1. MLE for Binomial:

$$L(p) = \prod_{i=1}^{n} \binom{k}{x_i} p^{x_i} (1-p)^{k-x_i}$$

$$= \left\{ \prod_{i=1}^{n} \binom{k}{x_i} \right\} \times p^{\Sigma x_i} (1-p)^{nk - \Sigma x_i}$$

The log-likelihood is:

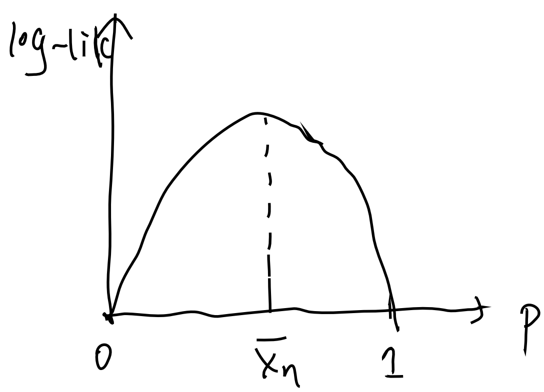
$$l(p) = \log \left\{ \prod_{i=1}^{n} \binom{k}{x_i} \right\} + \log p \cdot \Sigma x_i + \log(1-p) \cdot (nk - \Sigma x_i)$$

Taking the first derivative:

$$\frac{dl}{dp} = \frac{\Sigma x_i}{p} - \frac{nk - \Sigma x_i}{1-p} > 0$$

$$\Longleftrightarrow \quad p < \frac{\Sigma x_i}{nk}.$$

Therefore, the log-likelihood should look like this:



The MLE for p is:

$$\hat{p}_{MLE} = \frac{\bar{X}_n}{K}.$$

Since for Binomial distribution, $\mu = EX = kp$.

The MM estimator for p is also $\frac{\bar{X}_n}{K}$.

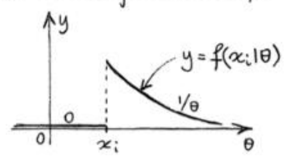
2. MLE for Uniform

$X_1, X_2, \dots, X_n \sim \text{Uniform}(0, \theta)$, i.e.

$$f(x_i|\theta) = \begin{cases} \frac{1}{\theta} & \text{for } 0 \leq x_i \leq \theta \\ 0 & \text{otherwise} \end{cases} \qquad (*)$$

Suppose we get sample values $x_1, x_2, \dots, x_n$, with each $x_i \geq 0$. Let's interpret $(*)$ as a function of θ:

$$f(x_i|\theta) = \begin{cases} \frac{1}{\theta} & \text{for } \theta \geq x_i \\ 0 & \text{otherwise} \end{cases}$$



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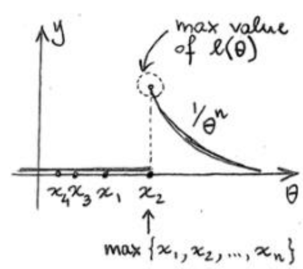
The likelihood, which is a function of θ, is

$$\ell(\theta) = f(x_1|\theta)\, f(x_2|\theta) \dots f(x_n|\theta),$$

which is 0 if, for any value of i, it is the case that $\theta \leq x_i$; otherwise $\ell(\theta) = \frac{1}{\theta^n}$.



In formulas:

$$\ell(\theta) = \begin{cases} \frac{1}{\theta^n} & \text{if } \theta \geq \max\{x_1, x_2, \dots, x_n\} \\ 0 & \text{otherwise} \end{cases}$$

In conclusion, $\hat{\theta}_{ML} = \arg\max_{\theta} \ell(\theta)$ is given by:

$$\boxed{\hat{\theta}_{ML} = \max\{x_1, x_2, \dots, x_n\}}$$

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