* U A - 3 7 0 2 *

UA-3702

First Year B. C. A. (Sem. I) Examination

March/April – 2012

102 - Mathematics

Time : Hours] [Total Marks : **70**

Instruction :

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **F. Y. B. C. A. (Sem. 1)**

Name of the Subject :

☛ **102 - Mathematics**

☛ Subject Code No. : **3 7 0 2** ☛ Section No. (1, 2,.....) : **NIL**

Seat No. :

Student's Signature

1 Answer the following questions : **10**

(1) If $A=\{a,b,c,d,e\}$ then write no. of subsets in powerset of A.

(2) If $A=\{4,5,6\}, B=\{1,2,3\}$ then find $(A-B)\cap(B-A)$.

(3) Define one-one function.

(4) If $f(x)=x^2-1$ then find $f(x+1)$.

(5) Define Boolean Algebra.

(6) Define contradiction.

(7) Find the equation of a line passing through orign and having slope 1/5.

(8) Find the value of k such that $3kx+5y+k=0$ passes through the point $(-1,4)$.

(9) If $A=\begin{bmatrix}-3 & 1\\ 2 & 4\end{bmatrix}$ then find $adj.A$.

(10) Define transpose of a matrix with illustration.

2 (a) State and prove De Morgan's law for Intersection. **4**

(b) Let $U=\{x \mid 3 \leq x \leq 13, x \in N\}$ **4**

$A=\{y \mid 1<y<8\}$

$Z=\{z \in N \mid z^2=25\}$ then (1) find A' and B'

(2) Verify that, $A-(B \cup C)=(A-B) \cap (A-C)$

(c) If $A=\{x \mid x^2-x-2=0\}$ and **4**

$B=\{x \mid x^2-5x+6=0\}$ then find $A \cup B$ and $A \cap B$.

OR

2 (a) In usual notation prove that **4**

$A \times (B \cap C)=(A \times B) \cap (A \times C)$

(b) If $A=\{x \mid 1<x<10, x \text{ is an even number}\}$ **4**

$B=\{x \mid 2 \leq x<9, x \text{ is an odd number}\}$

$C=\{x \in N \mid x^2=9\}$ then verify that,

$A \cap (B \cup C)=(A \cap B) \cup (A \cap C)$

(c) If $U=\{x \mid x \in N, \mid <x<10\}$ **4**

$A=\{x \mid 2 \leq x \leq 9\}$

$B=\{3, 5, 6\}$ then prove that

(1) $(A \cup B)'=A' \cap B'$ (2) $(A \cap B)'=A' \cup B'$

3 (a) The cost of manufacturing 50 pens is Rs. 15,000 and that of manufacturing 300 pens is Rs. 40,000. If the relationship between the number of units produced and its cost is linear, then find the relation. Also find the cost of manufacturing 400 pens. **4**

(b) If $f(x)=\dfrac{x^2-25}{x-5}$; $x \in Z-\{5\}$ and **4**

$g(x)=x+5$; $x \in Z$. Is $f(x)=g(x)$?

(c) If $f(x)=\frac{1}{x}$; $x \in Z-\{-1, 0, 1\}$ then P.T., 4

$f(x+1)-f(x-1)=2/1-x^2$.

OR

3 (a) Define domain and range of a function. If domain 4

of $f(x)=\sqrt{x^2+3}$ is $\{1, 2, 3, 4\}$ then find range of *f*.

(b) If $f(x)=\frac{x^2-5x+7}{x^2+2}$; $x \in Z$ then find $\frac{f(1)+f(-2)}{f(-1)+f(0)}$. 4

(c) If $y=f(x)=\frac{ax+b}{cx-a}$ then prove that $x=f(y)$. 4

4 (a) Let D_9 be the set of positive divisors of 9. 4

Define +, · &' on D_9 as follow :

$a+b=lcm$ of a & b

$a \cdot b=\gcd$ of a & b

$a'=\frac{9}{a}$ then verify that $(D_9, +, \cdot, ', 1, 9)$ is a Boolean algebra.

(b) Find the value of $\left(\left(x_1 \cdot x_2'\right)+x_3\right) \cdot x_2'$ if : 4

(1) $x_1=0,\ x_2=1,\ x_3=1$

(2) $x_1=0,\ x_2=0,\ x_3=1$.

(c) Using truth table prove that 4

$(p \wedge q) \wedge r=p \wedge (q \wedge r)$

OR

4 (a) Using truth table prove that : 4

$\sim(p \vee q)=(\sim p) \wedge (\sim q)$

(b) Simplify the Boolean expression 4

$x+x' \cdot (x+y)+y \cdot z$

(c) Find the product sum cannonical form of 4

$f(x_1, x_2)=x_1 \cdot x_2'+x_1' \cdot x_2+x_1' \cdot x_2'$

5 (a) If $A=\begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$ then find the value of A^2. **4**

(b) If $A=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ and $B=\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ then find AB and BA. Is $AB = BA$? **4**

(c) Solve the following system of equations using inverse of a matrix : **4**

$$x+y+z=3$$
$$x+2y+3z=6$$
$$3x+y+2z=6$$

OR

5 (a) If $A=\begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$ then show that $adj \cdot A = A$. **4**

(b) If $A=\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$ and $B=\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$ then verify that, **4**

$(AB)^{-1} = B^{-1}A^{-1}$.

(c) Solve the following equation using Cramer's rule : **4**

$$3x-2y+z=2$$
$$x+3y-2z=2$$
$$2x-y+z=2$$

6 (a) Prove that (2, 3), (6, 5) and (12, 8) are collinear. **4**

(b) Find the equation of a line passing through (4, 3) and making equal intercepts on the axes. Also find its slope. **4**

(c) Find the equation of a line whose X-intercept is 3 and y-intercept is 5. Also find the slope of this line. **4**

OR

6 (a) Find k if $A(3, 6), B(-k, 8)$ and $C(1, 2)$ are collinear. **4**

(b) Prove that (2, –1), (3, 4), (–2, 3) and (–3, –2) are the vertices of a rhombus. **4**

(c) If the line $x-7y-2=0$ and $kx+3y+1=0$ are parallel then find the value of k. **4**