Econ C142 - Section 2

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1 Covariances

Let X and Y be two random variables and $\{x_i\}_{i=1}^N$ and $\{y_i\}_{i=1}^N$ be their realizations. Show the following two identities:

$$E[(X - E(X))(Y - E(Y))] = E[XY] - E[X]E[Y]$$

$$\frac{1}{N}\sum_{i=1}^{N}(y_i - \bar{y})(x_i - \bar{x}) = \frac{1}{N}\sum x_i y_i - \frac{1}{N}\sum y_i \frac{1}{N}\sum x_i$$

2 First Order Conditions and Law of Iterated Expectations[1]

Consider the population regression of a variable y on a constant and another variable x,

$$y_i = \beta_0 + \beta_1 x_i + u_i$$

1. Show that the first order conditions for the coefficients β_0 and β_1 imply that:

$$E[y_i] = \beta_0 + \beta_1 E[x_i]$$

$$E[x_i y_i] = \beta_0 E[x_i] + \beta_1 E[x_i^2]$$

2. Suppose that $E[y_i|x_i] = x_i^2$ and in addition that x_i is random variable that is symmetrically distributed around 0 with $E[x_i] = 0$, $E[x_i^2] = \sigma_x^2$ and $E[x_i^3] = 0$. What are the coefficients β_0 and β_1?

3 Frisch Waugh theorem[2]

Consider a population regression model in which an outcome y_i is related to two covariates x_{1i},x_{2i} as follows:

$$y_i = \beta_1 x_{1i} + \beta_2 x_{2i} + u_i$$

1. State the Frisch Waugh theorem relating the population regression coefficient β_2 to a univariate regression model for y_i which does not include x_{1i}. *NOTE: do not prove FW. Just state it as carefully and as clearly as you can in this case.*

2. In the case where $x_{i1} = 1$ (i.e. the first regressor is a constant), *prove* that your answer in part 1. implies that:

$$\beta_2 = \frac{E\left[y_i(x_{2i} - \mu_2)\right]}{E\left[(x_{2i} - \mu_2)^2\right]}$$

where $\mu_2 = E[x_{2i}]$.

3. Assume (as above) that $x_{1i} = 1$ and suppose that $x_{2i} > 0$ (i.e. that x_{2i} is a random variable that only takes on positive values), and that $y_i = x_{2i}^{\rho}$. Find the values of β_2 for $\rho = 1$, $\rho = 0$, and $\rho = -1$. *Extra points*: prove that when $\rho = -1$, $\beta_2 < 0$. (Hint: Jensen's inequality).[3]

[1] This exercise comes from the 2014 Midterm.
[2] This exercise comes from the 2015 Midterm.
[3] This hint was not included on the midterm exam.