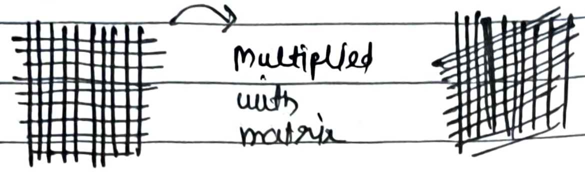
① Matrices

→ are linear transformation, which will transform the coordinate space

Ex



→ Transformation maybe squish, expand, rotate or even mixture of both.

Note: coordinate is made up of infinite vectors / pts.

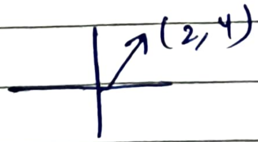
② Vectors multiplied with Matrix

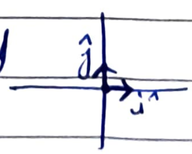
matrices are

→ Linear transformation, you can also say it is function

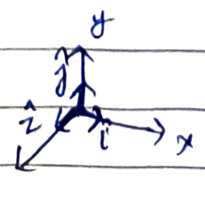
→ input vector in $f(x)$, you get another vector as output

Ⓔ Vectors with respect to unit vectors

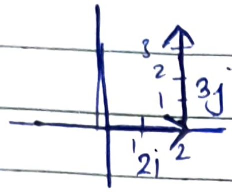
- 

- unit vectors in 2d 

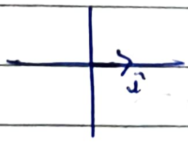
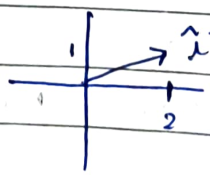
- magnitude of unit vectors is 1

- 3D 

- Vector (2,3) can be represent as $2\hat{i}+3\hat{j}$ ie 2 unit movement in x axis and 3 unit movement in y-axis

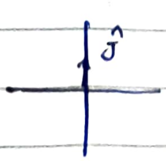
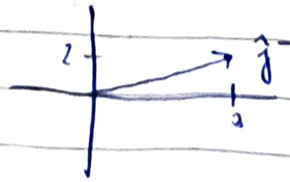
- 

Geometric intuition of vectors multiplied with matrix

- $\vec{x} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \rightarrow \vec{y}$
- The matrices no's have special meaning
- $\begin{bmatrix} \boxed{\begin{matrix} 2 \\ 1 \end{matrix}} & \begin{matrix} 3 \\ 2 \end{matrix} \end{bmatrix}$
- The (2, 1) numbers tells that your unit vector ($\hat{i}$) will reach at 2, 1
- Ex  î cap will reach (2, 1) 

- $\begin{bmatrix} \begin{matrix} 2 \\ 1 \end{matrix} & \boxed{\begin{matrix} 3 \\ 2 \end{matrix}} \end{bmatrix}$

- The set of (3,2) no's tell your $\hat{j}$ will reach at (3, 2)

Ex  ⇒ 

- These transformation is not only applied to unit vector, but all vectors (infinite vectors in coordinate)

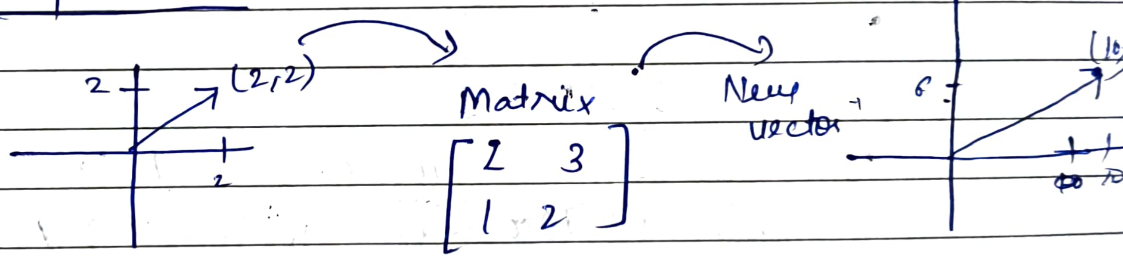
- Ex → $\vec{X}$ X matrix

- $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ = New vector (transformed)

  (Matrix) (2X2)   Vector (2X1)

- $= \begin{bmatrix} (2\times 2) + (2\times 3) \\ (2\times 1) + (2\times 2) \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \end{bmatrix}$ ⇒ New vector

- Geometric



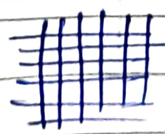
③ Matrix Multiplication

- $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ = A.B → $\begin{bmatrix} - & - \\ - & - \end{bmatrix}$

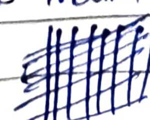
  A   B

Visual

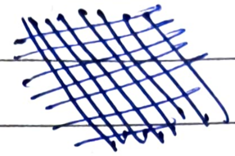
- We have coordinate space -



- A.B ⇒ first apply B matrix (linear tran) on coordinates →



- we got new coordinate space, then on that new apply A matrix, you will get new coordinate space called as A.B $\Rightarrow$



- Note: $A.B \neq B.A$

(4) Determinants

$$A = \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} \Rightarrow \begin{array}{l} 5 \times 2 - 3 \times 4 \\ = 10 - 12 = -2 \end{array}$$

- Determinant is a scalar value, calculated from square matrix
- Carries imp info about matrix
- and info is volume scaling factor for linear transformations

Example visually

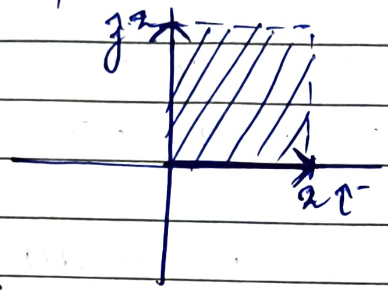
- we have coordinate space, where we have unit vectors & calculating area

- area is side² as it is square and of 1 area

• Apply transformation on coordinate space
ie $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

• Our whole coordinate system will get transformed including unit vectors

• Impact on unit vectors :-



• New area $\Rightarrow 2 \times 2 = 4$. (area')

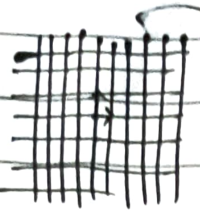
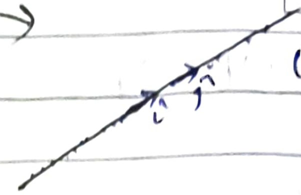
• Now $\boxed{Det(A) = \frac{\text{area}'}{\text{original area}}} = \frac{4}{1} = 4$

• Conclusion = Determinants tell how much area is squished or expand

Visually when determinant is zero

• $A = \begin{vmatrix} 2 & 4 \\ 2 & 4 \end{vmatrix} \Rightarrow 8 - 8 = 0$

• Our whole 2D coordinate system will come at one line (1D)

-  $\rightarrow \begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix} \rightarrow$  (Our whole coordinate system)

area = 0

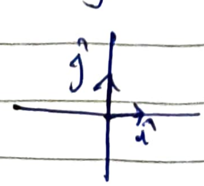
- Determinant = $\dfrac{\text{area}'}{\text{original area}}$
- Since, all pts are in one line area' = 0
- Det $\rightarrow \dfrac{0}{\text{original}} = 0$
- If you are in 2D, it becomes 1D
  " " " 3D, " " 2D

Q Why determinant is only possible for square matrix?

- When used non square matrix, the dimensions get changed, and for determinant (area scaling factor) is only meaningful when input and output spaces have same dimension

Q → What is meant by -ve determinant?

Ex - $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}$ $10 - 12 = -2$

- $\hat{i}$ & $\hat{j}$ relative positions get interchanged
-  $\rightarrow \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \Rightarrow$ 

5) **Inverse**

- Can't be visualized
- $A . A^{-1} = I$

- Ex: $AX = B$ → we have, find x matrix

  - $X = B/A$, → Division not possible in matrix

  - ∴ you multiply both sides with A^{-1}

  - $A^{-1} . AX = \cancel{B} A^{-1} . B$
  - $\boxed{IX = \cancel{B} A^{-1} B}$ $\quad (\because A . A^{-1} = I)$
  - $\boxed{X = \cancel{B} A^{-1} B}$ $\quad (\because XI = X)$

Ex: Solve this using matrix (find x & y)

- $x + 2y = 5$
  $3x + 5y = 6$

- Represent in matrix

- $$\underset{A}{\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{B}{\begin{bmatrix} 5 \\ 6 \end{bmatrix}} \quad \text{——— (1)}$$

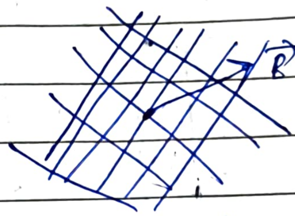
- $AX = B$
  $A^{-1} AX = A^{-1} B$
- $\boxed{X = A^{-1} B}$

Geometric Intuition of (1)

- $\underbrace{\begin{bmatrix}1 & 2\\ 3 & 5\end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix}x\\ y\end{bmatrix}}_{X} = \underbrace{\begin{bmatrix}5\\ 6\end{bmatrix}}_{B}$

- There is some vector X on which you applied matrix A, and you got another vector B

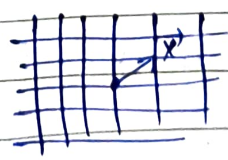
- Let's say the matrix transformation is →



- and in this you got ~~vector~~ $\vec{B}$ which is changed from $\vec{x}$ (unknown)

- To find $\vec{x}$ we have to trackback/revert the transformation, and the vector you will get will be $\vec{x}$

- Revert to original:



(1,1) ⇒ original vector (1,1)

$x=1,\ y=1.$

6) Transformation for non square matrix

- All the previous transformations were respect to square matrix

- What's with non square matrix ?
- Non square matrix is also the transformation, but it changes the dimensions of coordinate system

Ex1:
- Non square matrix
- $A = \begin{bmatrix} \boxed{1} & 2 \\ \boxed{3} & 4 \\ \boxed{5} & 6 \end{bmatrix} \Rightarrow$ Tall matrix (rows > col)

- We know set of (1, 3, 5) indicates unit vector ($\hat{i}$), moves in 1 unit right, 3 units in y direction and 5 in z direction.

- But, we have to 2d dat coordinate so, it will convert 2D to 3D

Ex2
- Wide matrix (col > rows)
- $\begin{bmatrix} \boxed{1} & 2 & 3 \\ \boxed{4} & 5 & 6 \end{bmatrix}_{2 \times 3}$

- We have 3d coordinate (3 col)
- We know (1, 4) set indicates $\hat{i}$ move in first 1 unit x direction and 4 units in y direction

- But didn't move in 2 direction
- i.e 3D space is converted into 2D
- But in square matrix, you remain in same direction.

7) Inverse only possible for square but not for non square matrix

- Reason is same, due to dimension change in non square matrix transformation
- refer to pdf.

8) Why inverse only possible for non singular

for matrix to have inverse, it must follow to constraint

1) Square matrix
2) Non singular (determinant should not be zero)

- When determinant gets zero, you move from 3D to 2D and you are working as non square matrix and we know inverse of non square matrix is not possible