Conditional probability

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$\downarrow$

probability of A given B

I am giving the information of $\underline{B}$ then calculate the probability of $\underline{A}$.

$P(\diamond \mid Red) \Rightarrow$ I am giving Red cards, I am asking the probability of $\diamond$

$$= \frac{13}{26} \Rightarrow \frac{P(\diamond \cap Red)}{P(Red)} = \frac{13/52}{26/52}$$

$$P(K \mid Pic) = \frac{4}{12} = \frac{P(K \cap Pic)}{P(Pic)} = \frac{4/52}{12/52}$$

$$P(A \mid (\sim \cup R)) = \frac{2}{20} = \frac{P(A \cap (\sim \cap R))}{P(\sim \cap R)} = \frac{2/52}{20/52}$$

Independent Events

If A and B are two events having information about one event is not influencing event then A & B are two independent events.

$P(\text{Red} \mid \text{Rainy day}) = 26/52$ Independent events.

$P(\text{Red} \mid \text{Sunday}) = 26/52$

Having information about one event is not influence outcome of another event.

→ If A and B are independent.

$$P(A \mid B) = P(A)$$

$P(\text{Red} \mid \text{Rainy day}) = P(\text{Red})$

$$\frac{P(A \cap B)}{P(B)} = P(A)$$

$$P(A \cap B) = P(A)\,P(B)$$

If A and B are two events, which are independent then $P(A \cap B) = P(A) \cdot P(B)$

Example: I am tossing two coins, what is the probability of P(H H)

$$\Omega = \{ (H_1 H_2), (H_1 T_2), (T_1 T_2), (T_1 H_2) \}$$

$$P(H\,H) = P(H \cap H) = \frac{1}{4} \rightarrow (1)$$

$$= P(H)\,P(H)$$

$$= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \rightarrow (2)$$

So tossing a coin is independent of each other.

⇒ Sampling with replacement ← $P(G) = \frac{10}{20}$ → independent

⇒ Sampling without replacement ← $P(G) = \frac{9}{19}$ → Dependent

Mutually exclusive events / Disjoint events.

If A & B are two disjoint events.

$P(A \cap B) = 0$

Disjoint : No common elements.

$P(H \cap T) = 0$ Tossing a coin, but we cannot get Head & tail at a time.

⇒ independent : 2 events happening at a time

⇒ mutually exclusive events: At a time is not possible

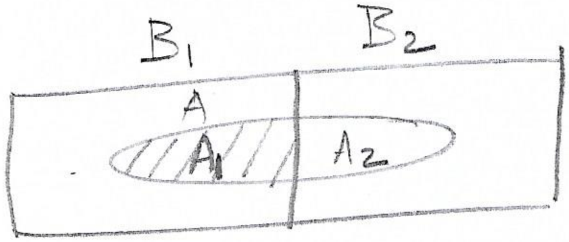
All the outcome's are mutually exclusive.

Boye's Low

1. Total probability
2. Conditional probability

1. Total probability:

B_1, B_2 } Disjoints
A_1, A_2 }



$P(A) = P(A_1 \cup A_2)$

$= P(A \cap B_1) + P(A \cap B_2) \rightarrow$ 5 $\rightarrow$ total probability

$P(B_1|A) = \frac{P(B_1 \cap A)}{P(A)}$ $\quad$ $P(B_2|A) = \frac{P(B_2 \cap A)}{P(A)}$

$P(B_1 \cap A) = P(B_1|A) \cdot P(A) \rightarrow$ 1 $\quad$ $P(B_2 \cap A) = P(B_2|A) \cdot P(A) \rightarrow$ 2

$P(A|B_1) = \frac{P(A \cap B_1)}{P(A)}$ $\quad$ $P(A|B_2) = \frac{P(A \cap B_2)}{P(A)}$

$P(B_1 \cap A) = P(A|B_1) \cdot P(A) \rightarrow$ 3 $\quad$ $P(B_2 \cap A) = P(A|B_2) \cdot P(A) \rightarrow$ 4

from 1 & 3

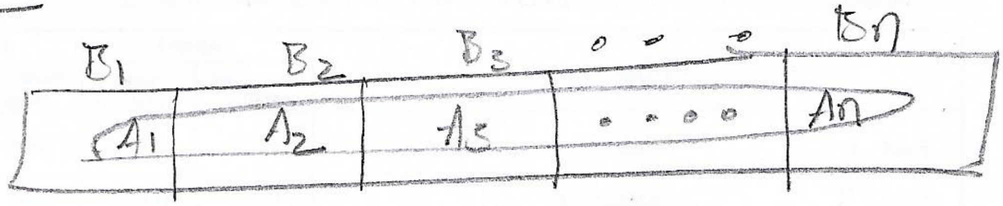
$P(A|B_1) \cdot P(B_1) = P(B_1|A) \cdot P(A)$

$$P(B_1|A) = \frac{P(A|B_1)\cdot P(B_1)}{P(A)} \longrightarrow 6$$

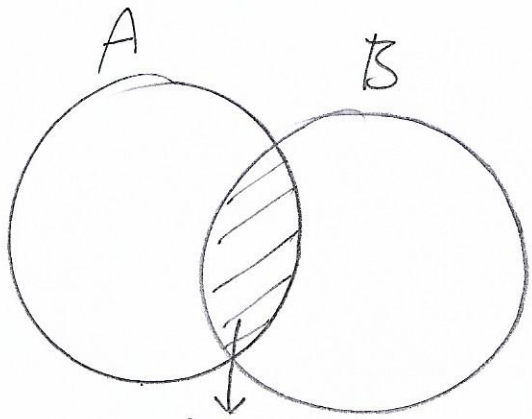
$$P(B_1|A) = \frac{P(A|B_1)\cdot P(B_1)}{P(A\cap B_1) + P(A \cap B_2)}$$

$$P(B_1|A) = \frac{P(A|B_1)\cdot P(B_1)}{P(A|B_1)\cdot P(B_1) + P(A|B_2)\, P(B_2)}$$

<u>in genral</u>



$$\boxed{P(B_i|A) = \frac{P(A|B_i)\cdot P(B_i)}{\Sigma\, P(A|B_i)\cdot P(B_i)}}$$ — Bay's Low



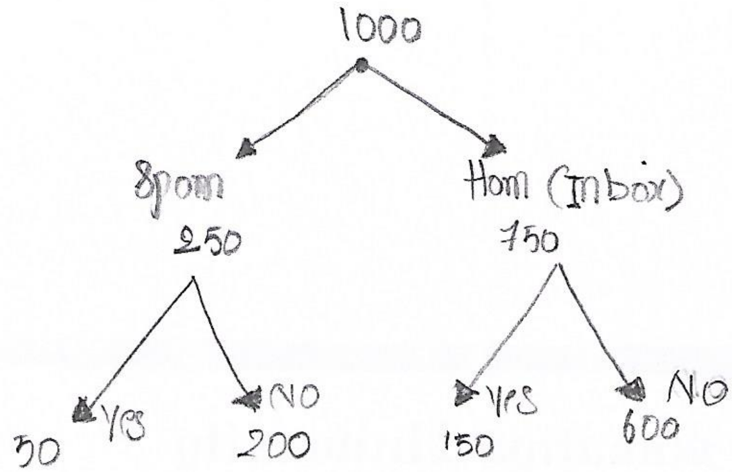
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If A and B are disjoint $\Rightarrow P(A \cap B) = 0$

$$P(A \cup B) = P(A) + P(B)$$

→ If A and B disjoint, we can add the probabilities

→ If A and B not disjoint, we can not add the probabilities problem 1:



the word "love"

Q: 1001 New message with 'love' where it will go Spam/Ham?

$$P(S|N) = \frac{P(N|S) \cdot P(S)}{P(N)} \qquad P(H|N) = \frac{P(N|H) \cdot P(H)}{P(N)}$$

$$P(N|S) = \frac{50}{250} \qquad P(N|H) = \frac{150}{750}$$

$$P(S) = \frac{250}{1000} \qquad P(H) = \frac{750}{1000}$$

$$P(N) = \frac{200\ (50+150)}{1000} \qquad P(N) = \frac{200}{1000}$$

$$= \frac{50/\cancel{250} \cdot \cancel{250}/\cancel{1000}}{200/\cancel{1000}} \qquad = \frac{\frac{150}{\cancel{750}} \times \frac{\cancel{750}}{\cancel{1000}}}{\frac{200}{\cancel{1000}}}$$

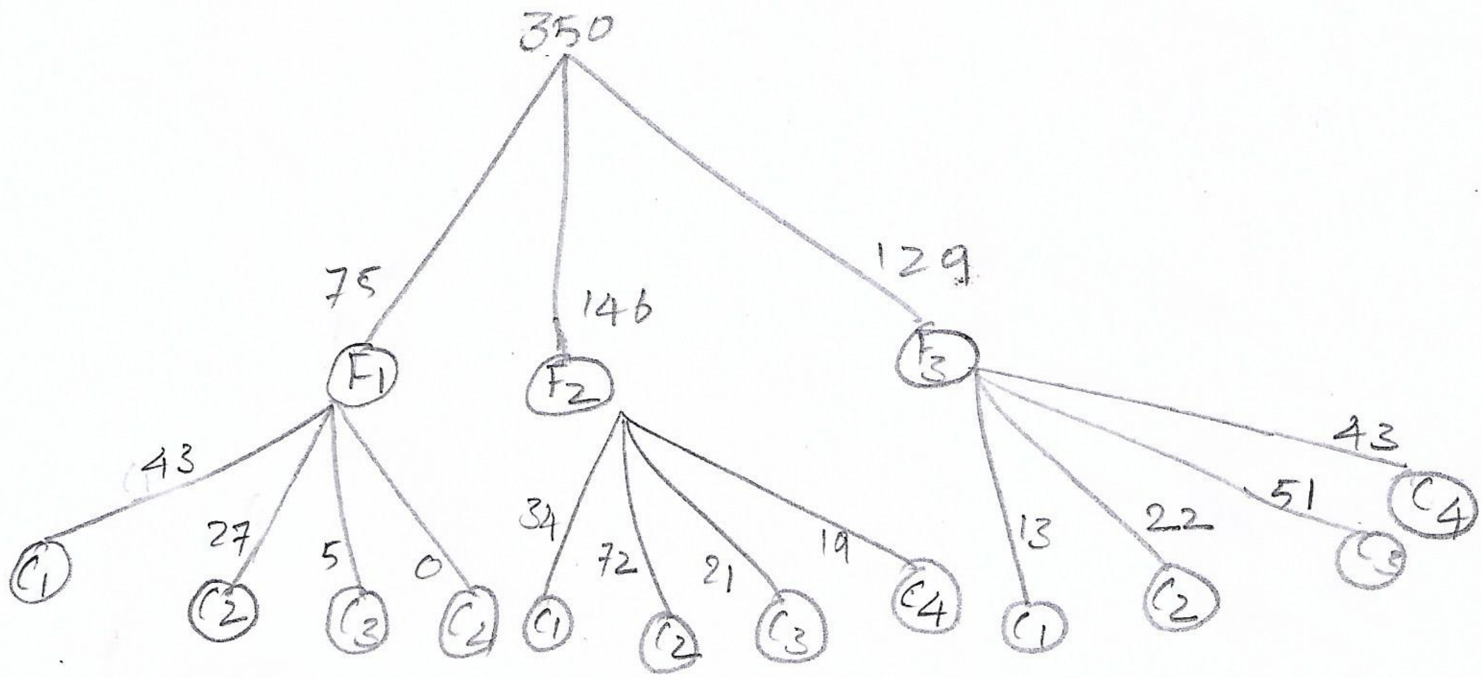
$$= \frac{50}{200} = \frac{1}{4} \qquad = \frac{150}{200} = \frac{3}{4}$$

Ans). The message has to be moved to <u>Ham</u>

Problem 2:

Collection of data

	F_1	F_2	F_3	Total
C_1	43	34	13	90
C_2	27	72	22	121
C_3	5	21	51	77
C_4	0	19	43	62
Total	75	146	129	350



Q, C_3 crime is happend in the city, which need to be investigated first?

$$\begin{array}{r}51\\21\\5\\ \hline 77\end{array}$$

1. $$P(F_1|C_3) = \frac{P(C_3|F_1)\cdot P(F_1)}{P(C_3)}$$

$$P(C_3|F_1) = \frac{5}{75}, \quad P(F_1) = \frac{75}{350}, \quad P(C_3) = \frac{5+21+51}{350}$$

$$= \frac{5/\cancel{75} \cdot \frac{\cancel{75}}{\cancel{350}}}{77/\cancel{350}} = \frac{5}{77}$$

2. $$P(F_2|C_3) = \frac{P(C_3|F_2)\cdot P(F_2)}{P(C_3)}$$

$$P(C_3|F_2) = \frac{21}{146}, \quad P(F_2) = \frac{146}{350} \qquad P(C_3) = \frac{77}{350}$$

$$= \frac{\frac{21}{\cancel{146}} \cdot \frac{\cancel{146}}{\cancel{350}}}{77/\cancel{350}} = \frac{21}{77}$$

3. $$P(F_3|C_3) = \frac{P(C_3|F_3)\cdot P(F_3)}{P(C_3)}$$

$$P(C_3|F_3) = \frac{51}{129}, \quad P(F_3) = \frac{129}{350} \qquad P(C_3) = \frac{77}{350}$$

$$= \frac{\frac{51}{\cancel{129}} \cdot \frac{\cancel{129}}{\cancel{350}}}{77/\cancel{350}} = \frac{51}{77}$$

Ans: F_3 has more probability so investigate F_3