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System Simulation

## Verification and Validation of Simulation Models


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## Verification of Simulation Models

Many commonsense suggestions can be given for use in the verification process.


1. Have the code checked by someone other than the programmer.
2. Make a flow diagram which includes each logically possible action a system can take when an event occurs, and follow the model logic for each action for each event type.



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## Verification and Validation of Simulation Models


- **Verification**: concerned with building the **model right**. It is utilized in the comparison of the conceptual model to the computer representation that implements that conception. It asks the questions: Is the model implemented correctly in the computer? Are the input parameters and logical structure of the model correctly represented?



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## Verification of Simulation Models


3. Closely examine the model output for reasonableness under a variety of settings of the input parameters. Have the code print out a wide variety of output statistics.
4. Have the computerized model print the input parameters at the end of the simulation, to be sure that these parameter values have not been changed inadvertently.



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## Verification and Validation of Simulation Models (cont.)

- **Validation**: concerned with building the **right model**. It is utilized to determine that a model is an accurate representation of the real system. Validation is usually achieved through the calibration of the model, an iterative process of comparing the model to actual system behavior and using the discrepancies between the two, and the insights gained, to improve the model. This process is repeated until model accuracy is judged to be acceptable.




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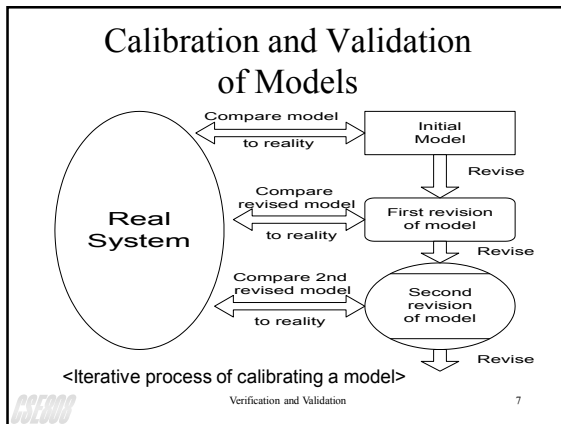
## Verification of Simulation Models

5. Make the computer code as self-documenting as possible. Give a precise definition of every variable used, and a general description of the purpose of each major section of code.

These suggestions are basically the same ones any programmer would follow when debugging a computer program.



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### Validation of Model Assumptions (cont.)

2. Interarrival times during a slack period
3. Service times for commercial accounts
4. Service times for personal accounts

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### Validation of Simulation Models

As an aid in the validation process, Naylor and Finger formulated a three-step approach which has been widely followed:

1. Build a model that has high face validity.
2. Validate model assumptions.
3. Compare the model input-output transformations to corresponding input-output transformations for the real system.

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### Validation of Model Assumptions (cont.)

The analysis of input data from a random sample consists of three steps:

1. Identifying the appropriate probability distribution
2. Estimating the parameters of the hypothesized distribution
3. Validating the assumed statistical model by a goodness-of fit test, such as the chi-square or Kolmogorov-Smirnov test, and by graphical methods.

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### Validation of Model Assumptions

- Structural involves questions of how the system operate (Example1)  
Data assumptions should be based on the collection of reliable data and correct statistical analysis of the data.  
Customers queueing and service facility in a bank (one line or many lines)
1. Interarrival times of customers during several 2-hour periods of peak loading ("rush-hour" traffic)

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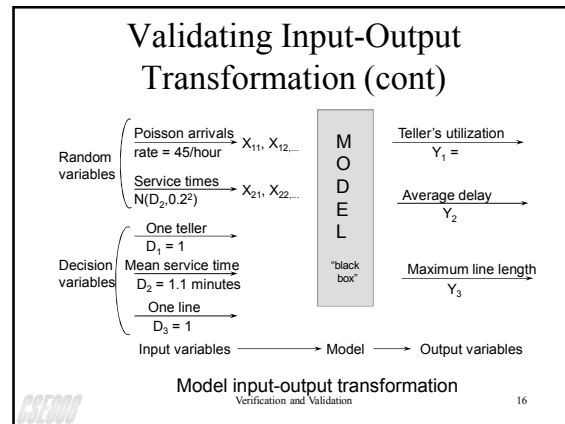
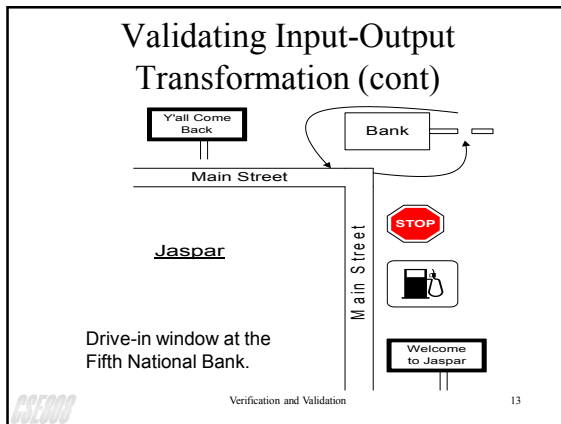
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### Validating Input-Output Transformation

(Example) : The Fifth National Bank of Jasper  
The Fifth National Bank of Jasper, as shown in the next slide, is planning to expand its drive-in service at the corner of Main Street. Currently, there is one drive-in window serviced by one teller. Only one or two transactions are allowed at the drive-in window, so, it was assumed that each service time was a random sample from some underlying population. Service times  $\{S_i, i = 1, 2, \dots, 90\}$  and interarrival times  $\{A_i, i = 1, 2, \dots, 90\}$

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### Validating Input-Output Transformation (cont)

were collected for the 90 customers who arrived between 11:00 A.M. and 1:00 P.M. on a Friday. This time slot was selected for data collection after consultation with management and the teller because it was felt to be representative of a typical rush hour. Data analysis led to the conclusion that the arrival process could be modeled as a Poisson process with an arrival rate of 45 customers per hour; and that service times were approximately normally distributed with mean 1.1 minutes and

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### Validating Input-Output Transformation (cont)

The uncontrollable input variables are denoted by  $X$ , the decision variables by  $D$ , and the output variables by  $Y$ . From the “black box” point of view, the model takes the inputs  $X$  and  $D$  and produces the outputs  $Y$ , namely

$$(X, D) \xrightarrow{f} Y$$

or

$$f(X, D) = Y$$

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### Validating Input-Output Transformation (cont)

standard deviation 0.2 minute. Thus, the model has two input variables:

1. Interarrival times, exponentially distributed (i.e. a Poisson arrival process) at rate  $\lambda = 45$  per hour.
2. Service times, assumed to be  $N(1.1, (0.2)^2)$

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### Validating Input-Output Transformation (cont)

Input Variables	Model Output Variables, $Y$
$D$ = decision variables	Variables of primary interest
$X$ = other variables	to management ( $Y_1, Y_2, Y_3$ )
Poisson arrivals at rate = 45 / hour	$Y_1$ = teller's utilization
$X_{11}, X_{12}, \dots$	$Y_2$ = average delay
	$Y_3$ = maximum line length

Input and Output variables for model of current bank operation (1)

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## Validating Input-Output Transformation (cont)

Input Variables	Model Output Variables, Y
Service times, $N(D_2, 0.2^2)$	Other output variables of secondary interest
$X_{21}, X_{22}, \dots$	$Y_4$ = observed arrival rate
$D_1 = 1$ (one teller)	$Y_5$ = average service time
$D_2 = 1.1$ min (mean service time)	$Y_6$ = sample standard deviation of service times
$D_3 = 1$ (one line)	$Y_7$ = average length of line

Input and Output variables for model of current bank operation (2)  
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## Validating Input-Output Transformation (cont)

$Z_2 = 4.3$  minutes, the model responses,  $Y_2$ . Formally, a statistical test of the null hypothesis

$$H_0 : E(Y_2) = 4.3 \text{ minutes}$$

versus ----- (Eq 1)

$$H_1 : E(Y_2) \neq 4.3 \text{ minutes}$$

is conducted. If  $H_0$  is not rejected, then on the basis of this test there is no reason to consider the model invalid. If  $H_0$  is rejected, the current version of the model is rejected and the modeler is forced to seek ways to improve the model, as illustrated in Table 3.

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## Validating Input-Output Transformation (cont)

Statistical Terminology	Modeling Terminology	Associated Risk
Type I : rejecting $H_0$ when $H_0$ is true	Rejecting a valid model	$\alpha$
Type II : failure to reject $H_0$ when $H_0$ is false	Failure to reject an invalid model	$\beta$

(Table 1) Types of error in model validation

Note: Type II error needs controlling increasing  $\alpha$  will decrease  $\beta$  and vice versa, given a fixed sample size. Once  $\alpha$  is set, the only way to decrease  $\beta$  is to increase the sample size.

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## Validating Input-Output Transformation (cont)

As formulated here, the appropriate statistical test is the  $t$  test, which is conducted in the following manner:

**Step 1.** Choose a level of significance  $\alpha$  and a sample size  $n$ . For the bank model, choose

$$\alpha = 0.05, \quad n = 6$$

**Step 2.** Compute the sample mean  $\bar{Y}_2$  and the sample standard deviation  $S$  over the  $n$  replications.

$$\bar{Y}_2 = \{1/n\} \sum_{i=1}^n Y_{2i} = 2.51 \text{ minutes}$$

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## Validating Input-Output Transformation (cont)

Replication	$Y_4$ (Arrivals/Hours)	$Y_5$ (Minutes)	$Y_2$ = avg delay (Minutes)
1	51	1.07	2.79
2	40	1.12	1.12
3	45.5	1.06	2.24
4	50.5	1.10	3.45
5	53	1.09	3.13
6	49	1.07	2.38
sample mean			2.51
standard deviation			0.82

(Table 2) Results of six replications of the First Bank Model

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## Validating Input-Output Transformation (cont)

and

$$S = \left\{ \sum_{i=1}^n (Y_{2i} - \bar{Y}_2)^2 / (n - 1) \right\}^{1/2} = 0.82 \text{ minute}$$

where  $Y_{2i}$ ,  $i = 1, \dots, 6$ , are shown in Table 2.

**Step 3.** Get the critical value of  $t$  from Table A.4. For a two-sided test such as that in Equation 1, use

$$t_{\alpha/2, n-1}; \text{ for a one-sided test, use } t_{\alpha, n-1} \text{ or } -t_{\alpha, n-1}$$

as appropriate ( $n-1$  is the degrees of freedom). From

Table A.4,  $t_{0.025, 5} = 2.571$  for a two-sided test.

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## Validating Input-Output Transformation (cont)

**Step 4.** Compute the test statistic

$$t_0 = (\bar{Y}_2 - \mu_0) / \{S / \sqrt{n}\} \quad \text{----- (Eq 2)}$$

where  $\mu_0$  is the specified value in the null hypothesis,  $H_0$ . Here  $\mu_0 = 4.3$  minutes, so that

$$t_0 = (2.51 - 4.3) / \{0.82 / \sqrt{6}\} = -5.34$$

**Step 5.** For the two-sided test, if  $|t_0| > t_{\alpha/2, n-1}$ , reject  $H_0$ . Otherwise, do not reject  $H_0$ . [For the one-sided test with  $H_1: E(Y_2) > \mu_0$ , reject  $H_0$  if  $t > t_{\alpha, n-1}$ ; with  $H_1: E(Y_2) < \mu_0$ , reject  $H_0$  if  $t < -t_{\alpha, n-1}$ ]

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## Validating Input-Output Transformation (cont)

Step 1. Choose  $\alpha = 0.05$  and  $n = 6$  (sample size).

Step 2. Compute  $\bar{Y}_2 = 4.78$  minutes,  $S = 1.66$  minutes -----> (from Table 3)

Step 3. From Table A.4, the critical value is  $t_{0.025,5} = 2.571$ .

Step 4. Compute the test statistic

$$t_0 = (\bar{Y}_2 - \mu_0) / \{S / \sqrt{n}\} = 0.710.$$

Step 5. Since  $|t| < t_{0.025,5} = 2.571$ , do not reject  $H_0$ , and thus tentatively accept the model as valid.

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## Validating Input-Output Transformation (cont)

Since  $|t| = 5.34 > t_{0.025,5} = 2.571$ , reject  $H_0$  and conclude that the model is inadequate in its prediction of average customer delay.

Recall that when testing hypotheses, rejection of the null hypothesis  $H_0$  is a strong conclusion, because  $P(H_0 \text{ rejected} | H_0 \text{ is true}) = \alpha$

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## Validating Input-Output Transformation (cont)

To consider failure to reject  $H_0$  as a strong conclusion, the modeler would want  $\beta$  to be small. Now,  $\beta$  depends on the sample size  $n$  and on the true difference between  $E(Y_2)$  and  $\mu_0 = 4.3$  minutes, that is, on

$$\delta = |E(Y_2) - \mu_0| / \sigma$$

where  $\sigma$ , the population standard deviation of an individual  $Y_{2i}$ , is estimated by  $S$ . Table A.9 and A.10 are typical operating characteristic (OC) curves, which are graphs of the probability of a

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## Validating Input-Output Transformation (cont)

Replication	$Y_4$ (Arrivals/Hours)	$Y_5$ (Minutes)	$Y_2 = \text{avg delay}$ (Minutes)
1	51	1.07	5.37
2	40	1.11	1.98
3	45.5	1.06	5.29
4	50.5	1.09	3.82
5	53	1.08	6.74
6	49	1.08	5.49
sample mean			4.78
standard deviation			1.66

(Table 3) Results of six replications of the REVISED Bank Model

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## Validating Input-Output Transformation (cont)

Type II error  $\beta(\delta)$  versus  $\delta$  for given sample size  $n$ . Table A.9 is for a two-sided  $t$  test while Table A.10 is for a one-sided  $t$  test. Suppose that the modeler would like to reject  $H_0$  (model validity) with probability at least 0.90 if the true means delay of the model,  $E(Y_2)$ , differed from the average delay in the system,  $\mu_0 = 4.3$  minutes, by 1 minute. Then  $\delta$  is estimates by

$$\bar{\delta} = |E(Y_2) - \mu_0| / S = 1 / 1.66 = 0.60$$

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## Validating Input-Output Transformation (cont)

For the two-sided test with  $\alpha = 0.05$ , use of Table A.9 results in

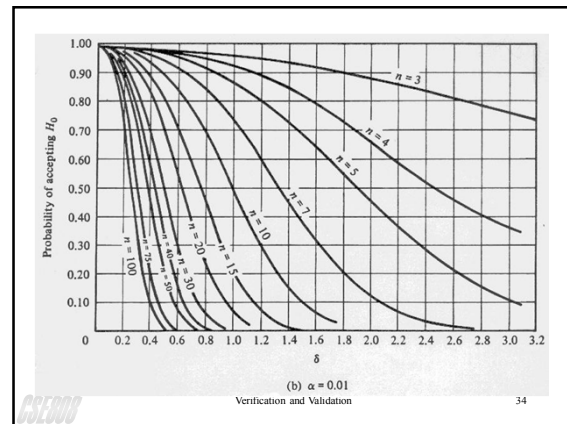
$$\beta(\bar{\delta}) = \beta(0.6) = 0.75 \quad \text{for } n = 6$$

To guarantee that  $\beta(\bar{\delta}) \leq 0.10$ , as was desired by the modeler, Table A.9 reveals that a sample size of approximately  $n = 30$  independent replications would be required. That is, for a sample size  $n = 6$  and assuming that the population standard deviation is 1.66, the probability of accepting  $H_0$  (model validity), when in fact the model is invalid

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## Validating Input-Output Transformation (cont)

( $|E(Y_2) - \mu_0| = 1$  minute), is  $\beta = 0.75$ , which is quite high. If a 1-minute difference is critical, and if the modeler wants to control the risk of declaring the model valid when model predictions are as much as 1 minute off, a sample size of  $n = 30$  replications is required to achieve a power of 0.9. If this sample size is too high, either a higher  $\beta$  risk (lower power), or a larger difference  $\delta$ , must be considered.

CS5000

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