How to calculate Eigen vectors and Eigen values.

calculate Eigen vector & values of this matrix ⇒ $\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}_{2\times 2}$ is linear transformation

$\boxed{A\vec{x} = \lambda \vec{x}}$ → Definition of eigen vector

A ⇒ matrix
$\vec{x}$ = vector
λ → scalar (eigen value)

$$\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}_{2\times 2}$$

⇒ $IA\vec{x} = \lambda \underline{I}\vec{x}$ where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

⇒ $A\vec{x} - \lambda I \vec{x} = 0$

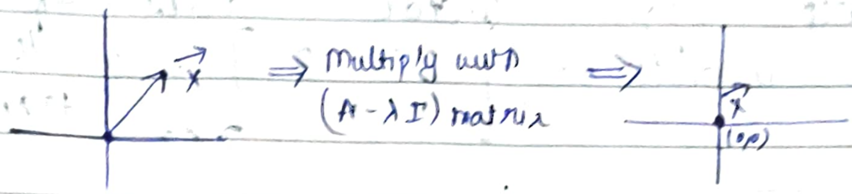
⇒ $\vec{x}(A - \lambda I) = 0$ ← (1)

⇒ Decoding (1)
⇒ A is matrix, λI is also matrix
→ Matrix - Matrix = Matrix
⇒ ∴ $(A - \lambda I)$ is matrix
⇒ & $\vec{x}$ is ~~scalar~~ vector

Decoded → On $\vec{x}$ vector, you are applying $(A - \lambda I)$ matrix transformation

and constraint is when you multiply them you should get zero vector. (0, 0)

- We need such matrix, when multiplied with $\vec{x}$ as transformation, that will make $\vec{x}$ to zero vector (0,0)



- And we know for zero constraint, your matrix should be non invertible ie determinant of matrix will be zero

- We want → $\det(A-\lambda I)=0$ —— (2)

- Basically, the whole coordinate will shift from 2D to 1D, and the vector $\vec{x}$ will be zero $\vec{x}$ (0,0)

- Solving eq (2)

- $$\det\left(\begin{bmatrix}2 & 3\\ 1 & 1\end{bmatrix}-\lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right)=0$$

- $\Rightarrow (2-\lambda)\cdot(1-\lambda)+0=0$

  $\Rightarrow$ ~~$\lambda^2-3\lambda+5=0$~~

  $\Rightarrow$ $\lambda=2,\ \lambda=1$ (Eigen values)

- ∴ There will be 2 eigen vectors 1 for $\lambda=2$, and for $\lambda=1$

put λ values in (1)

$$(A - \lambda I)\vec{X} = 0$$

for $\lambda = 2$,

$$\left(\underbrace{\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}}_{A} - \underbrace{\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}}_{\lambda I} \right) \underbrace{\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}}_{\vec{X}} = \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{\text{zero vector}}$$

$$\Rightarrow \underbrace{\begin{bmatrix} 0 & 3 \\ 0 & -1 \end{bmatrix}}_{2\times 2} \underbrace{\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}}_{2\times 1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0X_1 + 3X_2 \\ 0X_1 - X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \boxed{3X_2 = 0}$$

$$\boxed{X_2 = 0}$$

- Any vector whose X_2 is zero is a eigen vector

for $\lambda = 1$;

$$\left(\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$X_1 + 3X_2 = 0$$

$X_1 = -3X_2 \Rightarrow$ Take any value of X_2 you will get eigen vectors who will behave same

~~Ex $x_1 = 1, x_2 = -3$ ⇒ eigen vector $(1, -3)$~~

~~$x_1 = 2, x_2 = -6$ ⇒ " " $(2, -6)$~~

SUMMARY OF PCA with eigen vectors

Let's say we have 2 col & 1 o/p

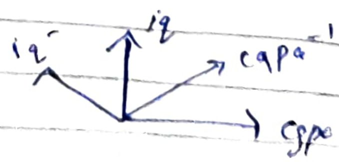
- cgpa iq lpa (data)
- We need to perform feature extraction
- PCA, finds the axis of maximum variance, & who is that axis?

- finding that axis
- If you find covariance matrix of data

$$\begin{bmatrix} cov(cgpa, cgpa) & cov(cgpa, iq) \\ cov(iq, cgpa) & cov(iq, iq) \end{bmatrix}$$

$n \times n$ $(2 \times 2) \rightarrow 2$ cols.

- which is a symmetric matrix $(A^T = A)$
- and for this ~~symmetric~~ cov matrix you will find eigen vectors
- Since, it is a symmetric matrix the eigen vectors will be orthogonal (90)
- Max eigen vectors can be n, min $\rightarrow 0$.
- ∴ We will get 2 eigen vectors that are orthogonal



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YOUVA

- You will select that eigen vectors whose variance should be maximum

- Because after solving the optimisation problem we get to know that the axis whose variance will be maximum will be eigen vector.