\ y = 0.7241$

8. **State True or False.**

Gauss-Seidel iteration converges only if the Coefficient matrix is diagonally dominant.

Solution : True

9. Compare Gauss-elimination and Gauss-Seidel methods.

(or)

State the merits and demerits of elimination methods and iterative methods for solving a system of equations.

1. Gauss elimination method has the advantage that it is finite and works in theorey for any non-singular set of equations.

2. Gauss Seidel iteration method converges only for special system of equations. For some systems, elimination is the only course available.

3. In general, the round off error is smaller in iteration methods, Iteration is a self-correcting method. Any errors made at any step in the computation are corrected in the subsequent iterations.

10. Compare Gauss-Jacobi and Gauss Seidel methods.

[A.U. April/May 2003]

Solution :

	Gauss-Jacobi method	Gauss-Seidel method
1.	Convergence rate is slow	The rate of convergence of Gauss-Seidel method is roughly twice that of Gauss-Jacobi.
2.	Indirect method	Indirect method
3.	Condition for convergence is the coefficient matrix is diagonally dominant	Condition for convergence is the coefficient matrix is diagonally dominant.

11. Is the iteration method, a self-correcting method always ?

[M.U. Oct. 1997

Solution : In general, iteration is a self correcting method, sinc the round off error is smaller.

12. Distinguish between direct and iterative methods of solving simultaneous equations.

Solution : There are numerical methods of solving simultaneous equations. They are particularly suited for computer operations. These numerical methods are of two types, direct or iterative. Direct methods involve a certain amount of fixed computation and they are exact solutions. Iterative or indirect methods are those in which the solution is got by successive approximations. But the method of iteration is not applicable to all systems of equations.

13. When will iteration method succeed ?

Solution : In order that the iteration may succeed, equation of the system must contain one large coefficient (much larger than the others in that equation) and the large coefficient must be attached to a different unknown in that equation. This requirement will be got when the large coefficients are along the leading diagonal of the matrix of the coefficient.

14. "In an iterative method, the amount of computation depends on the degree of accuracy required". Say whether this is 'true or false' ?

Solution : The statement is true.

15. Pick up the correct answer.

"As soon as a new value for a variable is found by iteration, it is used immediately in the following equation". This method is called

(a) Gauss-Seidel (b) Jocobi's
(c) Gauss-Jordan (d) Relaxation

Solution : (a) Gauss-Seidel.

16. Fill up the blank.

As soon as a new value for a variable is found by iteration, it is used immediately in the following equations. This is method is called **[A.U. Nov. 1995]**

....

Solution : Gauss-Seidel method of iteration

17. **State 'True or False'.**

The convergence in the Gauss-Seidel method is thrice as fast as in Jacobi's method. [A.U, April 1995]

Solution : The statement is false. In fact, the rate of convergence of Gauss-Seidel method is roughly twice that of Gauss-Jacobi.

18. **Why Gauss-Seidel method is a better method than Jacobi's iterative method ?**

Solution : Since the current value of the unknowns at each stage of iteration are used in proceeding to the next stage of iteration, the convergence in Gauss-Seidel method will be more rapid than in Gauss-Jacobi method.

19. **Distinguish between direct and iterative (indirect) method of solving simultaneous equations.**

Solution :

	Direct method	Iterative method
1.	We get exact solution	Approximate solution.
2.	Simple, take less time	Time consuming laborious.