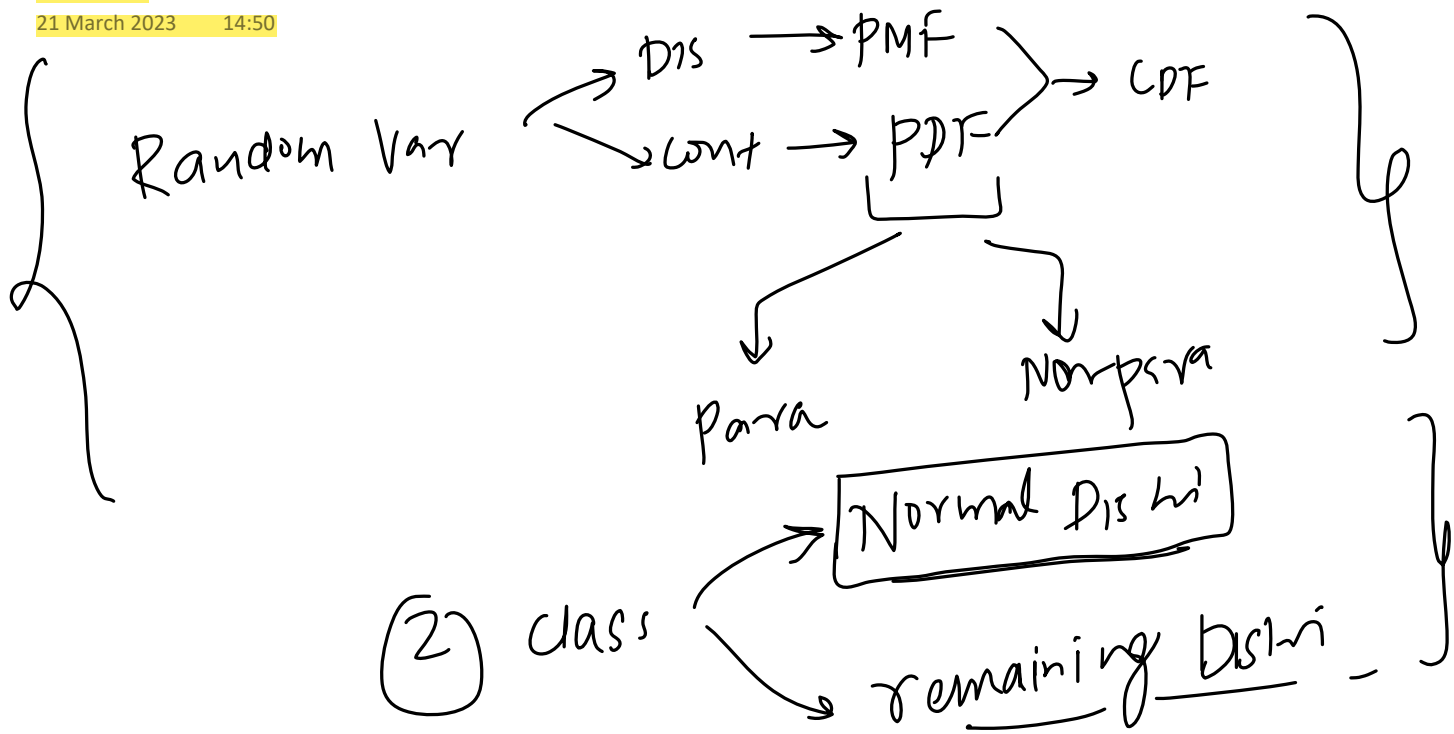


# Recap

21 March 2023 14:50



PDE  $\rightarrow$

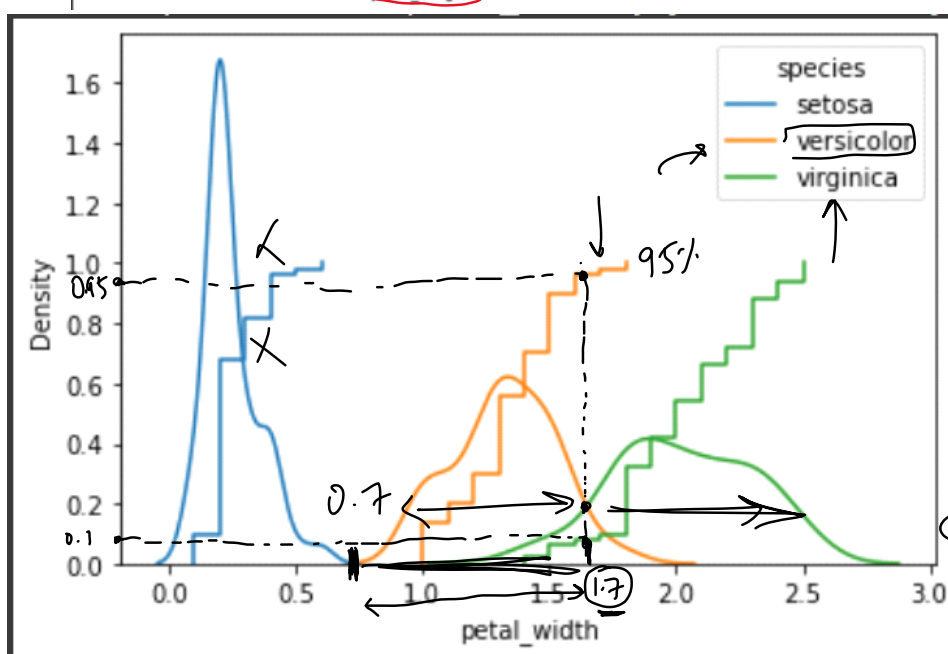
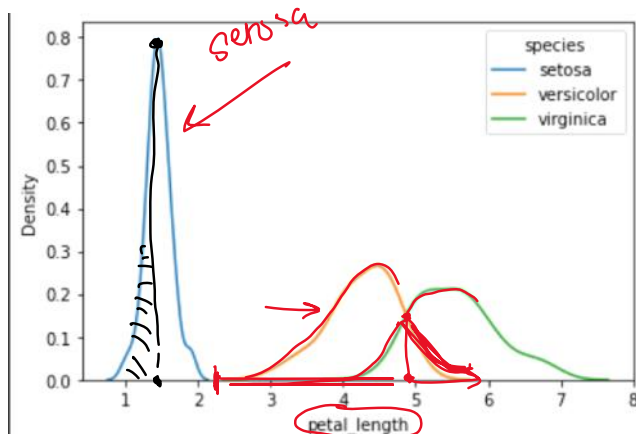
$C \hat{D} \hat{F}$

PDF  $P(X=x)$

$2.3 < pl < 5 \rightarrow \text{version}$

$$p(X \leq x)$$

$$p_1 < 2.3$$



95% of times

if  $0.7 < pw < 1.7$

versicolt

virgin

90%

10% virgin

## Benefit of using PDF

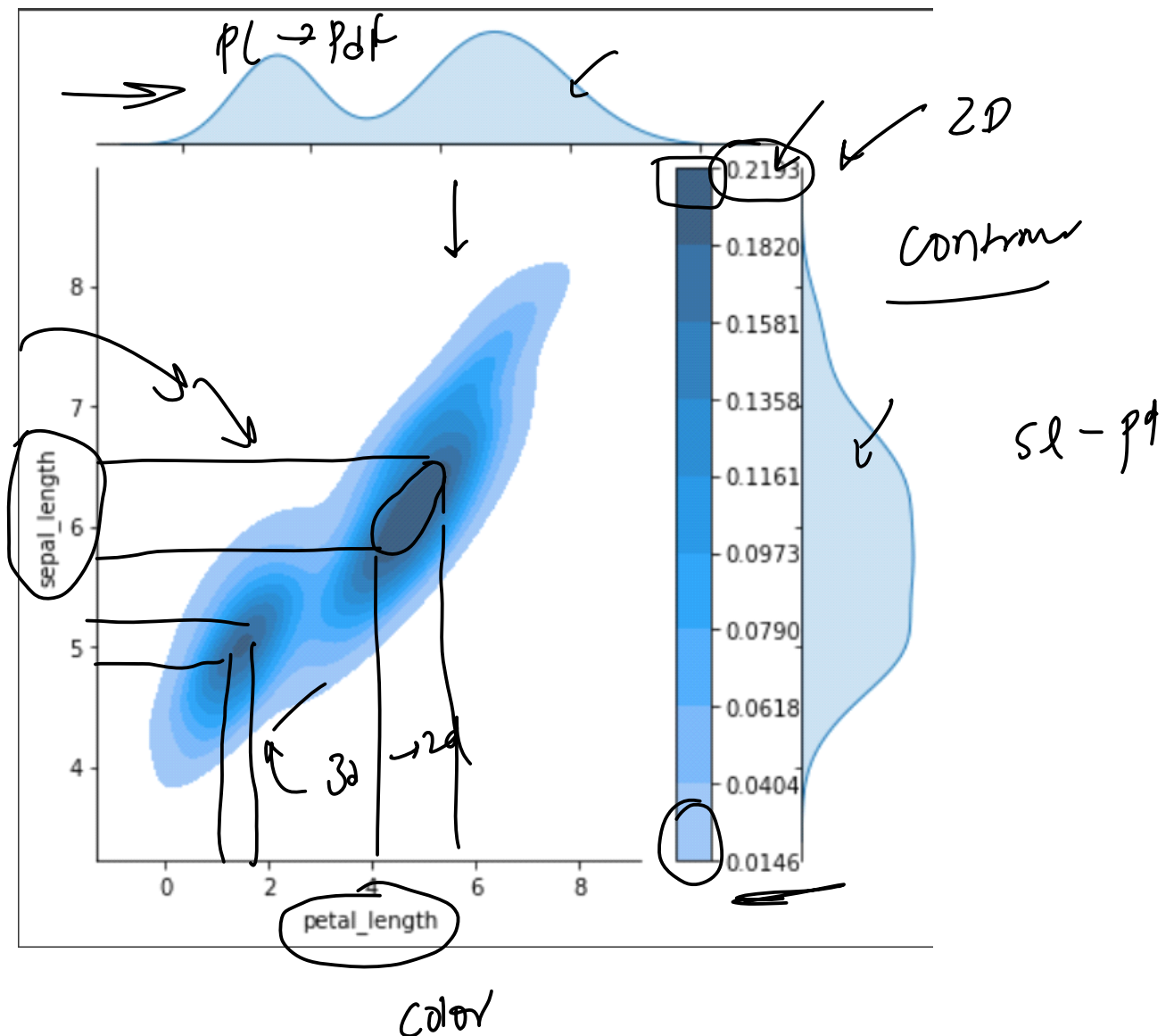
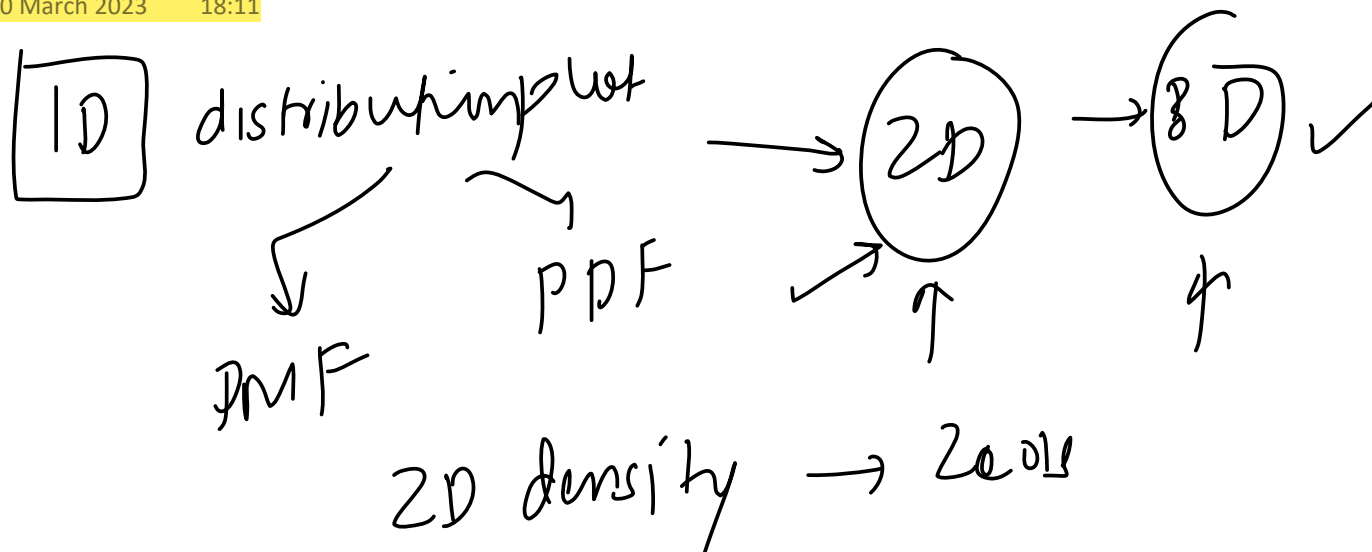
Thats the benefit of using pdf

## Benefit of using CDF

at point 1.7 we can see the cdf is 95 means if there are 100 virginica 95 will have petal width less than 1.7. at same point if we look at cdf of virginica it shows the cdf of 10 which means if there are out of 100 virginica only 10 will have the petal width less than 1.7 and 90 will have more than 1.7. Thats how you were able to quantify the decision making

## 2D Density Plots

20 March 2023 18:11



By these graphs you will know that which combination probability is highest. If we try interpreting we can say that the probability/density of having petal\_length greater than equal 4 and less than 6 and having sepal length greater than 5.8 and less than 6.5 is 21 percent which is the highest.

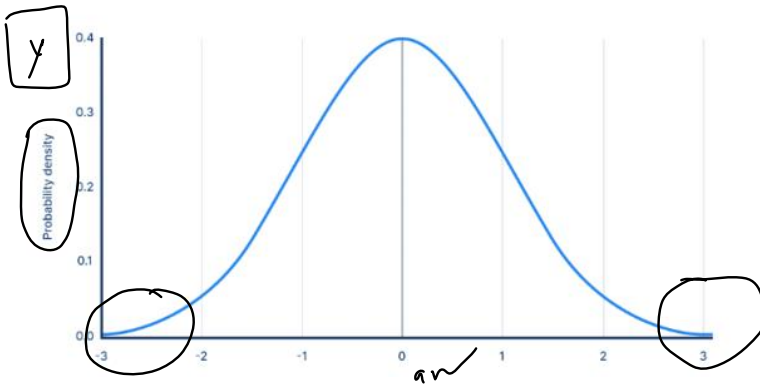
# Normal Distribution

20 March 2023 18:06

## 1. What is normal distribution?

Normal distribution, also known as Gaussian distribution, is a probability distribution that is commonly used in statistical analysis. It is a continuous probability distribution that is symmetrical around the mean, with a bell-shaped curve.

→ pdf



- Tail
- Asymptotic in nature
- Lots of points near the mean and very few far away

The normal distribution is characterized by two parameters: the mean ( $\mu$ ) and the standard deviation ( $\sigma$ ). The mean represents the centre of the distribution, while the standard deviation represents the spread of the distribution.

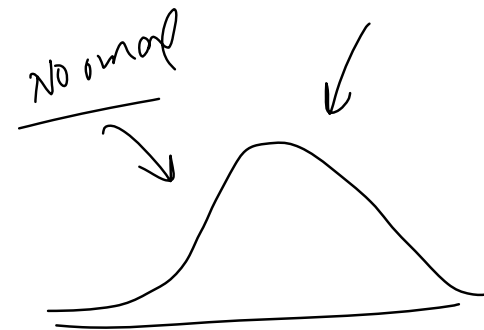
Denoted as:

$$X \sim N(\mu, \sigma)$$

$\mu \rightarrow \text{mean}$   
 $\sigma \rightarrow \text{std}$

### Why is it so important?

**Commonality in Nature:** Many natural phenomena follow a normal distribution, such as the heights of people, the weights of objects, the IQ scores of a population, and many more. Thus, the normal distribution provides a convenient way to model and analyse such data.



## PDF Equation of Normal Distribution

$$y = f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Handwritten annotations: A box around  $\mu, \sigma$  in red. An arrow points from 'data' to 'Normal' to a box containing  $\mu, \sigma$ . Another arrow points from 'Normal' to the same box. A label  $\sigma\sqrt{2\pi}$  is written next to the denominator.

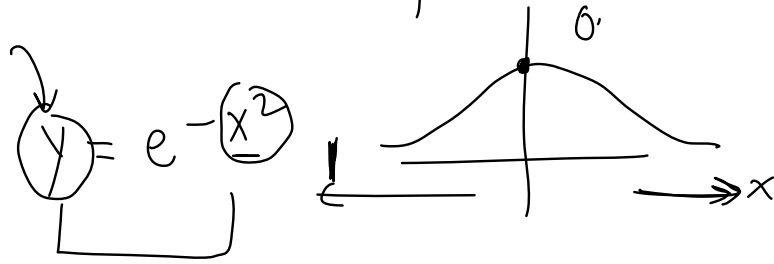
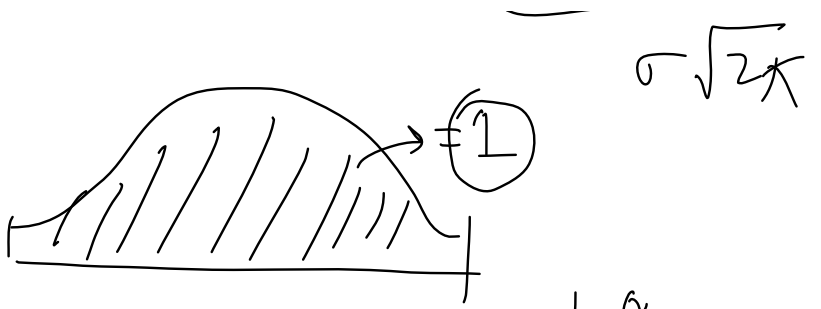
## Parameters in Normal Distribution

<https://samp-suman-normal-dist-visualize-app-lkntug.streamlit.app/>

### Equation in detail:

$$y = \frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}$$

To check impact of mean and std on normal distribution pdf check out the link on left



$$y = \frac{1}{e^{x^2}}$$

# Standard Normal Variate (Z) → Standard Normal distribution

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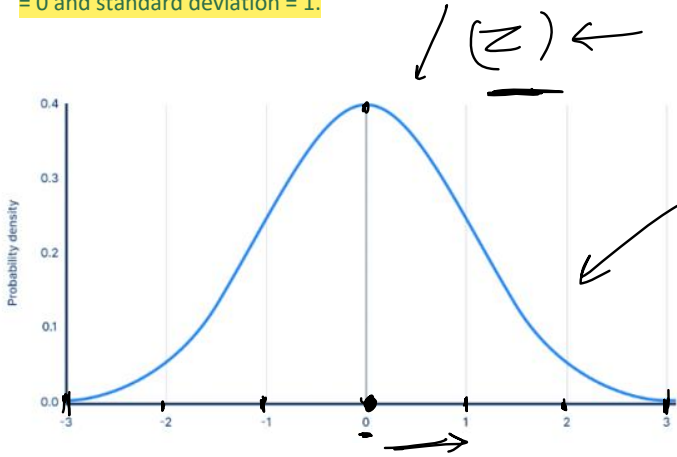
$$X \sim N(\mu, \sigma) \quad \mu=0 \quad \sigma=1$$

$$\downarrow$$

$$Z \sim N(0, 1)$$

## What is Standard Normal Variate

A Standard Normal Variate (Z) is a standardized form of the normal distribution with mean = 0 and standard deviation = 1.



Z is the special case of normal distribution where mean is 0 and std is 1

Importance of Standardization

Standardizing a normal distribution allows us to compare different distributions with each other, and to calculate probabilities using standardized tables or software.

Equation:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

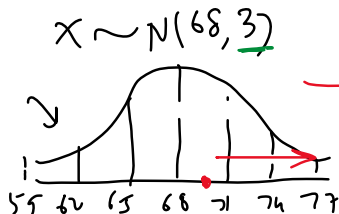
$X \sim N(5, 2.5)$

## How to transform a normal distribution to Standard Normal Variate

Refer Python code

Kya Fayda Standardize karne ka?

Suppose the heights of adult males in a certain population follow a normal distribution with a mean of 68 inches and a standard deviation of 3 inches. What is the probability that a randomly selected adult male from this population is taller than 72 inches?



## What are Z-tables

A z-table tells you the area underneath a normal distribution curve, to the left of the z-score

<https://www.ztable.net/>

For a Normal Distribution  $X \sim (\mu, \sigma)$  what percent of population lie between mean and 1 standard deviation, 2 std and 3 std?

$$X \sim N(\mu, \sigma) \quad \mu \rightarrow \quad \sigma \rightarrow$$

$$Z = \frac{X - \mu}{\sigma}$$

Z table gives area under the curve till point, means cdf, means probability of less than or equal to for all standardize points

$$X \sim N(\mu, \sigma)$$

$$M \rightarrow \checkmark$$

$$Z = \frac{X - \mu}{\sigma}$$

$$Z = \frac{\mu - \mu}{\sigma} = 0$$

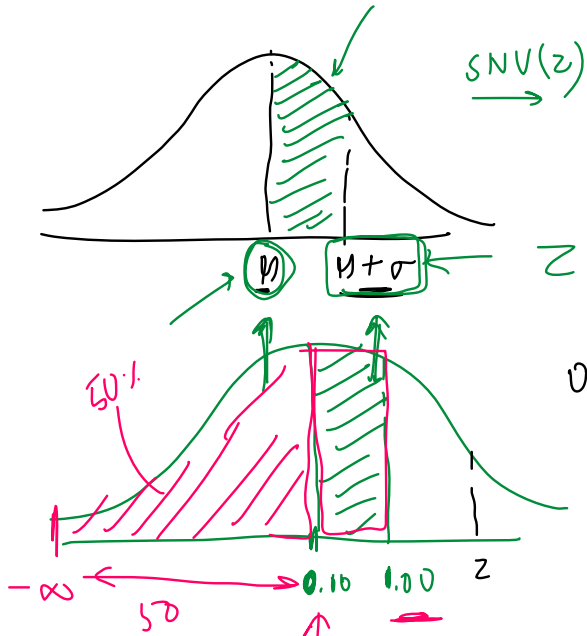
SNV(z)

$$Z = \frac{X - \mu}{\sigma} = \frac{\mu + \sigma - \mu}{\sigma}$$

0.9772  
97.72%

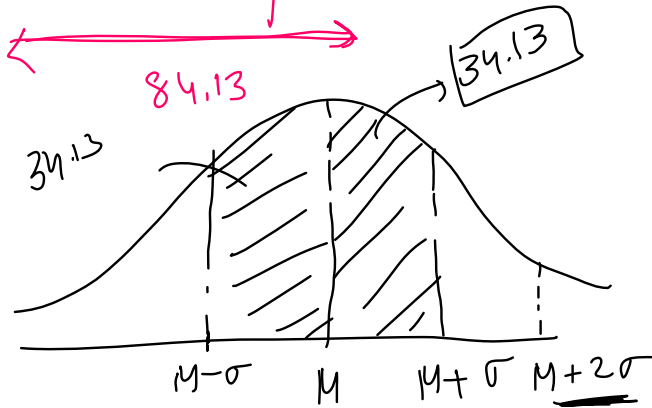
$$= \frac{\sigma}{\sigma} = 1$$

$$84.13 - 50 = 34.13$$



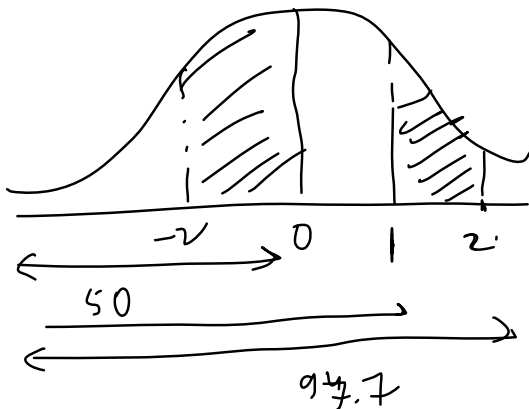
-1σ      +1σ

$$64.28\%$$



$$Z = \frac{\mu + 2\sigma - \mu}{\sigma} = 2$$

69.

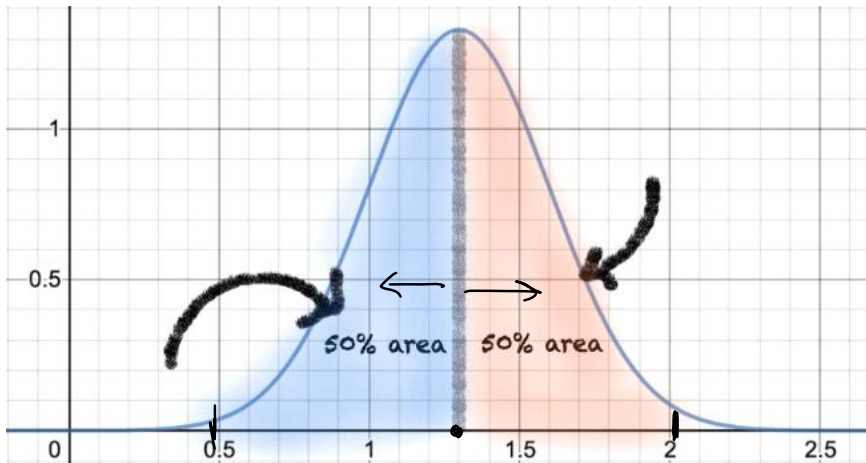


# Properties of Normal Distribution

20 March 2023 18:06

## 1. Symmetry

The normal distribution is symmetric about its mean which means that the probability of observing a value above the mean is the same as the probability of observing a value below the mean. The bell-shaped curve of the normal distribution reflects this symmetry.



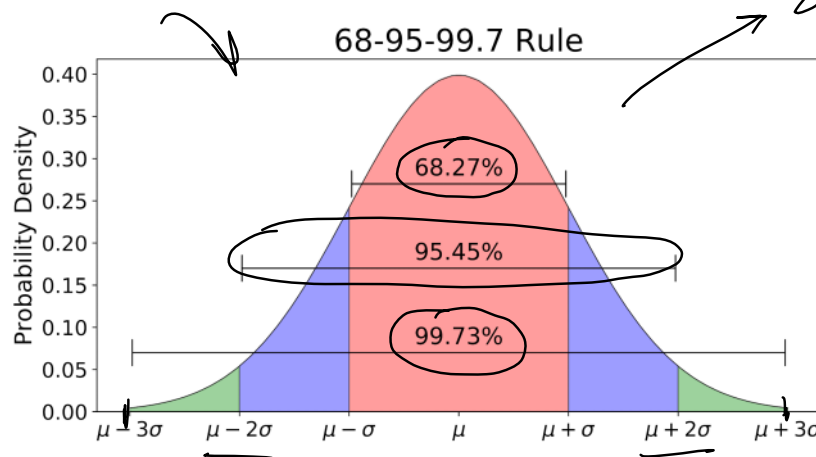
2. Measures of Central Tendencies are equal  $\rightarrow$  mean  $\rightarrow$  median  $\rightarrow$  mode

## 3. Empirical Rule

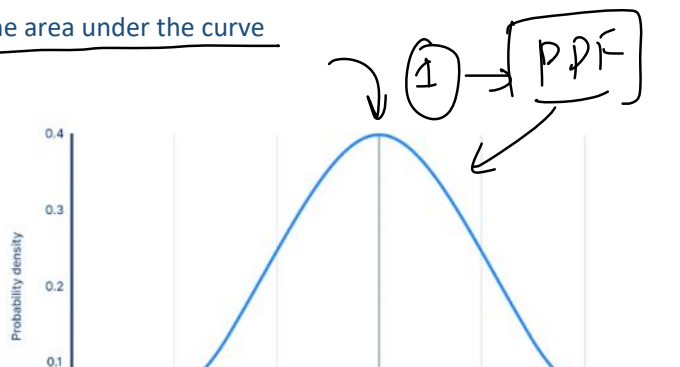
The normal distribution has a well-known empirical rule, also called the 68-95-99.7 rule, which states that approximately 68% of the data falls within one standard deviation of the mean, about 95% of the data falls within two standard deviations of the mean, and about 99.7% of the data falls within three standard deviations of the mean.

Standard deviation

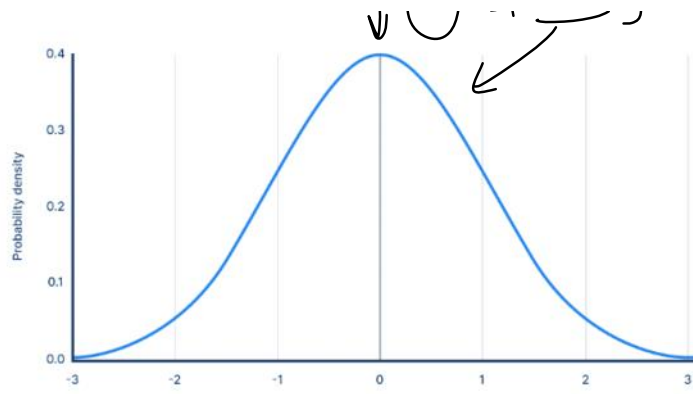
Z-table



## 4. The area under the curve





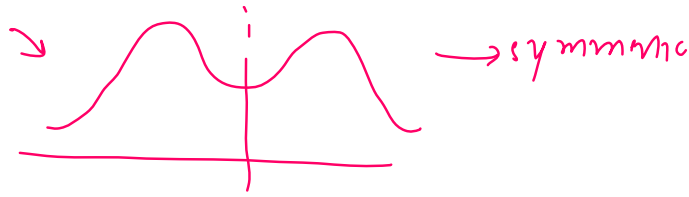


# Skewness

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## What is skewness?

A normal distribution is a bell-shaped, symmetrical distribution with a specific mathematical formula that describes how the data is spread out. Skewness indicates that the data is not symmetrical, which means it is not normally distributed.

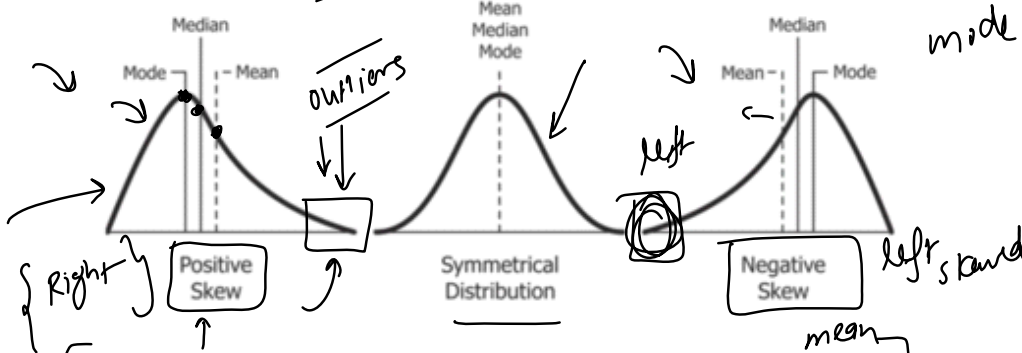


Skewness is a measure of the asymmetry of a probability distribution. It is a statistical measure that describes the degree to which a dataset deviates from the normal distribution.

In a symmetrical distribution, the mean, median, and mode are all equal. In contrast, in a skewed distribution, the mean, median, and mode are not equal, and the distribution tends to have a longer tail on one side than the other.

Skewness can be positive, negative, or zero. A positive skewness means that the tail of the distribution is longer on the right side, while a negative skewness means that the tail is longer on the left side. A zero skewness indicates a perfectly symmetrical distribution.

mode < median < mean  
tail event



moment  
mode > median > mean  
2nd moment - variance  
3rd moment - skewness  
4th - kurtosis

The greater the skew the greater the distance between mode, median and mean

## How skewness is calculated?

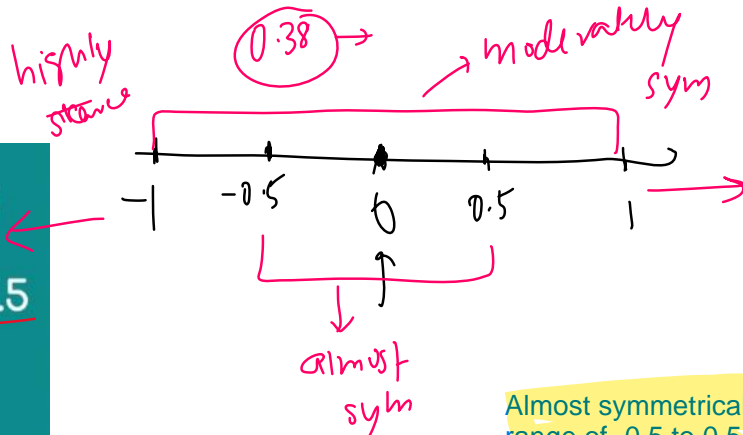
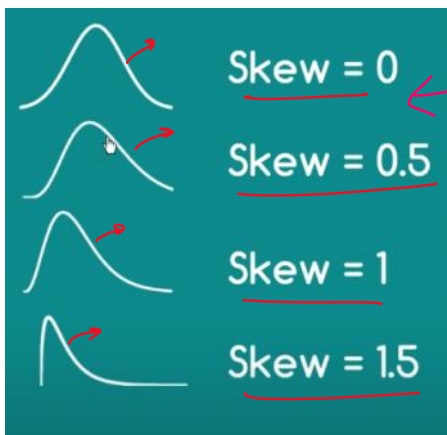
$$\frac{n}{(n-1)(n-2)} \sum \left( \frac{(x - \bar{x})}{s} \right)^3$$

moment  
sample skew

Formula for statistical moment (3rd)  
note that it is the formula for sample skew

## Python Example

## Interpretation

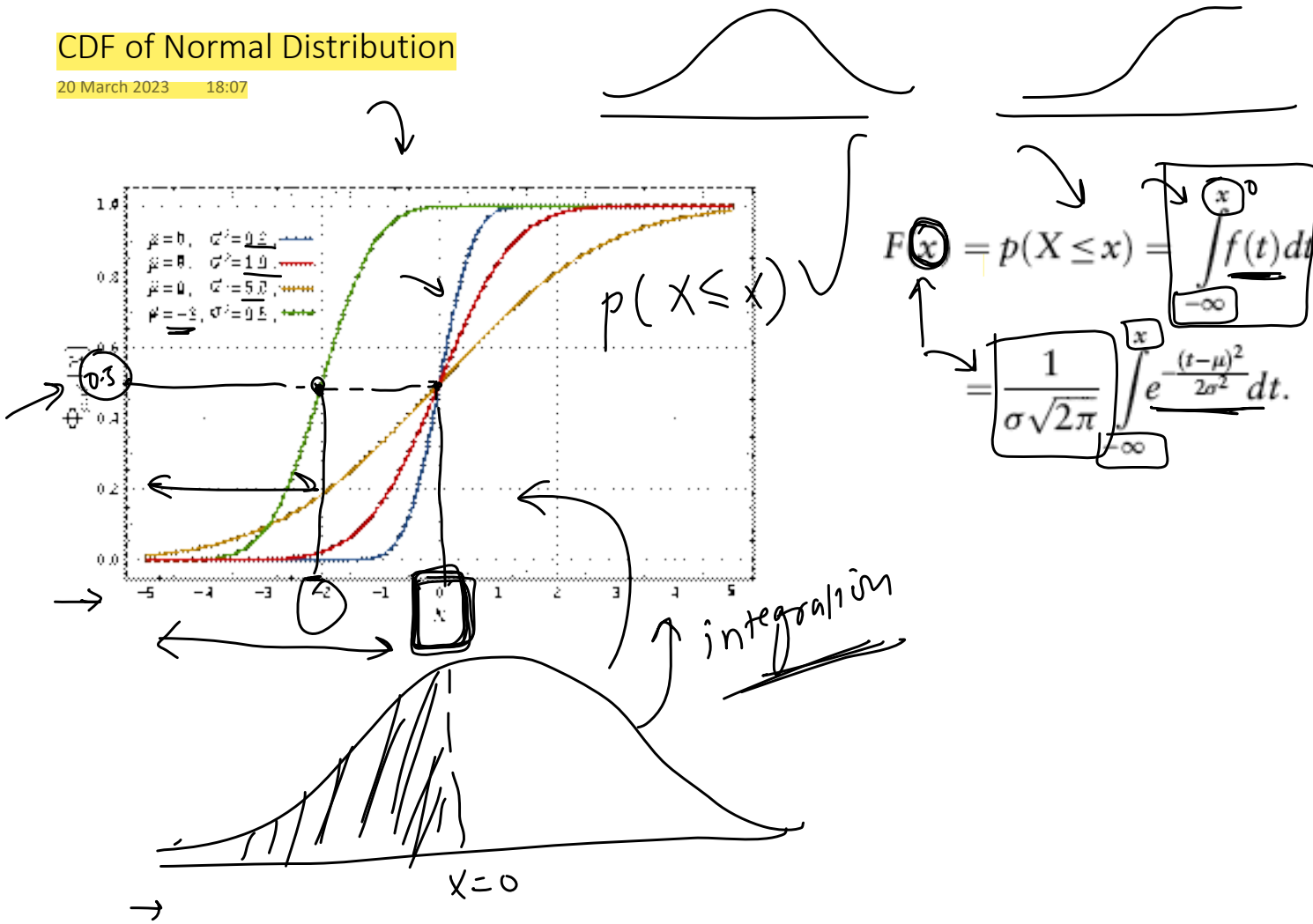


Almost symmetrical if in the skewness is in range of -0.5 to 0.5

Having skew between this range doesn't mean its normal it simply means its symmetrical. If the distribution looks like normal and have a skewness of 0.5 then you can assume that is normal distribution



# CDF of Normal Distribution

20 March 2023 18:07



# Use in Data Science

20 March 2023 18:08

- 
- 
- Outlier detection
  - Assumptions on data for ML algorithms -> Linear Regression and GMM
  - Hypothesis Testing
  - Central Limit Theorem