



**SB-1402**

**First Year B. C. A. (Sem. - I) Examination**

**March / April - 2011**

**Mathematics - I**

Time : 3 Hours]

[Total Marks : 70

**Instructions :**

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| <p>નીચે દર્શાવેલ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.<br/> <b>Fillup strictly the details of signs on your answer book.</b></p> <p>Name of the Examination :<br/> <div style="border: 1px solid black; padding: 2px; display: inline-block;"><b>F. Y. B. C. A. (SEM. - 1)</b></div></p> <p>Name of the Subject :<br/> <div style="border: 1px solid black; padding: 2px; display: inline-block;"><b>MATHEMATICS - 1</b></div></p> <p>Subject Code No. : <div style="border: 1px solid black; padding: 2px; display: inline-block; width: 20px; text-align: center;">1</div> <div style="border: 1px solid black; padding: 2px; display: inline-block; width: 20px; text-align: center;">4</div> <div style="border: 1px solid black; padding: 2px; display: inline-block; width: 20px; text-align: center;">0</div> <div style="border: 1px solid black; padding: 2px; display: inline-block; width: 20px; text-align: center;">2</div> Section No. (1, 2,.....) : <div style="border: 1px solid black; padding: 2px; display: inline-block;"><b>NIL</b></div></p> | <p>Seat No. :<br/> <div style="display: flex; justify-content: space-around; width: 100%;"> <div style="border: 1px solid black; width: 20px; height: 20px;"></div> <div style="border: 1px solid black; width: 20px; height: 20px;"></div> <div style="border: 1px solid black; width: 20px; height: 20px;"></div> <div style="border: 1px solid black; width: 20px; height: 20px;"></div> <div style="border: 1px solid black; width: 20px; height: 20px;"></div> <div style="border: 1px solid black; width: 20px; height: 20px;"></div> </div> <div style="border: 1px solid black; border-radius: 10px; height: 80px; margin-top: 10px; display: flex; align-items: center; justify-content: center;"> <p>Student's Signature</p> </div> </p> |
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- (2) All questions are compulsory.
- (3) Figures to the right indicates full marks.

**1 Answer the following questions : 10**

- (1) Prove that  $(A')' = A$
- (2) Define equivalent set will illustration.
- (3) 'Every function is a relation' or 'Every relations is a function'. Which is true ? Justify your answer.
- (4) If  $f(x) = 3x^2 + mx + 5$  and  $f(2) = 27$  then find the value of  $m$ .
- (5) Explain idempotent law in Boolean Algebra.
- (6) 'Every skew symmetric matrix is a Diagonal matrix' true or false ? Justify your answer.

(7) If  $\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$  then find the values of  $a$  and  $b$ .

(8) Find the equation of line with slope  $\frac{1}{2}$  and passing through  $(5, 4)$ .

(9) Give the truth table of  $(p \Rightarrow q)$  and  $(p \Leftrightarrow q)$ .

(10) Define orthogonal matrix.

**2** (a) If  $A = \{x \mid x \text{ is a positive integer between 1 and 4}\}$  **12**

$B = \{x \mid x \text{ is a natural no. between 1 and 5}\}$

$C = \{x \mid x \text{ is an odd natural no. less than 6}\}$

$D = \{x \mid x \text{ is an even natural no. less than 5}\}$

then verify

$$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D).$$

(b) If  $A = \{x \mid x^2 - 2x - 3 = 0; \ x \in R\}$

$B = \{x \mid x^3 = x; \ x \in Z\}$

$C = \{x \mid x^3 = x; \ x \in N\}$

then verify  $A \times (B - C) = (A \times B) - (A \times C)$

- (c) If  $A = \{1, 2, 3, 4\}$ ,  $B = \{1, 2, 3\}$ ,  $C = \{2, 4\}$  find all possible sets  $X$  which satisfy the following conditions :
- (i)  $X \subset B$ ;  $X \not\subset C$
- (ii)  $X \subset B$ ,  $X \not\subset C$ ,  $X \neq B$
- (iii)  $X \subset A$ ,  $X \subset B$ ,  $X \not\subset C$ .

**OR**

- 2** (a) In usual notation prove that  $A - (A - B) = A \cap B$ . **12**

- (b) If  $U = \{x | x \in N; x \leq 10\}$

$$A = \{x | x \in N, x^2 < 10\}$$

$$B = \{2, 4, 6\}$$

$$C = \{x | x^3 - 3x^2 - 4x = 0\}$$
 then verify that

(i)  $A \cap (B - C) = (A \cap B) - (A \cap C)$

(ii)  $A' - B' = B - A$

- (c) In a housing society, 50 residents have scooters, 20 home cars and 15 have both types of vehicles. If there are 60 residents in the society, how many of them have neither scooter nor car ?

- 3** (a) The supply function of a commodity is  $S = ap^2 + bp + c$  **12**

the respective value of price and supply are given by the ordered pairs (2, 12), (3, 38), (4, 74) then determine the supply function and find the price when supply is 120 units.

(b)  $f = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ ,  $A = \{1, 2, 3, 4\}$ ,  $B = 2$ . If

$f: A \rightarrow B$  is a function such that  $f(x) = ax + b$  then find  $a$  and  $b$ .

(c) If  $f(x) = x^2 + x - 1$  then find the value of

$$f(x+1) - 3f(x-1) + 2f(x).$$

3 (a) If  $f(x) = \frac{1}{x} + \frac{2}{x-3}$ ;  $x \in \mathbb{R} - \{0, 3\}$  then find  $f(1)$ ,  $f(2)$ , **12**

$$f\left(\frac{1}{3}\right) \text{ and } f(-3)$$

(b) A function is defined as,  $f(x) = 2x + 3$ ;  $x \in [-2, 0]$   
 $= 4 - 3x$ ;  $x \in (0, \infty)$

then find the value of  $\frac{f(-2) - f(-1)}{f(2) + f(1)}$

(c) The cost function of an item is  $C(x) = 4x + 770$  and the selling price per item is Rs. 15. Find the break even point. If the profit is Rs. 1100, find the number of units produced.

4 (a) Show that  $D_{10}$  (Divisor of 10) is a Boolean Algebra **12**

where,  $\forall a, b \in D_{10}$

$$a + b = \text{lcm of } a, b$$

$$a \cdot b = \text{gcd of } a, b \text{ and}$$

$$a' = \frac{10}{a}$$

(b) Is the argument in following example valid ?

Hypothesis :  $S_1 : p \Rightarrow q$ ,  $S_2 : q \Rightarrow r$ , conclusion  $S : p \Rightarrow r$

(use truth table)

(c) Construct the input/output table for

(i)  $f(x_1, x_2) = x_1 \cdot x_2$

(ii)  $f(x_1, x_2, x_3) = (x_1 \cdot x_2') \cdot x_3$

**OR**

4 (a) Show that  $D_{21}$  is a boolean algebra where  $\forall a, b \in D_{21}$  **12**

$$a + b = \text{lcm of } a, b$$

$$a \cdot b = \text{gcd of } a, b \text{ and}$$

$$a' = \frac{21}{a}$$

(b) Is the argument given below is logically valid ?

Hypothesis :  $S_1 : p$ ,  $S_2 : p \wedge q \Rightarrow r \vee s$ ,  $S_3 : q$ ,  $S_4 : \sim s$  and

conclusion :  $S : r$ .

(c) Construct the input/output table for

(i)  $f(x_1, x_2) = x_1 \cdot x_2'$

(ii)  $f(x_1, x_2) = (x_1 \cdot x_2)' + x_2$

5 (a) Prove that 
$$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = 4a^2b^2c^2$$
 **12**

(b) If  $A = \begin{bmatrix} 7 & 3 & -5 \\ 0 & 4 & 2 \\ 1 & 5 & 4 \end{bmatrix}$  and  $B = 3A$ ,  $C = -B$  then find

$$2A - B + C.$$

(c) Find the inverse of the matrix  $A = \begin{bmatrix} 1 & 3 & 2 \\ 1 & -4 & 4 \\ 1 & 3 & -3 \end{bmatrix}$

**OR**

**5** (a) Show that  $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz(x-y)(y-z)(z-x)$  **12**

(b) Solve using Crammer's rule :

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \quad \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1 \quad \text{and}$$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

(c) If  $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & a \\ 4 & b \end{bmatrix}$  and  $(A+B)^2 = A^2 + B^2$  then find  $a, b$ .

**6** (a) If the distance between  $(a, -5)$  and  $(2, a)$  is 13 units **12**  
then find the value of  $a$ .

(b) Find the equation of a line passing through  $(-2, 3)$  and parallel to the line joining  $(1, 7)$  and  $(-2, -5)$ .

- (c) Show that the points  $(-3, 2)$ ,  $(1, 2)$  and  $(-3, 5)$  form a right angled triangle.

**OR**

- 6** (a) Find the equation of a line joining the point  $(1, -3)$  **12**  
and the point of intersection of the line  $x + y + 1 = 0$  and  $3x + y + 5 = 0$ .
- (b) Prove that the line  $4x + 3y + 2 = 0$  and  $6x - 8y + 11 = 0$  are perpendicular to each other.
- (c) Find the equation of the parallel to the line joining  $(3, 2)$  and  $(4, 0)$  and passing through the point  $(5, 7)$ .
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