Economics 152, Lecture 2: The Neoclassical Model of Labor Supply

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The Neoclassical Model of Labor Supply

- The **neoclassical model of labor supply** isolates the factors that determine whether a person works, and if so, how much
- This model is used to analyze the choice between:
  - Leisure, L
  - Consumption of other goods, C
- People prefer leisure time to working
- But working generates income that can be used to purchase goods
- The model predicts the effects of changes in economic conditions and government policies on work incentives, in view of this tradeoff

Utility Function

- A person has a utility function, $U(C, L)$
- This function transforms leisure and consumption into an index of happiness
- Marginal utilities:

$$MU_L = \frac{\partial U}{\partial L} \quad MU_C = \frac{\partial U}{\partial C}$$

- MU_L is the change in utility resulting from an additional hour of leisure
- Marginal utilities are positive: $MU_L > 0$ and $MU_C > 0$. More leisure and more goods are preferred to less
- Marginal utilities are decreasing: $\partial MU_L / \partial L < 0$ and $\partial MU_C / \partial C < 0$. Each unit is less valuable than the previous one.

The Marginal Rate of Substitution

- The **marginal rate of substitution** between leisure and consumption is

$$MRS_{LC} = \frac{MU_L}{MU_C}$$

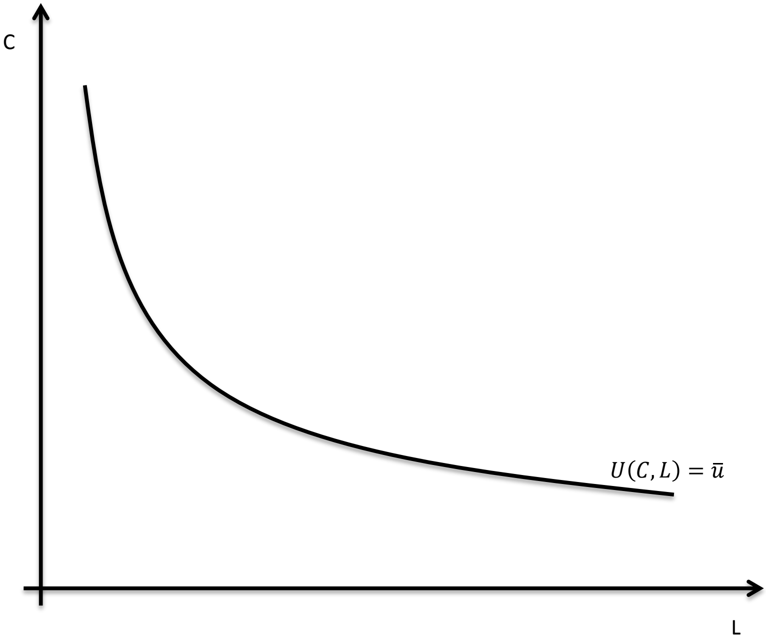
- The MRS is how much consumption the person is willing to give up to get another hour of leisure

Indifference Curves

- An **indifference curve** is a set of leisure/consumption combinations that yield the same utility
- Recall their key properties:
  1. Downward sloping
  2. Higher ones indicate higher utility
  3. Do not intersect
  4. Convex to the origin
- The slope of an indifference curve is minus the marginal rate of substitution:

$$\frac{\Delta C}{\Delta L} = -MRS_{LC}$$

- Convexity to the origin means diminishing MRS_{LC} as we move down an indifference curve


C
L
$U(C, L) = \bar{u}$

The Budget Constraint

- Consumption of goods is constrained by the person's income
- The budget constraint is:

$$C \leq wh + V$$

- C: Goods consumption (price normalized to one)
- h: Hours spent working
- w: The hourly wage
- V: Nonlabor income
- Assume there are T hours available for work or leisure, so $T = h + L$. Then the budget constraint is

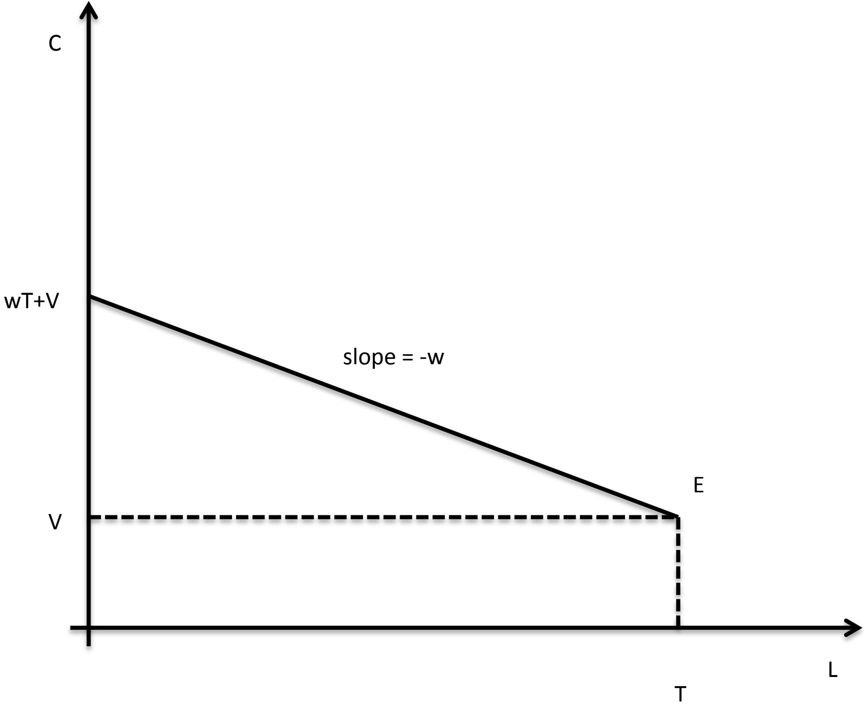
$$C \leq w(T - L) + V$$

The Budget Constraint

- The budget constraint can be re-written

$$C + wL \leq wT + V$$

- This defines a line with intercept $wT + V$ and slope $-w$
- Looks like a standard budget constraint, with income $(wT + V)$ and price of leisure w
- Note the difference between this and the usual budget constraint: The wage, w, determines both the price of leisure and the person's income
- The **endowment point**, E, is $(L = T, C = V)$
- Starting from this point, the person can sell an hour of leisure and increase consumption by w


C
wT+V
slope = -w
E
V
L
T

Utility Maximization

- Putting together the utility function and the budget constraint, the person's maximization problem is:

$$\max_{C,L} U(C, L)$$

s.t.

$$C + wL \leq wT + V$$

Lagrangean

- Write down a Lagrangean, with multiplier λ:

$$\mathcal{L} = U(C, L) + \lambda \cdot (wT + V - C - wL)$$

- First-order conditions: Set partial derivatives of $\mathcal{L}$ with respect to C, L, and λ equal to zero

$$\frac{\partial \mathcal{L}}{\partial C} = 0 : \quad MU_C = \lambda$$

$$\frac{\partial \mathcal{L}}{\partial L} = 0 : \quad MU_L = \lambda w$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 : \quad C + wL = wT + V$$

Solution

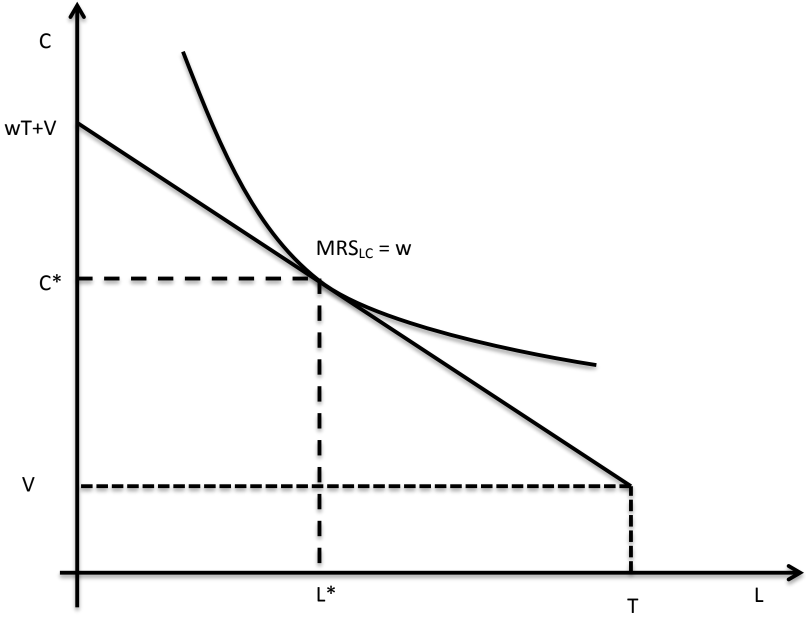
- Take the ratio of the first two conditions:

$$\frac{MU_L}{MU_C} = w$$

- In other words:

$$MRS_{LC} = w$$

- The MRS is the amount of consumption the person is willing to give up to get another hour of leisure
- The wage is the amount of consumption she is forced to give up
- At an interior solution, these must be equal
- Graphically: tangency between indifference curve and budget line


C
wT+V
MRS$_{LC}$ = w
C*
V
L*
T
L

Solution

- Another way of writing the tangency condition:

$$\frac{MU_L}{w} = MU_c$$

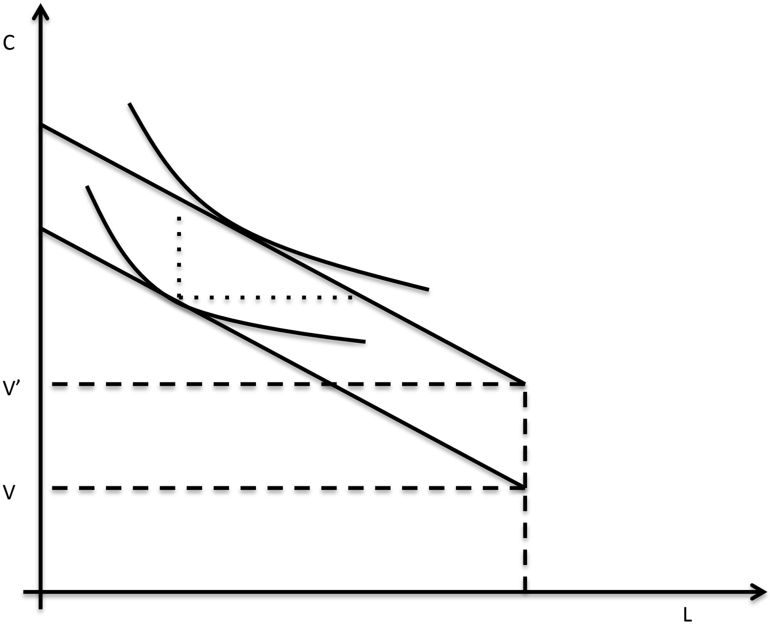
- The right-hand side is the utility gained from an extra dollar spent on consumption
- The left-hand side is the utility gained from an extra dollar "spent" on leisure
- Bang-per-buck must be the same for both goods
- Otherwise, the person could increase utility by spending more on the good with higher bang-per-buck

Comparative Statics

- The neoclassical model of labor supply can be used to predict the effects of changes in the budget set:
    - Changes in non-labor income, V
    - Changes in the wage, w

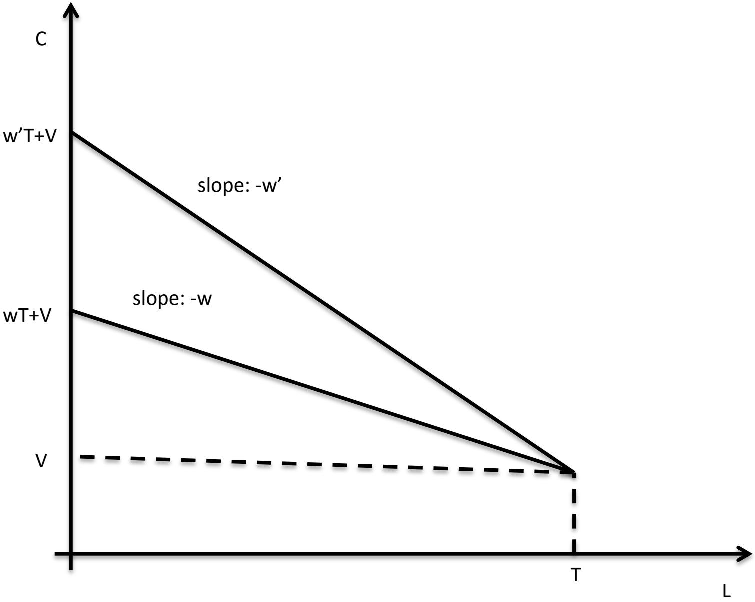
Changes in Non-Labor Income

- An increase in V induces a parallel shift of the budget line
- If L and C are both normal goods, consumption of both will increase
- Works just like an increase in income in standard consumer theory


C
V’
V
L

Changes in the Wage

- An increase in w induces a rotation of the budget line
- This is like an increase in the price of leisure
- Crucial difference from standard consumer theory: Value of endowment depends on w
- An increase in the wage therefore *expands* the budget set, rather than shrinking it


C
w'T+V
slope: -w'
slope: -w
wT+V
V
T
L

Income and Substitution Effects

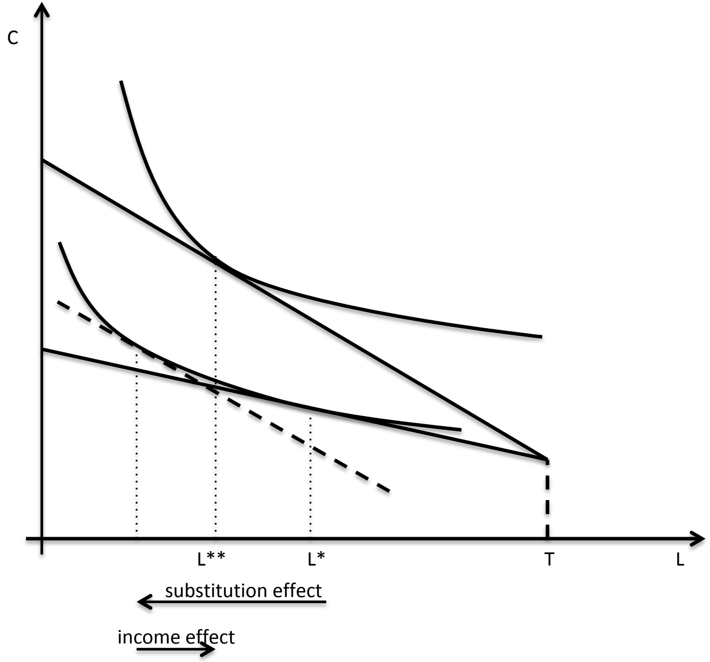
- Break the response to a wage increase into two parts:
  1. **Income effect:** The person is now effectively richer – her real income has risen
  2. **Substitution effect:** Leisure is now more expensive relative to consumption

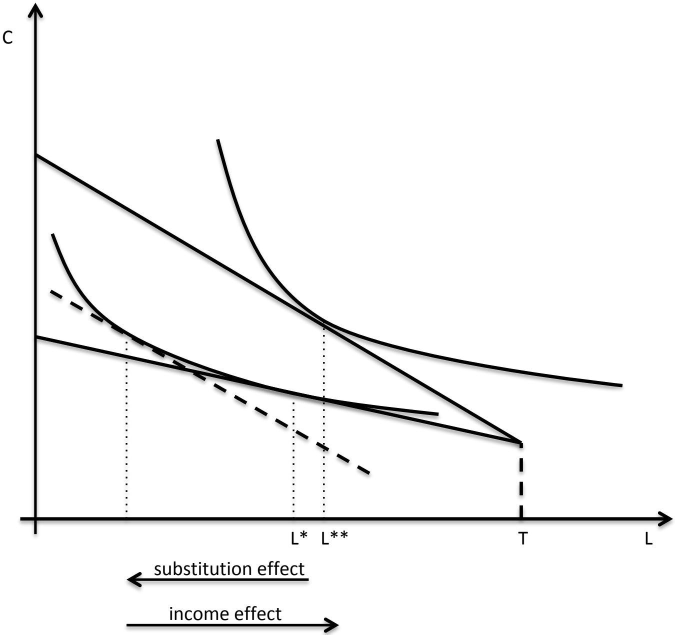
Income and Substitution Effects

- The person is endowed with T hours of leisure, and can sell them to increase consumption
- An increase in w is an increase in the sales price
- This makes the person richer, so she may want more of everything (income effect)
- But it also increases the value of selling an hour (substitution effect)
- Income and substitution effects therefore work in *opposite* directions

Income and Substitution Effects

- Graphically, represent the substitution effect with a line parallel to the *new* budget line, tangent to the *old* indifference curve
    - This increases consumption and reduces leisure
- Then represent the income effect as a parallel shift to the new budget line
    - This increases both consumption and leisure
- Total effect:
    - Consumption goes up
    - Leisure may go up or down


C
L**
L*
T
L
substitution effect
income effect

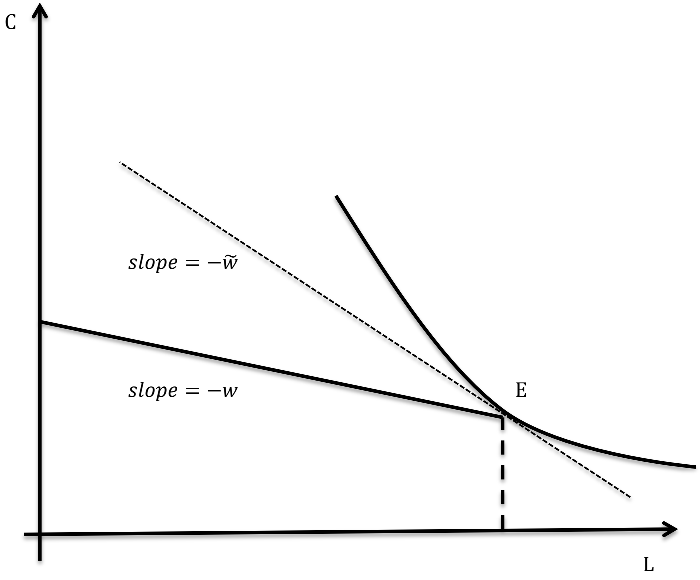

C
L* L** T L
substitution effect
income effect

The Participation Decision

- So far we've looked at decisions for people who work some of the time (interior solutions)
- In the real world, some people work zero hours
- This is a **corner solution**, with the person consuming the maximum possible amount of leisure: $L = T$
- What factors determine the decision to participate in the labor force?

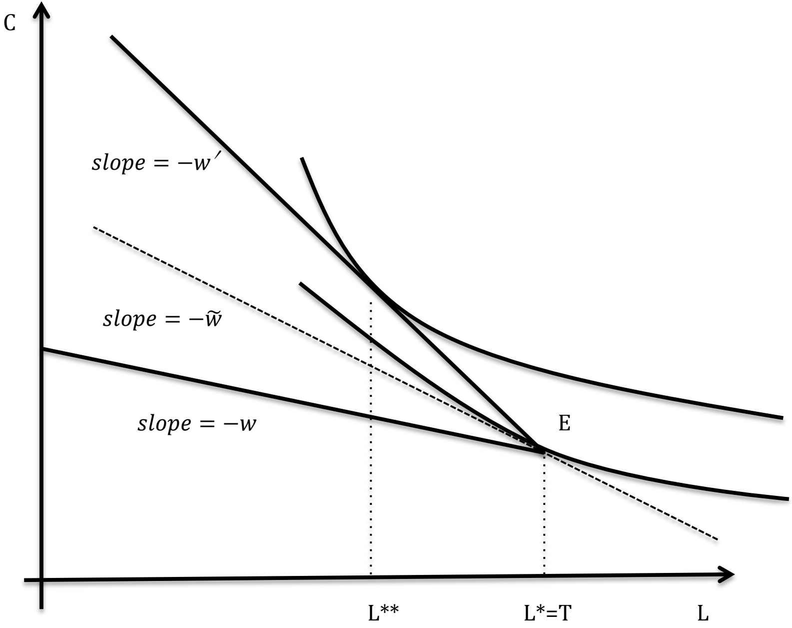
The Reservation Wage

- Consider the indifference curve through the endowment point E, where $L = T$ and $C = V$
- If the budget line is flatter than the IC at this point, the person chooses not to work
- If the budget line is steeper than the IC, she works
- The MRS at the endowment point therefore determines participation
- This MRS is called the **reservation wage**, $\tilde{w}$
- At a wage of $\tilde{w}$, the person is exactly indifferent between not working and entering the labor force
- The decision to work is based on a comparison of the market wage and reservation wage


C
$slope = -\widetilde{w}$
$slope = -w$
E
L

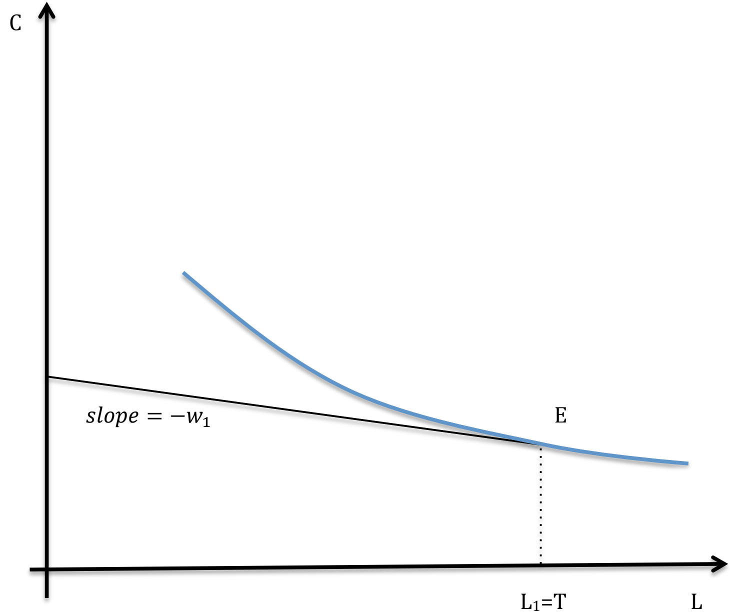
Income and Substitution Effects for Non-Workers

- An increase in w increases the chances that $w > \tilde{w}$, so increases the chances the person will work
- Contrast this with the predicted effect of a wage increase for someone who is already working (ambiguous)
- Key difference: A wage increase generates an income effect only if the person is already working
- The worker has no extra money if she remains at her initial choice – her real income has not increased
- The substitution effect pushes in the direction of working more

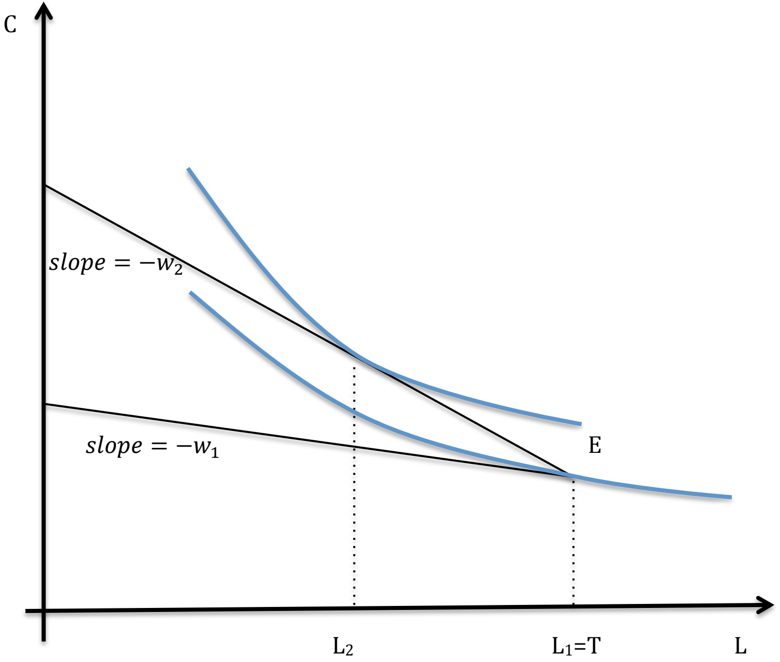

C
$slope = -w'$
$slope = -\widetilde{w}$
$slope = -w$
E
L**
L*=T
L

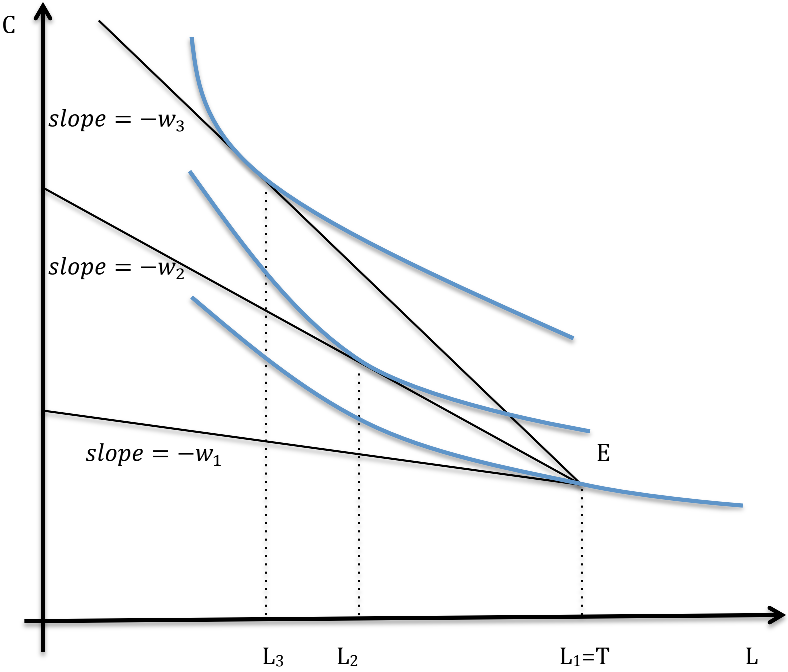
The Labor Supply Curve

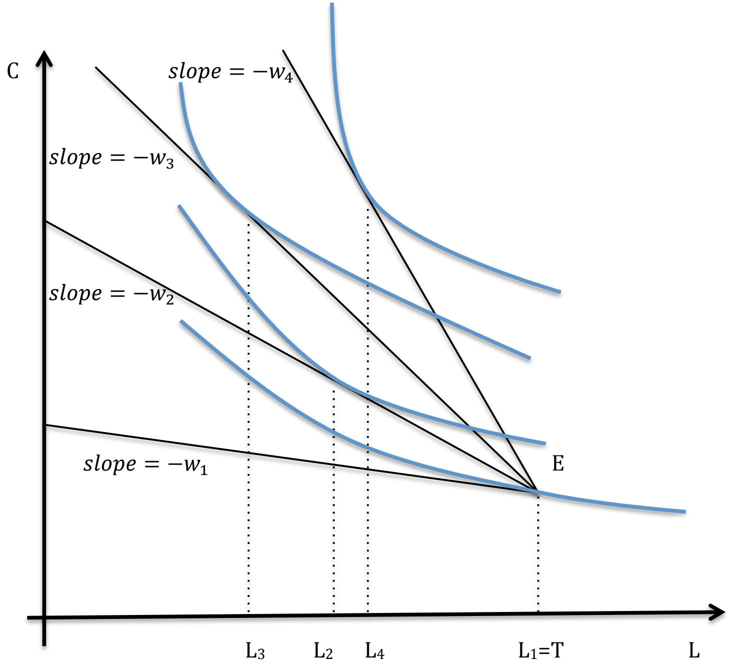
- The **labor supply curve** is the relationship between hours of work and the wage rate
- Our model of labor/leisure choice can be used to trace out this relationship
- Thinking about increasing the wage, starting from $w = 0$
- At first $w < \tilde{w}$ and the person doesn't work
- At $w = \tilde{w}$, the person starts working
- For wages just above $\tilde{w}$, the substitution effect dominates, and hours increase
- At higher wages income effects get stronger – hours may decrease
- This creates the possibility of a "backward-bending" labor supply curve

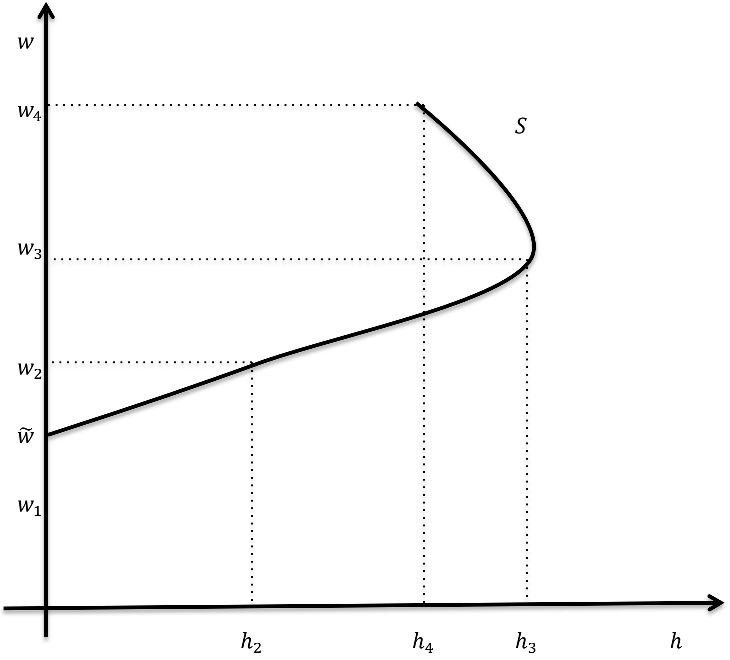


C
$slope = -w_1$
E
L_1=T
L


C
$slope = -w_2$
$slope = -w_1$
E
L_2
L_1=T
L


C
$slope = -w_3$
$slope = -w_2$
$slope = -w_1$
E
L_3
L_2
$L_1 = T$
L


C
slope = −w4
slope = −w3
slope = −w2
slope = −w1
E
L3
L2
L4
L1=T
L



w
w_4
S
w_3
w_2
$\widetilde{w}$
w_1
h_2
h_4
h_3
h

The Elasticity of Labor Supply

- The **elasticity of labor supply** measures the labor supply response to a wage change
- This elasticity is defined as:

$$\sigma = \frac{\text{Percent change in hours of work}}{\text{Percent change in the wage rate}} = \frac{\Delta h/h}{\Delta w/w}$$

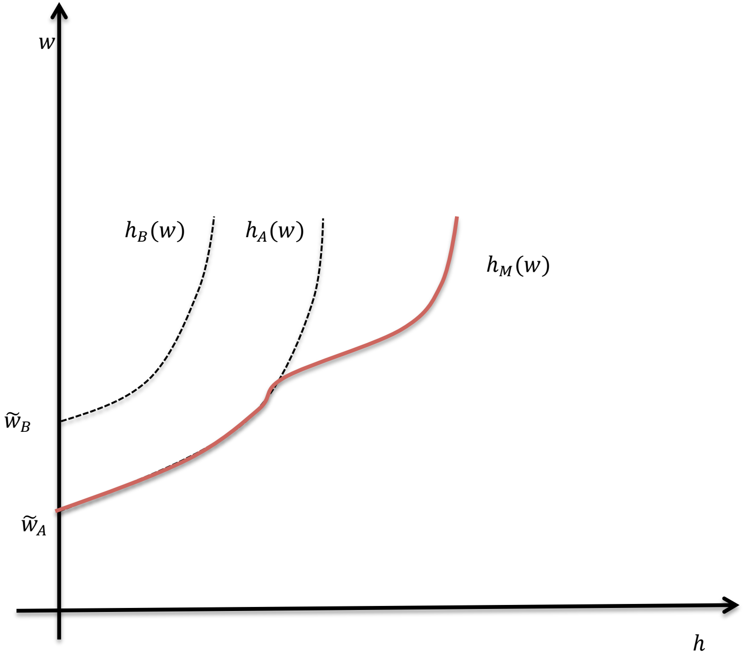
- Let $h(w)$ denote the hours a person will supply at a wage of w
- Then

$$\sigma = \frac{\partial h}{\partial w} \cdot \frac{w}{h}$$

The Market Labor Supply Curve

- The **market labor supply curve** adds up the supplies of workers at each wage
- Suppose there are two workers, Alice and Brenda
- Their reservation wages are $\tilde{w}_A$ and $\tilde{w}_B$; Brenda's is higher
- At a wage of w, they supply $h_A(w)$ and $h_B(w)$ hours
- Then the market supply curve, $h_M(w)$, is:

$$h_M(w) = \begin{cases} 0, & w < \tilde{w}_A \\ h_A(w), & \tilde{w}_A \leq w < \tilde{w}_B \\ h_A(w) + h_B(w), & w \geq \tilde{w}_B \end{cases}$$



w
$h_B(w)$
$h_A(w)$
$h_M(w)$
$\tilde{w}_B$
$\tilde{w}_A$
h

Application: Welfare Programs

- Let's apply the neoclassical model of labor supply to study welfare programs
- The Aid to Families with Dependent Children (AFDC) program traditionally provided income assistance to needy families
- AFDC was a **negative income tax (NIT)**: A cash grant to families with no earnings, phased out for higher-earners
- Political debate over AFDC centered on its work incentive effects
- 1996: Congress passed the Personal Responsibility and Work Opportunity Reconciliation Act (PRWORA)
- PRWORA replaced AFDC Temporary Assistance for Needy Families (TANF)
- Introduced time limits, tightened eligibilty rules, added work requirements

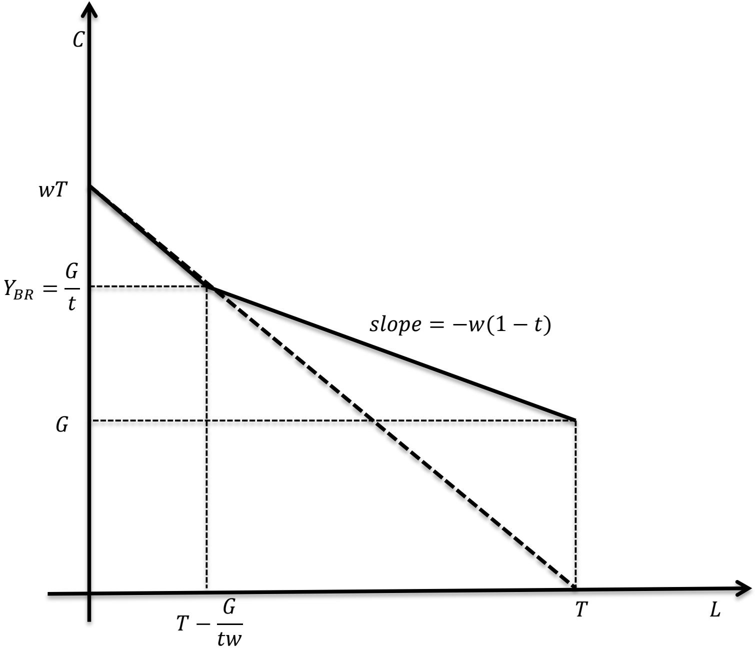
Negative Income Taxes

- An NIT provides a cash grant, G, to households with no income
- As a person earns more money, the guarantee is phased out at a tax rate t, until it hits zero
- Benefit formula:

$$B = \max\{G - t(wh + V), 0\}$$

- Break-even income:

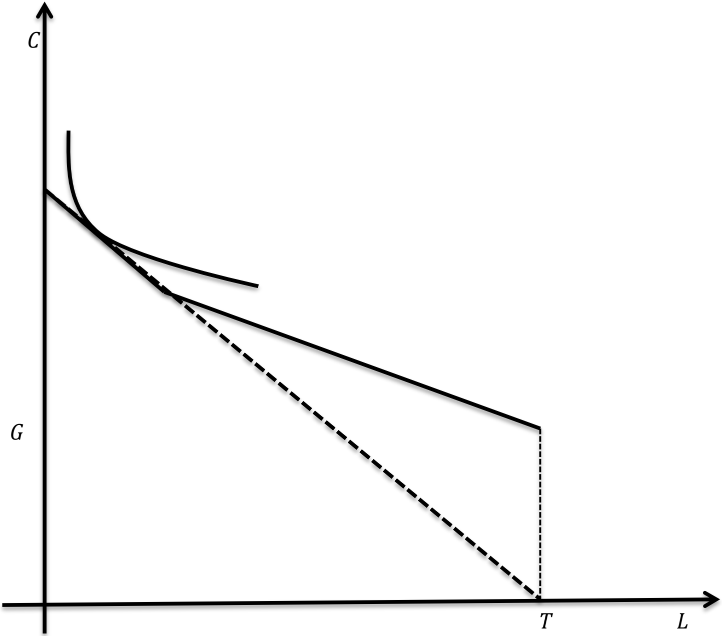
$$Y_{BR} = \frac{G}{t}$$

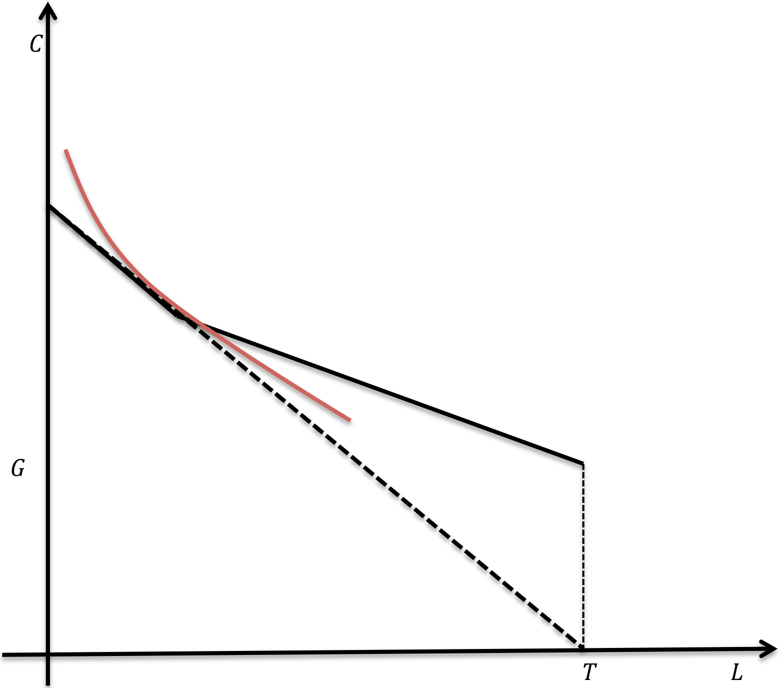


C
wT
$Y_{BR} = \frac{G}{t}$
$slope = -w(1-t)$
G
$T - \frac{G}{tw}$
T
L

Response to an NIT

- Think about labor supply response to introduction of an NIT for people who:
  1. Earn above Y_{BR}
  2. Work, but earn less than Y_{BR}
  3. Earn nothing (do not work)


C
G
T
L



C
G
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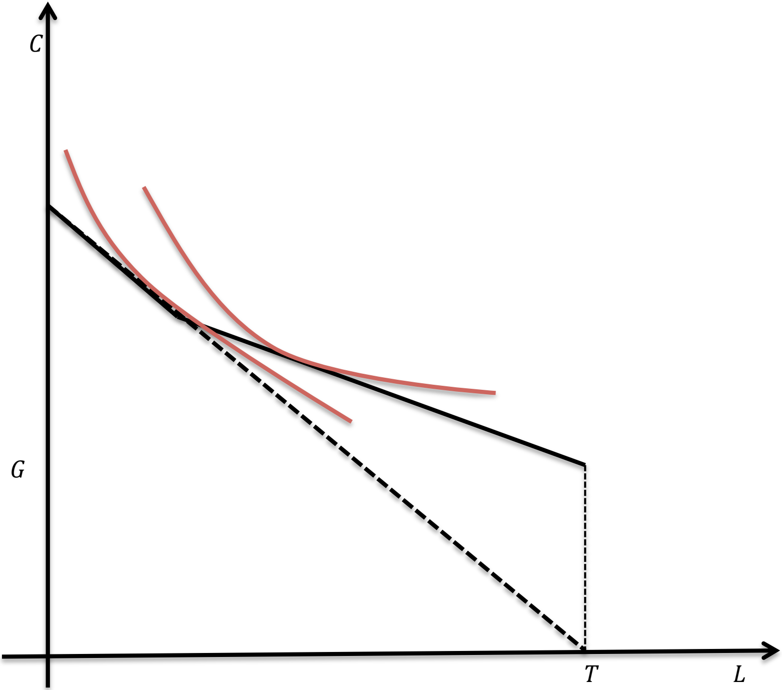
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G
T
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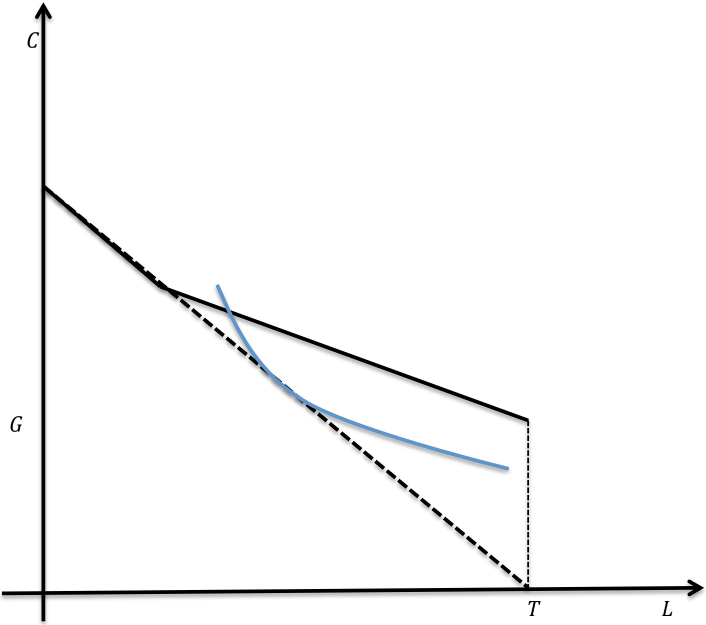

C
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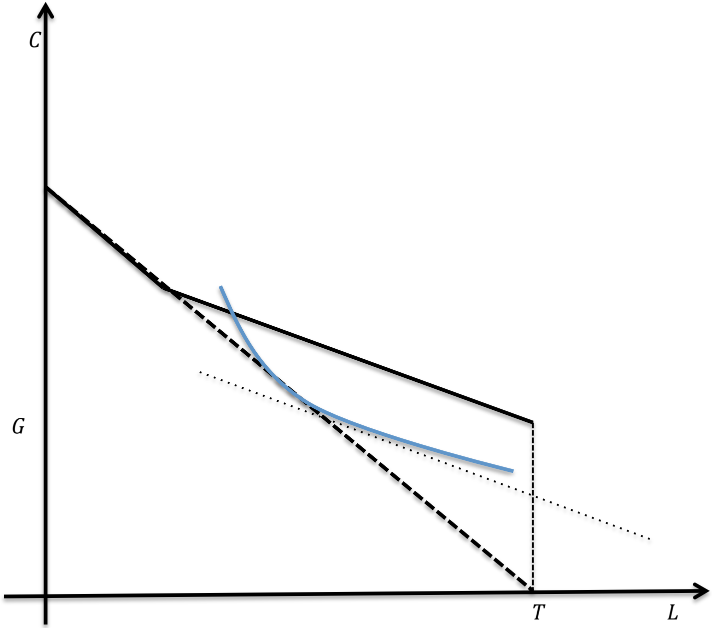

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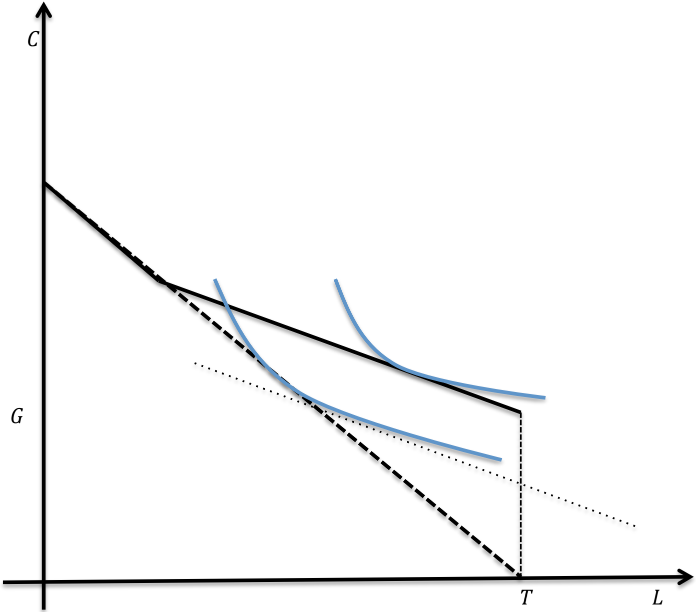
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G
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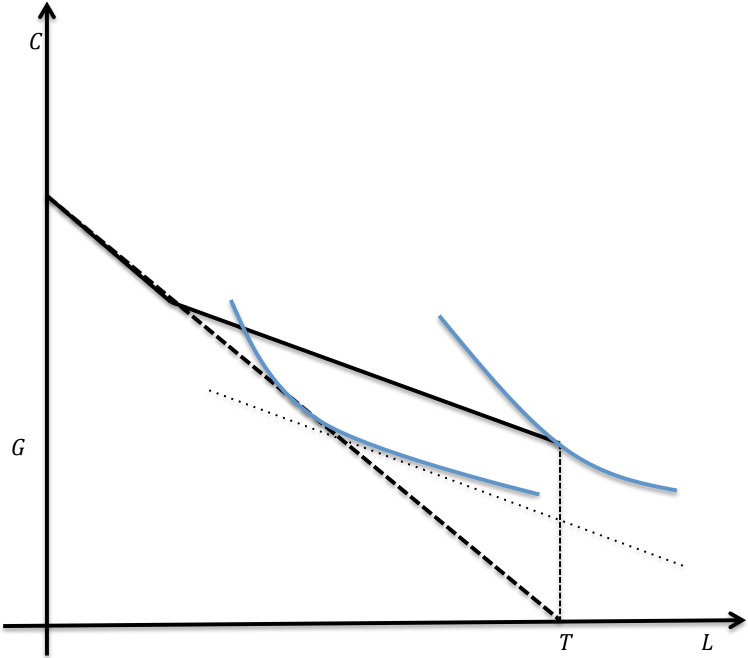

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G
T
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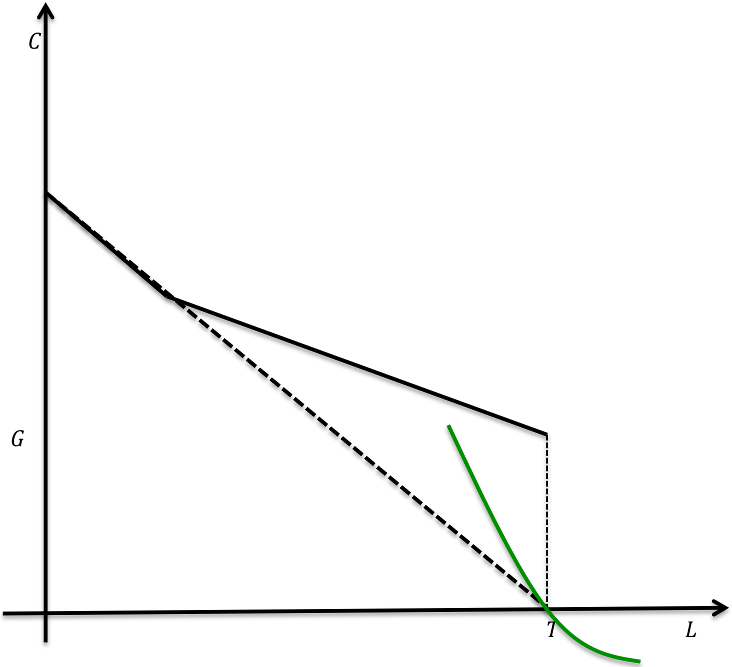

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G
T
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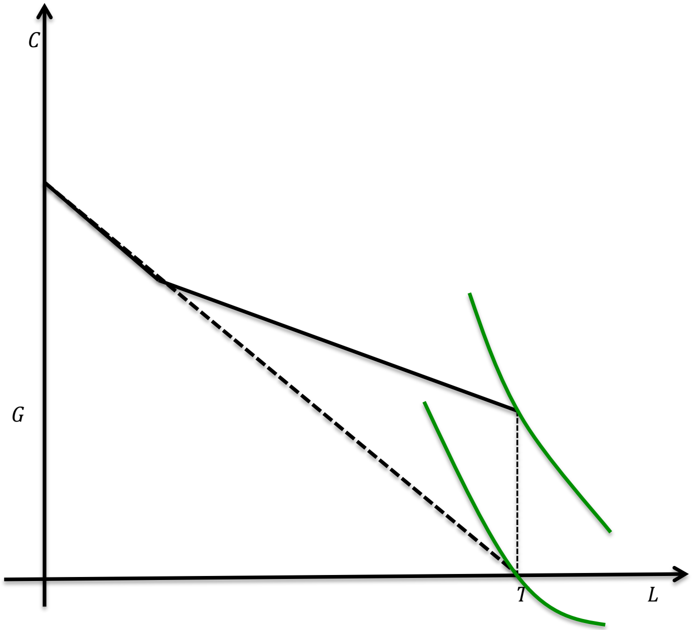

C
G
T
L

Response to an NIT

- Labor supply responses to introduction of an NIT for people who:
  1. Earn above Y_{BR}:
     - May not change
     - May reduce earnings to qualify for benefits
  2. Work, but earn less than Y_{BR}:
     - Reduce labor supply for sure
     - May exit the labor force
  3. Earn nothing (do not work)
     - No change. Will not start to work
- Next: Empirical evidence on labor supply responses to an NIT