Illustration 1 : If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 9, 11\}$, $C = \{2, 11, 18, 22$ find : $A \cap B$, $B \cap C$, $C \cap A$, $A \cap B \cap C$.
Also verify that : $(A \cap B) \cap C = A \cap (B \cap C)$,

Ans :

Here $A \cap B = \{1, 2, 3, 4\} \cap \{3, 4, 9, 11\}$

$\therefore$ $A \cap B = \{3, 4\}$

$B \cap C = \{11\}$

$C \cap A = \{2\}$

$A \cap B \cap C = \phi, \{\ \}$

Now we want to verify that $(A \cap B) \cap C = A \cap (B \cap C)$

$A \cap B = \{3, 4\}$, $C = \{2, 11, 18, 22\}$

$\therefore$ $(A \cap B) \cap C = \phi$

Also $A = \{1, 2, 3, 4\}$, $B \cap C = \{11\}$

$\therefore$ $A \cap (B \cap C) = \phi$

$\therefore$ $(A \cap B) \cap C = A \cap (B \cap C)$

Illustration 2 : If $A = \{x/x \in N, x \leq 5\}$ $B = \{x/x \in N, 2 \leq x \leq 8$ $C = \{x/x \in N, x \leq 3\}$, find $A \cap B$, $B \cap C$ and $C \cap A$

Ans : Here

$A = \{1, 2, 3, 4, 5\}$

$B = \{2, 3, 4, 5, 6, 7, 8\}$

$C = \{1, 2, 3\}$

$\therefore$ $A \cap B = \{2, 3, 4, 5\}$

$B \cap C = \{2, 3\}$

$C \cap A = \{1, 2, 3\}$

Illustration 4 : $A = \{x/x \in N, x^2 < 10\}$,
$B = \{x/x \in N, x \leq 1\}$,
$C = \{x/x \in N, 1 \leq x < 5\}$

Find $A \cup B$, $B \cup C$, $C \cup A$ and verify that $(A \cup B) \cup C = A \cup (B \cup C)$

Ans. :

$A = \{1, 2, 3\}$, $B = \{1\}$, $C = \{1, 2, 3, 4\}$,

$A \cup B = \{1, 2, 3\}$

$B \cup C = \{1, 2, 3, 4\}$

$C \cup A = \{1, 2, 3, 4\}$

Now, $(A \cup B) \cup C = \{1, 2, 3\} \cup \{1, 2, 3, 4\} = \{1, 2, 3, 4\}$

And, $A \cup (B \cup C) = \{1, 2, 3\} \cup \{1, 2, 3, 4\} = \{1, 2, 3, 4\}$

$\therefore (A \cup B) \cup C = A \cup (B \cup C)$

Illustration 9 : If $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{2, 3, 6\}$, $B = \{3, 5, 6\}$ then verify that (i) $(A \cup B)' = A' \cap B'$

Ans. (i) $A \cup B = \{2, 3, 5, 6\}$

$(A \cup B)' = \{1, 4\}$

$A' = U - A = \{1, 4, 5\}$

$B' = U - B = \{1, 2, 4\}$

$A' \cap B' = \{1, 4\}$

$\therefore (A \cup B)' = A' \cap B'$

... to prove that $(A \cup B)' = A' \cap B'$*

Illustration 12 : If $A = \{x/x \leq 9, x \in N\}$,
$B = \{y / 3 \leq y \leq 7$, and y is odd number$\}$
$C = \{z / 1 \angle z \angle 7$, and z is even number$\}$ then prove that
$A - (B \cup C) = (A - B) \cap (A - C)$

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$B = \{3, 5, 7\}$

$C = \{2, 4, 6\}$

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$B \cup C = \{2, 3, 4, 5, 6, 7\}$

L.H.S $= A - (B \cup C) = \{1, 8, 9\}$ ....(i)

Now, $A - B = \{1, 2, 4, 6, 8, 9\}$

$A - C = \{1, 3, 5, 7, 8, 9\}$

R.H.S $= (A - B) \cap (A - C) = \{1, 8, 9\}$ .... (ii)

From (i) and (ii) We have $A - (B \cup C) = (A - B) \cap (A - C)$

* Define : differ

Illustration 15 : If $A = \{x / x \in N, |x^3 - 2| \leq 25\}$,
$B = \{y / y \in N, 1 \angle y \angle 5\}$
$C = \{z / z \in N, Z^4 = 81\}$
Verify that $= A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Ans :

$A = \{x / x \in N, |x^3 - 2| \leq 25\}$

i.e. $\mathbf{A = \{1, 2, 3\}}$

$B = \{y / y \in N, 1 < y < 5\}$

i.e. $\mathbf{B = \{2, 3, 4\}}$

$C = \{z / z \in N, z^4 = 81\}$

i.e. $\mathbf{C = \{3\}}$

Now $A' = \{1, 2, 3\}$ $\qquad$ $A \cup B = \{1, 2, 3, 4\}$

$B \cap C = \{3\}$ $\qquad$ $A \cup C = \{1, 2, 3\}$

L.H.S $= A \cup (B \cap C) = \{1, 2, 3\}$

R.H.S. $= (A \cup B) \cap (A \cup C)$

$= \{1, 2, 3\}$

$\therefore$ L.H.S = R.H.S

Illustration 16 : If $A = \{a \,/\, a^2 - 1 \angle 10, a \in Z\}$,

$B = \{b \mid b - 1 \mid \angle 2, b \in N\}$

$C = \{c \,/ \mid c \mid \leq 1, c \in Z\}$

Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Ans :

Here $A = \{-3, -2, -1, 0, 1, 2, 3\}$,

$B = \{1, 2\}$

$C = \{-1, 0, 1\}$

Now $B \cap C = \{1\}$

L.H.S $= A \times (B \cap C) = \{(-3, 1), (-2, 1), (-1, 1), (0, 1), (1, 1), (2, 1), (3, 1)\}$

$A \times B = \{(-3, 1), (-2, 1), (-1, 1), (0, 1), (1, 1), (2, 1), (3, 1), (-3, 2), (-2, 2), (-1, 2), (0, 2), (1, 2), (2, 2), (3, 2)\}$

$A \times C = \{(-3, -1), (-2, -1), (-1, -1), (0, -1), (1, -1), (2, -1), (3, -1), (-3, 0), (-2, 0), (-1, 0), (0, 0), (1, 0), (2, 0), (3, 0), (-3, 1), (-2, 1), (-1, 1), (0, 1), (1, 1), (2, 1), (3, 1)\}$

R.H.S $= (A \times B) \cap (A \times C) = \{(-3, 1), (-2, 1), (-1, 1), (0, 1), (1, 1), (2, 1), (3, 1)\}$

$\therefore$ L.H.S = R.H.S

Illustration 17 : In a class of 42 students, each play atleast one of the three games Cricket, Hockey and Football. It is found that 14 play Cricket, 20 play Hockey and 24 play Football, 3 play both Cricket and Football, 2 play both Hockey and Football. None play all the three games. Find the number of students who play Cricket but not Hockey.

Ans.

Let C denote the set of students who play cricket $\Rightarrow n(C) = 14$.

Let H denote the set of students who play Hockey $\Rightarrow n(H) = 20$

Let F denote the set of students who play Football $\Rightarrow n(F) = 24$

Also, $n(C \cup H \cup F) = 42$

$C \cap F$ = {students who play both Cricket & Football} $\Rightarrow n(C \cap F) = 3$

$H \cap F$ = {students who play Hockey & Football} $\Rightarrow n(H \cap F) = 2$

$C \cap H \cap F$ = {students who play Cricket, Hockey & Football}

$\Rightarrow n(C \cap H \cap F \cap) = 0$

Here, we are required to find the number of students who play cricket but not Hockey i.e. $n(C \cap H')$

Now, $n(C \cup H \cup F) = n(C) + n(H) + n(F) - n(C \cap H) - n(H \cap F) - n(F \cap C) + n(C \cap H \cap F)$

$\therefore 42 = 14 + 20 + 24 - n(C \cap H) - 2 - 3 + 0$

$\therefore 42 = 53 - n(C \cap H)$

$\therefore n(C \cap H) = 11$

Now $n(C) = n(C \cap H') + n(C \cap H)$

$\therefore 14 = n(C \cap H') + 11$

$\therefore n(C \cap H') = 3$

$\therefore$ 3 students play Cricket but not Hockey.

Illustration 18 : If $U = \{x / x \in N, x \le 10\}$,

$A = \{x / x \in N, x^2 \angle 10\}$

$B = \{2, 4, 6\}$

$C = \{x / x^3 - 3x^2 - 4x = 0\}$

Verify that, (i) $A \cap (B - C) = (A \cap B) - (A \cap C)$

(ii) $A' - B' = B - A$

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 2, 3\}$, $B = \{2, 4, 6\}$, $C = \{0, -1, 4\}$

[$\therefore x^3 - 3x^2 - 4x = 0$,

$x(x^2 - 3x - 4) = 0$,

$x(x - 4)(x + 1) = 0$,

$x = 0$ or $x = 4$ or $x = -1$]

(i) $B - C = \{2, 6\}$

$\therefore A \cap (B - C) = \{2\}$ ....(i)

Now $A \cap B = \{2\}$, $A \cap C = \{\ \}$,

$(A \cap B) - (A \cap C) = \{2\}$ ....(ii)

From (i) and (ii)

$\therefore A \cap (B - C) = (A \cap B) - (A \cap C)$

(ii) $A' = \{4, 5, 6, 7, 8, 9, 10\}$

$B' = \{1, 3, 5, 7, 8, 9, 10\}$

L.H.S = A' – B' = {4, 6}

R.H.S = B – A = {4, 6}

∴ LHS = RHS

Illustration 19 : Examine the validity of the following statements and justify your answer :

(i) If $A = \{x / x^3 - 5x^2 + 6x = 0\}$

$B = \{x / x \in Z, |x| < 1\}$ then $A \cap B = \phi$

(ii) If $A - B = A$, Then $A \cap B = A$

(iii) If $A = \{\phi, a\}$ Then $\{\phi\} \in P(A)$

(iv) If $x \notin (A \cup B)$ Then $x \notin A$ or $x \notin B$

(v) If $A = \{a, b, c\}$, $B = \{2, 5, 7\}$

then $B \times A = \{2a, 5b, 7c\}$

Ans. :

(i) **False :** Here A = {0, 2, 3}, B = {0}

∴ $A \cap B = \{0\}$, and not ϕ

∴ Statement is false

(ii) **False :** A – B = A

i.e.No element of B is in A Hence $A \cap B = \phi$. The statement is therefore false.

(iii) **True :** because P(A) = {{}, {ϕ, a}, {ϕ}, {a}}

Thus $\{\phi\} \in P(A)$ is true

(iv) **False :** because If $x \notin (A \cup B)$

$\Rightarrow x \notin A$ and $x \notin B$.

(v) **False :** because

A = {a, b, c}, B = {2, 5, 7}

Then B × A = {(2, a), (2, b), (2, c), (5, a), (5, b), (5, c), (7, a), (7, b), (7, c)}

Illustration 20 : If A = [1, 3, a, {1}, {1, a}], state whether the following statements are true or false.

(i) $1 \in A$,

(ii) $\{1\} \in A$,

(iii) $\phi \in A$,

(iv) $\{1, a\} \subset A$

(v) $\phi \subset A$

(vi) $\{1, a\} \in A$

Ans. :

(i) $1 \in A$, True

(ii) $\{1\} \in A$, True

(iii) $\phi \in A$, False

(iv) $\{1, a\} \subset A$, True,

(v) $\phi \subset A$, True

(vi) $\{1, a\} \in A$, True

Illustration 21 : Prove that $(A')' = A$

We shall prove that

(i) $(A')' \subseteq A$ (ii) $A \subseteq (A')'$

(i) Let x be any element of $(A')'$ $\Rightarrow x \in (A')'$

$\Rightarrow x \notin A'$

$\Rightarrow x \in A$

$\Rightarrow (A')' \subseteq A$ .....(i)

(ii) Let y be any element of A $\Rightarrow y \in A$

$\Rightarrow y \notin A'$

$\Rightarrow y \in (A')'$

$\Rightarrow A \subseteq (A')'$ .....(ii)

From (i) & (ii), $(A')' = A$

Illustration 22 : Prove that : $A - (A - B) = A \cap B$

We shall prove that (i) $A - (A - B) \subseteq (A \cap B)$

(ii) $A \cap B \subseteq A - (A - B)$

(i) Let x be any element of $A - (A - B)$

$\Rightarrow x \in A - (A - B)$

$\Rightarrow x \in A$ but $x \notin (A - B)$

$\Rightarrow x \in A$ but $(x \notin A$ and $x \in B)$

$\Rightarrow x \in A$ and $x \in B$

$\Rightarrow x \in A \cap B$

$\Rightarrow A - (A - B) \subseteq A \cap B$

(i) Let y be any element of $A \cap B$ ....(i)

$\Rightarrow y \in (A \cap B)$

$\Rightarrow y \in A$ and $y \in B$

$\Rightarrow y \in A$ but $(y \notin A$ and $y \in B)$

$\Rightarrow y \in A$ but $y \notin (A - B)$

$\Rightarrow y \in A - (A - B)$

$\Rightarrow A \cap B \subseteq (A - (A - B)$ ......(ii)

From (i) & (ii)

$A - (A - B) = (A \cap B)$

EXERCISE