Camera Calibration and Analysis -Project 3

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April 2023

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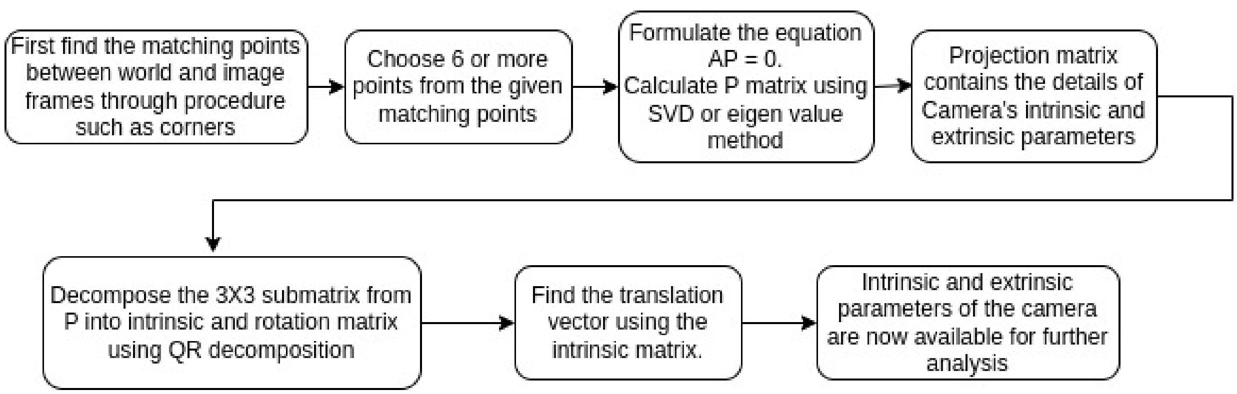
1.1 Problem 1.1

- **Approach**

  1. Camera calibration is the process of estimating the intrinsic and extrinsic parameters of the camera. Using these parameters the transformation from world frame to camera image plane frame becomes simple.
  2. A projection matrix(P) is the combination of the intrinsic and extrinsic matrices. the size of this matrix is $3X4$. So, while solving $AP = 0$, we have 12 unknown variables in the P vector which leads to 12 independent equations.
  3. To obtain these 12 points atleast 6 points from the image plane and world frame are required. Therefore, to perform or solve the camera calibration we need atleast **6 matching points from both image and world points**

1.2 Problem 1.2

- **Pipeline** The pipeline followed to solve the problem is below:



1.3 Problem 1.3

- **Mathematical Derivation to Find Intrinsic Matrix K:**

  1. The intrinsic camera matrix contains the information required for the transformation from camera projection frame to image plane where the image is captured. This matrix is a upper triangular matrix comprises of the focal length(f), projection center(cx,cy) and skew(generally taken as s= 1). The intrinsic matrix is calculated as below as below:

     $$P_{in} = \begin{bmatrix} f & s & cx \\ 0 & f & cy \\ 0 & 0 & 1 \end{bmatrix}$$

     The image points from above intrinsic matrix and camera point are calculated using below formula:

     $$P_i = P_{in} * P_c$$

     where P_i is the image plane coordinate, P_w is the world plane coordinates

  2. The extrinsic matrix is used for transformation of a world point to camera coordinate system. It is a combination of rotation and translation matrices. The calculation of a point in camera coordinate system from the world coordinate system is calcuated as below:

     $$P_c = P_{ext} * P_w$$

     $P_{ext} = \begin{bmatrix} R & T \end{bmatrix}$

     where P_ext is the camera's extrinsic matrix with a size of $3X4$, P_c is the point in camera coordinate system.

  3. Combining the intrinsic and extrinsic matrices gives ultimately the calibrated matrix called as projection matrix 'M' which is of size $3X4$.

     $$P_i = P_{in} * P_{ext} * P_w$$

     $$P_i = P_{in} * [R \;\; T] * P_w$$

  4. The projection matrix 'M' with 12 unknowns is written as:

     $$M = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}$$

  5. The 12 equations formed with 12 unknowns considering the world and image points gives rise to homogeneous linear system of form below as per ref[1]:

     $$A * M = 0$$

$$
\begin{bmatrix}
X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1Z_1 & -u_1 \\
0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1X_1 & -v_1Y_1 & -v_1Z_1 & -v_1 \\
 & & & & & & \vdots & & & & & \\
X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -u_nX_n & -u_nY_n & -u_nZ_n & -u_n \\
0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -v_nX_n & -v_nY_n & -v_nZ_n & -v_n
\end{bmatrix}
\begin{bmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{31} \\ m_{32} \\ m_{33} \\ m_{34} \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}
$$

6. Now, the matrix M is found by first computing the matrix $A^T * A$. The eigen vector corresponding to the minimum eigen value from the above computed matrix gives the projection matrix M.
7. Once the matrix M is computed, the left $3X3$ submatrix of M is taken out. This matrix is further used to get the intrinsic matrix P_{in} and rotation matrix R. This is obtained through QR decompostion.
8. The QR decomposition or gram Schmidt process is decomposing a matrix into upper triangular matrix and a orthogonal matrix. In this problem, we perform this operation using numpy's function 'qr' which gives us the upper triangular intrinsic matrix P_{in} and orthogonal rotation matrix R.
9. Finally, the translation matrix of the world frame is obtained by multiplying the inverse of intrinsic matrix with the last column of projection matrix according to ref[2].
10. Once we find the camera calibration parameters, we obtain the reprojection error for each point given using the projection matrix and world coordinate point.

1.4 Problem 1.4

- The Projection matrix thus resulted from above mathematical procedure is given as below

```
The projection matrix is:
[[ 2.87364445e+01 -1.75735412e+00 -7.00687538e+01  7.56890519e+02]
 [-2.01369011e+01  6.58890121e+01 -2.22140404e+01  2.13263797e+02]
 [-2.77042391e-02 -2.59559755e-03 -3.13888009e-02  1.00000000e+00]]
```

1.5 Problem 1.5

- The Intrinsic matrix, rotation matrix, translation matrix and reprojection errors calculated are below

```
The intrinsic matrix is given as:
[[ 338.46686596 -378.60686345 -430.53464237]
 [  -0.          510.75461595 -563.3383194 ]
 [  -0.           -0.            1.        ]]
The rotation matrix is given as :
[[-8.18945183e-01 -5.73870862e-01  1.01057196e-03]
 [ 5.73871209e-01 -8.18945560e-01  6.63468593e-05]
 [ 7.89528891e-04  6.34272593e-04  9.99999487e-01]]
The translation matrix is given as:
[[5.20906732]
 [1.52049946]
 [1.        ]]
The reprojection error for point 1 is  0.28561277320378464
The reprojection error for point 2 is  0.9725828090697887
The reprojection error for point 3 is  1.0360818174590143
The reprojection error for point 4 is  0.45408636463784796
The reprojection error for point 5 is  0.19089837357312717
The reprojection error for point 6 is  0.31899200483588097
The reprojection error for point 7 is  0.19594246762020287
The reprojection error for point 8 is  0.30829584584109465
The mean reprojectin error for all the points is 0.47031155703009275
```

2 Problem 2

2.1 Camera Calibration for given set of Images

- **Pipeline Approach**

  1. From the given original dimensions for checkerboard, each image is being looked for corners in pattern size of (9,6) where size of each square considered is 21.5mm .
  2. For each image below is the following procedure followed to find the corners, pixel locations of the corners and drawing the corners:
     – First, each image is converted to gray-scale image. On the gray-scale 'findChessboardCorners' function is used from opencv to find the corners for the given pattern size.
     – Now, to refine the corner location to get exactly the required corners on the chessboard 'cornerSubPix' from opencv is used. These corners are stored as image pixel location point for each image. Also, these corners are shown on the original image using 'drawChessBoardCorners' function.
  3. Now, to find the camera matrix or intrinsic matrix, calibrate camera function from opencv is used with given image size, world coordinate points of corners from each image and pixel locations of corners from each image. The calibrate camera function returns intrinsic matrix, rotation vectors, distortion coefficients and translation vectors.
  4. Finally, the re-projection errors for each image is calculated. This is performed by first calculating the projected pixel coordinate corners from the world point corners, intrinsic matrix, rotation and translation matrices. These are fed to projectPoints function in openCV giving calculated corners pixel locations of each image.

5. These corner points are subracted from the original chessboard corners using L2 norm. This is called as the re-projection error.

- **Results** The intrinsic matrix and other parameters calculated are shown in below figure:

```
The Intrinsic Matrix K is:
[[2.04272943e+03 0.00000000e+00 7.64360020e+02]
 [0.00000000e+00 2.03501640e+03 1.35902591e+03]
 [0.00000000e+00 0.00000000e+00 1.00000000e+00]]
The reprojection error for image 1 is  0.10805784499400187
The reprojection error for image 2 is  0.062355126713086954
The reprojection error for image 3 is  0.03201701753001409
The reprojection error for image 4 is  0.11822729646816064
The reprojection error for image 5 is  0.05727789035017854
The reprojection error for image 6 is  0.12290643223159095
The reprojection error for image 7 is  0.08916218986685945
The reprojection error for image 8 is  0.13817273307939035
The reprojection error for image 9 is  0.07488903455136782
The reprojection error for image 10 is  0.1159751002243462
The reprojection error for image 11 is  0.07120680804111207
The reprojection error for image 12 is  0.07547120291245601
The reprojection error for image 13 is  0.07290379963579774
Mean Total Reprojection error is : 0.08758634435372022
```

Figure 1: Results of Intrinsic Matrix, Re-projection error

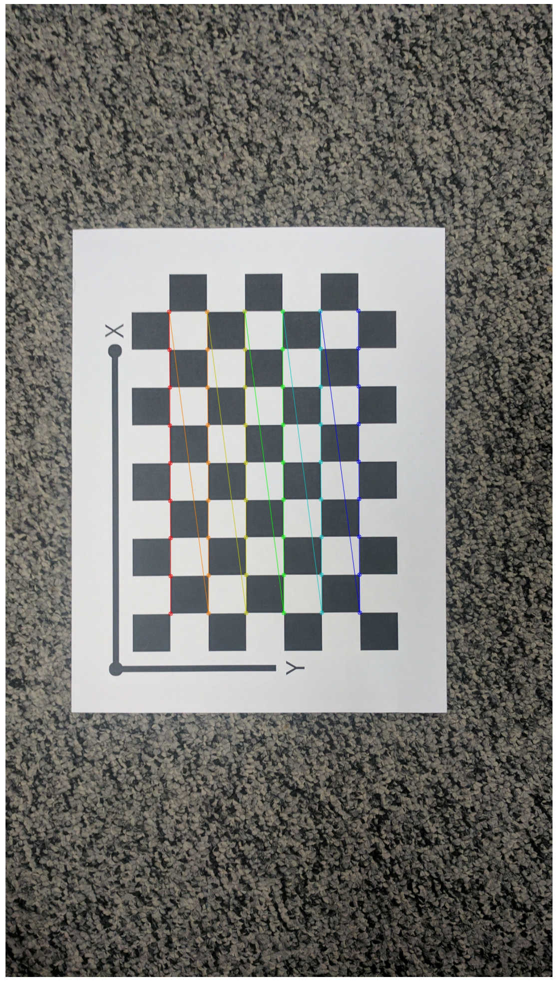


Figure 2: Corner Points in one of the image

2.2 Intrinsic Matrix Accuracy Improvement

1. For the current problem, the accuracy of the intrinsic matrix can be improved by follwing ways with ultimate goal of minimizing the re-projection error.

    - Capture more number of images and perform camera calibration with images taken at different angles and field of view. Also, accurate target will be helpful.
    - Use accurate camera model to account for distortions such as radial or tangential. A function from openCV such as 'optimalNewCameraMatrix' can be used to obtain more refined intrinsic matrix.Remove images with more distortions
    - Use more accurate initial estimates of the world points and perfrom re-calibration of the camera parameters.

2. Also, sometimes outlier corner points will be present in each image or some images are only considered as outliers where matching points are not accurate. Such images or points are removed using outlier rejection methods such as RANSAC, Total Least Squares.

3 References

1. Lecture Notes-Perception for Autonomous Robots, Dr. Samer Charifa.

2. ”First Principles of Computer Vision , Shree Nayar” - A youtube series

3. https://www.cc.gatech.edu/classes/AY2016/cs4476_fall/results/proj3/html/kshu6/index.html