Network Theory

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Website: www.pw.live

Email: support@pw.live

NETWORK THEORY

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1 BASIC CONCEPTS OF NETWORKS

1.1 Introduction

Network theory is the study of solving the problems of electric circuits or electric networks.

An electric circuit contains a closed path for providing a flow of electrons from a voltage source or current source. The elements present in an electric circuit will be in **series connection, parallel connection,** or in any combination of series and parallel connections and an electric network need not contain a closed path for providing a flow of electrons from a voltage source or current source. Hence, we can conclude that “all electric circuits are electric networks” but the converse need not be true.

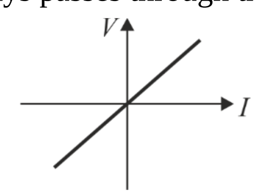
1.1.1. Types of Network Elements

Different types of network elements are

1. Linear Elements and Non-linear Elements.
2. Bilateral Elements and Unilateral Elements.
3. Active Elements and Passive Elements.
4. Time Invariant and Time Variant Elements.
5. Lumped and Distributed Elements.

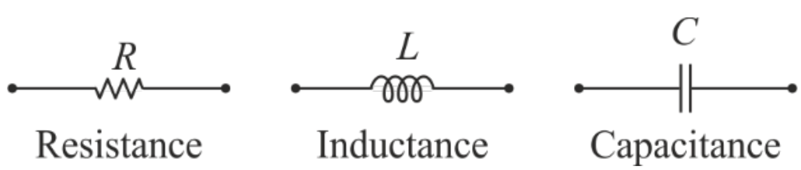
1. Linear Elements

Characteristics of linear elements always passes through the origin in the form of straight line.



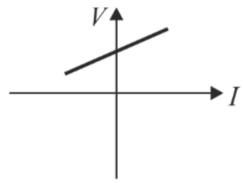
Linear

Example of linear elements:

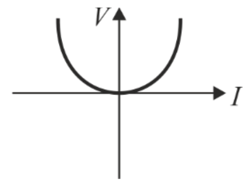


All basic electrical elements are linear (R, L, C).

2. Non-Linear Elements

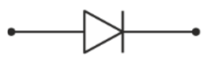


Non linear

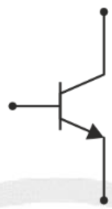


Non linear

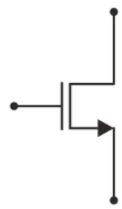
Example of non-linear elements :



Diode



BJT



MOSFET

All electronic devices are non-linear (Diode, MOSFET, JFET).

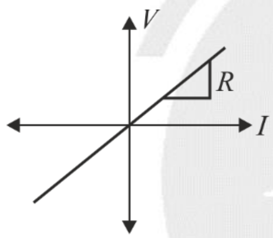


Fig. *V*-*I* plane

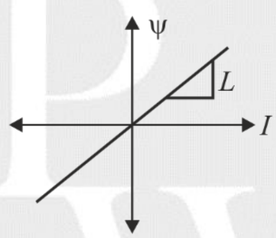


Fig. Ψ-*I* plane

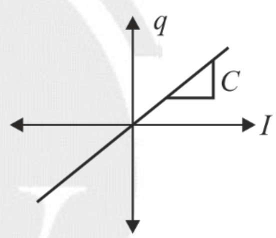
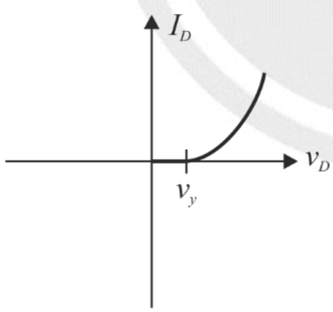
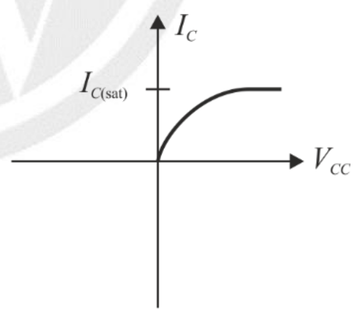


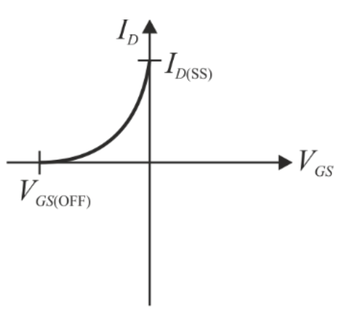
Fig. *q*-*v* plane



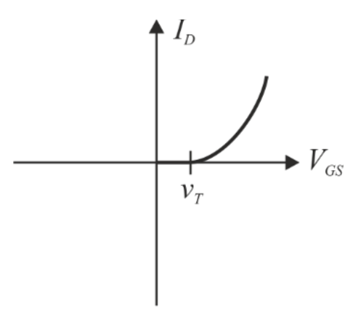
Feedback characteristics of diode



Output characteristics of BJT



Transfer characteristics of JFET



Forward characteristics of MOSFET

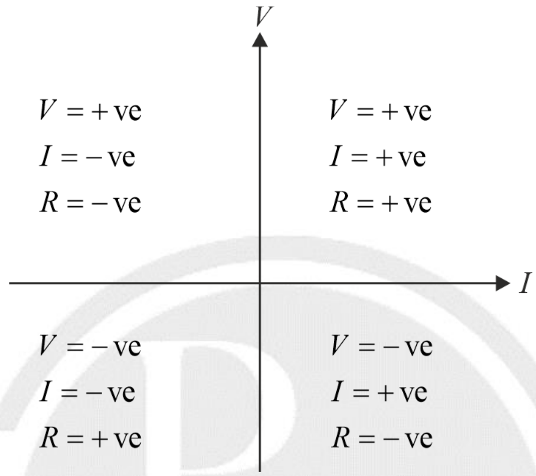
2. Bilateral and Unilateral Elements

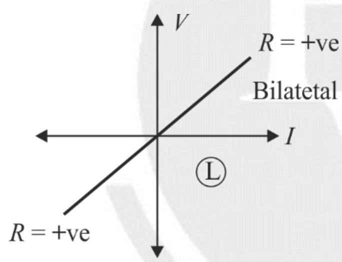
(i) In case of V-I plane, characteristics of **bilateral** elements offer same impedance throughout the characteristics.

(ii) In case of V-I plane, characteristics of **unilateral** elements offer different impedance in different origin.

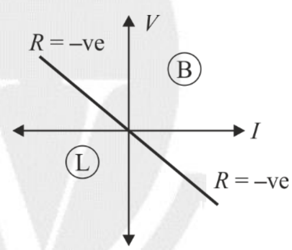
(iii) In case of generalized plane, characteristics of **bilateral** elements is always symmetrical about origin.

Characteristics are same in either I and III quadrant or II and IV quadrant.

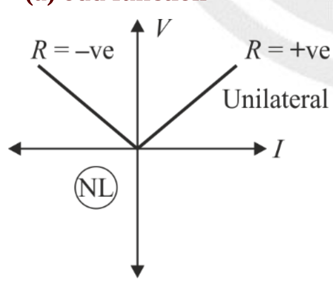




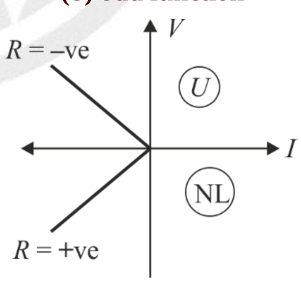
(a) odd function



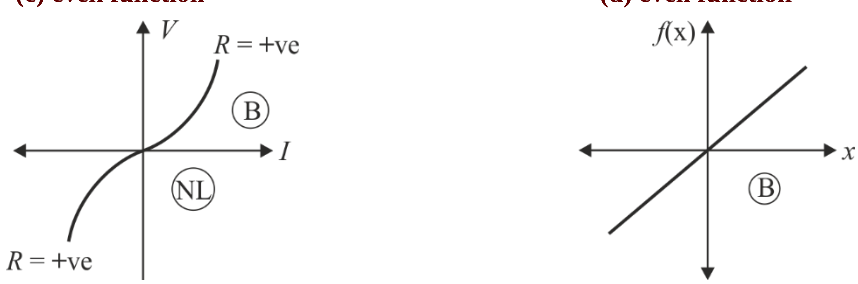
(b) odd function



(c) even function



(d) even function



(e) odd function

Note: L = Linear, NL = Non-linear, B = Bilateral, U = Unilateral

All linear elements are bilateral but reverse is not true.

Network element	Bilateral/Unilateral
R	Bilateral
L	Bilateral
C	Bilateral

Device Element	Bilateral/Unilateral
Diode	Unilateral
BJT	Unilateral
JFET	Unilateral

3. Active and Passive Elements

(i) In case of V-I plane, characteristics of passive element always have positive impedance where as active element offeres negative impedance.

(ii) Passive elements absorb the energy whereas active elements deliver the energy.

(iii) Active element controls, the flow of energy whereas passive elements dissipate or store the energy.

(iv) Elements having capability of delivering the energy is referred as active elements.

Examples of active elements :

(a) Voltage source

(b) Current source

(c) Generator

(d) Biased Transistors

(e) Operational Amplifier

Note: All dependent sources are considers as an active element.

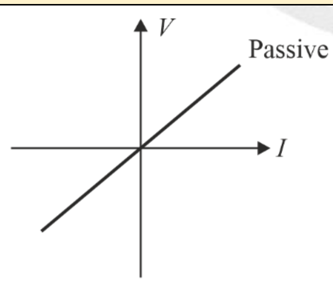


Fig. (a)

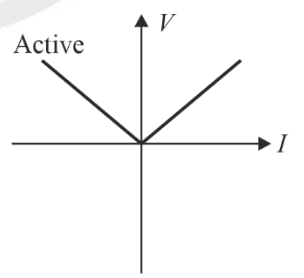


Fig. (b)

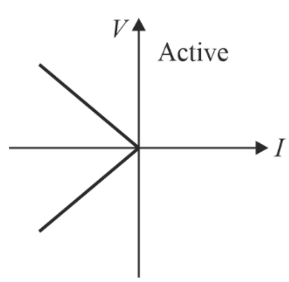


Fig. (c)

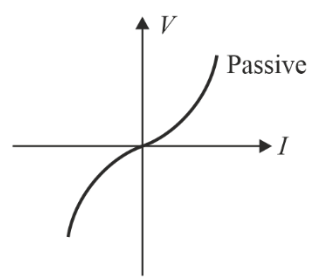
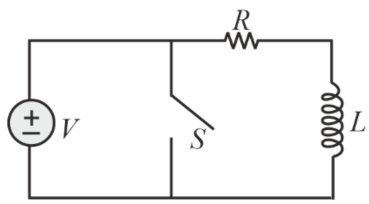


Fig. (d)

Generally, inductors and capacitors are passive elements. As they can not deliver the energy independently for long time? They can give energy only at the time of discharge.



Condition: At $t = 0$, S is closed

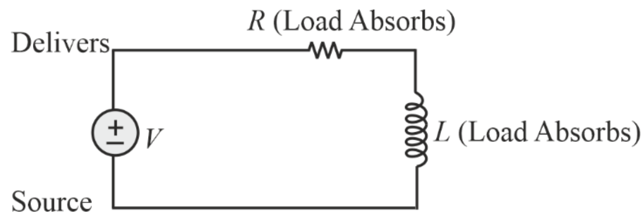


Fig. (i) $t < 0$
Source RL network
(charging RL network)

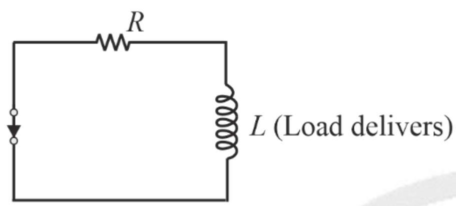


Fig. (iii) $t > 0$
Source RL network
(discharging RL network)

4. Lumped and Distributed Elements

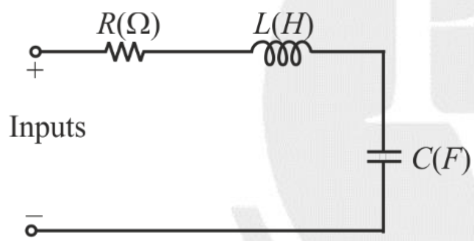


Fig. RLC Network

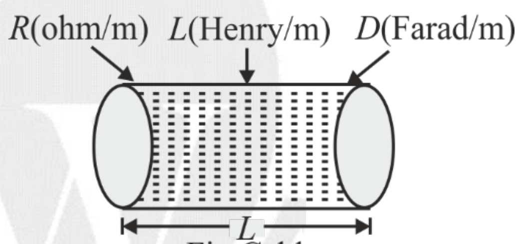


Fig Cable

(i) Physically separated elements in a network is referred as lumped elements.

(ii) If elements are distributed along the line, then it is referred as distributed elements.

(iii) Concept of circuit theory is based on lumped elements.

(iv) Concept of field theory is based on distributed elements.

Network Elements	Passive Condition	Active Condition
R	$R \geq 0$	$R < 0$
L	$L \geq 0$	$L < 0$
C	$C \geq 0$	$C < 0$

<table>
<tr>
<td>R = 2 Ω (Passive)
R = 4 kΩ (Passive)
R = –6 kΩ (NP)
L = 1 mH (Passive)
L = –2 μH (NP)</td>
<td>
<table>
<tr><th>Network Theory</th></tr>
<tr><td>$R \geq 0$</td></tr>
<tr><td>$L \geq 0$</td></tr>
<tr><td>$C \geq 0$</td></tr>
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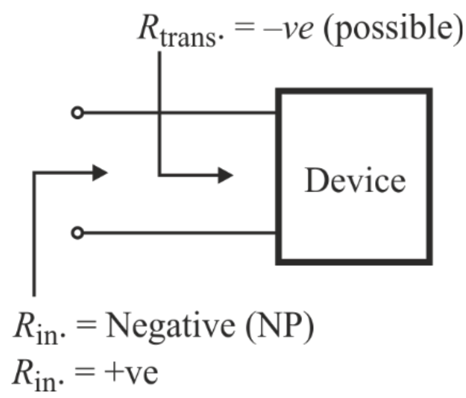


5. Time Invariant and Time Variant Element

(i) If characteristics of elements is varied with time, then it is called **Time Variant.**

(ii) If characteristics of elements is not varied with time, then it is called **Time Invariant.**

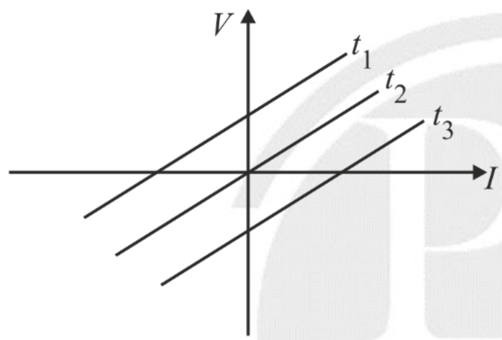


Fig. T-V characteristics

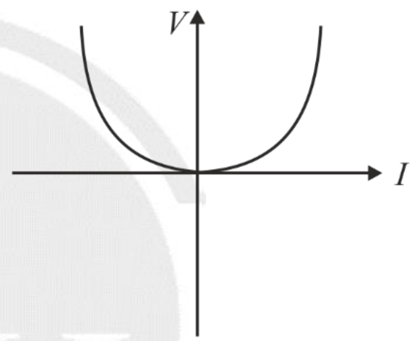


Fig. T-V characteristics

Network Element	TI/TV
R	TI
L	TI
C	TI

1.2. Analysis of Passive Elements

Resistor :

The main functionality of Resistor is either opposes or restricts the flow of electric current.

Relationship between resistance and resistivity is given by,

$$R = \frac{\rho l}{A}$$

$\rho =$ resistivity of the material

$l =$ length of wire

$A =$ area of cross section of the wire

According to Ohm's law, the voltage across resistor is the product of current flowing through it and the resistance of that resistor, provided the temperature is constant.

Mathematically, it can be represented as

$$V = IR$$

$$I = \frac{V}{R}$$

Where, R is the resistance of a resistor and unit of R is ohm (Ω).

From field theory of ohm's law, (Microscopic form of Ohm's Law)

$$J \propto E$$

$$J = \sigma E$$

$J =$ Current density

$E =$ Electric field intensity

$\sigma =$ Conductivity of materials

R can range from zero to infinity, the two extreme possible values. An element with $R = 0$ is called as short circuit, as shown in figure. For a short circuit, $V = IR = 0$.

Fig. (A) A short-circuit (R = 0) **Fig. (b) An open circuit (R = ∞)**

Similarly, an element with resistance $R = \infty$ is known as an open circuit, as shown in figure (b).
For an open circuit,

$$I = \frac{V}{R} = 0$$

Network Condition	I	V	R
SC	I_{sc}	0	0
OC	0	V_{sc}	∞

Current :

The current "I" flowing through a conductor is nothing but the time rate of flow of positive charge. Mathematically, it can be written as,

$$i(t) = \frac{dQ}{dt} \qquad [\text{C/sec = Amp}]$$

$$Q = \int i(t)\,dt$$

Q is the charge and its unit is Coulomb.

t is the time and its unit is Second.

I is the current and its unit is Ampere.

Voltage/Potential :

The voltage "V" is nothing but an electromotive force that causes the charge (electrons) to flow.

$$V = \frac{dW}{dQ}$$

$$W = \int V(Q)dQ$$

W is the potential energy and its unit is Joule.

Q is the charge and its unit is Coloumb.

V is the voltage and its unit is Volt.

Power :

The power "P" is nothing but the time rate of flow of electrical energy. Mathematically, it can be written as

$$P = \frac{dW}{dt}$$

$$P = \frac{dW}{dt} = \frac{dW}{dQ} \times \frac{dQ}{dt}$$

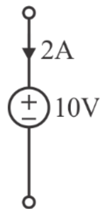
$$P = VI$$

Concept of Absorbed and Delivered Power :

1. If current enters into the positive terminal of voltage source, then it is referred as absorbed power (i.e. power is absorbed by voltage source).
2. If current leaves from positive terminal of voltage source, then it is referred as delivered power (i.e. power is delivered by voltage source).

Examples :

(i)



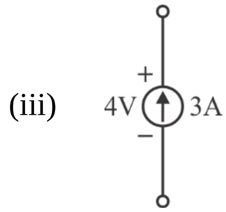
$$P_{absorbed} = 2 \times 10 = 20 \text{ watt}$$

$$P_{delivered} = -20 \text{ watt}$$

(ii)

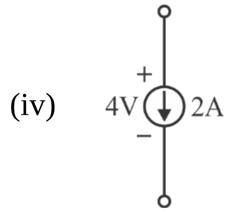
$$P_{delivered} = 2 \times 5 = 10 \text{ watt}$$

$$P_{absorbed} = -10 \text{ watt}$$



$P_{delivered} = 3 \times 4 = 12$ watt

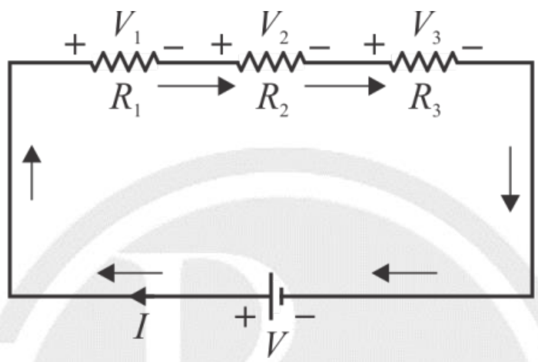
$P_{absorbed} = -12$ watt



$P_{absorbed} = 4 \times 2 = 8$ watt

$P_{delivered} = -8$ watt

Series Equivalent Circuit :



$R_{eq} = R_1 + R_2 + R_3$

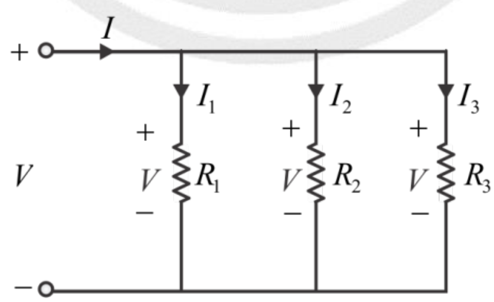
Concept :

(i) $I_1 = I_2 = I_3 = I$

(ii) $R_{eq} = R_1 + R_2 + R_3$

(iii) $R_{eq} > R_{max}$

Parallel Equivalent circuit :



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Concept :

(i) $V_1 = V_2 = V_3 = V$

(ii) $R_{eq} = \dfrac{R_1 R_2 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3}$

(iii) $R_{eq} < R_{min}$

Inductor :

Inductance is the property of inductor which opposes the change of current.

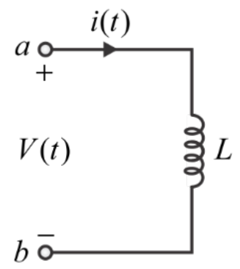
$$I_L(0^+) = I_L(0^-) = I_L(0)$$

$$\psi(t) \propto i(t)$$

$$\psi = Li$$

$$L = \frac{\psi}{i}$$

$$\psi = N\phi$$



$\psi =$ Total magnetic flux (Flux linkage of a coil)

$N =$ Number of turns in coil

$\phi =$ flux per turn

$$L = \frac{N\phi}{i}$$

$$\phi = \frac{MMF}{Reluctance} = \frac{Ni}{l/A\mu}$$

$$L = \frac{N^2 A\mu}{l}$$

$$L \propto N^2$$

$$V(t) = \frac{d\psi}{dt}$$

$\because$ $$\psi = Li$$

$$V(t) = L\frac{di(t)}{dt}$$

$$i(t) = \frac{1}{L}\int V(t)\,dt$$

Energy in inductors :

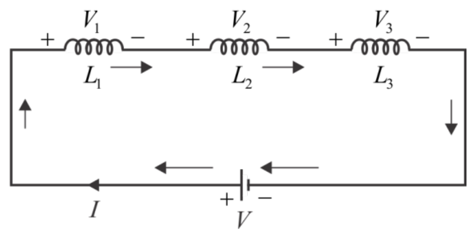
Energy, $$W = \int P(t)\,dt$$

$$W = \frac{1}{2}Li^2 \text{ Joule(J)}$$

Note :

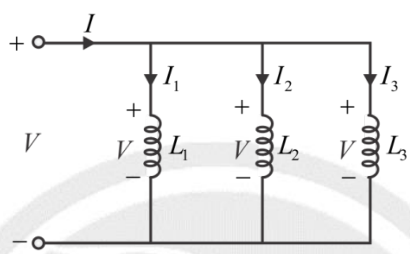
1. Inductor stores the energy in the form of magnetic fields.
2. It is referred as absorbing element like resistor.

Series Equivalent Circuit :



$$L_{eq} = L_1 + L_2 + L_3$$

Parallel Equivalent circuit :



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

Concept :

(i) $V_1 = V_2 = V_3 = V$

(ii) $L_{eq} = \dfrac{L_1 L_2 L_3}{L_1 L_2 + L_2 L_3 + L_3 L_1}$

(iii) $L_{eq} < L_{\min}$

Capacitor :

Capacitance is the property of capacitor that opposes the change of voltage $V(0^-) = V(0^+)$ of the movement time.

$$q \propto V$$

$$q = CV$$

$$C = \frac{q}{V} \left[\frac{\text{Columb}}{\text{Volt}} \frac{(\text{C})}{(\text{V})} \text{or Farad (F)} \right]$$

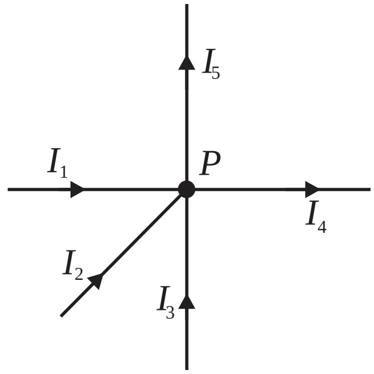
$$i(t) = \frac{dq}{dt}$$

$$i(t) = C\frac{dV(t)}{dt}$$

Kirchoff's Current Law (KCL) :

- In DC circuit KCL states that the algebraic sum of currents entering a node (or a closed boundary) is zero.
- Mathematically, KCL implies that

$$\sum_{n=1}^{N} i_n = 0$$



Applying KCL at node P,

$$-I_1 - I_2 - I_3 + I_4 + I_5 = 0$$

$$I_1 + I_2 + I_3 = I_4 + I_5$$

Sum of incoming currents = Sum of outgoing currents

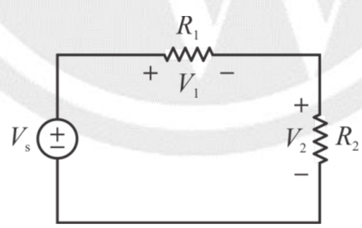
Nodal Analysis :

- It is combination of KCL and Ohm's law (KCL + Ohm's law).
- In Nodal analysis, we will consider the node voltages with respect to Ground. Hence, Nodal analysis is also called as Node-voltage method.

Kirchoff's Voltage Law (KVL) :

- KVL states that the algebraic sum of all voltages around a closed path (or loop) is zero.
- Mathematically, KVL states that

$$\sum_{m=1}^{M} v_m = 0$$



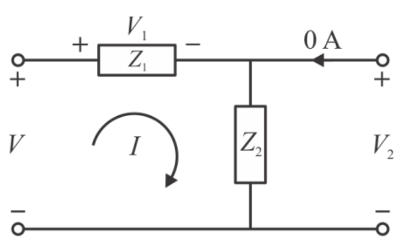
Applying KVL in the above circuit,

$$V_s - V_1 - V_2 = 0$$

$$V_s = V_1 + V_2$$

Sum of voltage drops = Sum of voltage rises

Voltage Division Rule [VDR]



It is used in series equivalent circuit for distribution of voltage.

$$V_1 = \frac{V \times Z_1}{Z_1 + Z_2}$$

$$V_2 = \frac{V \times Z_2}{Z_1 + Z_2}$$

Voltage division rule for Resistor :

$$Z[R_1] = R_1 \qquad Z[R_2] = R_2$$

$$\therefore \quad V_1 = \frac{V \times R_1}{R_1 + R_2} \qquad V_2 = \frac{V \times R_2}{R_1 + R_2}$$

Voltage division rule for Inductor :

$$Z[L_1] = j\omega L_1 \qquad Z[L_2] = j\omega L_2$$

$$\therefore \quad V_1 = \frac{V \times j\omega L_1}{j\omega L_1 + j\omega L_2} \qquad V_2 = \frac{V \times j\omega L_2}{j\omega L_1 + j\omega L_2}$$

$$V_1 = \frac{V \times L_1}{L_1 + L_2} \qquad V_2 = \frac{V \times L_2}{L_1 + L_2}$$

Voltage division rule for Capacitor:

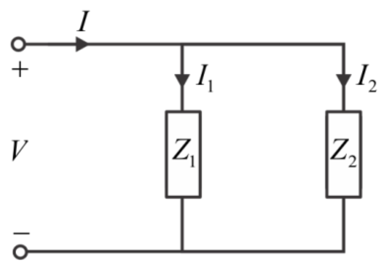
$$Z[C_1] = \frac{1}{j\omega C_1} \qquad Z[C_2] = \frac{1}{j\omega C_2}$$

$$\therefore \quad V_1 = \frac{V \times \frac{1}{j\omega C_1}}{\frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}} \qquad V_2 = \frac{V \times \frac{1}{j\omega C_2}}{\frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}}$$

$$V_1 = \frac{V \times C_2}{C_1 + C_2} \qquad V_2 = \frac{V \times C_1}{C_1 + C_2}$$

Current Division Rule [CDR]

CDR is applicable when passive elements and connected in parallel.



$$I_1 = \frac{I \times Z_2}{Z_1 + Z_2}, \quad I_2 = \frac{I \times Z_1}{Z_1 + Z_2}$$

Current division rule for Resistor :

$$Z[R_1] = R_1 \qquad Z[R_2] = R_2$$

$$\therefore \quad I_1 = \frac{I \times R_2}{R_1 + R_2} \qquad I_2 = \frac{I \times R_1}{R_1 + R_2}$$

Current division rule for Inductor :

$$Z[L_1] = j\omega L_1 \qquad Z[L_2] = j\omega L_2$$

$$\therefore \quad I_1 = \frac{I \times j\omega L_2}{j\omega L_1 + j\omega L_2} \qquad I_2 = \frac{I \times j\omega L_1}{j\omega L_1 + j\omega L_2}$$

$$I_1 = \frac{I \times L_2}{L_1 + L_2} \qquad I_2 = \frac{I \times L_1}{L_1 + L_2}$$

Current division rule for Capacitor :

$$Z[C_1] = \frac{1}{j\omega C_1} \qquad Z[C_2] = \frac{1}{j\omega C_2}$$

$$\therefore \quad I_1 = \frac{I \times \frac{1}{j\omega C_2}}{\frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}} \qquad I_2 = \frac{I \times \frac{1}{j\omega C_1}}{\frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}}$$

$$I_1 = \frac{I \times C_1}{C_1 + C_2} \qquad I_2 = \frac{I \times C_2}{C_1 + C_2}$$

Star to Delta Conversion [Y to Δ] or [T to π] Conversion

Star to Delta conversion [T to π] :

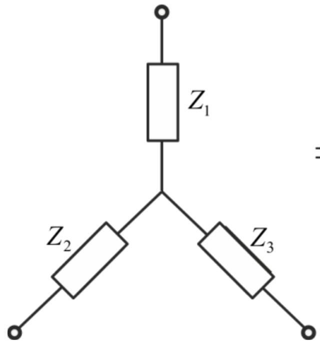


Fig. Star network

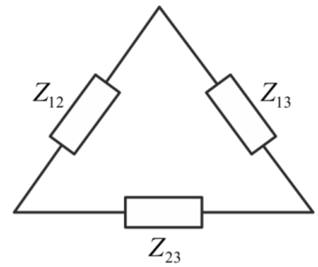


Fig. Delta network

Or

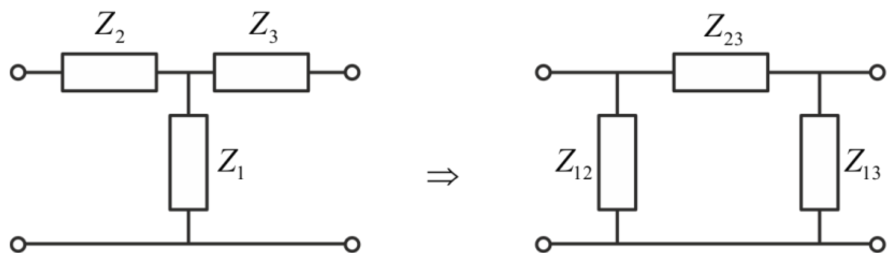


Fig. *T* - network **Fig. π- network**

$$Z_{12} = \frac{Z_1Z_2 + Z_2Z_3 + Z_1Z_3}{Z_3} \qquad Z_{23} = \frac{Z_1Z_2 + Z_2Z_3 + Z_1Z_3}{Z_1} \qquad Z_{13} = \frac{Z_1Z_2 + Z_2Z_3 + Z_1Z_3}{Z_2}$$

If $Z_1 = Z_2 = Z_3 = Z$ then $Z_{12} = Z_{23} = Z_{13} = 3Z$

$$Z_{eq} = 3Z \qquad R_{eq} = 3R \qquad L_{eq} = 3L \qquad C_{eq} = C/3$$

Delta to Star Conversion [Δ to Y] or [π to T] Conversion

Delta to Star conversion [π to *T*] :

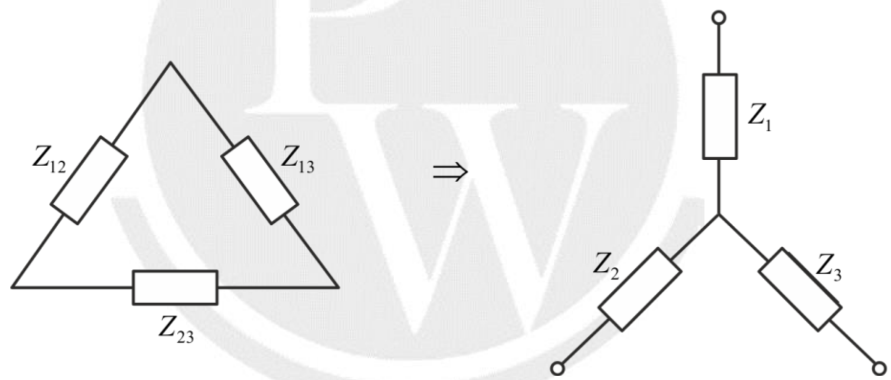


Fig. Delta network **Fig. Star network**

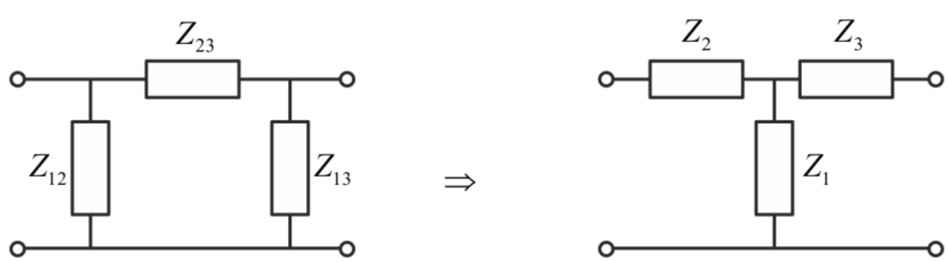


Fig.π- network **Fig. *T* - network**

$$Z_1 = \frac{Z_{12}Z_{13}}{Z_{12} + Z_{23} + Z_{13}} \qquad Z_2 = \frac{Z_{12}Z_{23}}{Z_{12} + Z_{23} + Z_{13}} \qquad Z_3 = \frac{Z_{23}Z_{13}}{Z_{12} + Z_{23} + Z_{13}}$$

If $Z_{12} = Z_{23} = Z_{13} = Z$ then $Z_1 = Z_2 = Z_3 = \dfrac{Z}{3}$

In case of same impedance, delta to star conversion decreases the impedance by factor of 3. (Increases if the element is capacitance by the same factor of 3).

$$Z_{eq} = \frac{Z}{3} \qquad R_{eq} = \frac{R}{3} \qquad L_{eq} = \frac{L}{3} \qquad C_{eq} = 3C$$

Sources

1. Ideal Voltage Source :

An ideal voltage source is a device which has a constant voltage independent of current delivered by it. Ideally, it has zero internal resistance.

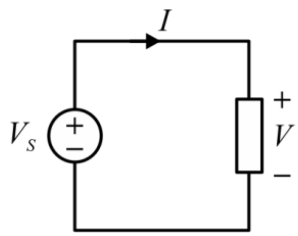


Fig. Ideal voltage source

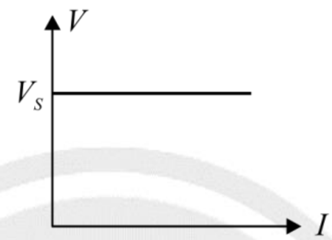


Fig. *V-I* characteristics

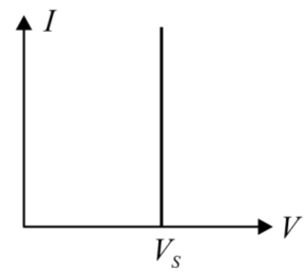


Fig. *I-V* characteristics

2. Practical Voltage Source :

An practical voltage source is a device which has a constant voltage with non-zero internal resistance dependent on the current supplied by the source. Practically its internal resistance should be as small as possible.

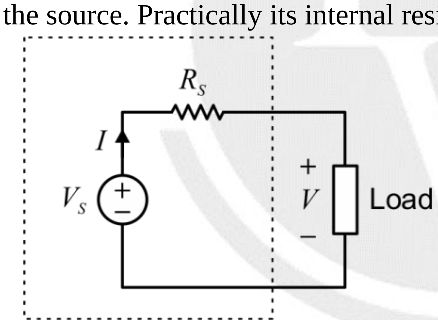


Fig. Practical voltage source

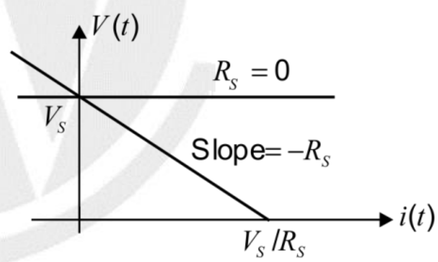


Fig. *V-I* characteristics

3. Ideal Current Source :

An ideal current source is a device which delivers a constant current to any load independent of the voltage across it.

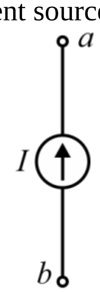


Fig. ideal voltage source

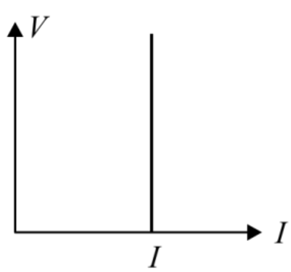


Fig. *V-I* characteristics

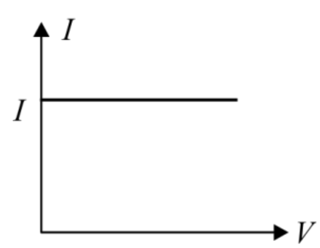


Fig. *I-V* characteristics

4. Practical Current Source :

A practical current source is a device which delivers a constant current to any load independent of the voltage across the source. A practical current source has finite internal resistance.

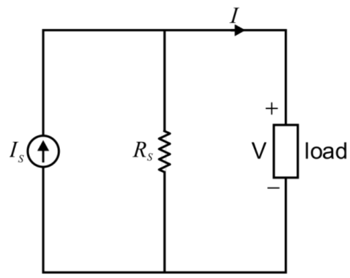


Fig. Practical current source

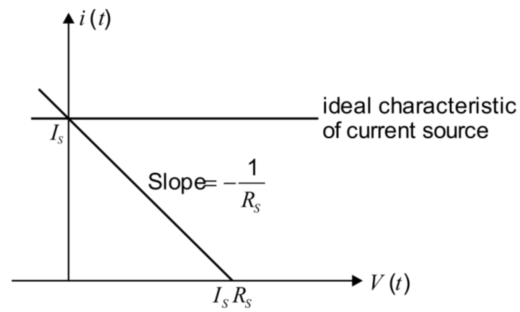
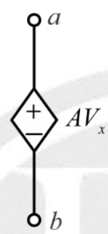


Fig. *I-V* characteristics

Dependent Source :

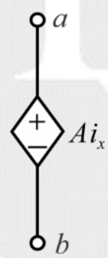
1. A voltage controlled voltage source (VCVS) :



Here, A is unitless

$$V = AV_x$$

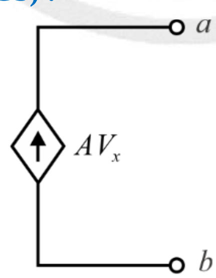
2. A current controlled voltage source (CCVS) :



Unit of A is Ohm.

$$V = Ai_x$$

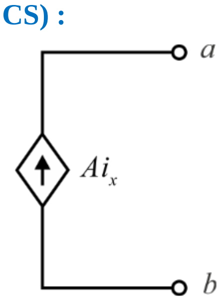
3. A voltage controlled current source (VCCS) :



Here, A is unitless

$$I = AV_x$$

4. A current controlled current source (CCCS) :



Unit of A is Mho.

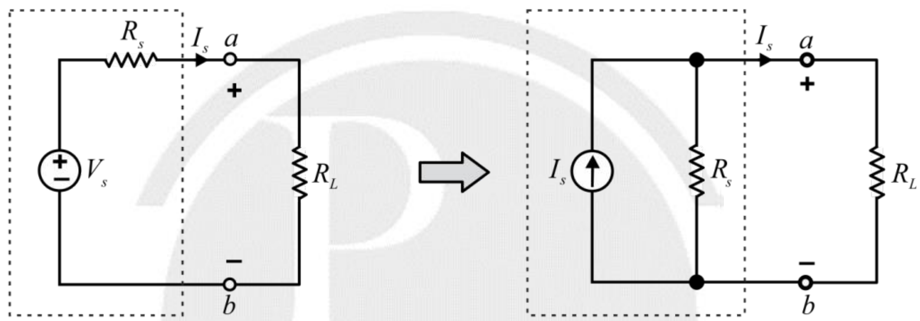
$$I = Ai_x$$

Source/device	Practically Internal resistance	Ideally Internal Resistance
1. Voltage source	$R_s = \text{small}$	$R_s = 0$
2. Current source	$R_s = \text{high}$	$R_s = \infty$
3. Voltmeter	$R_m = \text{high}$	$R_m = \infty$
4. Ammeter	$R_m = \text{small}$	$R_m = 0$

Source Transformation

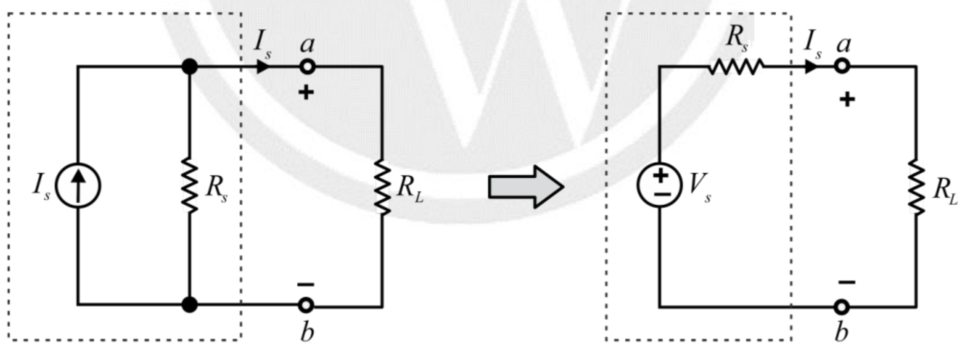
Practical voltage source into a practical current source :

- It states that an independent voltage source V_s in series with a resistance R_s is equivalent to an independent current source, $(I_s = V_s / R_s)$ in parallel with a resistance R_s.



Practical voltage source   Practical current source

Practical current source into a practical voltage source :



Practical current source   Practical voltage source

Note: Source transformation is not possible in Ideal Source.

Average and RMS Value of Periodic Waveform

Average Value / DC value / Mean value :

$$\text{Average } [x(t)] = \frac{1}{T}\int_0^T x(t)\,dt$$

Average value can be defined with the help of area,

$$\text{Average } [x(t)] = \frac{\text{Area in one period}}{\text{Time period}(T)}$$

RMS value / Effective value :

$$\text{RMS } [x(t)] = \text{ Effective } [\, x(t) \,] = \sqrt{\frac{1}{T}\int_0^T x^2\,(t)\,dt}$$

1. Average value can be negative, positive or zero but RMS value is always a positive number.

2. $$Avg\begin{bmatrix} A\sin\omega t \,/\, A\cos\omega t \,/\, A\sin 2\omega t \,/\, A\cos 2\omega t \,/\, A\sin n\omega t \,/\, A\cos n\omega t \,/ \\ A\sin\left(n\omega t \pm n\phi\right) / \,A\cos\left(n\omega t \pm n\phi\right) \end{bmatrix} = 0$$

3. $$RMS\begin{bmatrix} A\sin\omega t \,/\, A\cos\omega t \,/\, A\sin 2\omega t \,/\, A\cos 2\omega t \,/\, A\sin n\omega t \,/\, A\cos n\omega t \,/ \\ A\sin\left(n\omega t \pm n\phi\right) / \,A\cos\left(n\omega t \pm n\phi\right) \end{bmatrix} = \frac{A}{\sqrt{2}}$$

4. If $x(t) = DC + AC\sin\omega t$

$$RMS\left[x(t)\right] = \sqrt{(DC)^2 + \left(\frac{AC}{\sqrt{2}}\right)^2}$$

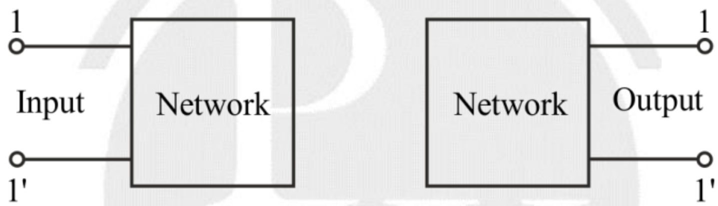
❑❑❑

2 TWO PORT NETWORK

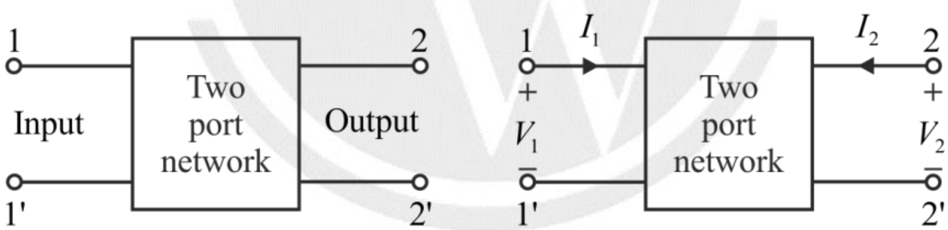
2.1. One Port Network

One port network is a two terminal electrical network in which, current enters through one terminal and leaves through another terminal. i.e. Resistors, inductors and capacitors are the one port network.

Example: Motor, Generator etc.



Two Port Network :



Maximum number of possible parameters for analysis of any port network is given by:

1. Z parameter (impedance parameter)
2. Y parameter (admittance parameter)
3. *h*-parameter (Hybrid parameter)
4. *g*-parameter (Inverse Hybrid parameter)
5. ABCD parameter (Transmission parameter)
6. $[ABCD]^{-1}$ parameter (Inverse transmission parameter)

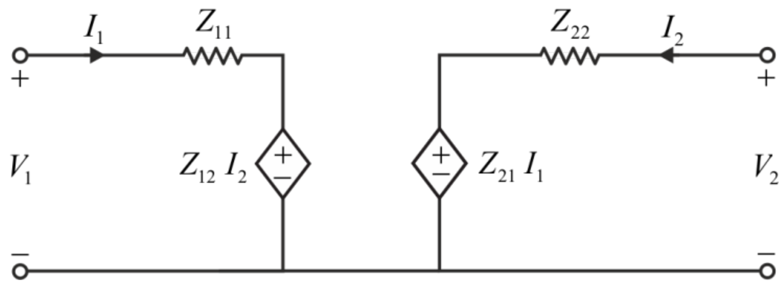
Z Parameter (Impedance Parameter)

In Z-parameter, voltage (V_1, V_2) is the dependent variable and current (I_1, I_2) is the independent variable, the equations of Z – parameter can be written by,

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad \text{...(i)}$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \quad \text{...(ii)}$$

Circuit diagram of Z parameter,



From equation (i) & (ii),

Case (i) When output port is open circuit, i.e. $I_2 = 0$,

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \text{Driving point input impedance } (\Omega)$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \text{Transfer output impedance } (\Omega)$$

Case (ii) When input port is open circuit, i.e. $I_1 = 0$,

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \text{Transfer input impedance } (\Omega)$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \text{Driving point output impedance } (\Omega)$$

Equation (i) & (ii) can be written in the matrix form,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}_{2\times1} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}_{2\times2} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}_{2\times1}$$

$$[V]_{2\times1} = [Z]_{2\times2}[I]_{2\times1}$$

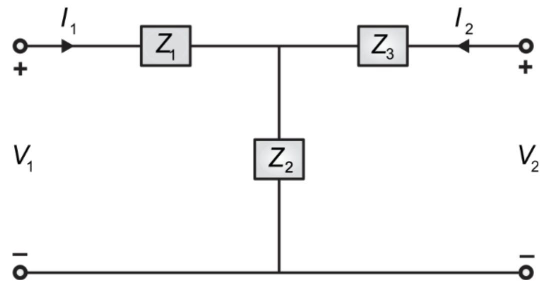
$$\boldsymbol{[Z]_{2\times2} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}}$$

Condition for Symmetry: $Z_{11} = Z_{22}$

Condition for Reciprocity: $Z_{12} = Z_{21}$

Z-parameter is referred as open circuit impedance parameter.

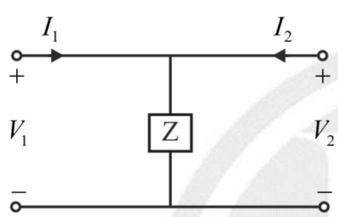
Standard Z Parameter Expression for T Network:



Hence,
$$[Z]=\begin{bmatrix} Z_1+Z_2 & Z_2 \\ Z_2 & Z_2+Z_3 \end{bmatrix}$$

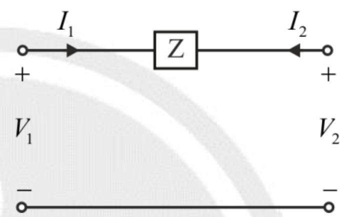
Standard Z Parameter Expression for Single Series and Shunt Element:

(i)



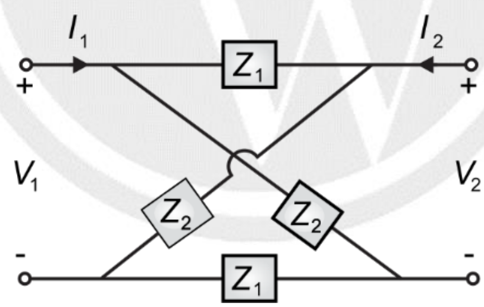
$$[Z]=\begin{bmatrix} Z & Z \\ Z & Z \end{bmatrix}$$

(ii)



$$[Z]=\begin{bmatrix} \infty & \infty \\ \infty & \infty \end{bmatrix}$$

Standard Z Parameter Expression for Symmetric Lattice Network :



$$[Z]=\begin{bmatrix} \dfrac{Z_1+Z_2}{2} & \dfrac{Z_2-Z_1}{2} \\ \dfrac{Z_2-Z_1}{2} & \dfrac{Z_1+Z_2}{2} \end{bmatrix}$$

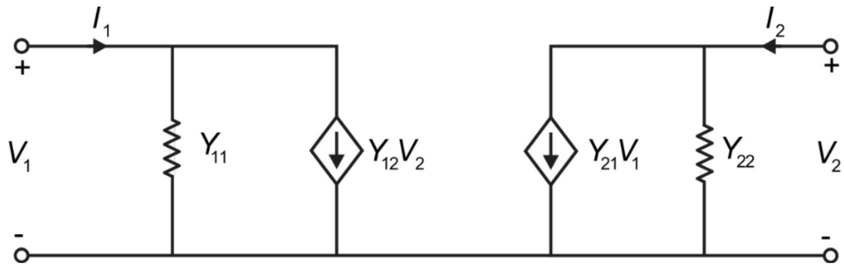
Y Parameter (Admittance Parameter)

In Y-parameter, current (I_1, I_2) is the dependent variable and voltage (V_1, V_2) is the independent variable, the equations of Y – parameter can be written by,

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad \text{...(i)}$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad \text{...(ii)}$$

Circuit diagram of Y parameter,



From equation (i) & (ii),

Case (i) When output port is short circuit, i.e. $V_2 = 0$,

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \text{Driving point input admittance } (℧)$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \text{Transfer output admittance } (℧)$$

Case (ii) When input port is short circuit, i.e. $V_1 = 0$,

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = \text{Transfer input admittance } (℧)$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \text{Driving point output admittance } (℧)$$

Equation (i) & (ii) can be written in the matrix form,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}_{2\times1} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}_{2\times2} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}_{2\times1}$$

$$[I]_{2\times1} = [Y]_{2\times2}[V]_{2\times1}$$

$$[Y]_{2\times2} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

Condition for Symmetry: $Y_{11} = Y_{22}$

Condition for Reciprocity: $Y_{12} = Y_{21}$

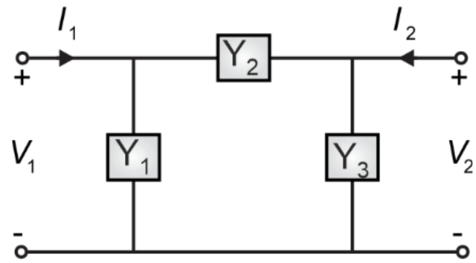
Relationship between Z and Y Parameter:

The relation between Z and Y parameter is given by,

$$[Z] = [Y]^{-1}$$

$$[Y] = [Z]^{-1}$$

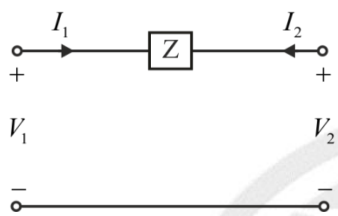
Standard Y Parameter for ∏-Network:



Hence, $$[Y]_{2\times 2} = \begin{bmatrix} Y_1 + Y_2 & -Y_2 \\ -Y_2 & Y_2 + Y_3 \end{bmatrix}$$

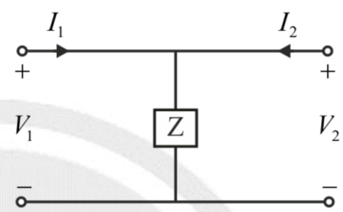
Standard Y Parameter for Single Series and Shunt Element:

(i)



$$[Y]_{2\times 2} = \begin{bmatrix} 1/Z & -1/Z \\ -1/Z & 1/Z \end{bmatrix}$$

(ii)



$$Y = \begin{bmatrix} \infty & \infty \\ \infty & \infty \end{bmatrix}$$

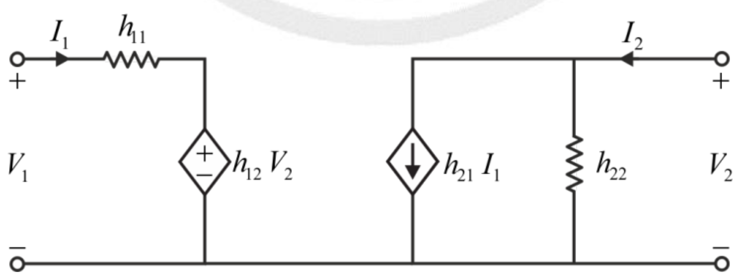
h-parameter (Hybrid Parameter)

In h-parameter, V_1, I_2 are the dependent variables and I_1, V_2 are the independent variables, the equations of h – parameter can be written by,

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{...(i)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{...(ii)}$$

Circuit diagram of *h*-parameter,



Case (i) When output port is short circuit, i.e. $V_2 = 0$,

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \text{Input impedance } (\Omega)$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \text{Forward current gain}$$

Case (ii) When input port is open circuit, i.e. $I_1 = 0$,

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \text{Reverse voltage gain}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \text{Output admittance } (\mho)$$

Equation (i) & (ii) can be written in the matrix form,

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix}_{2\times1} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}_{2\times2} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}_{2\times1}$$

Condition of reciprocity and symmetricity in h-parameter is:

Reciprocity: $h_{12} = -h_{21}$

Symmetricity: $\Delta h = 1$

h-Parameter in Terms of Z and Y Parameter:

Sr.	*Z*-Parameter	*Y*-Parameter	*h*-parameter
1.	$Z_{11} = \left.\frac{V_1}{I_1}\right\|_{I_2=0}$	$Y_{11} = \left.\frac{I_1}{V_1}\right\|_{V_2=0}$	$h_{11} = \left.\frac{V_1}{I_1}\right\|_{V_2=0} = \frac{1}{Y_{11}}$
2.	$Z_{12} = \left.\frac{V_1}{I_2}\right\|_{I_1=0}$	$Y_{12} = \left.\frac{I_1}{V_2}\right\|_{V_1=0}$	$h_{12} = \left.\frac{V_1}{V_2}\right\|_{I_1=0} = \frac{Z_{12}}{Z_{22}}$
3.	$Z_{21} = \left.\frac{V_2}{I_1}\right\|_{I_2=0}$	$Y_{21} = \left.\frac{I_2}{V_1}\right\|_{V_2=0}$	$h_{21} = \left.\frac{I_2}{I_1}\right\|_{V_2=0} = \frac{Y_{21}}{Y_{11}}$
4.	$Z_{22} = \left.\frac{V_2}{I_2}\right\|_{I_1=0}$	$Y_{22} = \left.\frac{I_2}{V_2}\right\|_{V_1=0}$	$h_{22} = \left.\frac{I_2}{V_2}\right\|_{I_1=0} = \frac{1}{Z_{22}}$

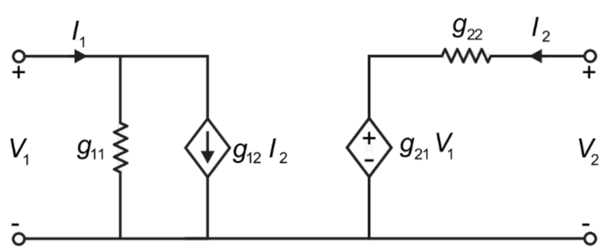
g-Parameter

In g-parameter, I_1, V_2 are the dependent variables and V_1, I_2 are the independent variables, the equations of g – parameter can be written by,

$$I_1 = g_{11}V_1 + g_{12}I_2 \quad \ldots\text{(i)}$$

$$V_2 = g_{21}V_1 + g_{22}I_2 \quad \ldots\text{(ii)}$$

Circuit diagram of g parameter,



Case (i) When output port is open circuit, i.e. $I_2 = 0$,

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} = \text{Input admittance } (\mho)$$

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0} = \text{Forward voltage gain}$$

Case (ii) When input port is short circuit, i.e. $V_1 = 0$,

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0} = \text{Reverse current gain}$$

$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0} = \text{Output impedance } (\Omega)$$

Equation (i) & (ii) can be written in the matrix form,

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix}_{2\times1} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}_{2\times2} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}_{2\times1}$$

Relationship between h and g Parameter:

The relation between h and g parameter is given by,

$$[h] = [g]^{-1}$$

$$[g] = [h]^{-1}$$

Condition for Symmetry: $\Delta g = 1$

Condition for Reciprocity: $g_{12} = -g_{21}$

ABCD Parameter (Transmission Parameter)

In ABCD-parameter, V_1, I_1 are the dependent variable and $V_2, -I_2$ is the independent variable, the equations of ABCD – parameter can be written by,

$$V_1 = AV_2 - BI_2 \qquad \ldots\text{(i)}$$

$$I_1 = CV_2 - DI_2 \qquad \ldots\text{(ii)}$$

Circuit diagram of T parameter cannot be drawn as both dependent variables are at input port.

Case (i) When output port is open circuit, i.e. $I_2 = 0$,

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \text{Reverse voltage gain}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \text{Transfer admittance } (\mho)$$

Case (ii) When output port is short circuit, i.e. $V_2 = 0$,

$$B = \left.\frac{-V_1}{I_2}\right|_{V_2=0} = \text{Transfer impedance } (\Omega)$$

$$D = \left.\frac{-I_1}{I_2}\right|_{V_2=0} = \text{Reverse current gain}$$

Equation (i) & (ii) can be written in the matrix form,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix}_{2\times1} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{2\times2} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}_{2\times1}$$

Condition for Symmetry: $A = D$

Condition for Reciprocity: $AD - BC = 1$

Inverse ABCD or Inverse T Parameter ($[ABCD]^{-1}/[T]^{-1}$)

In inverse ABCD-parameter, V_2, I_2 are the dependent variable and V_1, $-I_1$ is the independent variable, the equations of inverse ABCD – parameter can be written by,

$$V_2 = aV_1 - bI_1 \quad \text{...(i)}$$

$$I_2 = cV_1 - dI_1 \quad \text{...(ii)}$$

Case (i) When input port is open circuit, i.e. $I_1 = 0$,

$$a = \left.\frac{V_2}{V_1}\right|_{I_1=0} = \text{Forward voltage gain}$$

$$c = \left.\frac{I_2}{V_1}\right|_{I_1=0} = \text{Transfer admittance } (\mho)$$

Case (ii) When input port is short circuit, i.e. $V_1 = 0$,

$$b = -\left.\frac{V_2}{I_1}\right|_{V_1=0} = \text{Transfer impedance } (\Omega)$$

$$d = -\left.\frac{I_2}{I_1}\right|_{V_1=0} = \text{Forward current gain}$$

Equation (i) & (ii) can be written in the matrix form,

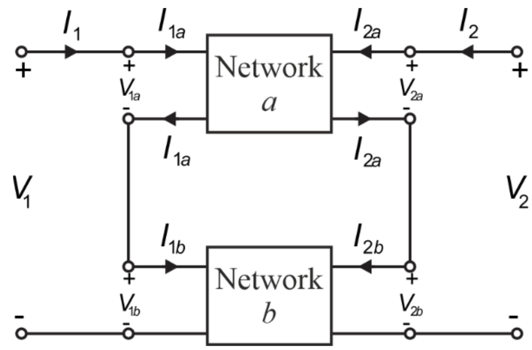
$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix}_{2\times1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times2} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}_{2\times1}$$

Condition for Symmetry: $a = d$

Condition for Reciprocity: $ad - bc = 1$

Interconnection of Two Port Network

1. Series - series Connection:

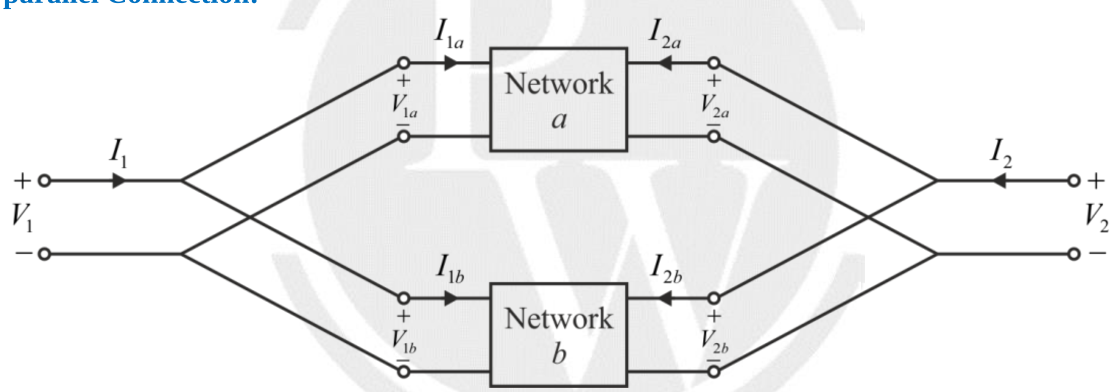


$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Z_{11a}+Z_{11b} & Z_{12a}+Z_{12b} \\ Z_{21a}+Z_{21b} & Z_{22a}+Z_{22b} \end{bmatrix} = \begin{bmatrix} Z_{11a} & Z_{12a} \\ Z_{21a} & Z_{22a} \end{bmatrix} + \begin{bmatrix} Z_{11b} & Z_{12b} \\ Z_{21b} & Z_{22b} \end{bmatrix}$$

$$[Z] = [Z]_a + [Z]_b$$

In case of series – series connection individual Z parameters are added. Care should be taken for series connection that some current should leave from second terminal of input and output port.

2. Parallel - parallel Connection:

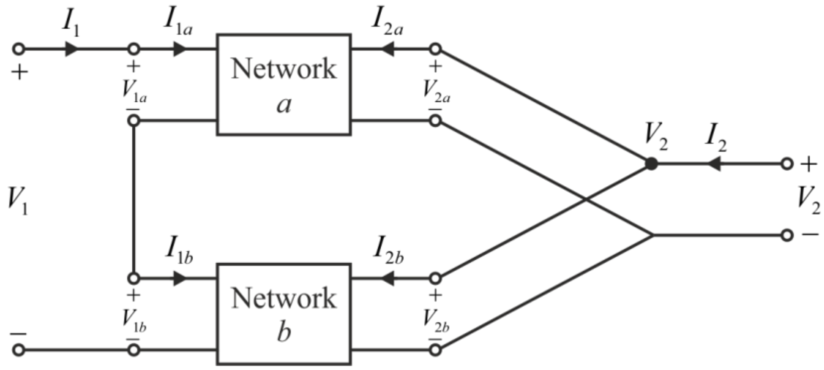


$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Y_{11a}+Y_{11b} & Y_{12a}+Y_{12b} \\ Y_{21a}+Y_{21b} & Y_{22a}+Y_{22b} \end{bmatrix} = \begin{bmatrix} Y_{11a} & Y_{12a} \\ Y_{21a} & Y_{22a} \end{bmatrix} + \begin{bmatrix} Y_{11b} & Y_{12b} \\ Y_{21b} & Y_{22b} \end{bmatrix}$$

$$[Y] = [Y]_a + [Y]_b$$

In case of parallel - parallel connection individual Y parameters are added.

3. Series Parallel Connection:

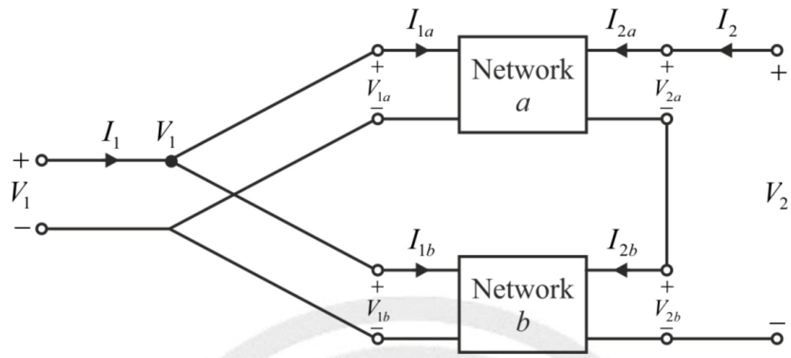


$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} h_{11a}+h_{11b} & h_{12a}+h_{12b} \\ h_{21a}+h_{21b} & h_{22a}+h_{22b} \end{bmatrix} = \begin{bmatrix} h_{11a} & h_{12a} \\ h_{21a} & h_{22a} \end{bmatrix} + \begin{bmatrix} h_{11b} & h_{12b} \\ h_{21b} & h_{22b} \end{bmatrix}$$

$$[h] = [h]_a + [h]_b$$

In case of series - parallel connection individual h parameters are added.

4. Parallel Series Connection :

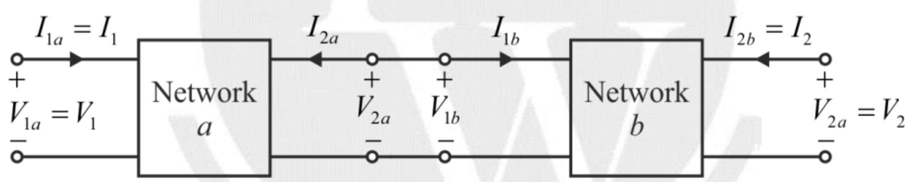


$$\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} = \begin{bmatrix} g_{11a}+g_{11b} & g_{12a}+g_{12b} \\ g_{21a}+g_{21b} & g_{22a}+g_{22b} \end{bmatrix} = \begin{bmatrix} g_{11a} & g_{12a} \\ g_{21a} & g_{22a} \end{bmatrix} + \begin{bmatrix} g_{11b} & g_{12b} \\ g_{21b} & g_{22b} \end{bmatrix}$$

$$[g] = [g]_a + [g]_b$$

In case of parallel - series connection individual g parameters are added.

5. Cascade Connection :



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}$$

$$[ABCD] = [ABCD]_a [ABCD]_b$$

In case of cascade connection individual T parameters matrix are multiplied.

Conclusion :

Interconnection of two port network	Equivalent parameter
Series connection	$[Z]=[Z]_a+[Z]_b$
Parallel connection	$[Y]=[Y]_a+[Y]_b$
Series - parallel connection	$[h]=[h]_a+[h]_b$
Parallel - series connection	$[g]=[g]_a+[g]_b$
Cascade connection	$[T]=[T]_a[T]_b$

3 NETWORK THEOREM

3.1. Thevenin's Theorem

Thevenin's theorem states that a linear two terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{th} (Thevenin's voltage) in series with a resistance R_{th} (Thevenin's resistance).

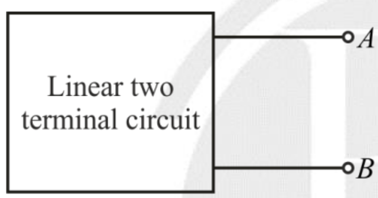


Fig.(a) General network

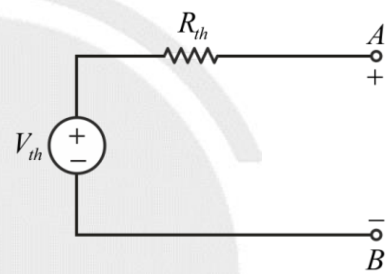
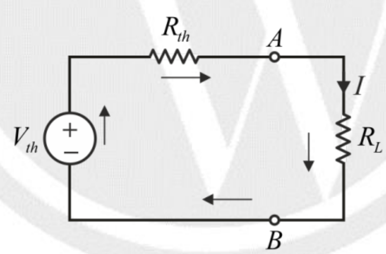


Fig.(b) Thevenin's equivalent



$$I = \frac{V_{th}}{R_{th} + R_L}$$

Case 1 : Circuit having only independent sources

Calculate the thevenin's resistance at load terminals by replacing all independent voltage sources by short circuit or by their internal resistance and all independent current sources by open circuit or by their internal resistances.

$$R_{th} = R_{eq}$$

Case 2 : Circuit having only dependent sources

By Keeping unchanged dependent sources

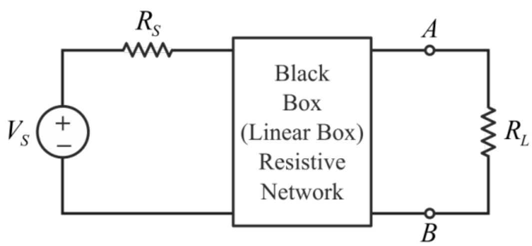
$$R_{th} = \frac{V_{dc}}{I_{dc}}$$

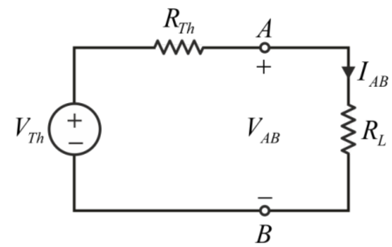
where, V_{dc} = Value of voltage source applied across load terminals

I_{dc} = Direct current supplied by DC voltage source

In this case, put thevenin's voltage,(V_{th})=0. (due to absence of independent source).

Example:





$$I_{AB} = \frac{V_{Th}}{R_{Th} + R_L}$$

$$V_{AB} = I_{AB} R_L = \frac{V_{Th} \times R_L}{R_{Th} + R_L}$$

Case 3 : Circuit having dependent as well as independent sources

Calculate the thevenin's resistance at load terminals by replacing all independent voltage sources by short circuit or by their internal resistance and all independent current sources by open circuit or by their internal resistances.

$$R_{th} = \frac{V_{dc}}{I_{dc}}$$

where, V_{dc} = Value of DC voltage applied across load terminals

I_{dc} = Direct current supplied by DC voltage source

3.2. Norton's Theorem

Norton's theorem states that a linear two terminal circuit can be replaced by an equivalent current source I_N (Norton's current) in parallel with a resistance R_N (Norton's resistance).

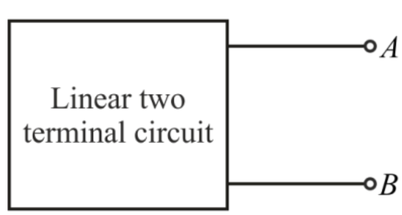


Fig.(a) General network

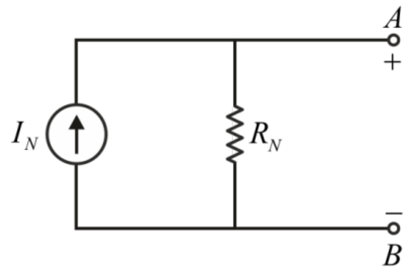
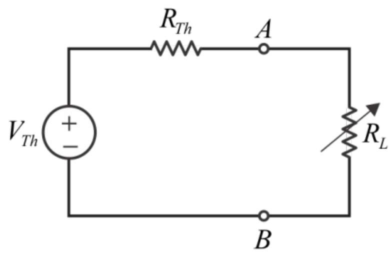


Fig.(b) Norton's equivalent

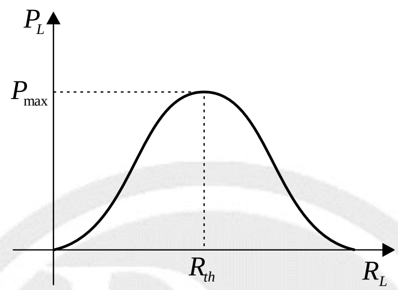
Relation between $V_{OC}\left(\text{or } V_{th}\right), I_{SC}\left(\text{or } I_N\right)$ and Z_{th}

$$I_{SC} = \frac{V_{OC}}{Z_{th}} \quad \Rightarrow \quad Z_{th} = \frac{V_{OC}}{I_{SC}} = \frac{V_{th}}{I_N}$$

3.2.1. Maximum Power Transfer Theorem



Power dissipated to load as a function of R_L is given by,



Maximum power transferred to the load is given by,

$$P_L = \frac{V_{th}^2}{4R_{th}} = \frac{V_{th}^2}{4R_L}$$

$$[R_{th} = R_L]$$

Maximum Power Transfer:

$$DC \div \boxed{P_{\max} = \frac{V_{th}^2}{4R_{th}}}$$

- $R_L = R_{th}$, if R_L varying
- $R_{th} = 0$, if R_L constant R_{th} vary

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Case 1 : $\boxed{Z_L = Z_S^*}$, $R_L = R_{th}$, $X_{th} = -X_L$

Case 2 : If only R_L is variable, then $X_L \to$ constant

$$\boxed{R_L = \sqrt{(R_{th})^2 + (X_L + X_{th})^2}}$$

Then $$\boxed{P_{\max} = \frac{V_{th}^2 \cdot R_L}{(R_{th} + R_L)^2 + (X_{th} + X_L)^2}}$$

Case 3 : If R_L is variable $(X_L = 0)$ or $Z_L = R_L$

$$\boxed{R_L = |Z_S| = \sqrt{R_{th}^{\ 2} + X_{th}^{\ 2}}}$$

Then, $$\boxed{P_{\max} = \frac{V_{th}^2 \cdot R_L}{(R_L + R_{th})^2 + X_{th}^{\ 2}}}$$

3.3. Superposition Theorem

Superposition theorem states that in any linear bilateral network containing two or more independent sources, the resultant current or voltage in any branch is the algebraic sum of currents or voltages caused by each independent source acting along with all are other independent sources being replaced by their internal resistances.

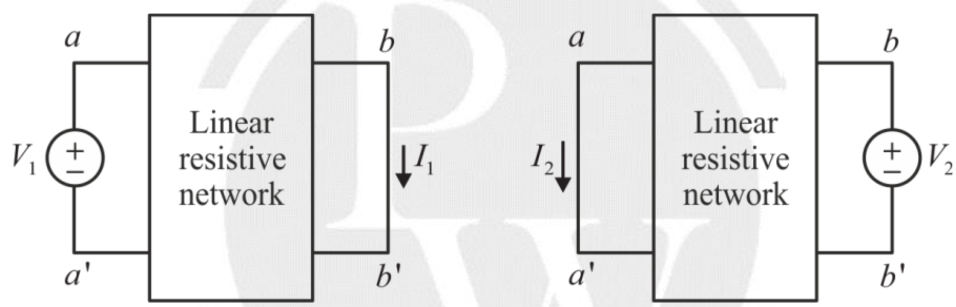
Note:

1. This Theorem is not valid for Non-linear quantities.
2. $P = \left(\sqrt{P_1} + \sqrt{P_2}\right)^2$

The dissipation of total power across any load,when two sources are working simultaneously.

3.4. Reciprocity Theorem

Reciprocity theorem states that in any linear bilateral network, if a source produces a certain current in any other branch, then the same source acting on the second branch produces the same current in the first branch.



$$\frac{V_1}{I_1} = \frac{V_2}{I_2}$$

3.5. Tellegen's Theorem

Tellegen's theorem states that an instantaneous power in an electrical network is zero.

Mathematically it is given by,

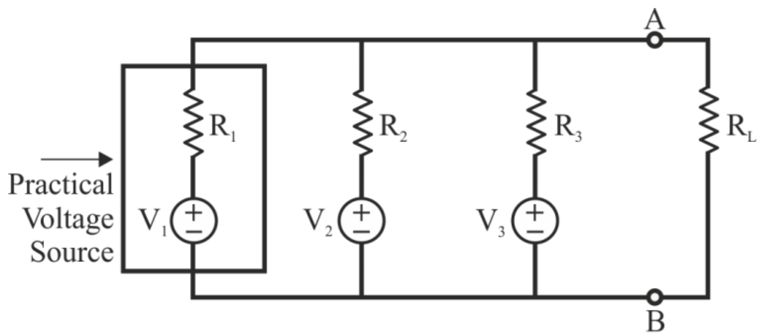
$$\sum_{K=1}^{n} P_K = 0$$

$$\sum_{K=1}^{n} V_K I_K = 0$$

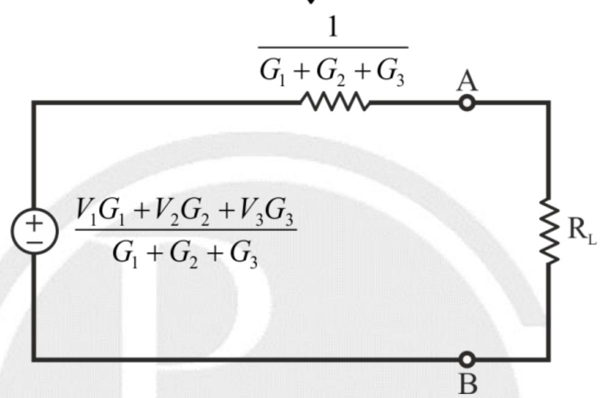
We point out it as Power Conservation theorem.

3.6. Millman's Theorem

Millman's theorem states that a number of voltage sources with internal resistance connected in parallel can be replaced by a single equivalent voltage source *V* in series with equivalent resistance *R*.

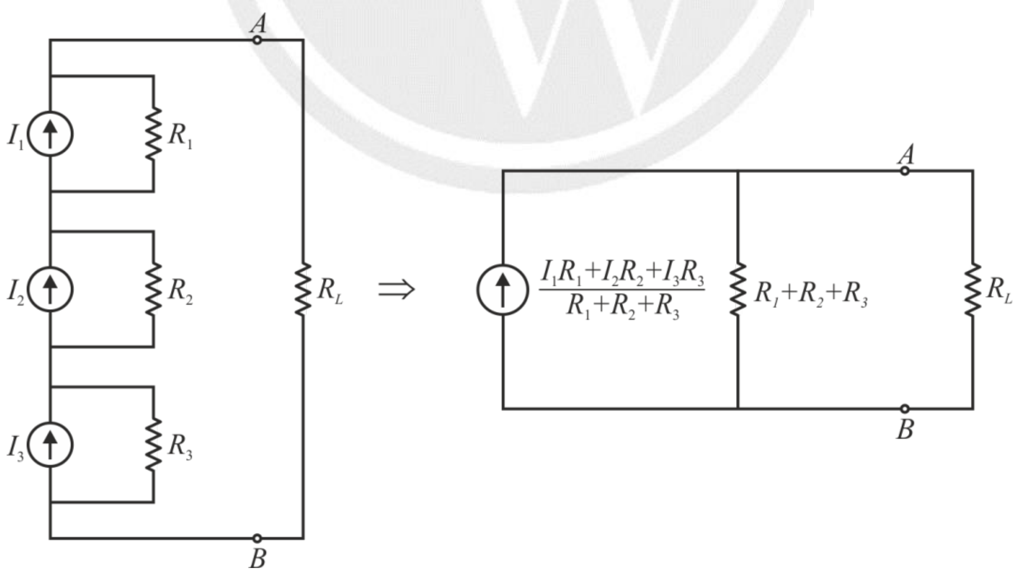


$$G_3 = \frac{1}{R_3} \quad \Downarrow \quad G_1 = \frac{1}{R_1},\ G_2 = \frac{1}{R_2}$$



3.6.1. Dual of Millman's Theorem

Dual of Millman's theorem state that a number of current sources with internal resistance connected in series can be replaced by a single equivalent current source in parallel with resistance.



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4 TRANSIENT ANALYSIS

4.1. Introduction

- Transient analysis is the analysis of the circuits during the time it changes from one steady-state condition to another steady-state condition
- The transient analysis will reveal how the currents and voltages are changing during the transient period.
- Transient will occur when at least one energy storage element is present in circuit.

4.2. Common Aspects of RC and RL Circuits

While doing transient analysis on simple RC and RL circuits, we need remember two facts:

1. The voltage across a capacitor as well as the current in an inductor cannot have a discontinuity i.e. the capacitor voltage must be continuous at time t = 0 and hence $v_{C(0^+)} = v_{C(0^-)}$ and the inductor current must be continuous at time t = 0 and hence $i_{L(0^+)} = i_{L(0^-)}$.
2. With DC excitation, at a steady state, the capacitor will act as an open circuit and the inductor will act as a short circuit.

4.2.1 Time Constant

It is the time required for the response to delay by a factor 1/e or 36.8% of its initial value. It is represented by τ.

- For a RC circuit, $\tau = RC$
- For a RL circuit, $\tau = \frac{L}{R}$

R is the Thevenin resistance across inductor or capacitor terminals.

4.2.2 General Method of Analysis

$$x(t) = x(\infty) + \left[x(t_0^+) - x(\infty)\right]e^{-(t-t_0)/\tau},\ t > 0$$

If switching is done at $t = t_0$,

$x(t_0^+)$ = Initial value of $x(t)$ at $t = t_0$

$x(\infty)$ = Final value of $x(t)$ at $t = \infty$

Follow these steps to solve numerical:

Step 1: Choose any voltage and current in the circuit which has to be determined.

Step 2: Assume circuit has reached steady state before switch was thrown at $t = t_0$.

Step 3: Draw the circuit at $t = t_0$ with capacitor replaced by open circuit and inductor replaced by short circuit. Solve for $V_c\left(t_0^-\right)$ and $i_L\left(t_0^-\right)$.

Step 4: Voltage across capacitor and inductor current cannot change instantaneously,

$$V_c\left(t_0^+\right) = V_c\left(t_0^-\right) = V_c\left(t_0\right)$$

$$i_L\left(t_0^+\right) = i_L\left(t_0^-\right) = i_L\left(t_0\right)$$

Step 5: Draw the circuit for $t = t_0^+$ with switches in new position.

Step 6: Replace a capacitor with a voltage source of value $v_c\left(t_0^-\right) = v_c\left(t_0^+\right)$ and inductor with a current source of value $i_L\left(t_0^+\right) = i_L\left(t_0^-\right)$. Solve for initial value of variable $x\left(t_0^+\right)$.

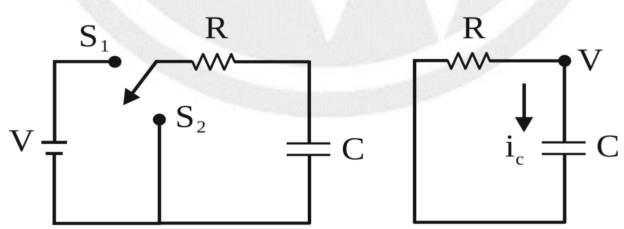
Step 7: Draw the circuit for $t = \infty$, in a similar manner as step- 2 and calculate $x(\infty)$. Calculate time constant of circuit

Step 8: $\tau = R_{th}C$ or $\tau = L / R_{th}$

Step 9: Substitute all value to calculate $x(t)$.

4.3. Source Free RC Circuit

A source free RC circuit occurs when its DC source is suddenly disconnected. The energy already stored in the capacitor is released to the resistors.



(a) (b)

State variable = V

Let, $V(0) = V_0$

Apply KCL in Fig (b)

$$\left(\frac{V}{R}\right) + C\frac{dV_c}{dt} = 0$$

$$C\frac{dV_c}{dt} = -\left(\frac{V}{R}\right)$$

$$\int \frac{dV_c}{dt} = -\int \frac{dt}{RC}$$

$$\ln(V) = -\frac{t}{RC} + \ln(A)$$

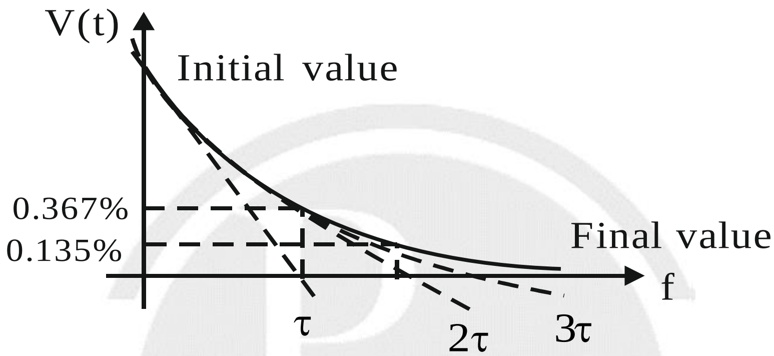
$$\ln\left(\frac{V_c}{A}\right) = -\frac{t}{RC}$$

$$V_c(t) = Ae^{-t/RC}$$

$$V = V_0 \quad \Rightarrow A = V_0$$

But at $t = 0$, $\quad V_c(t) = V_0 e^{-t/RC}$

for $t < 5\tau$, circuit will be in transient state and for $t \geq 5\tau$, circuit will be in steady state. Sometimes 4τ is also considered as settling time.

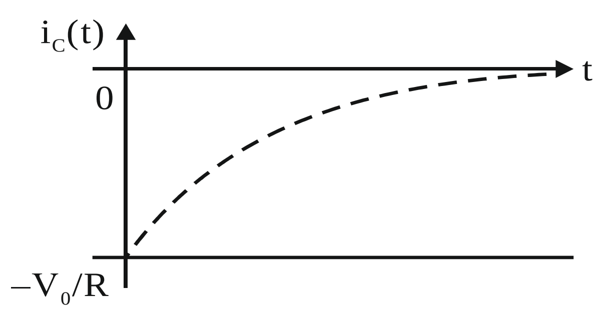


Expression for current through capacitor,

$$i_c = C\frac{dV_C}{dt} = C\frac{d}{dt}\left(V_0 e^{-t/RC}\right)$$

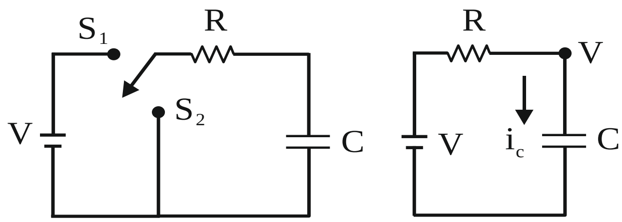
$$i_c(t) = -\left(\frac{V_0}{R}\right)e^{-t/\tau}\text{A}$$

and $\quad V_R(t) = V_0 e^{-t/\tau}\text{V}$



4.3.1 RC Circuit with Source:

Again, consider the circuit shown in below Figure. Let us say that the switch was in position S_2 long enough so that $v_c(t) = 0$ and $i_c(t) = 0$ i.e. all the energy in the capacitor is dissipated and the circuit is at rest. Now, the switch is moved to position S_1.



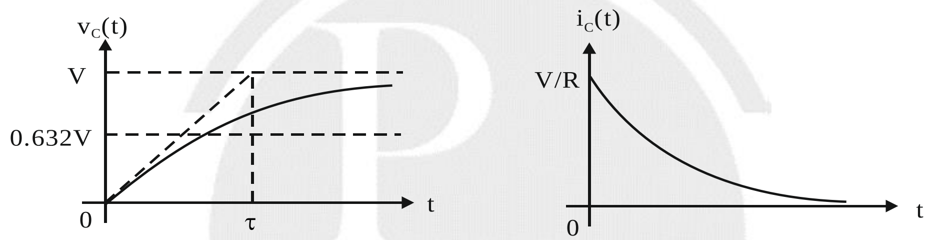
$$v_c(0^+) = v_c(0^-) = 0$$

The voltage through the capacitor is calculated as:

$$v_C(t) = V\left(1 - e^{-\frac{1}{RC}t}\right)$$

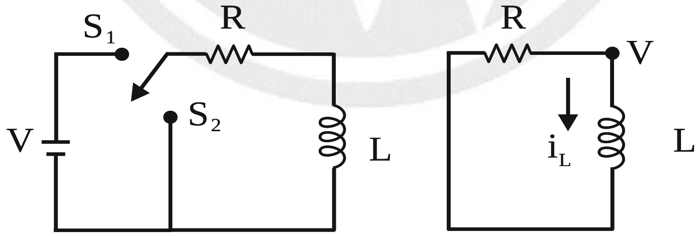
The current through the capacitor is calculated as:

$$i_C(t) = C\frac{dv_C}{dt} = \frac{V}{R}e^{-\frac{1}{RC}t}$$



4.4 Source Free RL Circuit

A source free RL circuit occurs when its dc source is suddenly disconnected. The energy already stored in the inductor is released to the resistors.



(a) **(b)**

Let, $i(0) = i_0$

Apply KVL, in Fig (b)

$$iR + L\frac{di}{dt} = 0$$

$$L\frac{di}{dt} = -iR$$

$$\int\frac{di}{i} = -\left(\frac{R}{L}\right)\int dt$$

$$\ln\left(\frac{i}{A}\right) = -\left(\frac{R}{t}\right)t$$

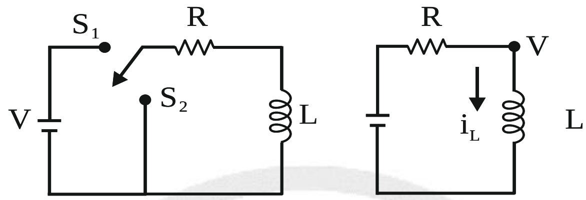
But at $t = 0$, $\quad i = I_0 \Rightarrow A = I_0$

$$i(t) = I_0 e^{-t/(L/R)} A$$

$$\tau = \frac{L}{R} \text{ (Time constant of R-L Circuit)}$$

4.4.1 With Source

Consider the circuit shown in Fig After the circuit has attained the steady state with the switch in position S_2, the switch is moved to position S_1 at time $t = 0$. We like to find the inductor current for time $t > 0$.



The current through the capacitor is calculated as:

$$i_L(t) = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

The voltage through the capacitor is calculated as:

$$v_L(t) = L\frac{di}{dt} = Ve^{-\frac{R}{L}t}$$

4.5. Comparison Table

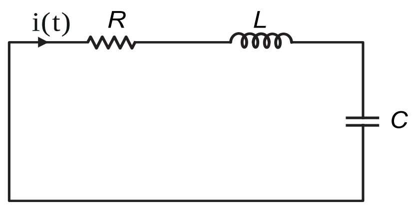
RC Circuit	RL Circuit
1. $\tau = RC$ (Time constant)	1. $\tau = \frac{L}{R}$ (Time constant)
2. With DC, at steady state capacitor acts as open circuit	2. With DC, at steady state inductor acts as short circuit
3. If $v_C(0) \neq 0$; $v_C(\infty) = 0$ Then $v_C(t) = v_C(0)e^{-\frac{1}{RC}t}$	3. If $i_L(0) \neq 0$; $i_L(\infty) = 0$ Then $i_L(t) = i_L(0)e^{-\frac{R}{L}t}$
4. If $v_C(0) = 0$; $v_C(\infty) \neq 0$ Then $v_C(t) = v_C(\infty)e^{-\frac{1}{RC}t}$	4. If $i_L(0) = 0$; $i_L(\infty) \neq 0$ Then $i_L(t) = i_L(\infty)e^{-\frac{R}{L}t}$
5. If $v_C(0) \neq 0$; $v_C(\infty) \neq 0$ Then $v_C(t) = v_C(\infty) + \left[v_C(0) - v_C(\infty)e^{-\frac{1}{RC}t}\right]$	5. If $i_L(0) \neq 0$; $i_L(\infty) \neq 0$ Then $i_C(t) = i_C(\infty) + \left[i_C(0) - i_C(\infty)e^{-\frac{R}{L}t}\right]$

4.6. Series RLC Circuit

Without source:

The voltage across Capacitor in the circuit shown below can be given as;

$$V(0) = \frac{1}{C}\int_{-\infty}^{0} i(t)dt = V_0$$



Characteristic equation of series RLC circuit is given as:

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$S_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$S_2 = -\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$S_1, S_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2};$$

$$\alpha = \frac{R}{2L};\ \omega_0 = \frac{1}{\sqrt{LC}}$$

1. If $\alpha > \omega_0$ roots are real and unequal (over-damped),

   $$i(t) = A e^{s_1 t} + B e^{s_2 t}$$

2. If $\alpha = \omega_0$ roots are real and equal (critically damped),

   $$i(t) = (A + Bt)e^{-\alpha t}$$

3. If $\alpha < \omega_0$, roots are complex conjugate (under damped),

   $$i(t) = e^{-\alpha t}(A\cos\omega_d t + B\sin\omega_d t)$$

   $$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

With a source:

$$v(t) = V_s + \left(A e^{s_1 t} + B e^{s_2 t}\right) \Rightarrow (\text{Over-damped})$$

$$v(t) = V_s + (A + Bt)e^{-\alpha t} \Rightarrow (\text{Critically damped})$$

$$v(t) = V_s + e^{-\alpha t}(A\cos\omega_d t + B\sin\omega_d t) \Rightarrow (\text{Under damped})$$

4.7 Parallel RLC Circuit

Without source:

$$i(0) = \frac{1}{L}\int_{-\infty}^{0} v(t)dt$$

Characteristic equation of parallel RLC circuit is given as:

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

$$S_1, S_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2};$$

$$\alpha = \frac{1}{2RC};\ \omega_0 = \frac{1}{\sqrt{LC}}$$

1. If $\alpha > \omega_0$ roots are real and unequal (over-damped),

$$v(t) = A\,e^{s_1 t} + Be^{s_2 t}$$

2. If $\alpha = \omega_0$ roots are real and equal (critically damped),

$$v(t) = (A + Bt)e^{-\alpha t}$$

3. If $\alpha < \omega_0$, roots are complex conjugate (under damped),

$$v(t) = e^{-\alpha t}\left(A\cos\omega_d t + B\sin\omega_d t\right)$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

With step input:

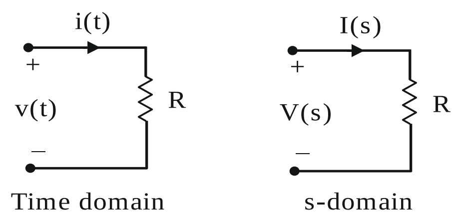
$$i(t) = I_s + \left(A\,e^{s_1 t} + Be^{s_2 t}\right) \Rightarrow \text{(Over-damped)}$$

$$i(t) = I_s + (A + Bt)e^{-\alpha t} \Rightarrow \text{(Critically damped)}$$

$$i(t) = I_s + e^{-\alpha t}\left(A\cos\omega_d t + B\sin\omega_d t\right) \Rightarrow \text{(Under damped)}$$

4.8 Representation of Circuit Elements in s-domain

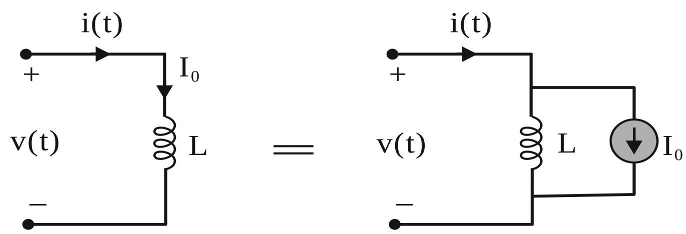
Resistor:



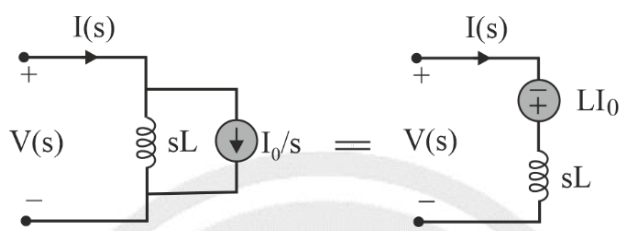
1. In Time domain V(t) = R.i(t)
2. In s-domain V(s) = R.I(s)

Inductor 'L' with initial current 'I_0'

In time domain:



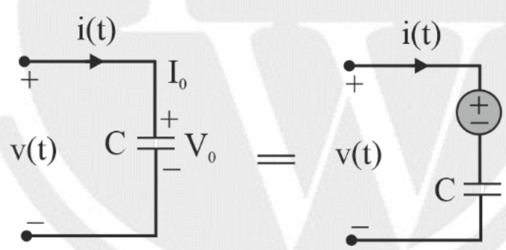
In frequency domain:



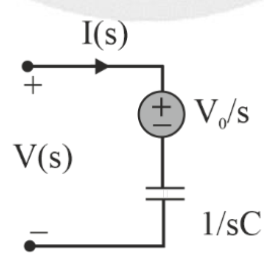
$$V(s) = sLI(s) - LI_0 \quad \text{or} \quad I(s) = \frac{V(s)}{sL} + \frac{I_0}{s}$$

Capacitor 'C' with initial Voltage 'V_0'

In time domain:



In frequency domain:



$$I(s) = sCV(s) - CV_0 \quad \text{or} \quad V(s) = \frac{I(s)}{sC} + \frac{V_0}{s}$$

4.9 Transient Analysis using Laplace Transform

Laplace Transform Method:

Step 1: Choose any voltage and current in the circuit which has to be determined.

Step 2: Assume circuit has reached steady state before switch was thrown at $t = t_0$.

Step 3: Draw the circuit at $t = t_0$ with capacitor replaced by open circuit and inductor replaced by short circuit. Solve for $V_c\left(t_0^-\right)$ and $i_L\left(t_0^-\right)$.

Step 4: Laplace-transform a circuit, including components with non-zero initial conditions.

Step 5: Analyze a circuit in the s-domain and Check the s-domain answers using the initial value theorem (IVT) and final value theorem (FVT).

Step 6: Inverse Laplace-transform the result to get the time domain solutions.

Following equations can be used to find response of RL and RC circuits after switching:

1. If the switching occurs at $t = 0$.

$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right]e^{-t/\tau} \qquad v(t) = v(\infty) + \left[v(0) - v(\infty)\right]e^{-t/\tau}$$

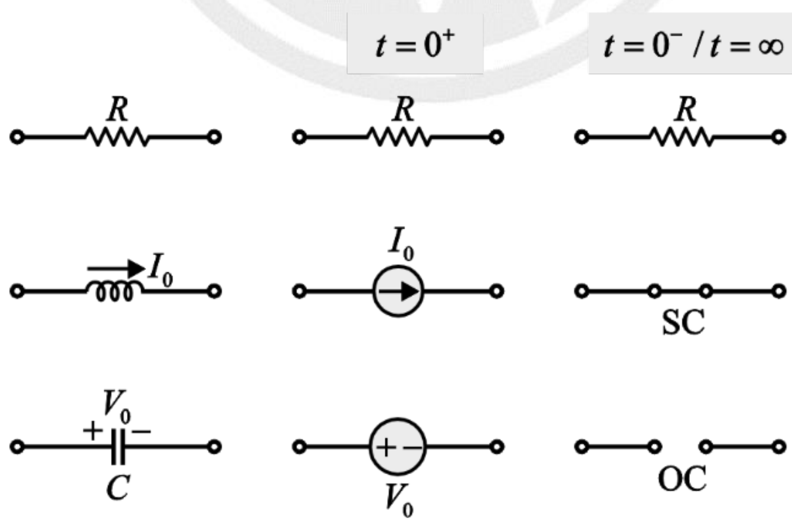
2. If the switching occurs at $t = t_0$.

$$i(t) = i(\infty) + \left[i(t_0) - i(\infty)\right]e^{-(t-t_0)/\tau} \qquad v(t) = v(\infty) + \left[v(t_0) - v(\infty)\right]e^{-(t-t_0)/\tau}$$

4.10. Limitations of Transient Equation

1. It is not applicable in circuits having two energized capacitors in series.
2. It is not applicable in circuits having two energized inductors in parallel.
3. If time dependent source is present in the circuit then transient equation is not applicable.
4. For inductor or capacitor if excitation is impulse signal then transient equation is not applicable.
5. If the source is sinusoidal in nature, transform domain approach is not valid.

Behavior of R, L and C at $t = 0^-$, $t = 0^+$ & $t = \infty$:



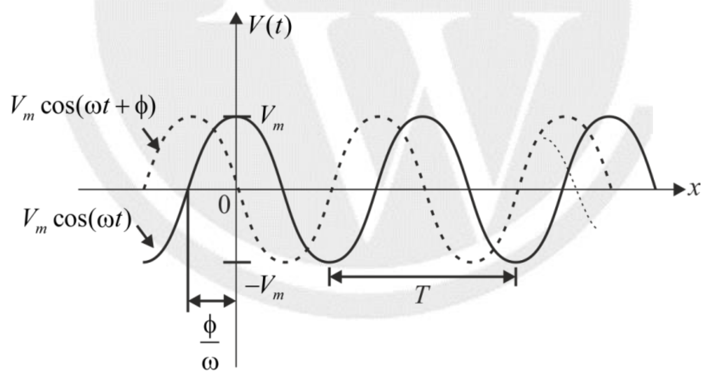
5 SINUSOIDAL STEADY STATE ANALYSIS

5.1. Introduction

- When we apply sinusoidal across any reactive network or complex network, then all responses either current or voltage are referred as sinusoidal steady state response or steady state response or sinusoidal response.
- These is no concept of open circuit and short circuit for sinusoidal input.
- In case of dc source, at steady state, capacitor is replaced by open circuit and inductor is replaced by short circuit.
- The sinusoidal varying voltage can be written as,

$$V(t) = V_m \cos(\omega t + \phi)$$

- To aid discussion of the parameters of the sinusoidal voltage equation, below is the figure.

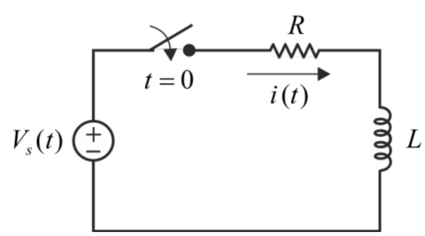


where, V_m = Amplitude

ϕ = Phase angle

ω= Angular frequency which is related to time period T as $\omega = \frac{2\pi}{T}$. The argument ωt changes 2π radians (360^0) in one period.

Example : Series RL circuit,



$$i_{tr}(t) = -\frac{V_m \cos\left(\phi - \tan^{-1}\frac{\omega L}{R}\right)}{\sqrt{R^2+\omega^2 L^2}} e^{-\frac{R}{L}t} \quad \text{[Transient response, dies out as } t \to \infty]$$

$$i_{ss}(t) = -\frac{V_m}{\sqrt{R^2+\omega^2 L^2}} \cos\left(\omega t + \phi - \tan^{-1}\frac{\omega L}{R}\right) \quad \text{[Steady state response, lasts even } t \to \infty]$$

5.2. Phasor

The phasor is a constant complex number that carries the amplitude and phase angle information of a sinusoidal function.

A sinusoidal function can be represented by the real part of a phasor times the "Complex carrier".

$$V_m \cos(\omega t + \phi) = V_m \operatorname{Re}\left\{e^{j(\omega t+\phi)}\right\} = \operatorname{Re}\left\{(V_m e^{j\phi}) e^{j\omega t}\right\}$$

$$= \operatorname{Re}\left\{\underset{\substack{\uparrow \\ Phasor}}{V}\ \underset{\substack{\uparrow \\ Carrier}}{e^{j\omega t}}\right\}$$

(i) Polar form

$$V = V_m e^{j\phi} = V_m \angle\phi$$

(ii) Rectangular form

$$V = V_m \cos\phi + jV_m \sin\phi$$

$$H^2 = \text{real}^2 + \text{Img}^2$$

$$H^2 = V_m^2 \cos^2\phi + V_m^2 \sin^2\phi$$

$$H^2 = V_m^2(\cos^2\phi + \sin^2\phi)$$

$$H = V_m$$

Impedances of the Passive Circuit Element

1. Generalize resistance to impedance.
2. Impedance of R, L, C
3. In phase and quadrature.

(i) In a resistor, the ratio of voltage $V(t)$ to the current $i(t)$ is a real constant *R*.

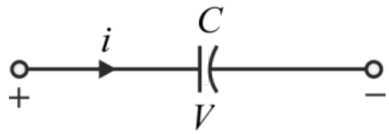
$$R = \frac{V(t)}{i(t)} \quad \ldots\text{(Resistance)}$$

(ii) For two terminals of a linear circuit driven by sinusoidal source, the ratio of voltage phasor V to the current phasor *I* is a complex constant *Z*.

$$Z = \frac{V}{I} \quad \ldots\text{(Impedance)}$$

Relation between *i, v* and Impedance of a Capacitor :

Let us take, $$V(t) = V_m \cos(\omega t + \theta_v)$$



The time t_0 is referred as switching time or transient free time

$V_{in}(t)$	s	Condition of transient free
$V_m \sin(\omega t + \phi)$	$t = 0$	$\phi - \tan^{-1}\dfrac{\omega L}{R} = 0 \quad \left[\tau = \dfrac{L}{R}\right]$ $\tau = RC$ for RC circuit $\phi - \tan^{-1}\omega RC = 0$
$V_m \sin(\omega t + \phi)$	$t = t_0$	$\omega t_0 + \phi - \tan^{-1}\dfrac{\omega L}{R} = 0$
$V_m \cos(\omega t + \phi)$	$t = 0$	$\phi - \tan^{-1}\dfrac{\omega L}{R} = \dfrac{\pi}{2}$
$V_m \cos(\omega t + \phi)$	$t = t_0$	$\omega t_0 + \phi - \tan^{-1}\dfrac{\omega L}{R} = \dfrac{\pi}{2}$

$$t_0 = \frac{\tan^{-1}\frac{\omega L}{R} - \phi_{rad}}{\omega(rad/sec)} \quad \text{(sin excitation)}$$

$$t_0 = \frac{\frac{\pi}{2} + \tan^{-1}\frac{\omega L}{R} - \phi}{\omega} \quad \text{(cosine excitation)}$$

1. Transient free response is only possible for ac excitation. It is not possible for dc excitation.
2. Transient free response is not applicable for series RLC as well as parallel RLC network (2nd order network)
3. Consider 2nd order system with complex conjugate roots $s = -\alpha \pm j\beta$

$$(t) = \underbrace{e^{-\alpha t}\left[K_1 \cos\beta t + K_2 \sin\beta t\right]}_{\text{Transient}} + \underbrace{\frac{V_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \sin\left[\omega t + \phi - \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)\right]}_{\text{Steady state}}$$

For Transient part to be 0, $K_1 \cos\beta\, t + K_2 \sin\beta\, t$ should be 0, which is not possible at same time because of sin and cos.

Transient free response is possible only for series and parallel first order RL and RC circuits, with ac excitation.

❑❑❑

6 RESONANCE

6.1 Resonance

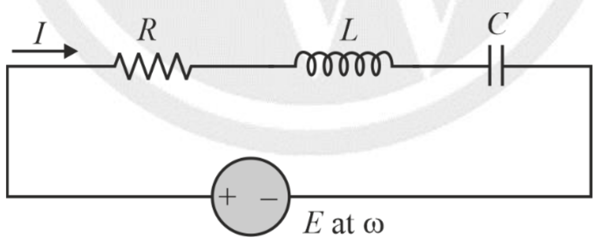
A.C Circuits made up of resistors, inductors and capacitors are said to be resonant circuits when the current drawn from the supply is in phase with the applied sinusoidal voltage. Then

1. The resultant Reactance or Susceptance is zero.
2. The circuit behaves as a resistive circuit.
3. The power factor is unity.

6.2. Series Resonance

Figure represents a series resonant circuit. Resonance can be achieved by

1. Varying frequency ω
2. Varying the inductance L
3. Varying the capacitance C



The current in the circuit is

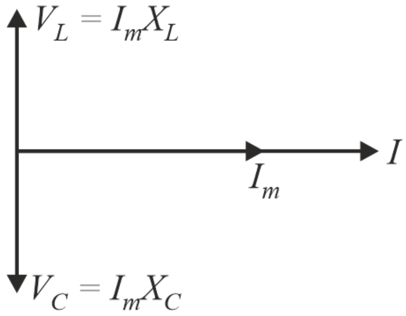
$$I = \frac{E}{R + j(X_L - X_C)} = \frac{E}{R + jX}$$

At resonance. X is zero. If ω_0 is the frequency at which resonance occurs, then $\omega_0 L = \frac{1}{\omega_0 C}$ or $\omega_0 = \frac{1}{\sqrt{LC}}$ = resonant frequency.

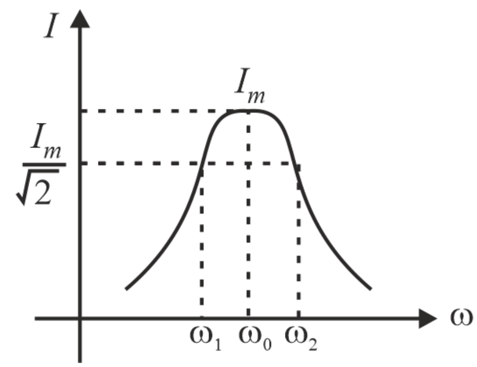
The current at resonance is $I_m = \frac{V}{R}$ = maximum current.

The phasor diagram for this condition is shown in Fig. (a).

The variation of current with frequency is shown in Fig. (b).



(a)



(b)

Points to be remembered:

At Series Resonance,

- The voltage across inductor and capacitor are equal in magnitude i.e. $|V_L| = |V_C|$ and 180° out of phase.
- Imaginary part of input impedance is equal to zero.
- The net impedance is Minimum i.e. $Z_{in}|_{\omega=\omega_0} = \text{R}(\text{Minimum})$
- The net current flow in the circuit is Maximum i.e. $I|_{\omega=\omega_0} = \frac{V_S}{R}(\text{Maximum})$

Impedance of a Series Resonant Circuit:

The impedance of a series RLC circuit is given by,

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right) \qquad |Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

Variation of impedance with change in frequency is shown in figure below,

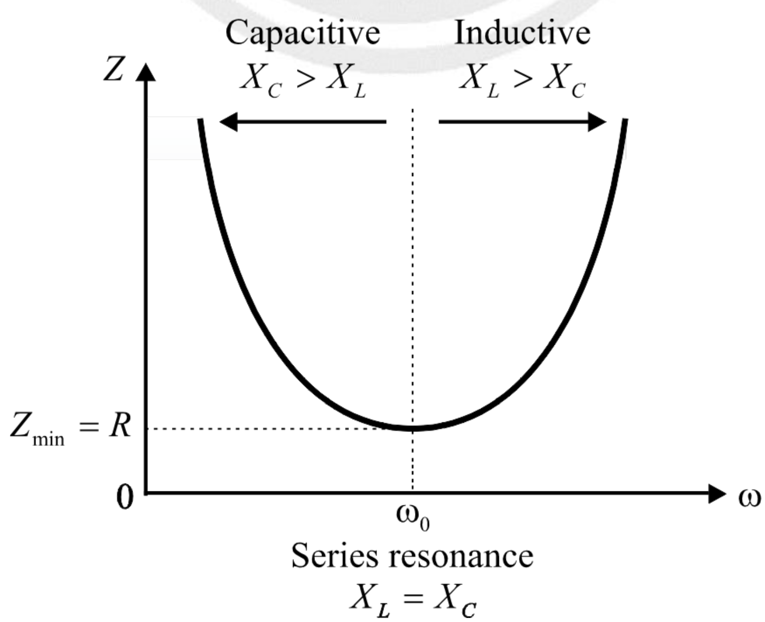


Fig. Impedance vs frequency curve for series RLC circuit

6.1.1 3dB Frequency

The range of frequencies at which the current drawn by network becomes 0.707 or $\frac{1}{\sqrt{2}}$ times of its maximum value.

Higher and lower 3dB cut off frequencies of series RLC circuit are given by,

$$\omega_H = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \text{ rad/sec} \quad \ldots\text{(i)}$$

$$\omega_L = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \text{ rad/sec} \quad \ldots\text{(ii)}$$

Quality factor (Q-factor):

- The "sharpness" of the resonance in a resonant circuit is measured quantitatively by the quality factor Q
- The Q factor is also known as the voltage amplification factor OR current amplification factor and it's value must be high for any tuned circuit.
- *Q*-factor describes the energy storage capability of inductor and capacitor in RLC network.

$$Q[L] = 2\pi \times \frac{\text{Energy stored by inductor}}{\text{Energy dissipated by resistance per cycle}}$$

$$Q[L] = \frac{2\pi \times \frac{1}{2} L I_0^2}{\frac{I_0^2 R}{2} \cdot T_0}$$

$$Q[L] = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Similarly, $$Q[C] = 2\pi \times \frac{\text{Energy stored by capacitor}}{\text{Energy dissipated by resistance per cycle}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

The relationship between bandwidth, frequency of resonance and quality factor is given by,

$$Q = \frac{\omega_0}{\text{BW}}$$

Note:

For any tuned network the Q-factor must be high and the bandwidth should be small and therefore the resistance used in the network should be small as $Q \propto \frac{1}{BW}$ & $BW \propto R$

6.1.2 Parallel RLC Resonance Circuit

The Dual of a series resonant circuit is often considered as a parallel resonant circuit and it is as shown in figure.

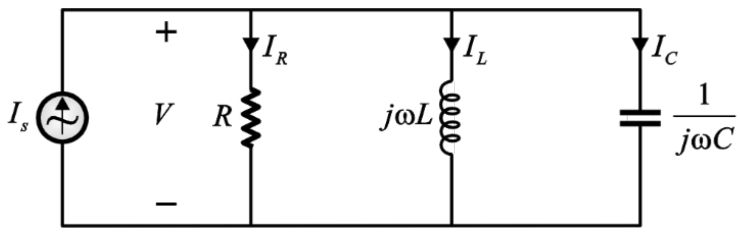


Fig. A parallel resonance circuit

The voltage across the parallel combination of RLC is given by,

$$V = \frac{I_s}{\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)} \qquad |V| = \frac{|I_s|}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}}$$

The value of ω that satisfies the condition of parallel resonance is called resonant frequency.

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec}$$

At resonance, $\omega L = 1/\omega C$, the admittance will be minimum and voltage will be maximum as given below,

$$|V_R| = |I_s| R$$

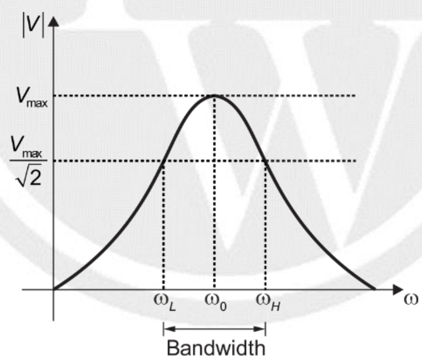


Fig. Voltage variation in a parallel resonant circuit

Admittance of the parallel RLC circuit given by,

$$Y = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

From above equation, admittance is dominated by inductive term at lower frequencies and by capacitive term at higher frequencies. The plot of Y against frequency is shown below.

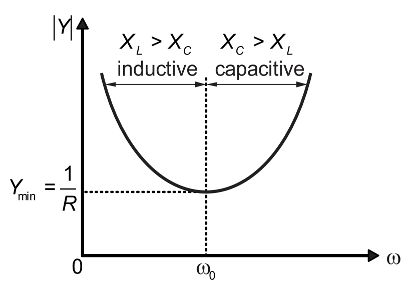


Fig. Admittance vs frequency curve in parallel RLC circuit

3dB Bandwidth

3 dB Bandwidth is given by,

$$BW = \omega_H - \omega_L \qquad BW = \frac{1}{RC} \text{ rad/sec} \qquad BW = \frac{1}{2\pi RC} \text{ Hz}$$

To obtain the relationship between bandwidth and frequency of resonance, multiply equation (i) and (ii),

$$\therefore \qquad \omega_H . \omega_L = \frac{1}{LC} \qquad \omega_H . \omega_L = \omega_0^2$$

$$\therefore \qquad \omega_0 = \sqrt{\omega_H . \omega_L} \text{ rad/sec}$$

3dB Frequencies

Higher 3dB cut frequencies of parallel RLC circuit are given by,

$$\omega_H = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \text{ rad/sec} \qquad \ldots\text{(i)}$$

Lower 3dB cut frequencies of parallel RLC circuit are given by,

$$\omega_L = \frac{-1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \text{ rad/sec} \qquad \ldots\text{(ii)}$$

3 dB Frequency in terms of bandwidth is given by

$$\omega_H = \frac{BW}{2} + \sqrt{\left(\frac{BW}{2}\right)^2 + \omega_0^2} \qquad \omega_L = -\frac{BW}{2} + \sqrt{\left(\frac{BW}{2}\right)^2 + \omega_0^2}$$

Quality factor for parallel RLC circuit:

- The "sharpness" of the resonance in a resonant circuit is measured quantitatively by the quality factor Q
- *Q*-factor describes the energy storage capability of inductor and capacitor in RLC network.

$$Q[L] = \frac{I_L}{I_R} = Q[C] = \frac{I_C}{I_R} = R\sqrt{\frac{C}{L}}$$

The relationship between bandwidth and quality factor is given by,

Quality factor, $$Q = \frac{\omega_0}{BW}$$

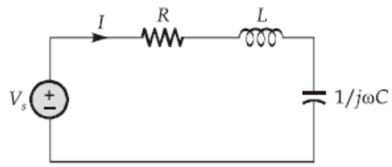
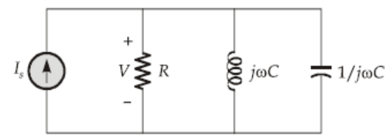
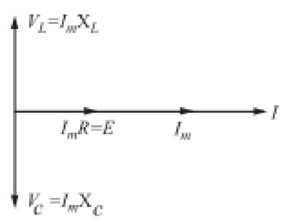
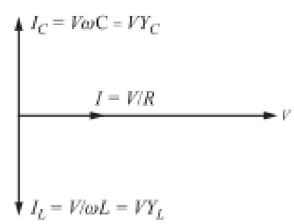
6.1.3 Resonance in Series RLC vs Parallel RLC

Parallel resonant circuit is the **DUAL** of the series resonant circuit and hence the results of parallel resonant circuit can be obtained just by following replacement in the results of series resonant circuit,

$$R \to \frac{1}{R}$$

$$L \to C, C \to L$$

The key differences between series resonance and parallel resonance are given in the following table −

Parameters	Series Resonance	Parallel Resonance
Circuit Diagram		
Impedance	The impedance of a series RLC circuit becomes minimum at series resonance. The imaginary part of the impedance is 0 i.e. $X_L - X_C = 0$ $X_L = X_C$	The impedance of a parallel RLC circuit becomes maximum at parallel resonance.
Admittance	Series resonance has maximum admittance.	Parallel resonance has minimum admittance. The imaginary part of the impedance is 0 i.e. $Y_L - Y_C = 0$ $Y_L = Y_C$
Phasor Diagrams	 Figure 6.2	
Upper cut-off frequency	$\omega_H = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$ rad/sec	$\omega_H = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$ rad/sec
Lower cut-off frequency	$\omega_L = -\left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]$ rad/sec	$\omega_H = -\left[\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}\right]$ rad/sec
Bandwidth	$BW = \omega_H - \omega_L = \frac{R}{L}$ rad/sec	$BW = \omega_H - \omega_L = \frac{1}{RC}$ rad/sec
Behave of the circuit	It acts as a voltage amplifier circuit or acceptor circuit.	It acts as current amplifier circuit or rejector circuit.
Current	The series resonance results in the maximum current through the circuit.	The current in circuit at parallel resonance is minimum.
Filter Characteristics	It works as a Band Pass Filter	It works as a Band Stop Filter and Band Reject Filter.

Parameters	Series Resonance	Parallel Resonance
Magnification Feature	The series resonance magnifies the voltage in the circuit.	The parallel resonance magnifies the current in the circuit.
Equation of effective impedance	The effective impedance is given by, Z = R	The effective impedance is given by, Z = L/CR
Quality factor (Q-factor)	$Q[L]=\frac{\omega_0 L}{R}=\frac{1}{R}\sqrt{\frac{L}{C}}$	$Q[L]=\frac{R}{\omega_0 L}=R\sqrt{\frac{C}{L}}$
Applications	The series resonance is widely used in tuning, oscillator circuits, voltage amplifiers, high-frequency filters, etc.	The parallel resonance is used in current amplifiers, induction heating, filters, radio-frequency amplifiers, etc.

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7 COMPLEX POWER

7.1. Introduction

Complex power is "the complex sum of real and reactive powers", it is also termed as apparent power, measured in terms of volt-amps (or), in kilo volt-amps (KVA).

The rectangular form of complex power is given below,

$$S = P + jQ$$

$$S = \sqrt{P^2 + Q^2} \angle \tan^{-1}\left(\frac{Q}{P}\right)$$

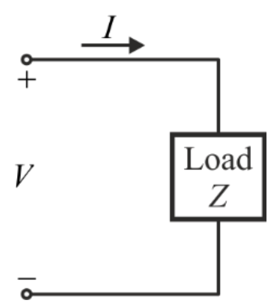
$$S = VI^*$$

where, I^* is the complex conjugate of current.

In case of AC circuit, complex power is given by, $S = V_{rms} I^*_{rms}$ VA.

7.2. Analysis of Complex Power

Consider the AC load figure in below.



$$S = V_{rms} I^*_{rms}$$

$$V_{rms} = V_{rms} \angle \theta_v = \frac{V_m}{\sqrt{2}} \angle \theta_v$$

The expression of RMS current in phasor form is given by;

$$I_{rms} = I_{rms} \angle \theta_i = \frac{I_m}{\sqrt{2}} \angle \theta_i$$

$$I_{rms}^* = I_{rms} \angle -\theta_i = \frac{I_m}{\sqrt{2}} \angle -\theta_i$$

$$S = V_{rms} I_{rms} \angle \theta_v - \theta_i \quad \text{[in polar form]}$$

$$S = V_{rms} I_{rms} e^{j(\theta_v - \theta_i)} \quad \text{[in exponential form]}$$

$$S = V_{rms} I_{rms} \cos(\theta_v - \theta_i) + jV_{rms} I_{rms} \sin(\theta_v - \theta_i)$$

$$S = \frac{V_m I_m}{2} \cos\phi + j\frac{V_m I_m}{2} \sin\phi$$

$$\phi = \angle \theta_v - \theta_i$$

7.3. Real Power or Reactive Power

The complex power may be expressed in terms of the load impedance Z.

The load impedance Z may be written as,

$$Z = \frac{V}{I} = \frac{V_{rms}}{I_{rms}} = \frac{V_{rms}}{I_{rms}} \angle \theta_v - \theta_i$$

$$V_{rms} = Z \times I_{rms}$$

$$S = I_{rms}^2 Z = \frac{V_{rms}^2}{Z^*} = V_{rms} I_{rms}^*$$

Since, $Z = R + jX$

$$S = I_{rms}^2 (R + jX) = P + jQ$$

$$S = I_{rms}^2 (R + jX)$$

$$S = P + jQ = I_{rms}^2 (R + jX)$$

where P and Q are the real and imaginary parts of the complex power, that is

$$P = \text{Re}(S) = I_{rms}^2 R$$

$$Q = \text{Im}(S) = I_{rms}^2 X$$

- P is the average or real power and if depend S on the loads resistance R.

- Q depend on the loads reactance X and is called the reactive (or quadrature) power.

Comparing equations (i) and (ii), we notice that,

$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

$$Q = V_{rms} I_{rms} \sin(\theta_v - \theta_i)$$

Points to be Remembered:

- The real power P is the average power in watts delivered to a load, it is the only useful power, it is actual power dissipated by the load.
- The reactive power Q is a measure of the energy exchange between the source and the reactive part of the load.

The nature of reactive power for different types of loads:

Reactive Power	Power factor	Load Types
Q = 0	Unity	Resistive
Q < 0	Leading	Capacitive
Q > 0	Lagging	Inductive

7.4. Power Triangle

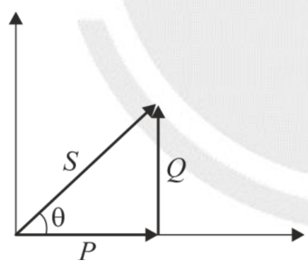


Fig.(a)

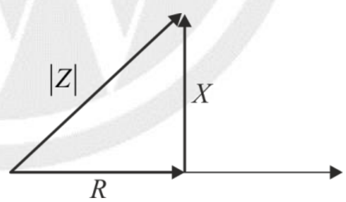


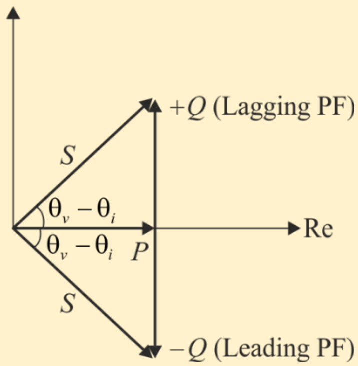
Fig.(b)

$$|S| = \sqrt{P^2 + Q^2}$$

$$\cos\phi = \frac{P}{S} = \text{ Power factor.}$$

$$\sin\phi = \frac{Q}{S} = \text{ Reactive factor}$$

Note:



1. $P = VI$ (In DC circuit)
2. $P = VI\cos\phi$ (In single phase AC circuit)
3. $P = \sqrt{3}V_L I_L \cos\phi$ (In three phase AC circuit)
4. $P = \sqrt{(S^2 - Q^2)}$
5. $P = \sqrt{(\text{VA})^2 - (\text{VAR})^2}$
6. $Q = VI\sin\phi$
7. Reactive power $= \sqrt{(\text{Apparent power})^2 - (\text{True power})^2}$
8. $Q = \sqrt{S^2 - P^2}$
9. $\text{KVAR} = \sqrt{(\text{KVA})^2 - (\text{KW})^2}$

Where: $P =$ Power in watt

$V =$ Voltage in volt

$I =$ Current in amperes

$\cos\phi =$ Power factor (phase angle difference)

$V_L =$ Line voltage

$I_L =$ Line current

$S =$ Apparent power in VA (volt ampere)

$Q =$ Reactive power in VAR (volt ampere reactive).

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8 MAGNETIC COUPLING

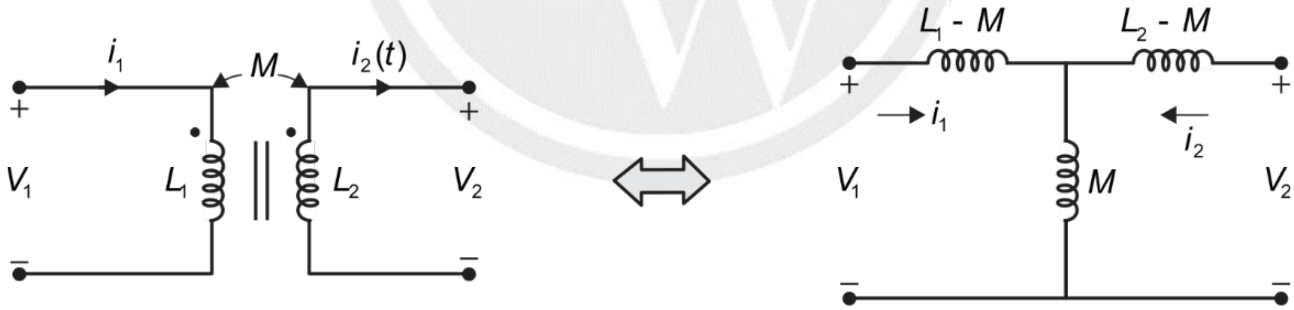
8.1 Introduction

When two loops with or without contacts between them affect each other through the magnetic field generated by one of them, they are said to be magnetically coupled. Whenever a current flows through a conductor, a magnetic field is generated (magnetic flux). When time varying magnetic field generated by one loop penetrates a second loop, a voltage induced between the ends of the second wire.

8.1.1 Important Points to Remember

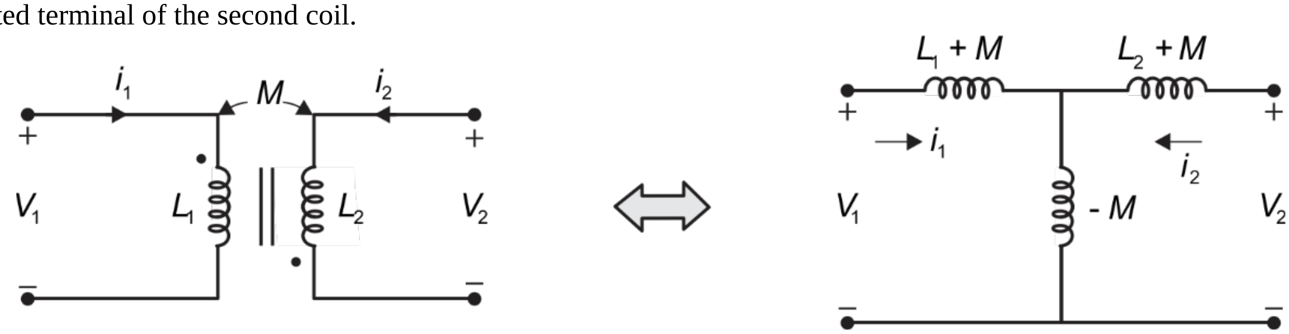
Magnetic Aiding:

If a current enters the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is positive at the dotted terminal of the second coil.



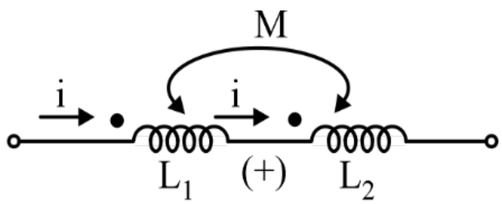
Magnetic Opposition:

If a current leaves the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is negative at the dotted terminal of the second coil.



Coupled inductors in series:

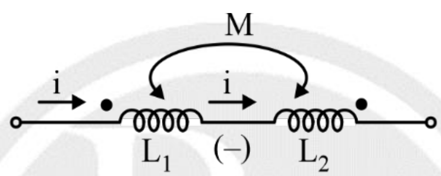
(i) Aiding



(a)

$$L = L_1 + L_2 + 2M \quad \text{(Series-aiding connection)}$$

(ii) Opposing



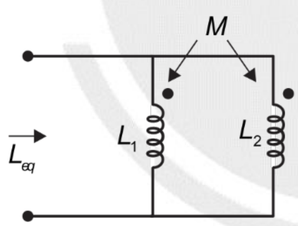
(b)

$$L = L_1 + L_2 - 2M \quad \text{(Series-opposing connections)}$$

Coupled inductors in parallel:

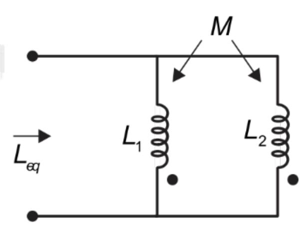
(i) Aiding

$$L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$$



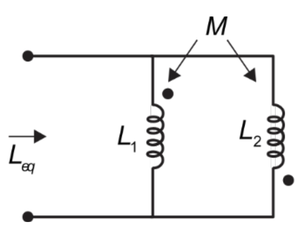
OR

$$L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$$



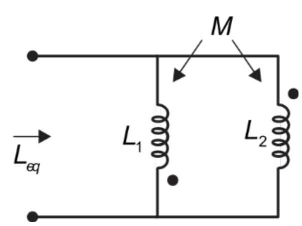
(ii) Opposing

$$L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$$



OR

$$L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$$



8.2. Coupling Coefficient, *k*

- A measure of the amount of magnetic coupling between two inductors

$$0 \le k \le 1$$

- k = 0: completely un-coupled inductors
- k = 1: perfect coupling – all magnetic flux generated by one inductor penetrates the coil of the other inductor.

- Relationship to mutual inductance, M:

$$M = k\sqrt{L_1 L_2}$$

8.1.2 Energy of Coupled Coil

Case (i): Magnetic Opposing Circuit

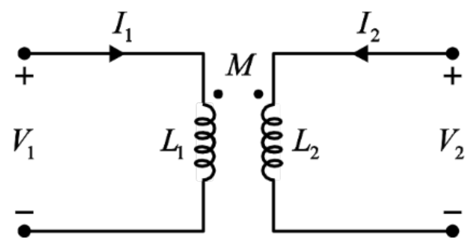


Fig. (a)

$$W = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + M I_1 I_2$$

Case (ii): Magnetic Adding Circuit

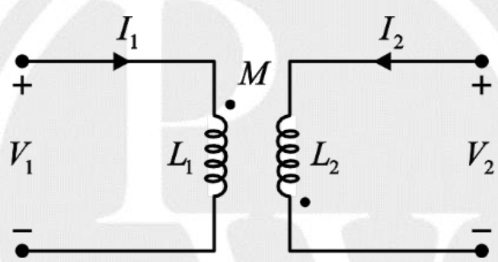


Fig. (b)

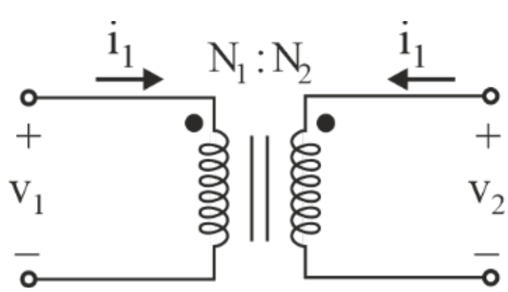
$$W = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 - M I_1 I_2$$

8.1.3 Dot Convention in Transformer

Transformation ratio or turn ratio is given by,

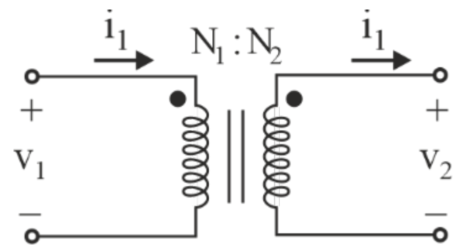
$$n = \frac{N_1}{N_2}$$

Case 1:



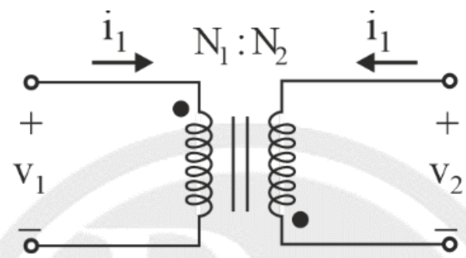
$$\frac{v_1}{v_2} = \frac{N_1}{N_2} \quad \frac{i_1}{i_2} = -\frac{N_2}{N_1}$$

Case 2:



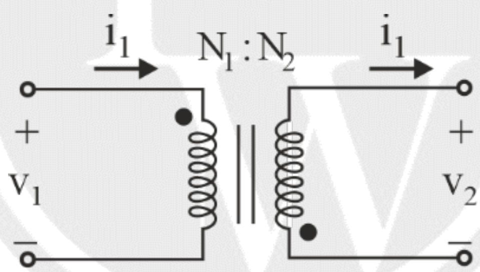
$$\frac{v_1}{v_2} = \frac{N_1}{N_2} \quad \frac{i_1}{i_2} = \frac{N_2}{N_1}$$

Case 3:



$$\frac{v_1}{v_2} = -\frac{N_1}{N_2} \quad \frac{i_1}{i_2} = \frac{N_2}{N_1}$$

Case 4:



$$\frac{v_1}{v_2} = -\frac{N_1}{N_2} \quad \frac{i_1}{i_2} = -\frac{N_2}{N_1}$$

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For more questions, kindly visit the library section: Link for web: https://smart.link/sdfez8ejd80if

PW Mobile APP: https://smart.link/7wwosivoicgd4