Economics 152, Lecture 4: Intertemporal Labor Supply

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Intertemporal Labor Supply

- The neoclassical model of labor supply looks at a person's labor supply decision in a single time period
- In fact, people make labor supply decisions repeatedly over many years
- By working more and saving, a worker can "trade" some leisure time today for additional consumption tomorrow
- Models of **intertemporal labor supply** (also known as "life cycle" labor supply) analyze a forward-looking person's sequence of labor supply decisions over time

A Model of Intertemporal Labor Supply

- Consider a person who makes labor supply decision in time periods $t = 0...A$, where A is retirement age
- C_t and L_t denote consumption and leisure at time t
- In each period the person's utility function is $U(C_t, L_t)$, assumed to have all the usual properties
- The person discounts future utility at rate $\beta < 1$
- In period t, the wage is w_t and non-labor income is V_t
- The person can freely borrow and save at a constant interest rate r

Lifetime Utility Function

- We can write the person's lifetime utility as:

$$\sum_{t=0}^{A} \beta^t U(C_t, L_t)$$

Lifetime Budget Constraint

- As usual, the person maximizes utility subject to a budget constraint
- The budget constraint says that the present discounted value (PDV) of lifetime income must pay for lifetime consumption
- With an interest rate of r, \$1 tomorrow is worth $\$\frac{1}{(1+r)}$ today
- PDV of income:

$$\sum_{t=0}^{A} \frac{w_t h_t + V_t}{(1+r)^t}$$

- PDV of consumption:

$$\sum_{t=0}^{A} \frac{C_t}{(1+r)^t}$$

Lifetime Budget Constraint

- The lifetime budget constraint is:

$$\sum_{t=0}^{A} \frac{C_t}{(1+r)^t} \leq \sum_{t=0}^{A} \frac{w_t h_t + V_t}{(1+r)^t}$$

- Assuming as usual that T hours are available in each period, this is

$$\sum_{t=0}^{A} \frac{C_t}{(1+r)^t} \leq \sum_{t=0}^{A} \frac{w_t(T-L_t)}{(1+r)^t} + \sum_{t=0}^{A} \frac{V_t}{(1+r)^t}$$

- Critical assumption: The person can save or borrow freely at rate r
- If borrowing/saving weren't allowed, there would be a separate budget constraint for every period, and we're back to the static neoclassical model

Lifetime Budget Constraint

- Putting the pieces together, the person's utility maximization problem is:

$$\max_{C_0 \ldots C_A, L_0 \ldots L_A} \sum_{t=0}^{A} \beta^t U(C_t, L_t)$$

s.t.

$$\sum_{t=0}^{A} \frac{C_t}{(1+r)^t} \leq \sum_{t=0}^{A} \frac{w_t(T-L_t)}{(1+r)^t} + \sum_{t=0}^{A} \frac{V_t}{(1+r)^t}$$

- How do we solve this?

Lagrangean

- Let's write down a Lagrangean:

$$\mathcal{L} = \sum_{t=0}^{A} \beta^t U(C_t, L_t)$$

$$+\lambda \cdot \left(\sum_{t=0}^{A} \frac{w_t(T - L_t)}{(1+r)^t} + \sum_{t=0}^{A} \frac{V_t}{(1+r)^t} - \sum_{t=0}^{A} \frac{C_t}{(1+r)^t} \right)$$

- We now have first-order conditions for every C_t and every L_t, as well as λ

First-Order Condition

- First-order conditions:

$$\left[\frac{\partial \mathcal{L}}{\partial C_t} = 0\right] : \ \beta^t MU_C\left(C_t, L_t\right) - \frac{\lambda}{(1+r)^t} = 0$$

$$\implies MU_C(C_t, L_t) = \lambda \cdot \frac{1}{\left(\beta\left(1+r\right)\right)^t}$$

$$\left[\frac{\partial \mathcal{L}}{\partial L_t} = 0\right] : \ \beta^t MU_L(C_t, L_t) - \frac{\lambda w_t}{(1+r)^t} = 0$$

$$\implies MU_L(C_t, L_t) = \lambda \cdot \frac{w_t}{(\beta^t (1+r)^t)}$$

$$\left[\frac{\partial \mathcal{L}}{\partial \lambda} = 0\right] : \ \sum_{t=0}^{A} \frac{w_t(T - L_t)}{(1+r)^t} + \sum_{t=0}^{A} \frac{V_t}{(1+r)^t} - \sum_{t=0}^{A} \frac{C_t}{(1+r)^t} = 0$$

Simplifying Assumptions

- To get intuition about this model, let's make two simplifying assumptions
- First, utility is **additively separable** in consumption and leisure:

$$U(C_t, L_t) = u_1(C_t) + u_2(L_t)$$

- This means that MU_C only depends on C_t and MU_L only depends on L_t
- Second, let's assume the person's discount rate is

$$\beta = \frac{1}{1+r}$$

- This eliminates complications that arise if the person's patience is very different than the rate of return on savings

Intertemporal Intuition

- Under these assumptions, our first-order conditions for C_t and L_t become

$$MU_C(C_t) = \lambda$$

$$MU_L(L_t) = \lambda w_t$$

- What do these conditions imply about:
    - Differences in consumption between time periods?
    - Differences in labor supply between periods with higher/lower wages?

Intertemporal Intuition: Consumption

$$MU_C(C_t) = \lambda$$

- The first-order condition for consumption is the same for every time period
- This means consumption will be the same in every period

Intertemporal Intuition: Consumption

$$MU_C(C_t) = \lambda$$

- Intution: With diminishing marginal utility, the person will want to "smooth" consumption across periods
- Two important implications:
  - The person will not consume more/less in periods when she works and earns more/less
  - A dollar in non-labor income in one period will be used to increase consumption a little in all periods
- How would these predictions change in a model without borrowing and saving?

Intertemporal Intuition: Labor Supply

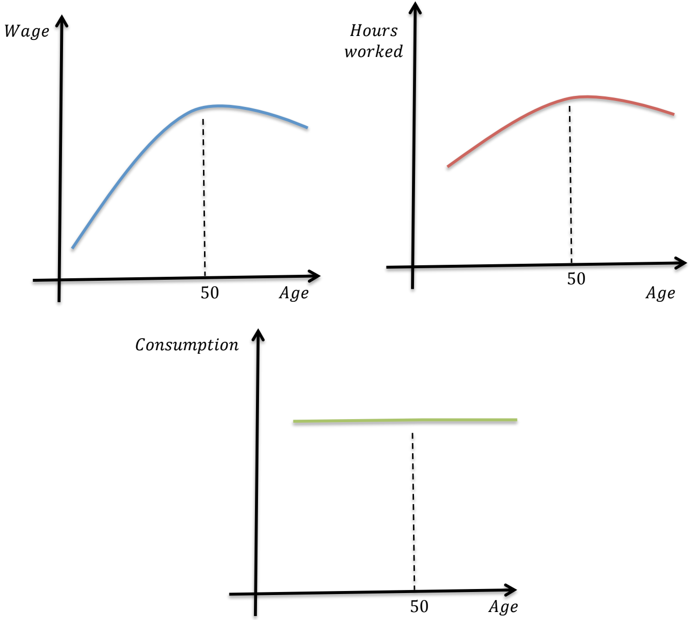
$$MU_L(L_t) = \lambda w_t$$

- This equation implies that differences in labor supply across periods are driven by the wage rate
- If the wage is the same in two periods, hours worked will also be the same
- The person will work more in periods when wages are high and less when wages are low

Intertemporal Intuition: Labor Supply

$$MU_L(L_t) = \lambda w_t$$

- Intuition: Leisure is expensive in a period when the wage is high – substitution effect pushes the person to work more in that period
- In contrast, the income effect of a higher wage is spread across all periods equally
- An anticipated change in the wage along a person's wage profile is known as an **evolutionary** wage change
- This model implies that wages and hours worked will move together as wages evolve over time



Intertemporal Labor Supply: Predictions

Wage Changes

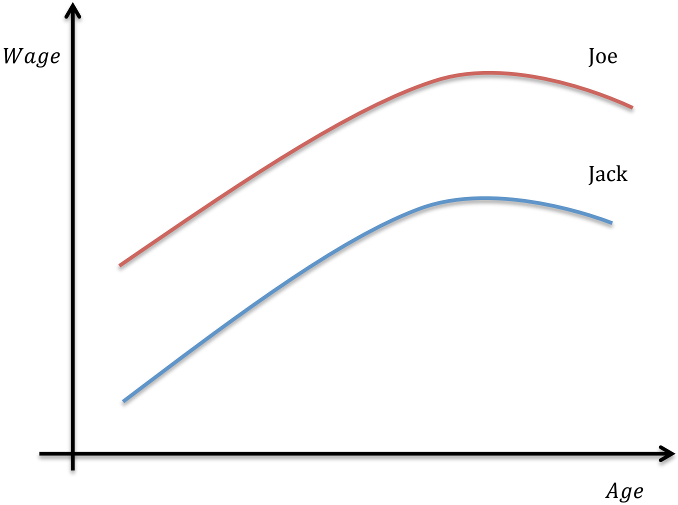
$$MU_L(L_t) = \lambda w_t$$

- Contrast the model's predicted effects of:
  - An expected evolutionary increase in the wage
  - An unexpected temporary increase in the wage
  - An unexpected permanent increase in the wage

Wage Changes

$$MU_L(L_t) = \lambda w_t$$

- The multiplier λ is the person's marginal utility of consumption. It reflects how "well" the person is doing in our big intertemporal optimization problem. A lower λ means the person is better off.
- An expected evolutionary increase has no effect on λ. The person will work more as the result of an evolutionary increase.
- An unexpected temporary increase causes a substitution effect, but the income effect is spread across all periods and there is little effect on λ. Similar to an evolutionary change, the person will work more.
- An unexpected permanent increase raises the present value of lifetime income, and reduces the value of λ. There are offsetting substitution and income effects, and hours worked may increase or decrease.



Suppose Joe and Jack have the same preferences. What does our model of intertemporal labor supply predict about

(a) each man's hours worked as he ages?
(b) Joe's hours vs. Jack's hours at a particular age?

The Intertemporal Substitution Elasticity

- Key concept in intertemporal labor supply: The **intertemporal substitution elasticity** (ISE)
- The ISE is defined as

$$\eta = \frac{\partial h_t}{\partial w_t} \cdot \frac{w_t}{h_t} | \lambda$$

- The ISE is the percentage change in hours worked caused by a 1% increase in the wage, *holding the multiplier λ constant*
- This is the elasticity that matters for the effects of anticipated evolutionary wage changes, and unanticipated temporary changes
- Our model predicts that the ISE must be positive

Intertemporal Evidence

- Next we'll consider empirical evidence on intertemporal labor supply
- In data that follows individuals over time, both wages and hours tend to increase and then flatten out as workers age, as the model predicts
- It's hard to say whether this reflects a positive ISE or something else (e.g. just the effects of age)
- It's also difficult to know which wage changes are expected vs. unexpected, and whether they are perceived as transitory vs. permanent
- Predictions of the intertemporal maximization framework are therefore difficult to verify

Fehr and Goette (2007): Intertemporal Evidence

- To get around these issues, Fehr and Goette (2007) run a randomized experiment
- Setting: Bike messengers in Zurich, Switzerland
  - Messengers deliver packages from place to place (messages, legal documents, digital media)
  - They are paid purely on commission, and can flexibly decide how many 5-hour shifts to work per week (max one per day)
- Fehr and Goette experimentally manipulate wages to measure the ISE

Fehr and Goette (2007): Setting

- Fehr and Goette study two Swiss messenger services: Veloblitz and Flash Delivery
- Messengers can commit to "fixed" shifts, and also choose "sign up" shifts
- Earnings are a percentage of daily revenues, not an hourly wage. If messengers work harder in a shift, they get paid more
- FG temporarily increase commission rates from 0.39 to 0.49 for men, and 0.44 to 0.54 for women (about 25%)
- Group A received the increase in September 2000 but not November; Group B received the increase in November but not September
- Experiment was announced in August 2000
- FG told messengers the study was about job satisfaction, not labor supply
- This is an anticipated, temporary increase in the wage, so can be used to credibly estimate the ISE

TABLE 1—DESCRIPTIVE STATISTICS

		Participating messengers		Difference groups A and B	Nonparticipating messengers, Veloblitz	Messengers, Flash
		Group A	Group B			
Four-week period prior to experiment	Mean revenues	3,500.67	3,269.94	241.67	1461.70	1637.49
		(2,703.25)	(2,330.41)	[563.19]	(1,231.95)	(1,838.61)
	Mean shifts	12.14	10.95	1.20	5.19	6.76
		(8.06)	(7.58)	[1.75]	(4.45)	(6.11)
	N	21	19		21	59
Treatment period 1	Mean revenues	4,131.33	3,005.75	1,125.59	844.21	1,408.23
		(2,669.21)	(2,054.20)	[519.72]	(1,189.53)	(1,664.39)
	Mean shifts	14.00	9.85	4.15	3.14	6.32
		(7.25)	(6.76)	[1.53]	(4.63)	(6.21)
	N	22	20		21	65
Treatment period 2	Mean revenues	2,734.03	3,675.57	−941.53	851.23	921.58
		(2,571.58)	(2,109.19)	[513.2]	(1,150.31)	(1,076.47)
	Mean shifts	8.73	12.55	−3.82	3.29	4.46
		(7.61)	(7.49)	[1.65]	(4.15)	(4.74)
	N	22	20		24	72

Notes: Standard deviations in parentheses, standard error of differences in brackets. Group A received the high commission rate in experimental period 1, group B in experimental period 2.
Source: Own calculations.

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TABLE 3—MAIN EXPERIMENTAL RESULTS
(*OLS regressions*)

	Dependent variable: Revenues per four-week period			Dependent variable: Shifts per four-week period		
	(1)	(2)	(3)	(4)	(5)	(6)
Observations are restricted to	Messengers participating in experiment	All messengers at Veloblitz	All messengers at Flash and Veloblitz	Messengers participating in experiment	All messengers at Veloblitz	All messengers at Flash and Veloblitz
Treatment dummy	1,033.6***	1,094.5***	1,035.8**	3.99***	4.08***	3.44**
	(326.9)	(297.8)	(444.7)	(1.030)	(0.942)	(1.610)
Dummy for nontreated at Veloblitz			−54.4			−0.772
			(407.4)			(1.520)
Treatment period 1	−211	−370.6	−264.8	−1.28	−1.57	−0.74
	(497.3)	(334.1)	(239.9)	(1.720)	(1.210)	(0.996)
Treatment period 2	−574.7	−656.2	−650.5**	−2.56	−2.63**	−2.19**
	(545.7)	(357.9)	(284.9)	(1.860)	(1.260)	(1.090)
Individual fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
R squared	0.74	0.786	0.753	0.694	0.74	0.695
N	124	190	386	124	190	386

Note: Robust standard errors, adjusted for clustering on messengers, are in parentheses.
*** Indicates significance at the 1-percent level.
** Indicates significance at the 5-percent level.
* Indicates significance at the 10-percent level.
Source: Own calculations.

Fehr and Goette (2007): ISE Estimate

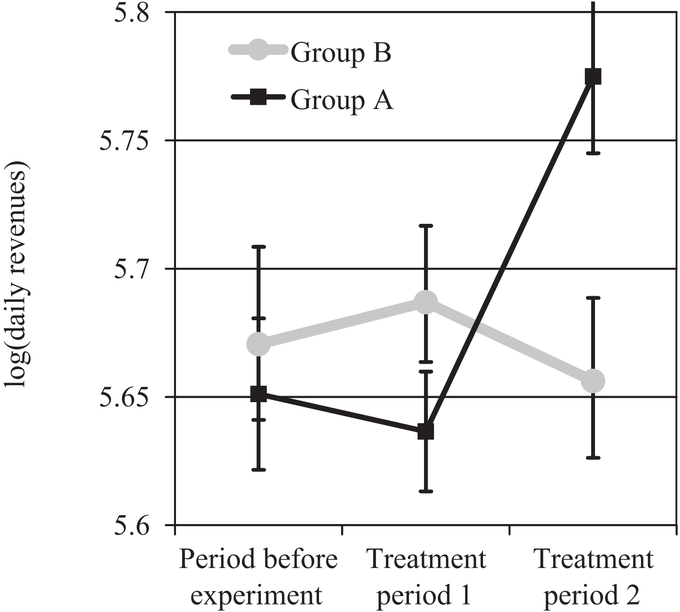
- On average, the higher wage induced workers to work four extra shifts
- The average number of shifts was 11.3
- This implies an hours elasticity of

$$\eta = \frac{\Delta h/h}{\Delta w/w}$$

$$= \frac{(4/11.3)}{0.25}$$

$$= 1.42$$

- Fehr and Geotte therefore find a very large ISE



A. Log of daily revenues

Fehr and Goette (2007): Conclusions

- Fehr and Goette also find that while the higher wage increased total revenues, these went up less than hours (revenue elasticity $= 1.2$)
- This implies less revenue per shift when the wage is high
- With high wages, people worked more shifts, but earned less per shift
- Interpretation?

Fehr and Goette (2007): Conclusions

- Fehr and Goette interpret this as evidence of non-standard "loss-averse" preferences – workers slack off once they reach an earnings target
- Alternative explanations:
  - Biking is tiring, so messengers become less productive when they work more shifts
  - Workers select highest-revenue shifts, and have to choose "worse" shifts when they want to work more
- Bottom line: The basic experimental results reveal large positive earnings and hours elasticities – suggests very large ISE
- How relevant is this estimate outside the context of bycicle messengers?