STAT 135 CONCEPTS OF STATISTICS
QUIZ 2, LAB 102

July 15, 2021

Instructions: *You have 35 minutes to complete the quiz and upload it on bCourses. This quiz is open book and you may use a calculator, but all work must be shown in order to receive full credit.*

Problem 1 (5 points total). Let $X_1, \ldots, X_n$ be i.i.d random variable with probability density function

$$f(\theta x^{\theta-1}) = \begin{cases} \theta x^{\theta-1}, & \text{if } 0 \leq x \leq 1, \\ 0, \text{otherwise}, \end{cases}$$

in which $\theta \in [0, 1]$ is unknown.

(1) What is the MLE $\hat{\theta}_{MLE}$ of θ? Is $\hat{\theta}_{MLE}$ sufficient?

(2) Write the asymptotic Normal distribution of $\hat{\theta}_{MLE}$.

(3) Given $\hat{\theta}_{MLE} = 2.1$ and $n = 120$. Give a 95% bootstrap confidence interval for θ.

(4) Calculate $E(\hat{\theta}_{MLE})$ and $\text{Var}(\hat{\theta}_{MLE})$, and compare the variance to the CR lower bound. Which has lower value?

(*Hint*: (i)$E(aX) = aE(X)$, $\text{Var}(aX) = a^2\text{Var}(X)$; (ii) $-\log X_i \overset{\text{iid}}{\sim}$ Gamma$(1, \theta)$; (iii) If $W \sim \text{Gamma}(\alpha, \beta)$, $E\left(\frac{1}{W}\right) = \frac{\beta}{\alpha-1}$ and $\text{Var}\left(\frac{1}{W}\right) = \frac{\beta^2}{(\alpha-1)^2(\alpha-2)}$.)