Characteristics of Finite Queue, Multiple Server Poisson Queue Model IV [(M/M/s): (k/FIFO) Model]

1. *Values of P_0 and P_n*

For the Poisson queue system, P_n is given by

$$P_n = \frac{\lambda_0 \lambda_1 \cdots \lambda_{n-1}}{\mu_1 \mu_2 \cdots \mu_n} \cdot P_0, \; n \geq 1, \tag{1}$$

where $$P_0 = \left\{1 + \sum_{n=1}^{k} \left(\frac{\lambda_0 \lambda_1 \cdots \lambda_{n-1}}{\mu_1 \mu_2 \cdots \mu_n}\right)\right\}^{-1} \tag{2}$$

For this $(M/M/s)$: $(k/FIFO)$ model,

$$\lambda_n = \begin{cases} \lambda, & \text{for } n = 0, 1, 2, \cdots, k-1 \\ 0, & \text{for } n = k, k+1, \cdots \end{cases}$$

$$\mu_n = \begin{cases} n\mu, & \text{for } n = 0, 1, 2, \cdots, s-1 \\ s\mu, & \text{for } n = s, s+1, \cdots \end{cases}$$

Using these values of λ_n and μ_n in (2) and noting that $1 < s < k$, we get

$$P_0^{-1} = \left\{1 + \frac{\lambda}{1!\,\mu} + \frac{\lambda^2}{2!\,\mu^2} + \cdots + \frac{\lambda^{s-1}}{(s-1)!\,\mu^{s-1}}\right\} + \left\{\frac{\lambda^s}{(s-1)!\,\mu^{s-1} \cdot \mu s}\right.$$

$$\left. + \frac{\lambda^{s+1}}{(s-1)!\,\mu^{s-1} \cdot (\mu s)^2} + \cdots + \frac{\lambda^k}{(s-1)!\,\mu^{s-1} \cdot (\mu s)^{k-s+1}}\right\}$$

$$= \sum_{n=0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{\lambda^s}{s!\mu^s}\left[1 + \frac{\lambda}{\mu s} + \left(\frac{\lambda}{\mu s}\right)^2 + \cdots + \left(\frac{\lambda}{\mu s}\right)^{k-s}\right]$$

$$= \sum_{n=0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s \sum_{n=s}^{k}\left(\frac{\lambda}{\mu s}\right)^{n-s} \tag{3}$$

$$P_n = \begin{cases} \dfrac{1}{n!}\left(\dfrac{\lambda}{\mu}\right)^n P_0, & \text{for} \quad n \le s \\ \dfrac{1}{s!s^{n-s}} \cdot \left(\dfrac{\lambda}{\mu}\right)^n \cdot P_0, & \text{for} \quad s < n \le k \\ 0, & \text{for} \quad n > k \end{cases} \tag{4}$$

2. *Average queue length or average number of customers in the queue*

$$E(N_q) = E(N-s) = \sum_{n=s}^{k} (n-s)P_n$$

$$= \frac{P_0}{s!}\sum_{n=s}^{k} (n-s)\left(\frac{\lambda}{\mu}\right)^n / s^{n-s} \text{ [using (4)]}$$

$$= \frac{\left(\frac{\lambda}{\mu}\right)^s \cdot P_0}{s!} \sum_{x=0}^{k-s} x \cdot \left(\frac{\lambda}{\mu s}\right)^x$$

$$= \frac{\left(\frac{\lambda}{\mu}\right)^s \cdot P_0 \rho}{s!} \sum_{x=0}^{k-s} x\,\rho^{x-1} \text{ where } P = \frac{\lambda}{\mu s}$$

$$= \left(\frac{\lambda}{\mu}\right)^s \cdot \frac{P_0 \rho}{s!} \sum_{x=0}^{k-s} \frac{d}{d\rho}(\rho^x)$$

$$= \left(\frac{\lambda}{\mu}\right)^s \cdot \frac{P_0 \rho}{s!} \frac{d}{d\rho}\left\{\frac{1-\rho^{k-s+1}}{1-\rho}\right\}$$

$$= \left(\frac{\lambda}{\mu}\right)^s \cdot \frac{P_0 \rho}{s!}\left[\frac{-(1-\rho)(k-s+1)\rho^{k-s} + \left(1-\rho^{k-s+1}\right)}{(1-\rho)^2}\right]$$

$$= \left(\frac{\lambda}{\mu}\right)^s \cdot \frac{P_0 \rho}{s!}\left[\frac{-(k-s)(1-\rho)\rho^{k-s} - (1-\rho)\rho^{k-s} + 1 - \rho^{k-s+1}}{(1-\rho)^2}\right]$$

$$= P_0 \left(\frac{\lambda}{\mu}\right)^s \frac{\rho}{s!}\left[\frac{-(k-s)(1-\rho)\rho^{k-s}+1-\rho^{k-s}(1-\rho+\rho)}{(1-\rho)^2}\right]$$

$$= P_0 \cdot \left(\frac{\lambda}{\mu}\right)^s \frac{\rho}{s!(1-\rho)^2}\,[1-\rho^{k-s}-(k-s)(1-\rho)\,\rho^{k-s}],$$

where $\rho = \dfrac{\lambda}{\mu s}$ (5)

3. *Average number of customers in the system*

$$E(N) = \sum_{n=0}^{k} nP_n = \sum_{n=0}^{s-1} nP_n + \sum_{n=s}^{k} nP_n$$

$$= \sum_{n=0}^{s-1} nP_n + \sum_{n=s}^{k} (n-s)\,P_n + \sum_{n=s}^{k} sP_n$$

$$= \sum_{n=0}^{s-1} nP_n + E(N_q) + s\left\{\sum_{n=0}^{k} P_n - \sum_{n=0}^{s-1} P_n\right\}$$

$$= E(N_q) + s - \sum_{n=0}^{s-1} (s-n)P_n \quad \left(\because \sum_{n=0}^{k} P_n = 1\right) \tag{6}$$

Obviously $\left\{s - \sum_{n=0}^{s-1} (s-n)P_n\right\} \neq \dfrac{\lambda}{\mu}$, so that step (6) represents Little's formula.

In order to make (6) to assume the form of Little's formula, we define the *overall effective arrival rate* λ' or λ_{eff} as follows:

$$\frac{\lambda'}{\mu} = s - \sum_{n=0}^{s-1} (s-n)P_n$$

i.e., $$\lambda' = \mu\left[s - \sum_{n=0}^{s-1}(s-n)\,P_n\right] \tag{7}$$

With this definition of λ', step (6) becomes

$$E(N) = E(N_q) + \frac{\lambda'}{\mu} \tag{8}$$

which is the modified Little's formula for this model.

4. *Average waiting time in the system and in the queue:*

By the modified Little's formulas,

$$E(W_s) = \frac{1}{\lambda'}\,E(N) \tag{9}$$

and $$E(W_q) = \frac{1}{\lambda'} E(N_q)$$

where λ' is the effective arrival rate, given by step (7).

Example 17

A 2-person barber shop has 5 chairs to accommodate waiting customers. Potential customers, who arrive when all 5 chairs are full, leave without entering barber shop. Customers arrive at the average rate of 4 per hour and spend an average of 12 min in the barber's chair. Compute P_0 P_1, P_7, $E(N_q)$ and $E(W)$.

The situation in this problem is one of finite capacity, multiserver Poisson queue models.

$$\lambda = 4 \text{ per hour}, \mu = 5 \text{ per hour}, s = 2, k = 2 + 5 = 7$$

(a) $$P_0 = \left[\sum_{n=0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s \sum_{n=s}^{k}\left(\frac{\lambda}{\mu s}\right)^{n-s}\right]^{-1}$$

[by formula (3) of model IV]

$$= \left[\sum_{n=0}^{1} \frac{1}{n!}\left(\frac{4}{5}\right)^n + \frac{1}{2}\cdot\left(\frac{4}{5}\right)^2 \sum_{n=2}^{7}\left(\frac{2}{5}\right)^{n-2}\right]^{-1}$$

$$= \left[1 + \frac{4}{5} + \frac{8}{25}\left\{1 + \frac{2}{5} + \left(\frac{2}{5}\right)^2 + \left(\frac{2}{5}\right)^3 + \left(\frac{2}{5}\right)^4 + \left(\frac{2}{5}\right)^5\right\}\right]^{-1}$$

$$= \left[\frac{9}{5} + \frac{8}{25}\left\{\frac{1-(0.4)^7}{1-0.4}\right\}\right]^{-1} = 0.4287$$

(b) $P_n = \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_0$, for $n \le s$ [by formula (4) of model IV]

$$\therefore \qquad P_1 = \left(\frac{4}{5}\right) \times 0.4287 = 0.3430$$

(c) $P_n = \frac{1}{s!.s^{n-s}}\left(\frac{\lambda}{\mu}\right)^n \cdot P_0$, for $s < n \le k$ [by formula (4) of model IV]

$$\therefore \qquad P_7 = \frac{1}{2 \times 2^{7-2}} \times \left(\frac{4}{5}\right)^7 \times 0.4287$$

$$= 0.0014$$

(d) $E(N_q) = P_0\left(\frac{\lambda}{\mu}\right)^s \cdot \frac{\rho}{s!(1-\rho)^2}\,[1 - \rho^{k-s} - (k-s)(1-\rho)\,\rho^{k-s}]$,

where $\rho = \frac{\lambda}{\mu s}$ [by formula (5) of model IV]

$$= (0.4287).\,(0.8)^2 \cdot \frac{(0.4)}{2 \times (0.6)^2}\,[1 - (0.4)^5 - 5 \times 0.6 \times (0.4)^5]$$

$$= 0.15 \text{ customer}$$

(e) $E(N) = E(N_q) + s - \sum_{n=0}^{s-1} (s-n)\,P_n$ [by formula (6) of model IV]

$$= 0.1462 + 2 - \sum_{n=0}^{1} (2-n)P_n$$

$$= 2.1462 - (2 \times P_0 + 1 \times P_1)$$

$$= 2.1462 - (2 \times 0.4287 + 1 \times 0.3430)$$

$$= 0.95 \text{ customer}$$

$E(W) = \frac{1}{\lambda'} \cdot E(N)$ [by formula (9) of model IV]

where $\lambda' = \mu\left[s - \sum_{n=0}^{s-1}(s-n)\,P_n\right]$ [by formula (7) of model IV]

$$= 4[2 - (2 \times 0.4287 + 1 \times 0.3430)]$$

$$= 3.1984$$

$$\therefore E(W) = \frac{0{\cdot}9458}{3{\cdot}1984} = 0.2957 \text{ h or } 17.7 \text{ min}$$

Example 18

At a port there ar 6 unloading berths and 4 unloading crews. When all the berths are full, arriving ships are diverted to an overflow facility 20 kms down the river.Tankers arrive according to a Poisson process with a mean of 1 every 2 h. It takes for an unloading crew, on the average, 10 h to unload a tanker, the unloading time following an exponential distribution. Find

(a) how many tankers are at the port on the average?
(b) how long does a tanker spend at the port on the average?
(c) what is the average arrival rate at the overflow facility?

$$\lambda = \frac{1}{2} \text{ per hour}, \mu = \frac{1}{10} \text{ per hour}, s = 4, k = 6$$

(a) $$P_0 = \left[\sum_{n=0}^{s-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!} \left(\frac{\lambda}{\mu}\right)^s \sum_{n=s}^{k} \left(\frac{\lambda}{\mu s}\right)^{n-s} \right]^{-1}$$

[by formula (3) of model IV]

$$= \left[\left(1 + 5 + \frac{1}{2} \times 5^2 + \frac{1}{6} \times 5^3\right) + \frac{1}{24} \times 5^4 \times \left\{ \left(\frac{5}{4}\right)^0 + \left(\frac{5}{4}\right)^1 + \left(\frac{5}{4}\right)^2 \right\} \right]^{-1}$$

$$= 0.0072$$

$$E(N_q) = P_0 \left(\frac{\lambda}{\mu}\right)^s \cdot \frac{\rho}{s!(1-\rho)^2} [1 - \rho^{k-s} - (k-s)(1-\rho)\rho^{k-s}],$$

where $\rho = \dfrac{\lambda}{\mu s}$ [by formula (5) of model IV]

$$= 0.0072 \times 5^4 \times \frac{1.25}{24 \times (.25)^2} [1 - (1.25)^2 - 2 \times (-.25)(1.25)^2]$$

$$= 0.8203 \text{ tanker}$$

$$E(N) = E(N_q) + s - \sum_{n=0}^{s-1} (s-n) P_n \text{ [by formula (6) of model IV]}$$

$$= 4.8203 - (4 P_0 + 3 P_1 + 2 P_2 + P_3)$$

$$= 4.8203 - \{4 \times .0072 + 3 \times 0.0360 + 2 \times 0.09 + 0.15\}$$

$$= 4.3535 \text{ tankers}$$

(b) $E(W) = \frac{1}{\lambda'} E(N)$ [by formula (9) of model IV]

where $\lambda' = \mu \left[s - \sum_{n=0}^{s-1} (s-n) P_n \right]$ [by formula (7) of model IV]

$$= \frac{1}{10} [4 - \{4P_0 + 3P_1 + 2P_2 + P_3\}]$$

$$= \frac{1}{10} [4 - 0.4668] = 0.3533$$

$\therefore$ $$E(W) = \frac{4.3535}{0.3533} = 12.32 \text{ h}$$

(c) When $N = 6$, i.e., when the number of tankers in the port is 6, overflow occurs.

$P(N = 6) = \frac{1}{s!s^{n-s}} \left(\frac{\lambda}{\mu}\right)^n P_0$, for $n = k$ [by formula (4) of model IV]

$$= \frac{1}{24 \times 4^2} \times 5^6 \times 0.0072$$

$$= 0.2930$$

Average arrival rate at the overflow facility = (Average arrival rate at the port) × (Probability that overflow occurs)

$$= \frac{1}{2} \times 0.2930 = 0.586 \text{ per hour}$$

Example 19

A car servicing station has 2 bays where service can be offered simultaneously. Because of space limitation, only 4 cars are accepted for servicing. The arrival pattern is Poisson with 12 cars per day. The service time in both the bays is exponentially distributed with $\mu = 8$ cars per day per bay. Find the average number of cars in the service station, the average number of cars waiting for service and the average time a car spends in the system.

$$\lambda = 12 \text{ per day}, \mu = 8 \text{ per day}, s = 2, k = 4$$

(a) $P_0 = \left[\sum_{n=0}^{s-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!} \left(\frac{\lambda}{\mu}\right)^s \sum_{n=s}^{k} \left(\frac{\lambda}{\mu s}\right)^{n-s} \right]^{-1}$

[by formula (3) of model IV]

$$= \left[1 + \frac{1.5}{1} + \frac{1}{2} \times (1.5)^2 \left\{1 + (.75) + (.75)^2\right\}\right]^{-1}$$

$$= 0.1960$$

$$E(N_q) = P_0 \left(\frac{\lambda}{\mu}\right)^s \cdot \frac{\rho}{s!(1-\rho)^2} [1 - \rho^{k-s} - (k-s)(1-\rho)\rho^{k-s}],$$

where $\rho = \dfrac{\lambda}{\mu s}$ [by formula (5) of model IV]

i.e., $E(N_q) = 0.1960 \times (1.5)^2 \times \dfrac{0\cdot75}{2 \times (0\cdot25)^2} \times$

$$[1 - (0.75)^2 - 2 \times 0.25 \times (0.75)^2]$$

$$= 0.4134 \text{ car}$$

(b) $E(N)$ = Average number of cars in the service station

$$= E(N_q) + s - \sum_{n=0}^{s-1} (s-n) P_n \text{ [by formula (6) of model IV]}$$

$$= 0.4134 + 2 - \sum_{n=0}^{1} (2-n) P_n$$

$$= 2.4134 - (2P_0 + P_1)$$

$$= 2.4134 - (2 \times 0.1960 + 1.5 \times 0.1960)$$

$$= 1.73 \text{ cars}$$

(c) $E(W) = \dfrac{1}{\lambda'} E(N)$ [by formula (9) of model IV]

where $\lambda' = \mu \left[s - \sum_{n=0}^{s-1} (s-n) P_n\right]$ [by formula (7) of model IV]

$$= 8[2 - (2P_0 + P_1)]$$

$$= 10.512$$

$$\therefore E(W) = \frac{1.73}{10.512} = 0.1646 \text{ day}$$

[(M/G/1) : (∞/GD) Model]

So far we have discussed Markovian queue models in which the inter-arrival and inter-service times were assumed to follow exponential distributions with parameters λ and μ. When the arrivals and departures do not follow Poisson distributions, the discussion of the queueing models is tedious. However we can derive the characteristics of a particular non-Markovian model $(M/G/1) : (\infty/GD)$, where M indicates that the number of arrivals in time t follows a Poisson process, G indicates that the service time follows a general (arbitrary) distribution and GD indicates general queue discipline (viz., any kind of queue discipline).

The average number L of customers in the $M/G/1$ queueing system is given by a formula, known as Pollaczek-Khinchine formula, which is derived below:

Pollaczek-Khinchine Formula

Let N and N' be the numbers of customers in the system at times t and $t + T$, when two consecutive customers have just left the system after getting service.

Thus T is the random service time, which is a continuous random variable. Let $f(t)$, $E(T)$, Var (T) be the pdf, mean and variance of T. Also let M be the number of customers arriving in the system during the service time T.

Hence
$$N' = \begin{cases} M, & \text{if } N = 0 \\ N - 1 + M, & \text{if } N > 0 \end{cases}$$

where M is a discrete random variable, taking the values 0, 1, 2, ...

Equivalently,
$$N' = N - 1 + M + \delta \tag{1}$$

where
$$\delta = \begin{cases} 1, & \text{if } N = 0 \\ 0, & \text{if } N > 0 \end{cases}$$

$\therefore$
$$E(N') = E(N) - 1 + E(M) + E(\delta) \tag{2}$$

When the system has reached the steady-state, the probability of the number of customers in the system will be a constant

Hence
$$E(N) = E(N') \text{ and } E(N^2) = E(N'^2) \tag{3}$$

Using this in (2), we get
$$E(\delta) = 1 - E(M) \tag{4}$$

Squaring both sides of (1), we have
$$N'^2 = N^2 + (M-1)^2 + \delta^2 + 2N(M-1) + 2(M-1)\delta + 2N\delta \tag{5}$$

Now
$$\delta^2 = \delta \; (\because \delta^2 = 0 \text{ or } 1, \text{ according as } \delta = 0 \text{ or } 1)$$

and
$$N\delta = \begin{cases} 0 \times 1, & \text{if } N = 0 \\ N \times 0, & \text{if } N > 0 \end{cases}$$
$$= 0$$

Using these values in (5), we have

$$N'^2 = N^2 + M^2 + 2N(M-1) + (2M-1)\,\delta - 2M + 1$$

i.e.,
$$2N(1-M) = N^2 - N'^2 + M^2 + (2M-1)\delta - 2M + 1$$

$\therefore$
$$2E\{N(1-M)\} = E(N^2) - E(N'^2) + E(M^2) + E\{(2M-1)\delta\} - 2E(M) + 1$$

i.e.,
$$2E(N)\{1 - E(M)\} = E(M^2) + \{2E(M) - 1\}\,E(\delta) - 2E(M) + 1$$

{by independence and by (3)}

$\therefore$
$$E(N) = \frac{E(M^2) + \{2E(M) - 1\}\{1 - E(M)\} - 2E(M) + 1}{2\{1 - E(M)\}}, \text{ by (4)}$$

$$= \frac{E(M^2) - 2E^2(M) + E(M)}{2\{1 - E(M)\}} \tag{5}$$

Since the number M of arrivals in time T follows a Poisson process with parameter λ, say, then $E(M) = \lambda T$ and Var $(M) = \lambda T$ or $E(M^2) = (\lambda T)^2 + \lambda T$

Now
$$E(M) = E\{E(M/T)\}$$
$$= E(\lambda T) = \lambda E(T) \tag{6}$$
$$E(M^2) = E\{E(M^2/T)\} = E\{\lambda^2 T^2 + \lambda T\}$$
$$= \lambda^2\{\text{Var}(T) + E^2(T)\} + \lambda E(T) \tag{7}$$

Using (6) and (7) in (5), we have

$$L_s = E(N) = \frac{\lambda^2 V(T) + \lambda^2 E^2(T) + \lambda E(T) - 2\lambda^2 E^2(T) + \lambda E(T)}{2\{1 - \lambda E(T)\}}$$

$$= \lambda E(T) + \frac{\lambda^2\{V(T) + E^2(T)\}}{2\{1 - \lambda E(T)\}}$$

Note

1. *The other characteristics* $L_q = E(N_q)$, $E(W_s)$ *and* $E(W_q)$ *of this model can be obtained by using Little's formulas.*
2. $\lambda E(T)$ *must be less than 1, otherwise* L_s *becomes negative, which is meaningless.*
3. *In this M/G/1 model, if* $G \equiv M$, *viz., the service time T follows an exponential distribution with parameter* μ, *then*

$$E(T) = \frac{1}{\mu} \text{ and } V(T) = \frac{1}{\mu^2} \text{ and hence}$$

$$L_s = \frac{\lambda}{\mu} + \frac{\lambda^2\left\{\frac{1}{\mu^2} + \frac{1}{\mu^2}\right\}}{2\left(1 - \frac{\lambda}{\mu}\right)} = \frac{\lambda}{\mu - \lambda},$$

which has already been derived for M/M/1 model.

Example 21

A one-man barber shop takes exactly 25 minutes to complete one hair-cut. If customers arrive at the barber shop in a Poisson fashion at an average rate of one every 40 minutes, how long on the average a customer spends in the shop? Also find the average time a customer must wait for service.

The service time T is a constant = 25 min viz., T follows a distribution with $E(T) = 25$ and $V(T) = 0$. Also $\lambda = \frac{1}{40}$.

∴ By Pollaczek-Khinchine formula,

$$E(N_s) = \lambda E(T) + \frac{\lambda^2 \{V(T) + E^2(T)\}}{2\{1 - \lambda E(T)\}}$$

$$= \frac{25}{40} + \frac{\frac{1}{40^2}\{0 + 25^2\}}{2\left\{1 - \frac{1}{40} \times 25\right\}}$$

$$= \frac{5}{8} + \frac{25/64}{2 \times (3/8)} = \frac{55}{48}$$

By Little's formula,

$$E(W_s) = \frac{1}{\lambda} E(N_s) = 40 \times \frac{55}{48} = 45.8 \text{ minutes}$$

$$E(W_q) = E(W_s) - \frac{1}{\mu} = E(W_s) - E(T) = 20.8 \text{ min.}$$

i.e., a customer has to spend 45.8 minutes in the shop and has to wait for service for20.8 minutes on the average.

Example 22

A patient who goes to a single doctor clinic for a general check-up has to go through 4 phases. The doctor takes on the average 4 minutes for each phase of the check-up and the time taken for each phase is exponentially distributed. If the arrivals of the patients at the clinic are approximately Poisson at the average rate of 3 per hour, what is the average time spent by a patient (i) in the examination? (ii) waiting in the clinic?

Let X_1, X_2, X_3, X_4 denote the times required for the 4 phases of the check-up. Each X_r is exponential with mean 4 min or with parameter $\frac{1}{4}$.

Since the X_r's independent, $(X_1 + X_2 + X_3 + X_4)$ follows an Erlang's distribution with parameters 'λ' $= \frac{1}{4}$ and 'k' $= 4$ [Refer to problem (43) in Exercise 5(B)]

The mean and variance of Erlang's distribution with parameters 'λ' and 'k' are $\frac{k}{\lambda}$ and $\frac{k}{\lambda^2}$.

Thus if T represents the service time for a patient,

$$E(T) = \frac{k}{\lambda} = \frac{4}{1/4} = 16$$

and $$V(T) = \frac{k}{\lambda^2} = \frac{4}{1/16} = 64.$$

$\therefore$ Average time for examination of each patient = 16 min. If λ_c represents the arrival rate in the clinic, then by $P - K$ formula,

$$E(N_s) = \lambda_c\, E(T) + \frac{\lambda_c^2\,\{V(T) + E^2(T)\}}{2\{1 - \lambda_c\, E(T)\}}$$

$$= \frac{1}{20} \times 16 + \frac{\frac{1}{400}\{64 + 256\}}{2\left\{1 - \frac{1}{20} \times 16\right\}} \qquad \left(\because \lambda_c = 3/\text{hour or } \frac{1}{20}/\text{min}\right)$$

$$= \frac{4}{5} + \frac{\frac{4}{5}}{2 \times \frac{1}{5}} = \frac{14}{5}$$

By Little's formula

$$E(W_s) = \frac{1}{\lambda_c} E(N_s) = 20 \times \frac{14}{5} = 56 \text{ minutes}$$

and $$E(W_q) = E(W_s) - \frac{1}{\mu} = 56 - \frac{1}{1/E(T)} = 56 - 16 = 40 \text{ min.}$$

i.e., a patient has to wait 40 minutes for check-up in the clinic.