Einführung in die Numerik Partieller Differentialgleichungen I (WS 16/17)

Homework 9

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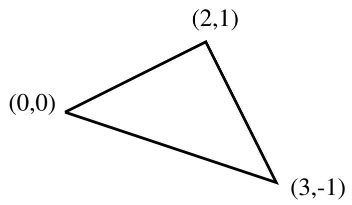
Deadline for submission:	23.12. **before** the lecture
Return of corrected assignments:	in the practical sections

Exercise 20: 2+2=4P

a) Compute the local stiffness matrix of a finite element discretization of the Poisson equation for the triangle with vertices $a_1 = (0,0)$, $a_2 = (3,-1)$, $a_3 = (2,1)$ and the triangle with vertices $a_1 = (0,0)$, $a_2 = (3,1)$, $a_3 = (2,1)$.

b) Estimate the asymptotic behavior of $\|B\|_2\|B^{-1}\|_2$ for $h \to 0$ for both triangle shapes (you may use some software to perform the computations).



Exercise 21: 2+2+3=7P

Consider the fourth order boundary value problem:
Find $u \in C^4(\Omega) \cap C^1(\overline{\Omega})$ *such that*

$$\begin{cases} \Delta^2 u - \nabla \cdot (K(x)\nabla u) = f & \text{in } \Omega\,, \\ \partial_\nu u = u = 0 & \text{on } \partial\Omega\,. \end{cases}$$

a) Choose V and derive a variational formulation:
Find $u \in V$ *such that* $\quad a(u,v) = b(v) \quad \forall v \in V\,.$

b) Show that a is continuous with regard to the norm of V, i.e., $\exists M > 0 :$ $|a(u,v)| \le M\|u\|\|v\|$, $\forall u, v \in V$.

c) Show that a is V-elliptic with regard to the norm of V, i.e., $\exists \alpha > 0 : a(u,u) \ge \alpha\|u\|^2$, $\forall u \in V$. Give some condition for $K(x)$, if necessary.