Brief Introduction to Bond Math

Pre-requisite: Wall St. Prep course

Suggested Reference: Fixed Income Mathematics, Fabozzi 4th Edition

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Bond Math – Session 1, Chapter 1

Introduction

Introduction

Why a bond math course?

- Concepts are broadly relevant to all finance
  - e.g. TVM is fundamental to equity analysis as well
- Large and challenging problem domain
  - Core knowledge of bond math is essential to building expertise
- Complexity of fixed income analysis

Ultimately, it underlies everything we do and sell in Fixed Income Group!

Introduction

Role of Fixed Income in capital markets:

- Transfer of consumption over time
  - Principal role historically
- Alternate capital structure to equity
- Transfer of risk
  - Interest rate risk
    - Optionality
    - Rates derivatives
  - Credit risk
    - Structured products
    - Credit Derivatives

Introduction

Highlights in the history of Fixed Income

1600s – Earliest corporate debt

1693 – First government bond: a tontine to fund England's war v. France

19th Century – Rapid growth of corporate bond market

1930s – Establishment of GSEs

1968 – First mortgage pass-through security (GNMA)

1983 – First Collateralized Mortgage Obligation (Solomon Bros, First Boston)

1987 – First Collateralized Debt Obligation (Drexel)

1994 – First Credit Default Swap (JPM)

Introduction

- Bonds were once very simple
  - Simple cash flow structure
  - Stable rate environment
  - One question: Will the issuer be able to make the promised payments?

- Today bonds are exceedingly complex to analyze
  - Complex cash flow structure
  - Volatile markets
  - “If there were no computers, there would be no CMOs.”

Introduction

- The Fallacy of I.R.R.
  - We value each cash flow assuming the same interest rate, regardless of when the cash flow is received.
  - We assume the buyer holds the bond to maturity.
  - We implicitly assume that all cash flows are reinvested at the quoted yield.

Even an analytic result as basic and standard as yield has serious shortcomings and must be combined with other information in order to give the investor a complete picture.

Bond Math – Session 1, Chapter 2

Overview of Fixed Income Securities and Derivatives

Overview of Fixed Income Securities and Derivatives

- General Features of Bonds
  - Term to Maturity
  - Par Value
  - Coupon Rate
    - Fixed vs. Floating
  - Provisions for Paying off Bonds
    - Bullet
    - Amortizing
    - Call

Overview of Fixed Income Securities and Derivatives

- Bonds
  - U.S. Treasuries
    - Backed by the US Government
    - Bills, Notes, Bonds
    - TIPS
    - Zero Coupon Bonds
  - Federal Agency Securities
    - GSE’s: FHLMC, FHLBB, FNMA
    - MBS and Debentures

Overview of Fixed Income Securities and Derivatives

- Bonds (continued)
  - Corporate Bonds
    - Utilities, Transportations, Industrials, Banks and Finance
    - Rating Agencies: Standard & Poor’s, Moody’s, Fitch
  - Municipal Securities
    - City, County, School Districts
    - Taxable vs. Tax-exempt
    - GO vs Revenue

Overview of Fixed Income Securities and Derivatives

- Securitized Products
  - Mortgage-Backed Securities (MBS)
    - Residential
      - Agency vs. Non-Agency (“Whole Loans”)
      - CMO’s, STRIPs
    - Commercial (CMBS)
      - Non-recourse
      - Call protection (Prepay Lockout, Prepay Penalty, Yield Maintenance)

Overview of Fixed Income Securities and Derivatives

- Securitized Products (continued)
  - Asset-Backed Securities
    - – Autos, Student Loans
    - – Credit Cards
      - Revolving Period; Amortization Period
  - Collateralized Debt Obligations
    - – Heterogeneous portfolio of collateral
    - – Managed vs. Static
    - – Balance-sheet vs. Arbitrage
    - – Senior, Mezzanine & Equity

Overview of Fixed Income Securities and Derivatives

- Preferred Stock
  - Equity Security
  - Perpetuity
  - Cumulative vs. Non-cumulative
  - Fixed vs. Floating

Overview of Fixed Income Securities and Derivatives

- Interest Rate Derivatives
  - Interest Rate Futures
    - Exchange traded
    - “Cheapest-to-deliver”’; Delivery Options
  - Forward Contracts
    - OTC
  - Forward Rate Agreements
    - OTC; Forward Borrowing Agreement

Overview of Fixed Income Securities and Derivatives

- Interest Rate Derivatives (continued)
  - Interest Rate Swaps
    - Pay Fixed/Receive Float or Pay Float/Receive Fixed
    - Fixed Rate or “Swap Rate”
    - Swap Spread
    - Variations: Basis Swap, FX Swap, Amortizing Swap, Swaption
  - Options
    - Physicals vs. Futures
  - Caps & Floors

Overview of Fixed Income Securities and Derivatives

- Credit Derivatives
  - Credit Default Swap
    - Protection Buyer, Protection Seller
    - Protection Premium
    - Reference Obligation or Reference Entity
    - Credit Event
    - Variations: Portfolio Default Swap, Basket Default Swap, Synthetic CDO

Bond Math – Session 1, Chapter 3

Future Value

Future Value

- Compound Interest is “interest on interest”

  $1,000 x 1.07

  $1,000 x 1.07 x 1.07

  $1,000 x 1.07 x 1.07 x 1.07

  …

  $\$1{,}000 \times (1.07)^8 = \$1{,}718.19$

- It is common to compute the future value of $1, then multiply by the “original amount” or “face value” of the investment
- We can assume a change in interest (investment) rates [Fabozzi, p. 32]
- We can assume a different rate in each period

Future Value

- Using observable market information such as Treasury Bond yields and prices, it is possible to construct an "appropriate" sequence of projected rates – The Forward Curve [Fabozzi, Ch. 8]
- Fractional periods are OK
- Multiple periods per year are OK
- The number of periods per year is known as the "Compounding Frequency"

Future Value

- An “Ordinary Annuity” – regular payments at fixed intervals, beginning one period from now
- There is a “Closed Form” formula for the Future Value of this common type of cash flow stream

$$FV = A\left[\frac{(1+i)^{N}-1}{i}\right]$$

Bond Math – Session 1, Chapter 4

Present Value

Present Value

Rearranging terms in the Future Value formula, we can solve for the "present value".

$$PV = FV\left[\frac{1}{(1+i)^N}\right]$$

This formula tells us, assuming a certain rate of interest on invested cash, what the present value of a future payment is in today's "dollars".

Present Value

Properties of Present Value

- The higher the rate of interest assumed, the less the present value of the future cash flow.
- The further out in the future a payment is, the less it is worth today.

Note also, that the present value of the sum of a stream of future cash flows, is simply the sum of the present values.

Present Value

The Present Value of an Ordinary Annuity

- An **annuity** is a stream of equal payments, occurring at a regular intervals, say annually or semi-annually.
- An **ordinary annuity** is an annuity in which the first payment occurs exactly one period from now.

$$PV = A\left[\frac{1-\left[\frac{1}{(1+i)^{N}}\right]}{i}\right]$$

There are many examples of ordinary annuities in the fixed income markets.

Present Value

The Present Value of a Perpetuity

- A **perpetuity** is a perpetual annuity.
- The present value of a perpetuity is interesting mathematically.

$$\lim_{N\to\infty} A\left[\frac{1-\left[\dfrac{1}{(1+i)^N}\right]}{i}\right] = \frac{A}{i}$$

Present Value

Other things to consider

- For payment frequencies other than annually, we use ‘n’ rather than ‘N’
- We can use different discount rates for payments which occur at different times. [Fabozzi, Chapter 8]