Oil seed procurement network design
IE 714: Quantitative Models for Supply Chain Management

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Introduction

We need to set up a system for extracting oil from oil-seeds and transport the oil-seeds, seed cakes and extracted oil. We are provided with different costs: fixed costs, processing costs and the transportation costs in different levels of the supply chain.
For setting up our oil extraction system, we can have different objectives as per our requirements. The underlying criteria is that we need to process and extract all the oil from the oilseeds grown in all growing area. Following objectives make sense in the oil seed problem:

- Minimize total cost
- Maximize oil extraction
- Maximize total profit
- Minimize the unit cost of oil (total cost/total oil extracted)

We will discuss all the 4 objectives in detail.

Assumptions

Given in the problem

- A plant of only 1 technology can be setup at a particular location
- Installation cost of any plant is ignored
- Production cost of oilseeds at the growing area is ignored
- all yield (oilseed) from one growing region will go to one oil technology plant
- Value of the by-product seedcake is ignored

Additional assumption taken:

- E_2 extraction plant (near refinery) is available but we have the choice to install the technology (Low, Medium, and High) as per our requirement.
- If E_2 extraction plant is required, we will consider the fixed cost of running it otherwise not.
- Multiple routes are available for transporting the oil from processing plant to refinery (e.g. G_3-Refinery or G_3-G_4-Refinery) and oil will be transported via most economical route.

Mathematical Model

Definations: Variables, sets and parameters

Sets

- Oil technology set $T = \{T_1, T_2, T_3\}$
- Growing region set $G = \{G_1, G_2, G_3, G_4, G_5, G_6, G_7, G_8\}$
- Possible oil technology region set $I = \{G_1, G_2, G_3, G_4, G_5, G_6, G_7, G_8\}$
- Seed cake oil extraction plant set $E = \{E_1, E_2\}$
- Seed cake oil extraction plant label set $L = \{L, M, H\}$
- set $K = \{G_1, G_2, G_3, G_4, G_5, G_6, G_7, G_8, E_1\}$

Variables

$$x_{t,i} = \begin{cases} 1, & \text{if technology } t \text{ open at } i \qquad t \in T, i \in I \\ 0, & \text{otherwise} \end{cases}$$

$$u_{t,i,g} = \begin{cases} 1, & \text{if oilseed transport from } g \text{ to } i \text{ where } t \text{ technology established} \qquad t \in T, i \in I, g \in G \\ 0, & \text{otherwise} \end{cases}$$

$$y_{l,e} = \begin{cases} 1, & \text{if label } l \text{ establish at } e \qquad l \in L, e \in E \\ 0, & \text{otherwise} \end{cases}$$

$$z_{l,e,i} = \text{seedcake recieved at extraction plant } e \text{ of label } l \text{ from } i \qquad l \in L, e \in E, i \in I$$

$$r_{l,i,t} = z_{l,e,i} * x_{t,i}$$

$$m_{i,k} = \begin{cases} 1, & \text{if oil transport from } i \text{ to refinary via } k \qquad i \in I, k \in K \\ 0, & \text{otherwise} \end{cases}$$

$$p_{t,i,g,k} = u_{t,i,e} * m_{i,k}$$

Parameters

- f_t^1 is fixed cost for oil technology plant, where $t \in T$
- p_t^1 is processing cost for oil technology plant, where $t \in T$
- $c_{i,g}^1$ is transportation cost of oilseed from growing region to oil technology plant, where $g \in G, i \in I$
- f_l^2 is fixed cost for Extraction plant, where $l \in L$
- p_l^2 is processing cost for extraction plant, where $l \in L$
- $c_{e,i}^2$ is transportation cost of seedcake from oil plant i to extraction plant e, where $e \in E, i \in I$
- $co_{i,k}^1$ is cost of tranporation from i to k, where $i \in I, k \in K$
- $co_{i,k}^2$ is cost of tranporation from k to refinary/E_2, where $i \in I, k \in K$
- cap_t^1 is capacity of oil technology plant, where $t \in T$
- cap_l^2 is capacity of Extraction plant, where $l \in L$
- q_t is percentage of oil in seed cake, where $t \in T$
- O_t is crude oil produced by oil technology plant t per unit oilseeds, where $t \in T$
- SC_t is seedcake amount in oil technology plant t per unit oilseeds, where $t \in T$

Objective Function 1: Minimize total cost

$$
\begin{array}{cl}
 & \sum_{i\in I,t\in T} f^1_t x_{t,i} + 1500 * (\sum_{i\in I,g\in G,t\in T} u_{t,i,g}p^1_t) + 1500 * (\sum_{i\in I,g\in G,t\in T} u_{t,i,g}c^1_{i,g}) \\
+ & \sum_{e\in E,l\in L} f^2_l y_{l,e} + \sum_{l\in L,e\in E,i\in I} z_{l,e,i}p^2_l + \sum_{l\in L,e\in E,i\in I} z_{l,e,i}c^2_{e,i} \\
+ & \sum_{i\in I,g\in G,t\in T} p_{t,i,g,k}O_t * (co^1 i, k + co^2 k, Plant) * 1.5 + \sum_{i\in I,l\in L,t\in T} q_t c^1_{'E1','E2'} r_{l.j,t}
\end{array}
$$

Constraints

$$\sum_{i\in I,t\in T} u_{t,i,g} = 1 \qquad \forall g \in G \tag{1}$$

$$\sum_{t\in T} x_{t,i} \leq 1 \qquad \forall i \in I \tag{2}$$

$$\sum_{g\in G} u_{t,i,g} \leq \frac{cap^1_t}{1500} x_{t,i} \qquad \forall t \in T \& i \in I \tag{3}$$

$$\sum_{l\in L} y_{l,e} \leq 1 \qquad \forall e \in E \tag{4}$$

$$\sum_{e\in E,l\in L} z_{l,e,i} = \sum_{t\in T,g\in G} u_{t,i,g} SC_t * 1.5 \qquad \forall i \in I \tag{5}$$

$$\sum_{i\in I} z_{l,e,i} \leq cap^2_l y_{l,e} \qquad \forall e \in E \& l \in L \tag{6}$$

$$r_{l,i,t} \leq cap^2_l x_{t,i} \qquad \forall l \in L, t \in T, i \in I \tag{7}$$

$$r_{l,i,t} - z_{l.'E1',i} \geq -(1 - x_{t,j}) cap^2_l \qquad \forall l \in L, t \in T, i \in I \tag{8}$$

$$r_{l,i,t} - z_{l.'E1',i} \leq (1 - x_{t,j}) cap^2_l \qquad \forall l \in L, t \in T, i \in I \tag{9}$$

$$P_{t,i,g,k} \leq m_{i,k} \qquad \forall g \in G, t \in T, i \in I, k \in K \tag{10}$$

$$P_{t,i,g,k} \leq u_{t,i,g} \qquad \forall g \in G, t \in T, i \in I, k \in K \tag{11}$$

$$P_{t,i,g,k} \geq m_{i,k} + u_{t,i,g} - 1 \qquad \forall g \in G, t \in T, i \in I, k \in K \tag{12}$$

constraint 1 ensures that all yield of any growing area goes to only one oil plant, constraint 3 and constraint 6 are the capacity constraints for oil plants and extraction plants respectively. constraint 5 ensures seedcake received at all the extraction plants from a particular oilplant is equals to the seed cake generated at that oilplant. contraints 7,8,9,10,11,12 are used to linearize the model.

We used AMPL (*gurobi*) for solving our model, and results are given below:

Solution of Objective 1: Minimize total cost

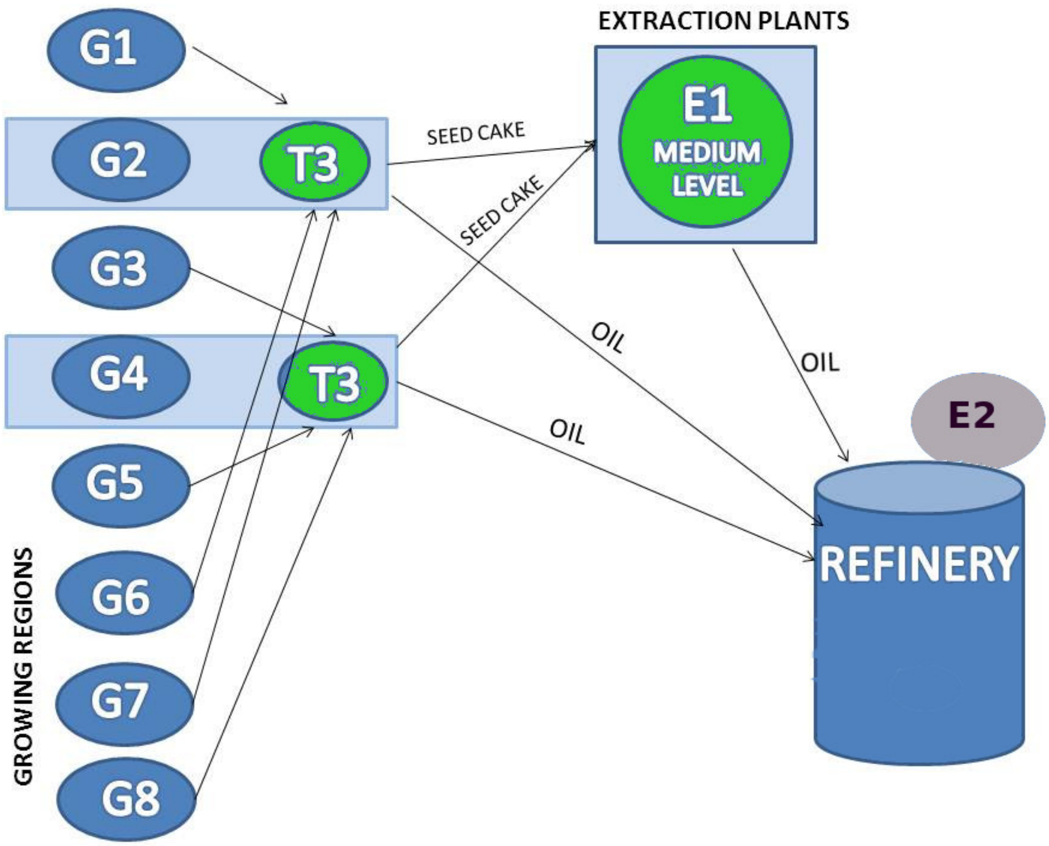

Figure 1: Schematic view of the oilseed procurement supply chain

- The minimum total cost for our system is Rs.1,60,26,008
- Two processing plants of T_3 technology were opened in total, one each in growing area G_2 and G_4
- E_1 extraction plant is opened and E_2 extraction plant is not required
- G_2 receives 6000 Tonnes of oil-seeds from growing area G_1, G_2, G_6, G_7
- G_4 receives 6000 Tonnes of oil-seeds from growing area G_3, G_4, G_5, G_8
- 5004 Tonnes of seedcake is received from each oil processing plant G_2 and G_4 at E_1 extraction plant
- 996 Tonnes of oil from each oil processing plant G_2 and G_4 is transported to refinery via direct route
- 1601.28 Tonnes of oil from E_1 extraction plant is transported to refinery

Objective 2: Maximize total oil output

Total oil output is the total oil extracted from the oil seeds at processing plant and further at extraction plant. We can see that the amount of oil extracted from the oil seeds depend only on the technology at the processing plant and is independent of the extraction plant technology. Total oil which will be extracted (per tonne of oil seeds) after using different processing plant technology:

1. $T_1 : 250 + (7.5\%)(750) = 306.25$ Kg
2. $T_2 : 150 + (12.5\%)(850) = 256.25$ Kg
3. $T_3 : 166 + (16\%)(834) = 299.44$ Kg

So, it is clear that using processing technology T_1 gives us the maximum oil output of 306.25 Kg per tonne of oil-seed.
Total oil-seed quantity(Tonnes) $= (1500) * 8 = 12,000$
Therefore, net oil output (Tonnes) $= (12,000) * (306.25) = 3675$

Results of maximum oil output:-

1. We will get a maximum of 3675 Tonnes of oil only by installing T_1 technology processing plant.
2. Location of T_1 processing plant does not matter.
3. Total number of T_1 processing plants does not matter.
4. Technology and location of extraction plant does not matter.

Now, we can minimize the total cost given the oil output is maximized. It can be solved similar to *objective function 1* with following change in constraint that only T_1 technology processing plant can be installed at any growing area. It will ensure that maximum oil is extracted.

Objective 3: Maximize total profit

Total Profit = Revenue Total Cost
Where, Revenue = (Total oil extracted)*(Unit price of oil)
We can assume the unit price of oil for solving this model. Total cost equation will be same as used in *objective function 1* and total oil output will be same as used in *objective function 4*. Solving this model similarly will give us the optimal answer.

Objective 4: Minimize Unit cost of Oil

Here we will minimize the unit cost of oil extracted.
where, Unit cost = Total cost/total oil output
All the equations and constraints are same as used in objective function 1. Total oil (in Tonnes) output is given by:

$$\boxed{\text{Total oil} = 1.5 * (\sum_{t\in T, i\in I, g\in G} u_{t,i,g} O_t) + \sum_{l\in L, t\in T, i\in I} r^1_{l,i,t} q_t + \sum_{l\in L, i\in I, t\in T} r^2_{l,i,t} q_t}$$

The objective function is non-linear and not smooth, so model is mixed integer non-linear programme. we tried solving this model using various available non-linear solver, but we failed to get the answer.