Vibration of Floating Structures

Autumn 2017 NA31007 3-1-0, 4 credits

Homework #1 Single DOF system : Free and Forced Vibration. Damping.

Reciprocating unbalance, Whirling of rotating Shafts, Transient Vibration.

Date Assigned :25/7/2017 Date Due : 22/8/2017

A) Free Vibration. (Chapter 2) Calculate the following:

Consider a single DOF system, of mass *m* kg, damping ratio ζ, stiffness *k* N/m.

Put *m* = last two digits of your roll number, *k* = the three digits of your hostel room number, ζ = 0, 0.02, 1, 1.2

Hard copy : show the derivation of the complimentary function (both frequency and amplitude).

MATLAB : Given the following initial conditions, plot the displacement, velocity, and acceleration as functions of time, for all damping ratios. Draw the *phase plane* (displacement vs. velocity/frequency).

(a) $x(0) = 1, \frac{dx(0)}{dt} = 0$

(b) $x(0) = 0, \frac{dx(0)}{dt} = -1$

(c) $x(0) = -0.5, \frac{dx(0)}{dt} = 1$

Calculate the energy loss due to damping in each case. Show the derivation in hard copy.

Now consider the damping to be unknown. If the amplitude becomes 10% of itself from n^{th} to $(n+k)^{th}$ cycle, what is the damping ratio? Plot the corresponding time series (MATLAB).

(a) n = 2, k = 3.

(b) n = 10, k = 6.

B) Forced Vibration. (Chapter 3)

1) Given a harmonic forcing of amplitude $F_0 = 1$, and tuning factor $\Lambda = 0.5, 1, 1.5.$ Hard copy : show the derivation of the particular integral (both amplitude and phase). MATLAB : Plot the response displacement, velocity, acceleration times-series, for the above three cases of initial conditions. The plots should be a combination of the complementary function and the particular integral. Plot the forcing (t) on the same plot for reference. Find the phase differences between the forcing and the response for each of the three tuning factor and damping ratios.
2) Considering the transient (Complementary Function) to be 'negligible' when it has actually decayed to 1% of its initial amplitude, after how many cycles is the steady-state achieved for ζ = 0.02, 0.05, 0.10, 0.20, 0.50? Show on hard copy and prove in MATLAB plots. Hard-copy : derive the logarithmic decrement from its definition.
3) Hard copy : How do the half power points change for changing damping ratio?
4) Hard-copy:Draw the *normalized* phasor diagram for the steady-state displacement, velocity, acceleration; and the forcing,.
5) Hard copy : At resonance, find the amplitude for each tuning factor. What is the energy loss due to damping?
6) Hard copy : What is the tuning factor after which the dynamic response amplitude is always less than the static deflection?
7) Hard copy : What is the damping ratio for which the dynamic response amplitude is always less than the static deflection, irrespective of the tuning factor.

C) Support motion (Chapter 3)

Suppose the 1 DOF system given in part (A) (with the stiffness and damper supporting it from below), is excited by its base displacement as $y(t) = Ycos(\omega t)$. MATLAB : Plot the base motion and body response due to the support motion as a times series (on the same plot). Here, tuning factor = 0.5, 1, 1.5. Amplitude $Y = 10^n$, $-2 < n\ (integer) < 2$. Show all derivations in hard copy.

D) Rotating unbalance (Chapter 3)

In the 1DOF system in Part (A), let the mass have a rotating unbalance mass $m_u = \frac{m}{p}$, $p = 10^n$, $-5 < n(integer) < 0$.The eccentricity of this mass from the rotating centre is $e = 0.1, 0.5, 1.$ The rotating unbalance causes unbalanced vertical forces, which causes the 1-DOF system to oscillate. MATLAB : Plot the response time-series at tuning factor = 0.5, 1, 1.5. How much force is transmitted to the base? Show all derivations in hard copy.

E) Transient Vibration (Chapter 4) : MATLAB

The initial displacement and velocity are zero. Plot the excitation and the response as functions of time on the same plot. Non-dimensionlize the time-axis by the rise time. Plot the Shock response spectrum (SRS). The forcing is as follows : A saw-tooth signal, which has a sweep time of τ, a rise time of α times τ, where α < 1. The force rises to a peak magnitude of P_0 in time ατ, decreases in time (1-α)τ, and then become a constant of 10% of P_0.

Steps (Euler's explicit scheme) :

- Convert the second order GDE into two first order equations. Assume $v(t) = \dot{x}(t),\ u(t) = x(t).$

  $$m\dot{v}(t) + c\,v(t) + k\,u(t) = F(t); \quad \dot{u}(t) = v(t).$$

- Use Initial conditions $v(0)=0,\ u(0) = 0.$
- Use Euler's explicit scheme to integrate the velocity and the displacement as functions of time.

  Use the forward difference formulae : $\dot{v}(t) = \dfrac{v^{n+1} - v^n}{\Delta t}$, $\dot{u}(t) = \dfrac{u^{n+1} - u^n}{\Delta t}$.

- Substitute the above in the two first-order GDEs. $m\dfrac{v^{n+1} - v^n}{\Delta t} + c\,v^n + k\,u^n = F(t^n);\ \dfrac{u^{n+1} - u^n}{\Delta t} = v^n$.
- Make the LHS as the (n+1)th time-step information, which is available from the information at the nth time step in the RHS. $v^{n+1} = v^n - \Delta t\dfrac{c}{m}v^n - \Delta t\dfrac{k}{m}u^n + \Delta t\dfrac{F(t)}{m}; \quad u^{n+1} = u^n + \Delta t v^n$.
- Set up the GDE in the matrix form :

  $$\begin{Bmatrix} v^{n+1} \\ u^{n+1} \end{Bmatrix} = \begin{bmatrix} \left(1 - \Delta t\dfrac{c}{m}\right) & -\Delta t\dfrac{k}{m} \\ \Delta t & 1 \end{bmatrix} \begin{Bmatrix} v^n \\ u^n \end{Bmatrix} + \begin{Bmatrix} \Delta t\dfrac{F(t)}{m} \\ 0 \end{Bmatrix}.$$

- Time-integrate the pair of equations. Plot u(t), which is the displacement as a function of time.
- Keep the time-step small to ensure convergence of the scheme, about 0.01 times of the rise time.