CS 763: Assignment 1

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Question 1

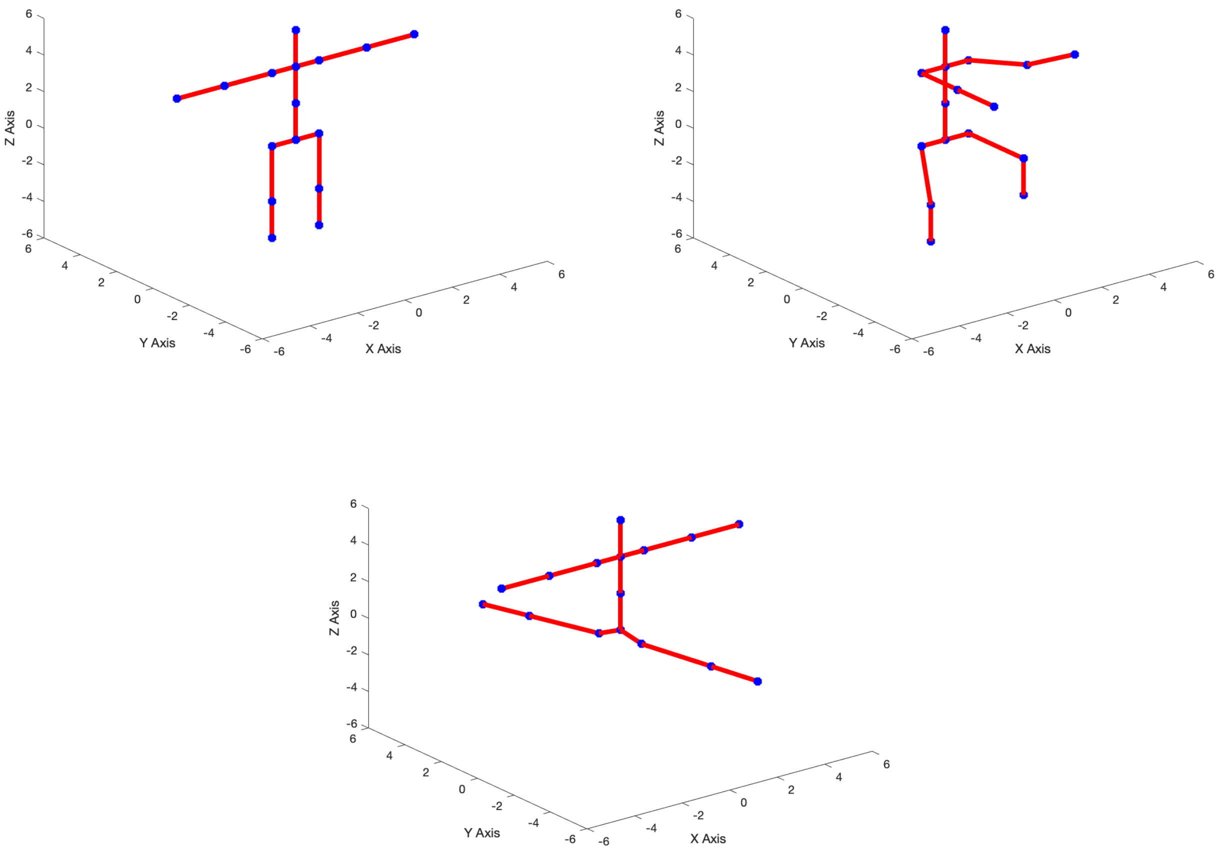


Figure 1: Base position, Sitting position and Forward split

Question 2

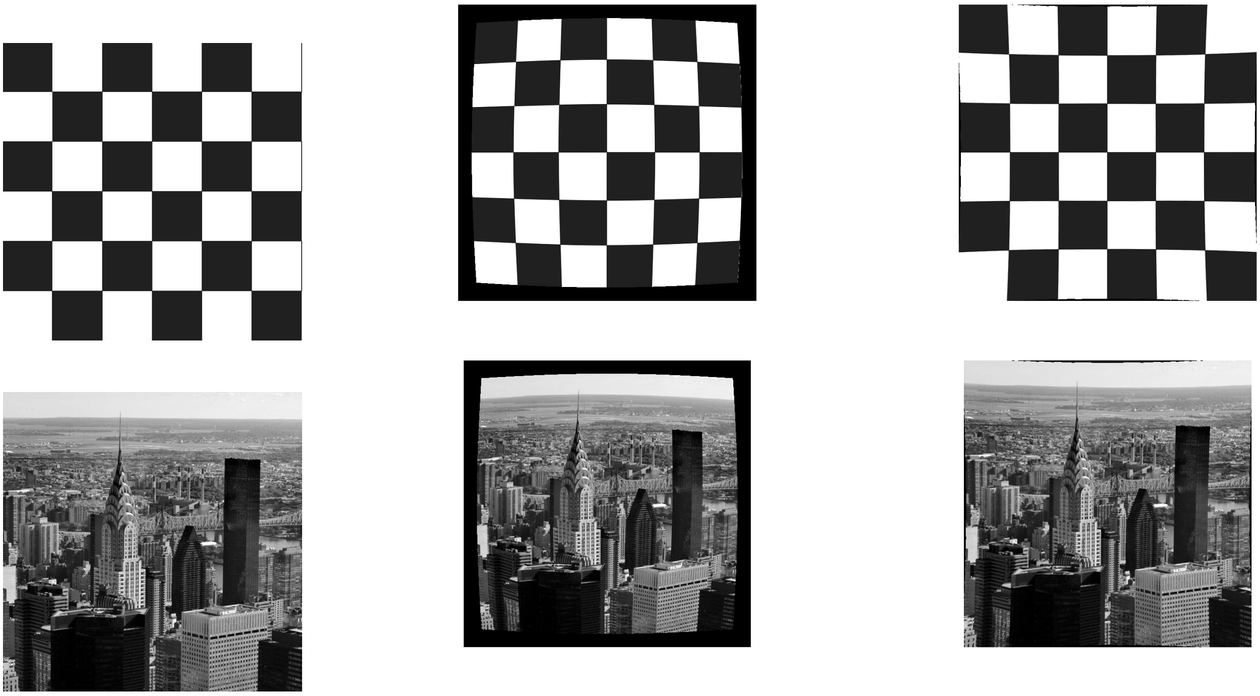


Figure 2: original, distorted and undistorted

Question 3

$$I = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} t \\ 1/t \\ 1 \end{bmatrix}$$

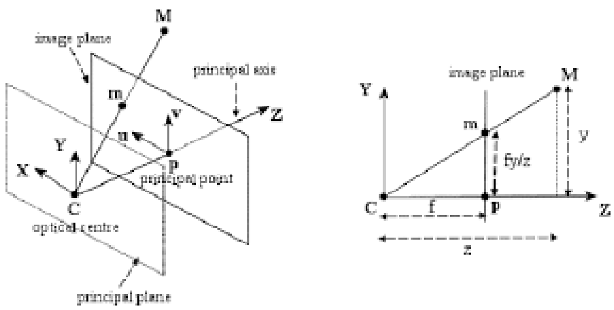
$$I = \begin{bmatrix} 1 \\ 1/t \\ t \end{bmatrix}$$

Therefore the required projection in real (Euclidean) coordinates is,

$$I_1 = \begin{bmatrix} 1/t \\ 1/t^2 \end{bmatrix}$$

We can see that this is the parametric equation of the parabola $y = x^2$. Therefore the projection of the hyperbola is a parabola.

Question 5



Assume the camera is at the origin and the image plane is at $z = f$.

(a)

From similar triangles property, $\mathbf{x} = f\frac{\mathbf{X}}{Z}$, where $\mathbf{x}$ is the projected point for $\mathbf{X}$. Consider a line with direction vector $\mathbf{D}$ passing through $\mathbf{A}$, its equation is given by,

$$\mathbf{X}(\lambda) = \mathbf{A} + \lambda\mathbf{D}$$

Consider the projection of the point for which $\lambda \longrightarrow \infty$,

$$= \lim_{\lambda\to\infty} f\frac{\mathbf{A} + \lambda\mathbf{D}}{A_Z + \lambda D_Z}$$

$$\mathbf{v} = f\frac{\mathbf{D}}{D_Z}$$

The point $\mathbf{v}$ is independent of $\mathbf{A}$, therefore the projection of two parallel lines have an intersection point, $\left(f\frac{D_X}{D_Z}, f\frac{D_X}{D_Z}, f\right)$, called the vanishing point.

(b)

Let the three direction vectors of the three sets of parallel lines be $\mathbf{D}_1 = (a,b,c)$, $\mathbf{D}_2 = (l,m,n)$, $\mathbf{D}_3 = (p,q,r)$. The three corresponding vanishing points are $\mathbf{A} = (f\frac{a}{c}, f\frac{b}{c}, f)$, $\mathbf{B} = (f\frac{l}{n}, f\frac{m}{n}, f)$ and $\mathbf{C} = (f\frac{p}{r}, f\frac{q}{r}, f)$. Since the lines lie on the same plane $\implies$ the box product of the direction vectors is zero (i-e) $[\mathbf{D}_1, \mathbf{D}_2, \mathbf{D}_3] = 0$, which yields us,

$$pbn - pcm + qlc - qan + ram - rbl = 0 \longrightarrow (1)$$

Consider $\mathbf{AB} \times \mathbf{CB}$,

$$= f^2\left(\frac{(lc-an)(mr-qn)-(lr-pn)(mc-bn)}{n^2cr}\right)\hat{\mathbf{k}}$$

$$= f^2\left(\frac{pcm - pbn + qan - qlc + rbl - ram}{ncr}\right)\hat{\mathbf{k}}$$

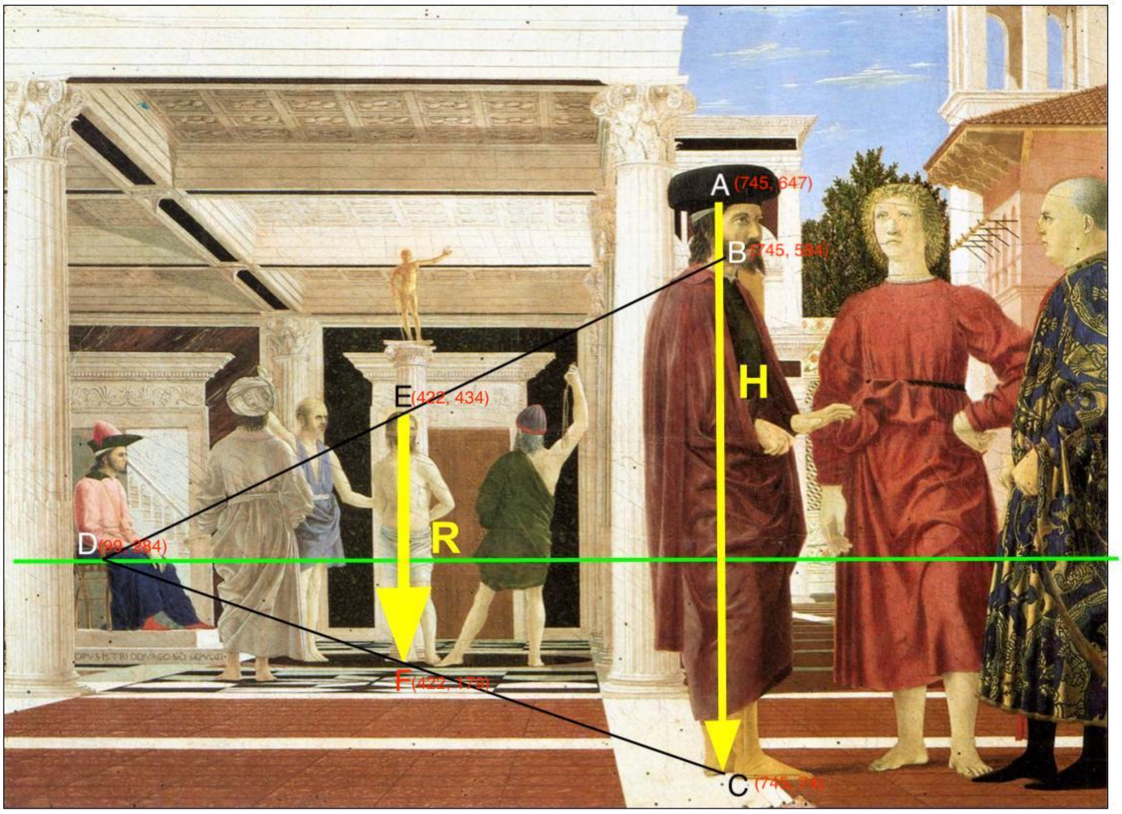
From equation (1),

$$\mathbf{AB} \times \mathbf{CB} = \mathbf{0}$$

$$\implies \mathbf{A,B,C} \textit{ are collinear}$$

Thus vanishing points corresponding to three different sets parallel lines on a 3D plane are collinear.

Question 6



With the help of MATLAB, we found out pixel positions of ends of both yellow arrows and position of green line.
Line joining C and F meets horizon line at D. Also line DE intersects arrow H at B. Since BD and CD intersects at the same point at horizon and this implies they are parallel lines. So actual length of $BC = 180cm$ (length of R). Also,

$$AC/BC = (647 - 74)/(584 - 74)$$

$$AC = 180 * 573/510$$

$$height, \mathbf{H} = 202.23cm$$