

# ANOVA

## One Way ANOVA

① Define null & alternate hypothesis

data  $\Rightarrow$

A	B	C
3	1	8
6	8	6
3	9	10

$\hookleftarrow$  Perform ANOVA

NULL Hyp (H<sub>0</sub>):  $\mu_A = \mu_B = \mu_C$  (All 3 pop mean are equal)

Alternative (H<sub>1</sub>): at least one of them is significantly different

② Calculate grand mean of all groups combined and mean of individual groups.

Given  $n = 9$  (total samples)  
 $k = 3$  (no of categories)

$\bar{x}$  grand mean =  $\frac{3+6+3+1+8+9+8+6+10}{9}$

$$\bar{y} = \frac{54}{9} \Rightarrow \boxed{\bar{x} = 6}$$

Individual mean:

$$\boxed{\bar{x}_A = 4}, \quad \boxed{\bar{x}_B = 6}, \quad \boxed{\bar{x}_C = 8}$$

③ Calculate "between group" and "within group" sum of square (SS)

Calculate the table.

i) SST → sum of square total

calculating spread of each point from grand mean

$$\sum (\text{each point} - \bar{X})^2$$

$$= (6-3)^2 + (6-6)^2 + (6-3)^2 + (6-1)^2 + (6-6)^2 + (6-9)^2 + (6-0)^2 + (6-6)^2 + (6-10)^2$$

$$\boxed{SST = 76}$$

(ii) SSW → sum of squares within

calculating variance of each group with its respective mean

A spread + B spread + C spread

$$[(4-3)^2 + (4-6)^2 + (4-3)^2]$$

$$+ [(6-1)^2 + (6-0)^2 + (6-9)^2]$$

$$+ [(0-0)^2 + (0-6)^2 + (0-10)^2]$$

$$= \boxed{SSW = 59}$$

(iii) Calculating degrees of freedom for SSW

$$\begin{aligned} &= n - k \\ &= 9 - 3 \\ &= 6 \end{aligned}$$

(iv) Calculating degrees of freedom for SST

$$\begin{aligned} &= n - 1 \\ &= 9 - 1 \\ &= 8 \end{aligned}$$

(4) SSB  $\rightarrow$  Sum of square between

Calculating spread of grand mean from each group mean and multiplied by ~~no~~ with their no of samples

$$\begin{aligned} &3X(\bar{X} - A_m)^2 + 3X(\bar{X} - B_m)^2 + 3X(\bar{X} - C_m)^2 \\ &= 3X(6 - 4)^2 + 3X(6 - 6)^2 + 3X(6 - 8)^2 \end{aligned}$$

$$\boxed{SSB = 24}$$

(ii) Degree of freedom for SSB

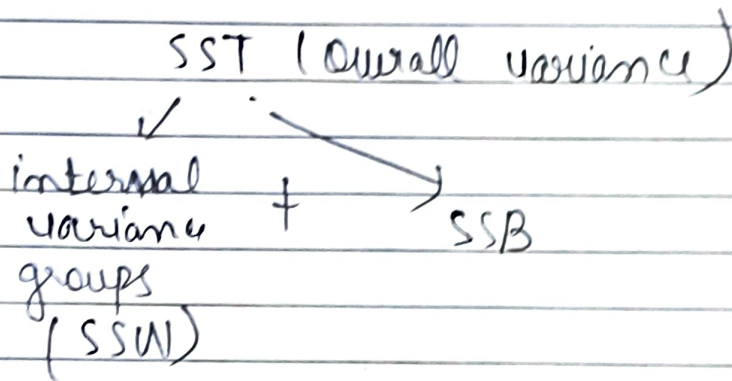
$$\begin{aligned} &k - 1 \\ &= 3 - 1 \\ &= 2 \end{aligned}$$



# TABLE

quantity	values	df
SSB	24	2
SSW	52	6
SST	76	8

Trick to verify  $\Rightarrow$   $SST = SSW + SSB$   
 $= 52 + 24$   
 $= 76$



④ Calculate F-ratio (Test Statistic)

$$\frac{MS \text{ between}}{MS \text{ within}}$$

where  $MS \text{ between} = \frac{SSB}{\text{its df}}$

2

$$MS \text{ within} = \frac{SSW}{\text{its df}}$$

$$F \text{ Ratio} = \frac{\frac{SSB}{df_{SSB}}}{\frac{SSW}{df_{SSW}}}$$

8 This will follow F-Distribution as you can see behind the scene

2  $\chi^2$  distributions are getting divided by their df. that's how you get F-distribution

$$F = \frac{\frac{24}{2}}{\frac{52}{6}} = 1.4$$

$$F = 1.4$$

5 Calculate P value

Use the python function

$$P\text{-value} = 0.31$$

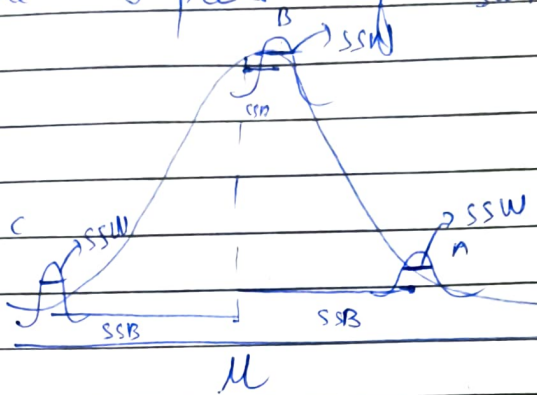
Since P value is more than  $\alpha$  (0.05) Null Hypothesis can't be rejected.

$\therefore$  all three pop means are equal

# GEOMETRIC INTUITION

Our Null hypo is  $\mu_A = \mu_B = \mu_C$

→ indirectly we are saying that these all are part of same population



SSB → distance from grand mean

SSW → distance mean ~~between~~ within

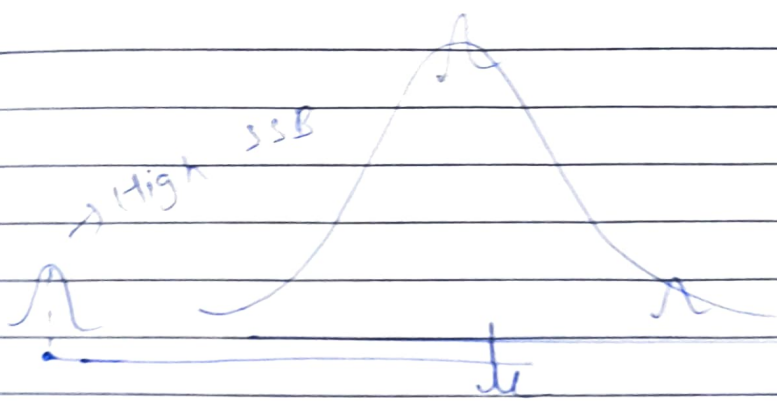
~~if~~ ~~if~~ F-value

Q When will f-stat increase  
 Ans → When SSW ↑ & SSB ↓

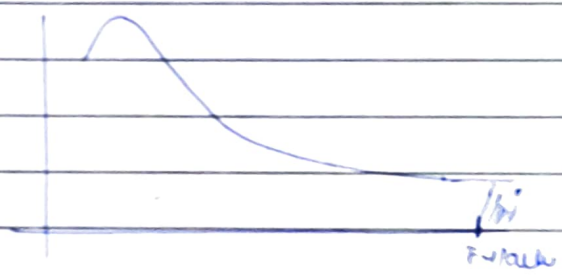
because  $F\text{-stat} = \frac{SSB}{df_{SSB}} \div \frac{SSW}{df_{SSW}}$

& if SSW increase this means distance from population mean is very high  
 or even it may not be a part of population EX -





∴ If high  $F$ -stat, that means  
 in  $F$  dist,  $F$ -value will lie here



Here  $P$ -value is very small, hence we  
 can reject null hypothesis

↳ if  $P$  is means atleast one of  
 the pop mean is not equal

Or we can say, all A, B, C are not  
 the part of same population.

if  ~~$P$ -value~~  $F$  stat very small, the  
 high  $P$ -value

