



DE-3704

First Year B. C. A. (Sem. I) Examination

March/April – 2016

Mathematics : Paper - 102

Time : 3 Hours]

[Total Marks : 70

Instructions :

(1)

<p>નીચે દર્શાવેલ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.  <b>Fillup strictly the details of signs on your answer book.</b></p> <p>Name of the Examination :  <div style="border: 1px solid black; padding: 2px; display: inline-block;">First Year B. C. A. (Sem. I)</div></p> <p>Name of the Subject :  <div style="border: 1px solid black; padding: 2px; display: inline-block;">Mathematics : Paper - 102</div></p> <p>Subject Code No. : <div style="border: 1px solid black; padding: 2px; display: inline-block; width: 20px; text-align: center;">3</div> <div style="border: 1px solid black; padding: 2px; display: inline-block; width: 20px; text-align: center;">7</div> <div style="border: 1px solid black; padding: 2px; display: inline-block; width: 20px; text-align: center;">0</div> <div style="border: 1px solid black; padding: 2px; display: inline-block; width: 20px; text-align: center;">4</div> Section No. (1, 2,...): <div style="border: 1px solid black; padding: 2px; display: inline-block; width: 40px; text-align: center;">Nil</div></p>	<p>Seat No. :  <div style="display: flex; justify-content: space-around; width: 100%;"> <div style="border: 1px solid black; width: 20px; height: 20px;"></div> <div style="border: 1px solid black; width: 20px; height: 20px;"></div> <div style="border: 1px solid black; width: 20px; height: 20px;"></div> <div style="border: 1px solid black; width: 20px; height: 20px;"></div> <div style="border: 1px solid black; width: 20px; height: 20px;"></div> <div style="border: 1px solid black; width: 20px; height: 20px;"></div> </div> <div style="border: 1px solid black; border-radius: 10px; height: 80px; margin-top: 10px; display: flex; align-items: center; justify-content: center;"> <p>Student's Signature</p> </div> </p>
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(2) All questions are compulsory.

(3) Figures to the right indicate full marks.

Q:1 Answer the following Questions:

[10]

1. Define proper and improper subsets with illustration.
2. When do you say that two functions are equal?

3. Find the value of  $\begin{vmatrix} 1 & 1 & 1 \\ bc & ca & ab \\ 1/a & 1/b & 1/c \end{vmatrix}$ .

4. Define conjunction and disjunction with illustration.
5. In a Boolean algebra  $B$ , show that  $\forall x \in B; (x')' = x$ .

Q: 2 (A) Verify Distributive law of intersection of  $A = \{x \leq 5; x \in N\}$  over the union of  $B = \{x : x^2 \leq 9; x \in Z\}$

and  $C = \{x : -1 \leq x \leq 4; x \in N\}$

[05]

OR

Q: 2 (A) In usual notations prove that  $A - (B \cup C) = (A - B) \cap (A - C)$ .

[05]

Q: 2 (B) Attempt any two:

[10]

(1) If  $A = \{1, 2, 3\}, B = \{2, 3, 4\}, S = \{1, 3, 4\}, T = \{2, 4, 5\}$ , then verify that  
 $(A \times B) \cap (S \times T) = (A \cap S) \times (B \cap T)$ .

(2) If  $U = \{x : x < 10; x \in N\}, A = \{x : x = 2y, y < 5; y \in N\}, B = \{3, 6, 9\}$   
 Then verify that  $(A \cap B)' = A' \cup B'$

(3) Prove that  $A \cap (B - C) = (A \cap B) - (A \cap C)$

(4) If  $A = \{x / x^2 - 17x + 16 = 0\}, B = \{x / x^2 - 7x + 12 = 0\}$  then prove that  $(A \cup B) - (A \cap B) = A \cup B$

**Q: 3 (A)** If  $f(x) = x(x+1)(2x+1)$  then prove that  $f(x) - f(x+1) = 6x^2$  [05]

OR

**Q: 3 (A)** It is observed that a quadratic function  $y = ax^2 + bx + c$  fits the points (1,9), (2,4) and (3,23) find the constants  $a, b, c$  and rewrite the function also find value of  $y$  when  $x = 4$ .

**Q: 3 (B) Attempt any two:** [10]

1. If  $f(x) = \frac{x^2 - 9x + 14}{x - 2}$ ;  $x \in \mathbb{Z} - \{2\}$  and  $g(x) = x - 7$ ,  $x \in \mathbb{Z} - \{2\}$  prove that  $f = g$ .

2. If  $f: \mathbb{N} \rightarrow \mathbb{N}$  and  $f(x) = \frac{x^2(x+1)}{4}$  then prove that  $f(x) - f(x-1) = x^3$ .

3. A Magazine publisher finds that the variable cost of each magazine is Rs.40 and the fixed cost is Rs. 18000. If the selling price is Rs. 50 then find the breakeven point.

4.. If  $f(x) = x(x-1)(2x-1)$  then prove that  $f(x+1) - f(x+2) = -6(x+1)^2$

**Q:4 (A)** Show that  $D_{24}$  is a Boolean Algebra where  $\forall a, b \in D_{24}$  [05]

$$a + b = \text{L.C.M. of } a, b$$

$$a \bullet b = \text{G.C.D. of } a, b$$

$$a' = 24 / a$$

OR

**Q:4 (A)** Prove that the argument in the following example is not logically valid [05]

$$\text{Hypothesis: } \begin{cases} S_1 : p \Rightarrow q \\ S_2 : q \Rightarrow r \end{cases} \quad \text{Conclusion: } S : p \Rightarrow r$$

**Q: 4(B) Attempt Any two** [10]

1. Show that the following pairs of propositions are logically equivalent.

$$(a) \sim q \Rightarrow \sim p \equiv p \Rightarrow q$$

$$(b) \sim (p \Leftrightarrow q) \equiv p \Leftrightarrow \sim q \equiv \sim p \Leftrightarrow q$$

2. Construct the input/output table for

$$(i) f(x_1, x_2) = x_1' \bullet x_2 \quad (ii) f(x_1, x_2, x_3) = (x_1 \bullet x_2') + x_3$$

3. Construct truth table for  $(\sim p \wedge (\sim q \wedge r)) \vee (q \wedge r) \vee (p \wedge r)$

4. In a Boolean algebra  $B$  prove that  $(x + y)' \bullet (x' + y') = x' \bullet y' \quad \forall x, y \in B$

**Q:5(A)** If  $A = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$  then prove that  $A^2 = I$  [05]

OR

**Q: 5(A)** Show that  $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$  satisfies the equation  $A^3 - 6A^2 + 9A - 4I = 0$ . **5**

**Q: 5(B) Attempt Any TWO**

**10**

1. If  $A^{-1} = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 4 \\ 1 & 1 & 3 \end{bmatrix}$ ,  $B^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \end{bmatrix}$  then find  $(A \cdot B)^{-1}$

2. Solve the following system of equations using crammer's rule

$$ax + by - ab = 0$$

$$bx + ay - ab = 0$$

3. Show that  $D_8$  is a Boolean Algebra where  $\forall a, b \in D_8$

$$a + b = \text{L.C.M. of } a, b$$

$$a \bullet b = \text{G.C.D. of } a, b$$

$$a' = 8 / a$$

4. Solve the following system of equations using crammer's rule

$$x + 2y - z = 2$$

$$3x + 6y + z = 1$$

$$3x + 3y + 2z = 3$$