

Types of Algorithms

Lecture 9

Algorithm classification

- Algorithms that use a similar problem-solving approach can be grouped together
- This classification scheme is neither exhaustive nor disjoint
- The purpose is not to be able to classify an algorithm as one type or another, but to highlight the various ways in which a problem can be attacked

A short list of categories

- Algorithm types we will consider include:
 - Simple recursive algorithms
 - Backtracking algorithms
 - Divide and conquer algorithms
 - Dynamic programming algorithms
 - Greedy algorithms
 - Branch and bound algorithms
 - Brute force algorithms
 - Randomized algorithms

Simple recursive algorithms I

- A simple recursive algorithm:
 - Solves the base cases directly
 - Recurs with a simpler subproblem
 - Does some extra work to convert the solution to the simpler subproblem into a solution to the given problem
- I call these “simple” because several of the other algorithm types are inherently recursive

Example recursive algorithms

- To count the number of elements in a list:
 - If the list is empty, return zero; otherwise,
 - Step past the first element, and count the remaining elements in the list
 - Add one to the result
- To test if a value occurs in a list:
 - If the list is empty, return false; otherwise,
 - If the first thing in the list is the given value, return true; otherwise
 - Step past the first element, and test whether the value occurs in the remainder of the list

Backtracking algorithms

- Backtracking algorithms are based on a depth-first recursive search
- A backtracking algorithm:
 - Tests to see if a solution has been found, and if so, returns it; otherwise
 - For each choice that can be made at this point,
 - Make that choice
 - Recur
 - If the recursion returns a solution, return it
 - If no choices remain, return failure

Example backtracking algorithm

- To color a map with no more than four colors:
 - color(Country n)
 - If all countries have been colored ($n > \text{number of countries}$) return success; otherwise,
 - For each color c of four colors,
 - If country n is not adjacent to a country that has been colored c
 - » Color country n with color c
 - » recursively color country n+1
 - » If successful, return success
 - Return failure (if loop exits)

Divide and Conquer

- A **divide and conquer algorithm** consists of two parts:
 - Divide the problem into smaller subproblems of the same type, and solve these subproblems recursively
 - Combine the solutions to the subproblems into a solution to the original problem
- Traditionally, an algorithm is only called divide and conquer if it contains two or more recursive calls

Examples

- Quicksort:
 - Partition the array into two parts, and quicksort each of the parts
 - No additional work is required to combine the two sorted parts
- Mergesort:
 - Cut the array in half, and mergesort each half
 - Combine the two sorted arrays into a single sorted array by merging them

Binary tree lookup

- Here's how to look up something in a sorted binary tree:
 - Compare the key to the value in the root
 - If the two values are equal, report success
 - If the key is less, search the left subtree
 - If the key is greater, search the right subtree
- This is *not* a divide and conquer algorithm because, although there are two recursive calls, only one is used at each level of the recursion

Fibonacci numbers

- To find the n^{th} Fibonacci number:
 - If n is zero or one, return one; otherwise,
 - Compute **fibonacci($n-1$)** and **fibonacci($n-2$)**
 - Return the sum of these two numbers
- This is an expensive algorithm
 - It requires **$O(\text{fibonacci}(n))$** time
 - This is equivalent to exponential time, that is, **$O(2^n)$**

Dynamic programming algorithms

- A **dynamic programming algorithm** remembers past results and uses them to find new results
- Dynamic programming is generally used for optimization problems
 - Multiple solutions exist, need to find the “best” one
 - Requires “optimal substructure” and “overlapping subproblems”
 - **Optimal substructure**: Optimal solution contains optimal solutions to subproblems
 - **Overlapping subproblems**: Solutions to subproblems can be stored and reused in a bottom-up fashion
- This differs from Divide and Conquer, where subproblems generally need not overlap

Fibonacci numbers again

- To find the n^{th} Fibonacci number:
 - If n is zero or one, return one; otherwise,
 - Compute, *or look up in a table*, **fibonacci($n-1$)** and **fibonacci($n-2$)**
 - Find the sum of these two numbers
 - Store the result in a table and return it
- Since finding the n^{th} Fibonacci number involves finding all smaller Fibonacci numbers, the second recursive call has little work to do
- The table may be preserved and used again later

Greedy algorithms

- An **optimization problem** is one in which you want to find, not just *a* solution, but the *best* solution
- A “greedy algorithm” sometimes works well for optimization problems
- A **greedy algorithm** works in phases: At each phase:
 - You take the best you can get right now, without regard for future consequences
 - You hope that by choosing a *local* optimum at each step, you will end up at a *global* optimum

Example: Counting money

- Suppose you want to count out a certain amount of money, using the fewest possible bills and coins
- A greedy algorithm would do this would be:
At each step, take the largest possible bill or coin that does not overshoot
 - Example: To make Rs 6.39, you can choose:
 - a Rs 5 bill
 - a Rs 1 bill, to make Rs 6
 - a 25ps coin, to make Rs 6.25
 - A 10ps coin, to make Rs 6.35
 - four 1ps coins, to make Rs 6.39
- the greedy algorithm always gives the optimum solution

Branch and bound algorithms

- **Branch and bound algorithms** are generally used for optimization problems
 - As the algorithm progresses, a tree of subproblems is formed
 - The original problem is considered the “root problem”
 - A method is used to construct an upper and lower bound for a given problem
 - At each node, apply the bounding methods
 - If the bounds match, it is deemed a feasible solution to that particular subproblem
 - If bounds do *not* match, partition the problem represented by that node, and make the two subproblems into children nodes
 - Continue, using the best known feasible solution to trim sections of the tree, until all nodes have been solved or trimmed

Example branch and bound algorithm

- Travelling salesman problem: A salesman has to visit each of n cities (at least) once each, and wants to minimize total distance travelled
 - Consider the root problem to be the problem of finding the shortest route through a set of cities visiting each city once
 - Split the node into two child problems:
 - Shortest route visiting city **A** first
 - Shortest route *not* visiting city **A** first
 - Continue subdividing similarly as the tree grows

Brute force algorithm

- A **brute force algorithm** simply tries *all* possibilities until a satisfactory solution is found
 - Such an algorithm can be:
 - **Optimizing**: Find the *best* solution. This may require finding all solutions, or if a value for the best solution is known, it may stop when any best solution is found
 - Example: Finding the best path for a travelling salesman
 - **Satisficing**: Stop as soon as a solution is found that is *good enough*
 - Example: Finding a travelling salesman path that is within 10% of optimal

Improving brute force algorithms

- Often, brute force algorithms require exponential time
- Various *heuristics* and *optimizations* can be used
 - **Heuristic**: A “rule of thumb” that helps you decide which possibilities to look at first
 - **Optimization**: In this case, a way to eliminate certain possibilities without fully exploring them

Randomized algorithms

- A **randomized algorithm** uses a random number at least once during the computation to make a decision
 - Example: In Quicksort, using a random number to choose a pivot
 - Example: Trying to factor a large prime by choosing random numbers as possible divisors