Vibration of Floating Structures

Autumn 2017 NA31007 3-1-0, 4 credits

Homework #2

Multi-DOF system, Vibration Absorption, Vibration of a Floating Euler-Bernoulli beam

Date Assigned : 03/10/2017 Date Due : 07/11/2017

A) Free Vibration of 3-DOF system. (Chapter 5)

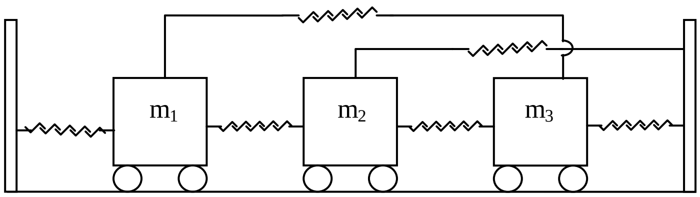


Fig. 1 Multi Degree of Freedom system (free vibration)

Consider the above 3 DOF system (Fig.1). The numerical values of masses (m_i) and stiffnesses (k_i) will be generated by a random number generator in your program (using function *rand*), and the subsequent results must follow.

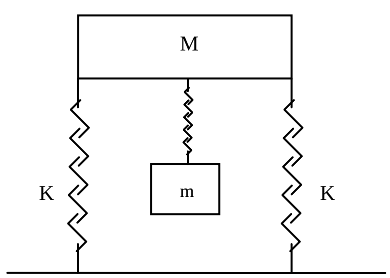
1) Establish the set of governing differential equation for free vibration in the matrix form.
2) Establish the frequencies of free vibration. (Eigen Value method)
3) Find the modeshapes (Eigen Vectors). Draw them.
4) Find the generalized masses and generalized stiffnesses.

B) Forced Vibration of 3-DOF system. (Chapter 5)

Consider a force $F_1 = f\cos(\omega t)$ acts on m_1, and $F_2 = f\sin(\omega t)$ acts on the m_2. No force acts on the m_3.

1) Find the generalized forcing vector. Plot as a function of time.
2) Find the principal coordinates as functions of time. Plot them.
3) Establish the expressions for the modal displacements. Plot them as function of time.

C) 2 DOF system : Vibration attenuator. (Chapter 5)



The vibration amplitude of Mass M (Fig.2) is found to be unacceptably large. A small mass m is attached through a spring to it, *in order to reduce its oscillations*. The mass 'm' is a variable. Plot the resultant **amplification factor** of M vs. the tuning factor of **m**. Test case : m/M = 0.01-0.2, k/K = 0.1-1.
Plot the resultant **amplification factor** of M vs. the **mass m;** for an external forcing $F(t) = F_0\cos(\omega_e t)$ acting on M.

Fig.2 Vibration attenuator.

D) Vibration of a Floating Euler-Bernoulli beam (Chapter 8)

1) Consider a Mild Steel ($\rho = 7850$ kg/m^3, E = 209 GPa, L = 100m) Euler-Bernoulli beam. For rectangular cross section : B = 2 m, D = 1m. Discretization : Use 100 points to define the length.
2) Generate the first 10 transverse vibration modeshapes for the given boundary conditions (Check attached EXCEL Sheet). Check their orthogonality.
3) Establish the inertia matrix. Mesh it. Establish the stiffness matrix. Mesh it.
4) **Write** the Modal GDE in the matrix form, for free vibration. Include 5% proportional damping.
5) Plot the principal coordinates as function of time (both C.F. and P.I., first separately and then combined)
6) Generate the total deflection as a function of space and time and mesh it.
7) Assume a harmonic Froude-Krylov forcing $F(t)\ \alpha\ \cos(kx-\omega t)$, obeying the deep water dispersion relation. The amplitude of the wave is derived from the wave-breaking limit (Derive the amplitude of the Froude-Krylov force). The wavelength equals the length of the beam. Establish the expression for the generalized forcing and plot them as a function of time. *All formulations must be shown in the hard copy.*
8) **Write** the Modal GDE in the matrix form, for forced vibration.
9) Plot the principal coordinates as function of time, including the **complimentary functions**. The initial conditions are given in the EXCEL Sheet provided. (Hint : since the initial conditions are for only one modeshape, the other modeshapes will not have any CF).
10) Generate the **total deflection** as a function of space and time and mesh it.
11) Generate the total **bending moment, bending stress, shear stress** as functions of space and time, and mesh them. Mention the location, instant and magnitude of its maximum.

E) Propeller Shaft Vibration due to rotating unbalance.

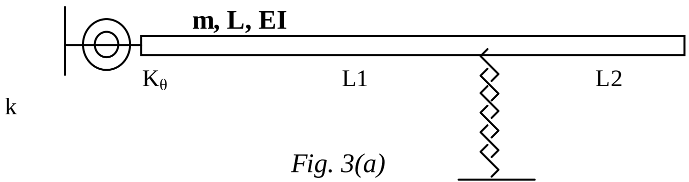


Fig. 3(a)

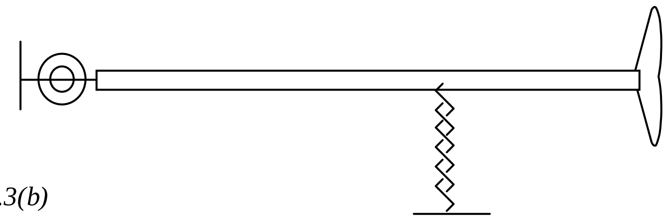
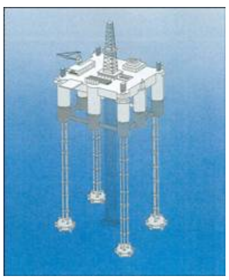


Fig.3(b)

Find the modeshape of the beam shown in *Fig.3(a)*. Plot the deflection, slope, bending moment, and shear force of the first 3 modeshapes of the beam. Keep the position of the elastic support user-friendly.

Consider a 4-bladed propeller of mass M, diameter D, rotating at a frequency ω rad/sec, attached to the free end of the beam, as shown in *Fig.3(b)*. One of the blades is heavier than the other three by 1 kg, acting at D/2 away from the shaft axis. This causes whirling of the shaft. What is the deflected shape of the beam due to this whirl? How much force is transmitted to the base of the elastic support?

F) Vortex-induced Vibration of compliant offshore structure (Fig.4) : tension leg platform (TLP)



BACKGROUND : The tension is caused by excess buoyancy. The platform is first ballasted to increase the draft and the tendons are attached to the sea bed. Deballasting reduces the weight, setting up axial tension in the tendons due to excess buoyancy. This set-up reduces excess motions of the platform. The heave DOF is completely restricted. Currents past the tendons lead to vortex shedding and vortex-induced cross-flow vibrations.

Fig.4

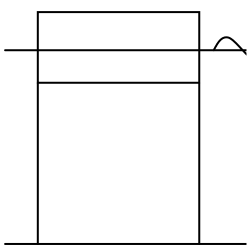


Fig.5

Consider a pontoon-type TLP as shown in **Fig. 5**, of dimensions 10 m by 8 m by 5 m depth. The draft is 3 m. Suppose the buoyancy is 10% greater than the weight. Model the 4 tendons as circular strings of mild steel, 500 m long, with D = 50cm diameter. What is the natural frequency of vibration of each tendon, including the added mass? The cross-section of the mooring line is circular. How does the natural frequency change when the platform is ballasted to provide buoyancy 5% and 15% greater than the weight?

Consider a current of U = 2 m/sec. For a Strouhal number of 0.2, what is the frequency of vortex shedding? The resonant frequency ± 5% must be avoided. What range of buoyancy must be avoided?

The lift force on the tendon due to the current, is expressed as $F_l = \frac{1}{2}\rho_{water}\, U^2 D\, C_L \sin(\omega_{vortex-shedding}\, t)$

The lift coefficient for a non-potential flow past a circular cylinder is given as $C_l = \frac{1}{2}\cos(\omega_{vortex-sheddin\,g}\, t)$

Find the amplitudes of string vibration at the half-power points, assuming 5% damping.