# some special matrices

## 1) Diognal Matrix

- Diagnol elements are non zero, rest are zeros
- square matrix

Ex - $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}_{2 \times 2}$

### Properties

a) Powers → $A^{100} = \begin{bmatrix} 1^{100} & 0 \\ 0 & 2^{100} \end{bmatrix}$

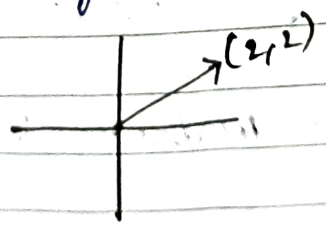
b) Eigen values: Diagnol values are eigen values

$A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}_{2 \times 2}$

c) Multiplication by vector:

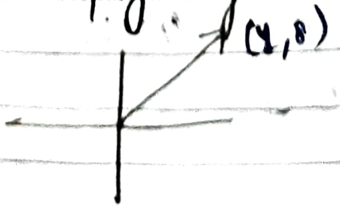
When vector multiplied by diagnol matrix the vector components will get scaled by diagnol values.

Ex




vector (2, 2)

- Multiply by $\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$




vector $(2 \times 2, 4 \times 2)$
$= (4, 8)$

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d) Multiplication by diagonal matrix with diagonal matrix

Ex →
$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \times \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix}$$

$$= \begin{bmatrix} ac & 0 \\ 0 & bd \end{bmatrix}$$

## ② orthogonal matrix -

- square matrix
- If matrix is orthognal, then the vectors will be at 90°. ~~to each other~~
- Ex →
$$\begin{bmatrix} \boxed{\begin{matrix} a \\ c \end{matrix}} & \boxed{\begin{matrix} b \\ d \end{matrix}} \end{bmatrix}$$
  - (a, c) vector 1
  - (b, d) vector 2
  - both vector will be at 90°
  - How to know ?
  - When you dot product, ~~the~~ it will be zero
- $a.b + cd = 0$ (dot product $= 0$)
- both vectors will be unit vectors
  ie $\sqrt{a^2 + c^2} = 1$ & $\sqrt{b^2 + d^2} = 1$
- Example of orthogonal matrix & checking the properties.

$$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \quad \theta = 30.$$

$$= \begin{bmatrix} \sqrt{3}/2 & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$v_1 \quad v_2$

I Property

$\Rightarrow a.b + c.d = 0$

$\frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2} = 0$

II property

both are unit vectors

$\rightarrow \sqrt{a^2+c^2} = 1 \ \& \ \sqrt{b^2+d^2} = 1$

$\rightarrow \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{-1}{2}\right)^2} = 1 \ \& \ \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = 1$

Hence the given matrix is orthogonal

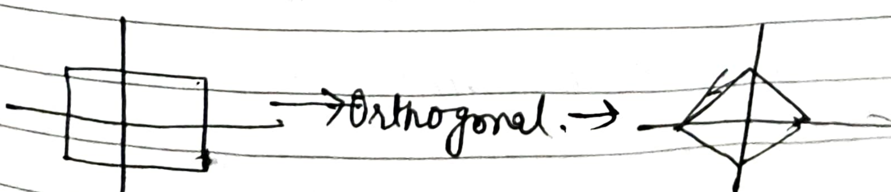
Properties of orthogonal

1) $A^T = A^{-1}$

2) when applied as linear transformation, it gives perfect rotation

Ex when use this orthogonal matrix ie $\begin{bmatrix} \sqrt{3}/2 & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

it will give perfect rotation of $\theta = 30°$



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③ Symmetric matrix

- square matrix whose transpose is same

→ $\boxed{A = A^T}$

Ex → $A = \begin{bmatrix} a & c \\ c & d \end{bmatrix} \rightarrow A^T = \begin{bmatrix} a & c \\ c & d \end{bmatrix}$

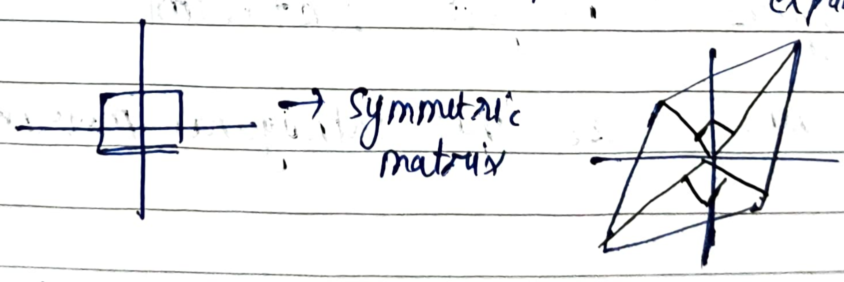
Properties

- 1) Eigen values of symmetric matrix are always real, not complex

2) eigen vectors of symmetric matrix are always orthognal

- Unlike, Diagnol & orthogonal matrix whose transformations are scaling and rotation, the transformation by symmetric is complex (At some side squish expand)

Ex




- It is a mixture of transformations

Ex 2 : Covariance matrix is symmetric matrix

# EIGEN DECOMPOSITION

- Before that, you must understand matrix decomposition
- & Before that, you must know <u>matrix composition</u>.
- We have already studied that :-
  - Ex : we have 2 matrices & multiply

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} k & l \\ m & n \end{bmatrix}$$

$$\text{Left} \longleftarrow \text{Right}$$

- Our result matrix made by first ~~left~~ right matrix multiplied / applied by then ~~right~~ left matrix applied

## * Matrix decomposition

- We have complex matrix, we are decomposing into simpler matrix
- Ex: $D = A.B.C$
- famous ways for matrix decomposition

Ways
- LU
- QR
- Eigen deco
- SVD

(Eigen deco, SVD) • used in Machine lear

## * Eigen Decomposition

- special type of matrix decomposition
- mathematical eqⁿ → A (matrix)
- 2 criterias for A
  1) Square matrix
  2) ~~Diagnosh~~ Diagonalizability

- If these 2 criterias are true, then you can write A matrix as

:→ $A = V \Lambda V^{-1}$

- V is a matrix whose columns are eign vectors of A
- Λ is diagnol matrix whose entries are

eigen values of A

- $V^{-1}$ is inverse of V

EX • $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

- It's eigen values $\lambda_1$ & $\lambda_2$
- and it's eigen vectors are :-

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \; \& \; \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

- $A = V \Lambda V^{-1}$

$$\underset{A}{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} = \underset{V}{\begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \end{bmatrix}} \underset{\Lambda}{\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}} \underset{V^{-1}}{\begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \end{bmatrix}^{-1}}$$

- Basically, you are decomposing A matrix into these 3 matrix, if given criterias are true.

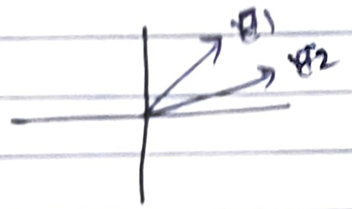
## * What is diagonalize

(n×n)

- Matrix is diagnizible when it should have n linearly independent vectors
- Meaning: let's say we have 2×2 matrix, then it is diagnizible when the 2 eigen vectors have different

direction.

Ex :




diagonizible

Non diagonizible

- You have already studied this

we know eigen vector ⇒ $\boxed{A\vec{v} = \lambda\vec{v}}$

Let's say we have to eigen vectors :-

- Eigen vector 1 → $A\vec{v_1} = \lambda_1\vec{v_1}$
- " " 2 → $A\vec{v_2} = \lambda_2\vec{v_2}$

- Basically, You can write this together in matrix form.

$$\boxed{AV = V\Lambda}$$

Where $V = [\vec{v_1} \;\; \vec{v_2}]$ & $v_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$, $v_2 = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$

$$V \Rightarrow \begin{array}{cc} \begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \end{bmatrix} \\ \quad\vec{v_1} \qquad \vec{v_2} \end{array}$$

$$\Lambda \Rightarrow \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

$\Rightarrow AV = V\Lambda$

Multiplied with $A^{-1}$

$\Rightarrow \boxed{A = V\Lambda V^{-1}}$ → Eigen decomposition

* What if that A is symmetric

- $A \rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ · square ✓ · symmetric ✓

- Eqn still remain true → $A = V\Lambda V^{-1}$
- But it is called spectral decomposition

- Specialities of components of A

- $V \rightarrow$ is matrix of vectors, and vectors are orthogonal
- $\Lambda \rightarrow$ is a diagnal matrix.
- $V^{-1} \rightarrow$ Orthogonal matrix

- That means, the symmetric metrics is decomposed into orthogonal and diagnal matrix

- ~~It means, that's why~~

- which defines the complex transformation of symmetric metrix

$A = V \Lambda V^{-1}$

Rotation ↓ (V), Scaling ↓ ($\Lambda$), Rotation ↓ ($V^{-1}$)

- ~~from left to right~~ ~~left to right~~
- from right to left

## * Advantages of Eigen decomposition

Ex(1) $A_{n \times n}$, (square matrix)

- You are tasked to find $A^{1000}$
- Multiple of matrix with itself is difficult
- Here, we can use eigen decomposition

$$A = V \Lambda V^{-1}$$

$$A^{100} = V\Lambda V^{-1} \times V \Lambda V^{-1} \cdots$$

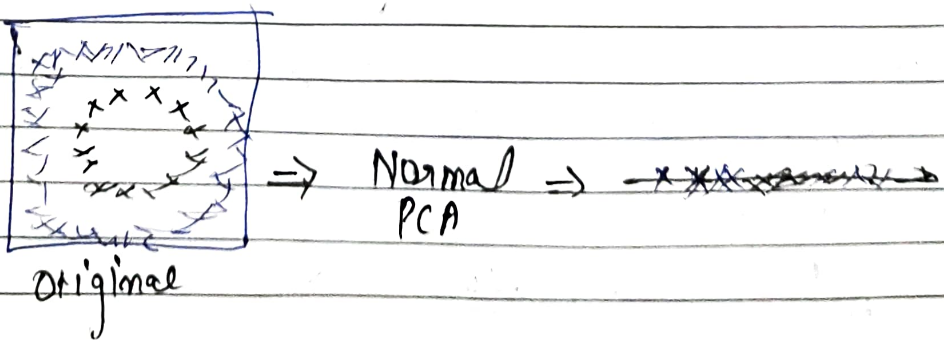
$$= V \Lambda^{100} V^{-1}$$

$= \quad \downarrow$ easy to find

- Conclusion - can be used for mathematical convenience.

## * Kernel PCA

- combination of PCA + SVM
- solves the problem of PCA
- kernel PCA, also works with non linear data, which PCA can't.



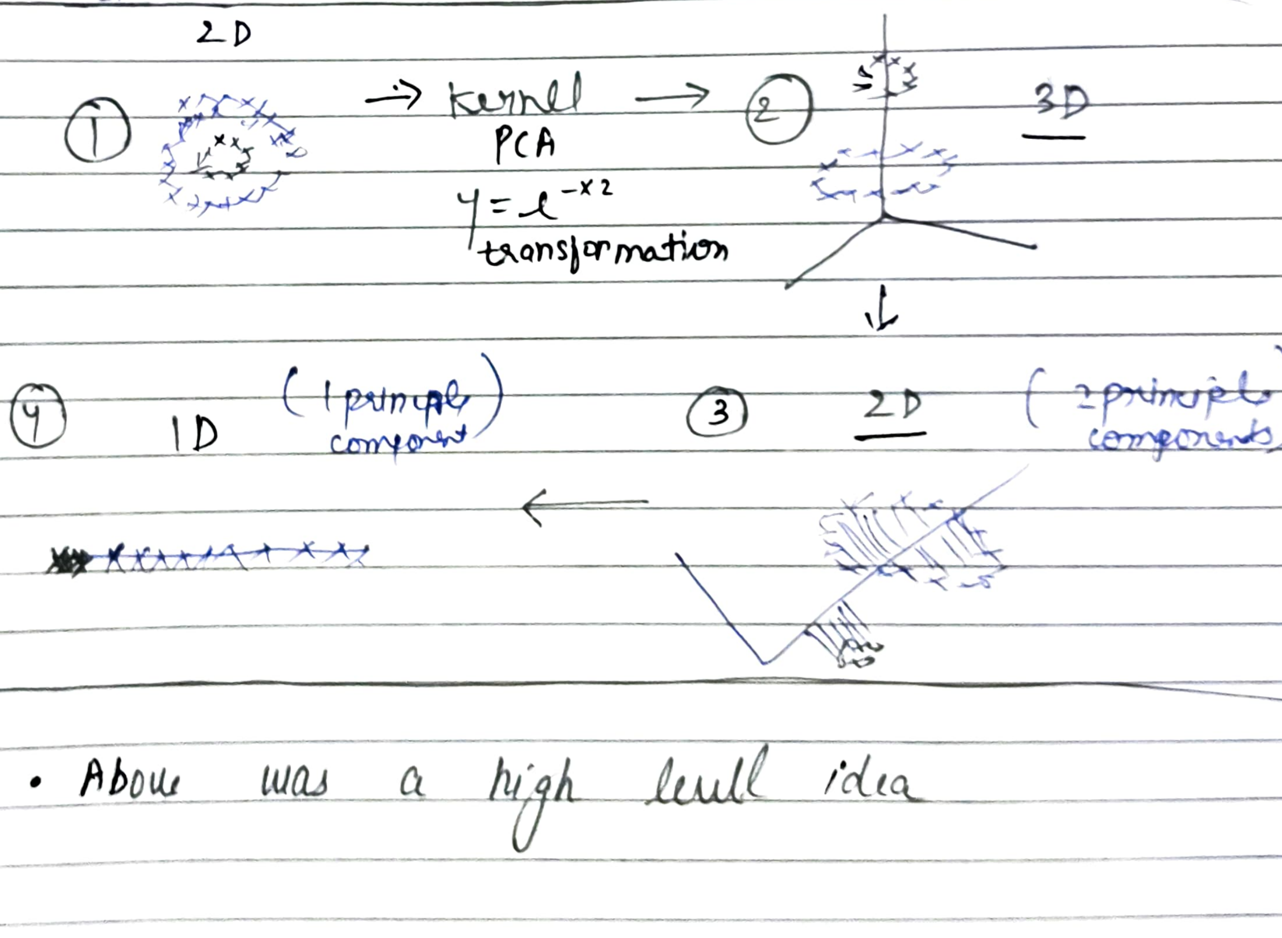
- Kernel PCA uses kernel trick of svm
- We will take 2D data to 3D then take to 1D

- On 2D, you apply mathematical $f^n$ which will take 2D to 3D, ie $y = e^{-x^2}$

AND

- It will take up the centre pts

Ex




- Above was a high level idea