Assignment 3
IE 709: IEOR for Health Care

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Answer 1:

Code for this is named as 'EX1'. Proper comments are written in code itself.

Assumptions:

- Every beam passes through centre of tumor (black region in below figure).
- Each beam is 1 unit wide.
- Each beam is uniform over its width.
- The radiation intensity diminishes to half every 4 units.

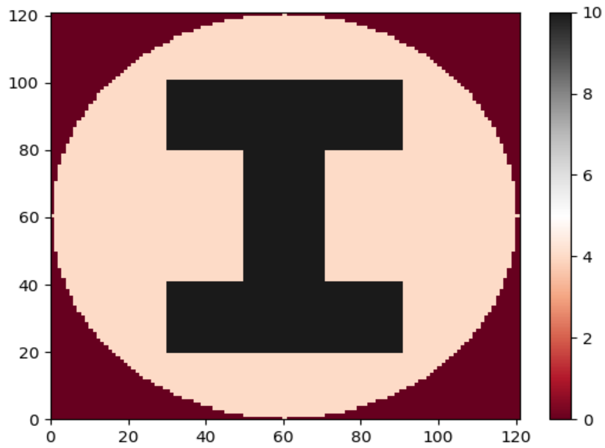

Figure 1: Given Setup

Figures below gives an idea about how an individual beam or a beam at an angle (here at 45 degree) is projected an how its intensity gets decayed with number of units travelled.

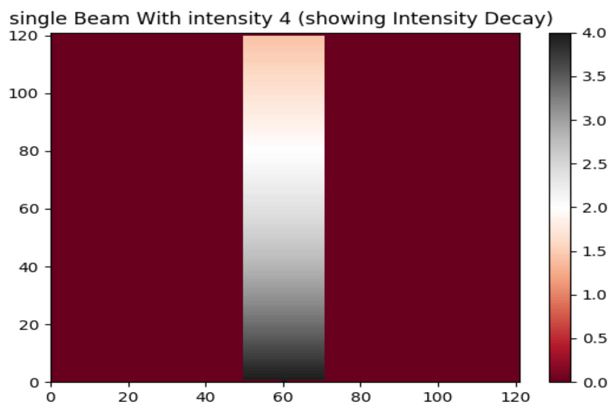

Figure 2: Showing one beam projected with decay.

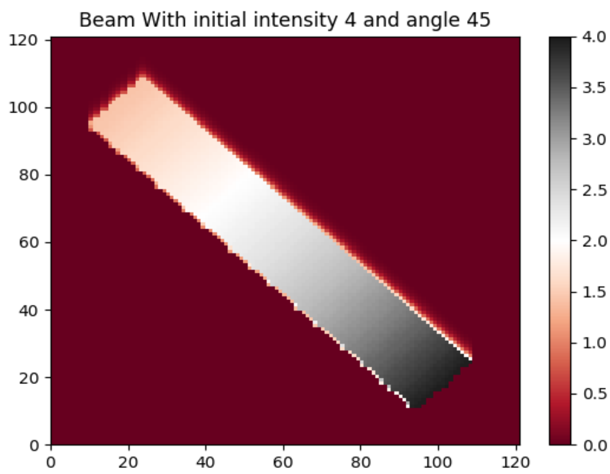

Figure 3: Showing beam projected at angle (45 degree).

Description: Beams were projected at different angles (in degrees) viz. 0, 30, 45, 135, 150, 180, 210, 225, 315, 330.

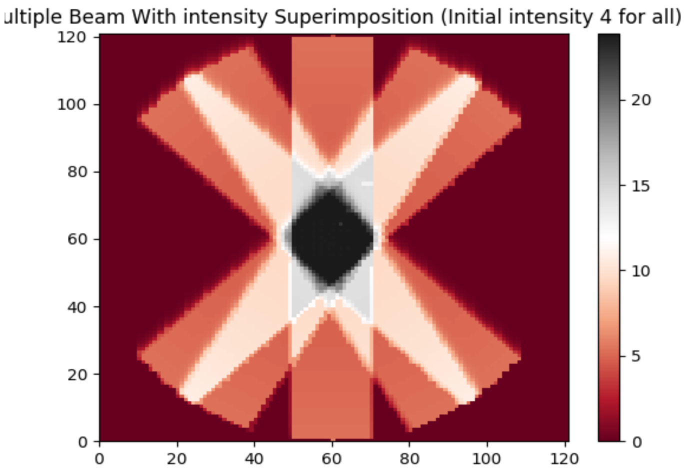

Figure 4: When 10 beams are projected at agnles mentioned above.

LP formulation:

- W_p = intensity of p^{th} beam, p= 1,...,10.
- D_{ij} = intensity at (i,j) pixel.

$$D_{ij} = \sum_{p=1}^{10} D_{ijp} W_p \qquad \forall i, j$$

- Where D_{ijp} = known intensity of p^{th} beam at pixel (i,j).

$$D_{lk} \geq \gamma \qquad (l, k) \in tumorpixel$$
$$D_{nm} \leqslant 4 \qquad (n, m) \in organpixel$$
$$W_p \geq 0 \qquad \forall p$$
$$D_{ij} \geq 0 \qquad \forall i, j$$

- where γ = 10 units as given in question.

The above LP is giving infeasible solution. Reasons for the same may be:

- Less number of beams considered.
- Instead of spliting beam into bemalets and control their intensity individually we considered intensity of beam to be uniform throughout the width.

Answer 2:

(Discussed with Inderjeet Singh).

Part 1:

Here **LP is:**

$$minimize : z^0 = \sum_{(u,v)\in A} x_{uv}$$

subjected to

$$\sum_{(u,v)\in A} R_{uv}x_{uv} = b$$

$$x_{uv} \geqslant 0, \qquad \forall(u,v) \in A$$

and after row transformation it gives an equivalent LP where each new column has one +1 in u^{th} row and one -1 in v^{th} row, and all other values are zero.

$$minimize : z^0 = \sum_{(u,v)\in A} x_{uv}$$

subjected to

$$\sum_{(u,v)\in A} R'_{uv}x_{uv} = b'$$

$$x_{uv} \geqslant 0, \qquad \forall(u,v) \in A$$

Now it is equivalent to network flow problem and its objective is same as that of original LP. And as elementary row operations were used to obtain the new system of linear equations, any solution x of $\sum_{(u,v)\in A} R_{uv}x_{uv} = b$ **(original LP)** would also be the solution of the new system of linear equations **(which correspond to network flow problem)** because elementary tranformations doesn't change the solution.

Coverse:

Converse is also true

- Proof:

Let x^* be the solution of network flow LP.
For the network flow system $Ax' = b$ apply row transformations as keep the first row as it is and from second row onwards add each row with the previous row (for e.g. $R_2 = R_2 + R_1$, $R_3 = R_3 + R_2$ and so on).
It would give the same constraints as original LP and objective function also be the same. Hence both will have same solution.