



**KA-3784**

**First Year B.C.A. (Sem. I) Examination**

**September/October – 2012**

**102 : Mathematics**

*(Old Course)*

Time : Hours]

[Total Marks :

**Instructions :**

(1)

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| <p>નીચે દર્શાવેલ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.<br/> <b>Fillup strictly the details of signs on your answer book.</b></p> <p>Name of the Examination :</p> <div style="border: 1px solid black; padding: 2px; display: inline-block;"> <b>FIRST YEAR B.C.A. (SEM. I)</b> </div> <p>Name of the Subject :</p> <div style="border: 1px solid black; padding: 2px; display: inline-block;"> <b>102 : Mathematics (Old Course)</b> </div> <p>Subject Code No. : <span style="border: 1px solid black; padding: 0 5px;">3</span> <span style="border: 1px solid black; padding: 0 5px;">7</span> <span style="border: 1px solid black; padding: 0 5px;">8</span> <span style="border: 1px solid black; padding: 0 5px;">4</span> Section No. (1, 2,...): <span style="border: 1px solid black; padding: 0 5px;">Nil</span></p> | <p>Seat No. :</p> <table border="1" style="width: 100%; height: 30px; border-collapse: collapse;"> <tr> <td style="width: 15%;"></td> <td style="width: 15%;"></td> <td style="width: 15%;"></td> <td style="width: 15%;"></td> <td style="width: 15%;"></td> <td style="width: 15%;"></td> </tr> </table> <div style="border: 1px solid black; height: 80px; margin-top: 10px; display: flex; align-items: center; justify-content: center;"> <p>Student's Signature</p> </div> |  |  |  |  |  |  |
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- (2) All questions are **compulsory**.  
 (3) Figures to the **right** indicate **full** marks.

1 Answer the following questions :

- (i) Define proper and improper subsets.
- (ii) If  $A = \{1, 3, 5, 7\}$ ,  $B = \{5, 7, 8, 9\}$  then find  $(A \cup B) - (A \cap B)$ .
- (iii) When do you say that given function is on to ?
- (iv) Find the domain for which the functions  $f(x) = 2x^2 - 1$  and  $g(x) = 1 - 3x$  are equal.
- (v) Define Tautology.
- (vi) What is the difference between a sentence and a statement ?
- (vii) If  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  then find  $A^{-1}$ .
- (viii) Define symmetric matrix.
- (ix) What is the difference between co-factors and minor ?
- (x) Find the equation of a line passing through origine and having slope  $\frac{1}{6}$ .

**2** (a) State and prove De'Morgans law for intersection. **12**

(b) If  $A = \{x | x \in R, x^2 - 3x - 4 = 0\}$  and  $B = \{x | x \in Z, x^2 = x\}$   
then find (i)  $A \cup B$  (ii)  $A \Delta B$ .

(c) If  $A = \{1, 2, 3\}$ ,  $B = \{2, 6, 7\}$ ,  $C = \{2, 7\}$  then prove that  
 $A \times (B - C) = (A \times B) - (A \times C)$ .

**OR**

**2** (a) State and prove distributive law of intersection over union. **12**

(b) If  $A = \{1, 2, 3\}$ ,  $B = \{3, 4\}$  and  $C = \{1, 3, 5\}$  then find

(i)  $A \times (B \cap C)$

(ii)  $A \times (B \cap C)$

(iii)  $(A \times B) \cup (A \times C)$

(c) If  $A = \{1, 2, 3, 4, 5\}$ ,  $B = \{1, 3, 5, 6\}$ ,  $C = \{1, 2, 3\}$  then verify

(i)  $(A - B) \cup B = A \cup B$

(ii)  $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$ .

**3** (a) The variable cost of processing 1 gm of coffee bean is Rs. 1 and fixed cost per day is Rs. 300. Give linear cost function and find the cost of processing 2000 gms of coffee beans in a day. **12**

(b) If  $f(x) = \frac{x^2 - 25}{x - 5}$ ;  $x \in Z - \{5\}$  and  $g(x) = x + 5$ ;  $x \in Z$ . Is

$f(x) = g(x)$  ?

(c) If  $f(x) = \frac{x^2 + 5x - 7}{x^2 + 2}$ ;  $x \in Z$  then find  $\frac{f(1) + f(-2)}{f(-1) + f(0)}$ .

**OR**

**3** (a) Define Domain and Range of the function. **12**

If  $f : A \rightarrow B$   $f(x) = 3x - 2$  and range  $R_f = \{-5, -8, 7\}$  then find Domain.

(b) If  $f(x) = \frac{1}{x} + \frac{3}{x-1}$ ;  $x \in R - \{0, 1\}$  then find

$$f(2), f(3), f\left(\frac{1}{2}\right), f(-2).$$

(c) If  $f(x) = \frac{1}{x}$ ;  $x \in Z - \{-1, 0, 1\}$  then prove that

$$f(x+1) - f(x-1) = \frac{2}{1-x^2}.$$

4 (a) Let  $D_{16}$  be the set of positive divisors of 16. Define  $+, \cdot, \& /$  on  $D_{16}$  as

$$a+b = \text{l.c.m. of } a \& b$$

$$a \cdot b = \text{g.c.d. of } a \& b$$

$a^{-1} = 16/a$  then verify that  $(D_{16}, +, \cdot, /, 1, 16)$  is a boolean Algebra.

(b) Verify using truth table  $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$ .

(c) In a boolean algebra  $B$ . Prove that  $(x+y) \cdot (x'+y') = x' \cdot y'$

**OR**

4 (a) Let  $D_9$  be the set of positive divisors of 9. Define  $+, \cdot, \& /$  on  $D_9$  as

$$a+b = \text{l.c.m. of } a \& b$$

$$a \cdot b = \text{g.c.d. of } a \& b$$

$a^{-1} = \frac{9}{a}$ . Then verify that  $(D_9, +, \cdot, /, 1, 9)$  is a boolean algebra.

(b) Verify using truth table.

$$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r).$$

(c) In a Boolean algebra prove that  $(x+y)' = x' \cdot y'$ .

- 5 (a) Without expanding prove that  $\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix} = 0$ .
- (b) If  $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & -2 & 4 \\ 0 & 3 & 5 \end{bmatrix}$ ,  $B = \begin{bmatrix} 0 & 2 & 5 \\ 7 & -1 & 1 \\ 4 & 3 & 4 \end{bmatrix}$  and  $3A + C = 4B$  then find  $C$ .
- (c) If  $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$  then prove that  $A^2 - 4A - 5I = 0$ .

**OR**

- 5 (a) Find the value of  $\begin{vmatrix} x+y & z & 1 \\ y+z & x & 1 \\ z+x & y & 1 \end{vmatrix}$ .
- (b) If  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$  then find  $B$  such that  $A + 2B = A^2$ .
- (c) Find the matrix  $A$  such that  $2A - 3B + 5C = 0$  where  $B = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$  and  $C = \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$ .
- 6 (a) Prove that  $(2, -1), (3, 4), (-2, 3)$  &  $(-3, -2)$  are the vertices of a rhombus.
- (b) Find the equation of a line passing through  $(1, 3)$  and  $(3, 4)$  and determine its intercept with co-ordinate axes.
- (c) Find the area of the triangle having the vertices  $A(-5, -5), B(-3, -5)$  &  $C(10, 5)$ .

**OR**

- 6 (a) Find the equation of a line passing through  $(3, 5)$  making equal intercept on axes also find slope.
- (b) Find the area of the quadrilateral generated by the points  $(1, 1), (3, 4), (5, -2)$  &  $(4, 7)$ .
- (c) Find the equation of a line whose X-intercept is 3 and Y-intercept is 5. Also find its slope.