

## Selection Sort Correctness

### Pseudocode:

```
for i = 1 to length(Array)-1;
    minElement = i
    for j = i+1 to length(Array);
        if Array[j] < Array[minElement]
            minElement = j
    temp = Array[i]
    Array[i] = Array[minElement]
    Array[minElement] = temp
```

### Inner Loop:

**Loop Invariant:** start of each iteration,  $\text{Array}[\text{minElement}]$  is less than or equal to elements in the subarray  $\text{Array}[i..j]$ . After first iteration  $i$ , the  $i$  min elements of Array are in sorted order in positions 1 to  $i$ .

**Initialization:** Before the first iteration of the inner loop,  $j = i$ , therefore the subarray to consider is  $\text{Array}[i..i]$  or simply  $\text{Array}[i]$ . Previously  $\text{minElement}$  was set to the index  $i$ ,  $\text{minElement}$  indexes the only element in  $\text{Array}[i]$ .

**Maintenance:** Before iteration  $j+1$ , we assume that  $\text{minElement}$  is the index of the smallest element in subarray  $A[i..j]$ . During iteration  $j+1$ , two cases :

1.  $\text{Array}[j+1] < \text{Array}[\text{minElement}]$
2.  $\text{Array}[j+1] \geq \text{Array}[\text{minElement}]$ .

When the inner loop completes,  $\text{minElement}$  refers to the index of the smallest element in  $\text{Array}[i+1..n]$ . Since  $\text{Array}[1..i]$  consists of the  $i$  smallest elements,  $\text{minElement}$  is the  $i+1$ st smallest element. We move this to  $\text{Array}[i+1]$  via the swap. After the swap, and at the end of the iteration  $\text{Array}[1..i]$  consists of the  $i$  smallest elements in positions 1 to  $i$ .

**Termination:** At the end of the inner loop,  $\text{minElement}$  is the index of an element less than or equal to all elements in  $\text{Array}[i..j]$ . So  $\text{minElement}$  is the index of the smallest element in  $\text{Array}[i..n]$ . When the outer loop terminates,  $i$  is the length of the array. Therefore we can conclude that  $\text{Array}[1..n]$  has all elements of Array, sorted order.

