CONFIDENCE INTERVAL

Z score (for 95%)

$$\boxed{CI = \text{point estimate} \pm Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}}$$

n = sample size
σ : population std.
$1-\alpha$ = 95% confidence interval
$1-\alpha = 95\%$

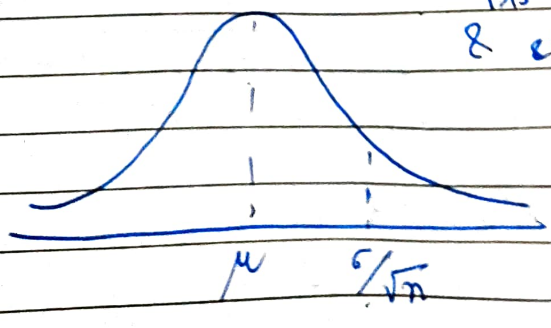
point estimate = $\bar{x}$ (sample mean if population is normal)

point estimate = mean of sampling distribution of mean
(CLT is applied as pop was not normal)

What is Z? & Intution of formula.

① Intution

→ Calculate point estimate
→ Use CLT, you will get sampling distrib which will be normal
its mean = pop mean
& std = pop std/$\sqrt{n}$

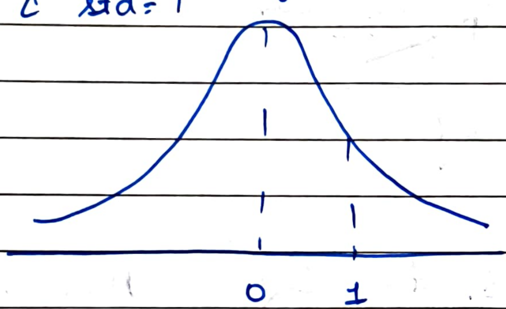


→ What if we convert into standard normal variate.

⇒ $Z = \bar{x} - \mu$ $\quad \dfrac{\text{value} - \text{mean}}{\text{std}}$.

$$\boxed{Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}}$$

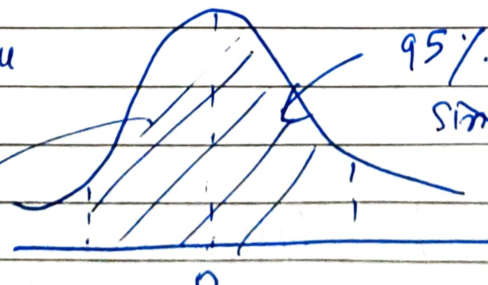
→ Now the sampling distribution mean will be 0 & std = 1



→ Previously we wanted range of $\bar{x}$ values, since it is converted into standard normal variate. Now we want range of z values, where we will be 95% sure that z will fall in that range.

Which means we have to create a area with area of $1 - \alpha = 95\% = 1 - \alpha$.

~~basically~~ we



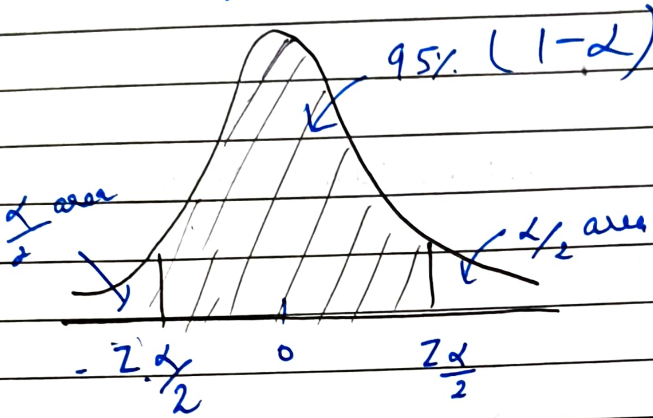
$95\% = 1 - \alpha$

~~Since this area covers 95% of area.~~

rest of ~~area~~ is ⇒ 8 1

If this area is $1 - \alpha$, then the rest of the area left is α

& since this a standard normal distribution which means it is symmetrical
$\therefore$ $\frac{\alpha}{2}$ is left at left and $\frac{\alpha}{2}$ is left at right.



Between these $\left(-Z_{\frac{\alpha}{2}} \text{ to } Z_{\frac{\alpha}{2}}\right)$ there is a probability of $(1-\alpha)$ (95%) the Z will fall in this range.

Since $1-\alpha = 95\%$
$\alpha = 5\% \Rightarrow \frac{\alpha}{2} = 2.5$

B/w $-Z_{2.5} \longleftrightarrow Z_{2.5}$

b/w this interval Z lie probability is 95%.

Conclusion.

P(Z) Probability of Z ~~belongs~~ lying b/w $-Z_{\frac{\alpha}{2}}$ & $Z_{\frac{\alpha}{2}}$ is 95% $(1-\alpha)$.

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$$P\left(-Z_{\alpha/2} < Z < Z_{\alpha/2}\right) = 1-\alpha \quad (1)$$

$\left(Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}\right)$ put in (1)

$$P\left(-Z_{\frac{\alpha}{2}} < \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < Z_{\alpha/2}\right) = 1-\alpha.$$

$$P\left(\left(-Z_{\frac{\alpha}{2}}\,\sigma/\sqrt{n}\right) < \bar{x}-\mu < \left(Z_{\alpha/2} \times \sigma/\sqrt{n}\right)\right) = 1-\alpha.$$

$$P\left(-\bar{x} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < -\mu < -\bar{x} + Z_{\frac{\alpha}{2}}\frac{\sigma}{n}\right) = 1-\alpha$$

$$\boxed{P\left(\bar{x} - Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right) = 1-\alpha}$$

Interpretation → The probability of μ between the range of these 2 is $1-\alpha$ (95%)

Now this not exactly probability

We know that our population mean is fixed, then why this range.

This range is due to $\bar{x}$ (sample mean)

maybe next time samples would be different, hence it is not probability it is confidence.

Right Interpretation

When we are infering population mean then the confidence of pop mean (μ) fall into this range is 95%.

Hence proved

$$CI = \mu = \bar{X} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

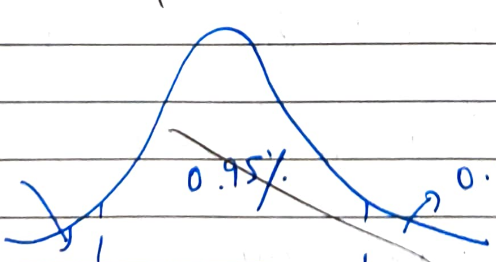
Given confidence interval = 95%

$$1-\alpha = 95\%$$
$$\alpha \Rightarrow 1-\alpha = 0.95$$
$$\alpha = 0.05$$

$$CI \Rightarrow \mu = \bar{X} \pm Z_{\frac{0.05}{2}} \frac{\sigma}{\sqrt{n}}$$

Intuition is done

② How to find Z.



0.025 percent area

Now you have to find Z value for this area.

~~find the Z value~~ where area = 0.025.

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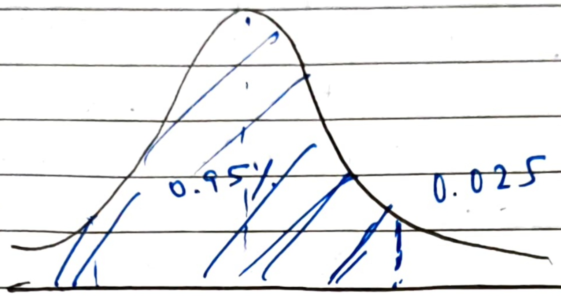


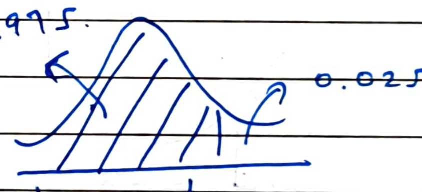
→ ~~If we want this~~

we know that z table gives area to the left.

If we want this z value then we we need area upto that pt

which will be

$1 - 0.025$
$= 0.975$



now look for z value where area is 0.975

z value for that ~~point~~ area will be 1.96.

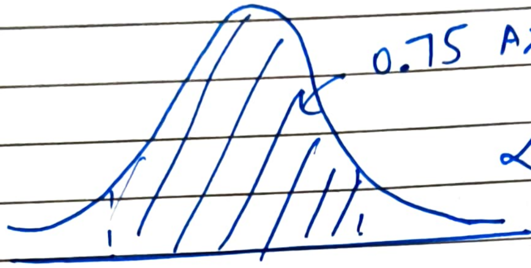
$$\mu = \bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}}$$

Example 2

find the confidence interval with confidence level of 75%.

Sol

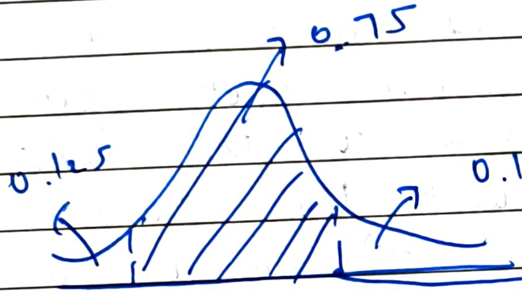
$1 - \alpha = 0.75$
$\alpha = 0.25$



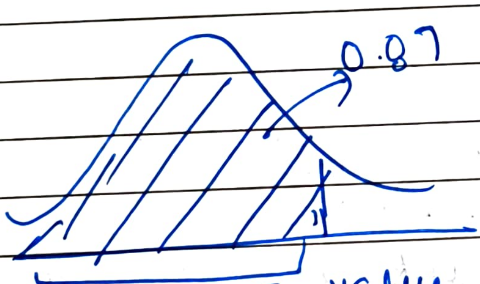
0.75 Area $(1-\alpha)=0.75$ $\quad \alpha = 1-0.75$

Rest of area = $\alpha = 1-0.75 = 0.25$

$\alpha = 0.25$



find z value for this



$1-0.125$
$= 0.87$

z value for area 0.87 will be 1.13.

$$\mu = \bar{x} \pm \left(1.13 \times \frac{\sigma}{\sqrt{n}}\right)$$

T-PROCEDURE

$$CI = \bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

$s \Rightarrow$ sample std

$n =$ sample size.