Stat 135
7/27/2021

Lab 11 Solution

1. One-Way ANOVA
   Rice 12.2.22: **Solution:** See the Shiny app.

2. Another Example
   Crysvita is a new medicine for the treatment of X-linked hypophosphatemia (a genetic disease), which was just approved by FDA in April 2018. Below is a simplified version of some test data for this pill where a high curing index corresponds to a positive effect. Construct a F-test assuming that each observation follows a Normal distribution with the same variances. Also construct 2 confidence intervals of the mean simultaneously using the Bonferonni method with an overall significance level of 5%. (A placebo has no effect on the patient, so it should be treated as a control group).

Pill	**n**	**Mean Curing Index**	**Standard Error of Mean**	**Sample SD**
Placebo	10	100	10	31.62
Crysvita	10	160	13	41.11

Solutions:
Here we have $I = 2$ and $J = 10$. Since we only have these summary statistics, we need to calculate SSb and SSw without the actual data. The last column can be calculated by multiplying the SE of mean by $\sqrt{10}$. Therefore,

$$SS_W = \sum (J-1)s_i^2 = 9 * (31.62^2 + 41.11^2) = 24208.71$$
$$\bar{Y}_{..} = \frac{\bar{Y}_{1.} + \bar{Y}_{2.}}{2} = \frac{100+160}{2} = 130$$
$$SS_B = J * \sum (\bar{Y}_{i.} - \bar{Y}_{..})^2 = 10 * (30^2 + 30^2) = 18000$$
$$F = \frac{SS_B/(I-1)}{SS_W/(I*(J-1))} = \frac{24208.71}{18000/18} = 24.209$$

According to the F-table, $F_{0.05,1,27} = 4.41 < 24.209$, so we reject the null that the new medicine has the same effect as a placebo (no effect).

Recall that the t statistic for 2 independent samples with size n is

$$t_{2n-2} = \frac{\bar{X} - \bar{Y} - 0}{s_p\sqrt{\frac{2}{n}}}$$

where

$$s_p = \sqrt{\frac{(n-1)(s_x^2 + s_y^2)}{2n-2}} = \sqrt{\frac{s_x^2 + s_y^2}{2}}$$

Let $a = \frac{\bar{X}-\bar{Y}}{2}$, so $\bar{X} - a = -(\bar{Y} - a) = b$

$$\begin{aligned}
\therefore (\bar{X} - \bar{Y})^2 &= [(\bar{X} - a) - (\bar{Y} - a)]^2 \\
&= b^2 + b^2 - 2b(-b) = 4b^2 \\
&= 2 * [(\bar{X} - a)^2 + (\bar{Y} - a)^2] \\
[(\bar{X} - a)^2 + (\bar{Y} - a)^2] &= \frac{(\bar{X} - \bar{Y})^2}{2}
\end{aligned}$$

Now let's rewrite our F statistic where $I = 2, J = n$

$$\begin{aligned}
F_{1,2(N-1)} &= \frac{J[(\bar{X} - a)^2 + (\bar{Y} - a)^2]/(I-1)}{(J-1)(s_x^2 + s_y^2)/(I*(J-1))} \\
&= \frac{\frac{n}{2}(\bar{X} - \bar{Y})^2}{s_p^2} = (t_{2n-2})^2
\end{aligned}$$