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I. Introduction

The objective of this coursework is to design a robust Model Predictive Control(MPC) controller to regulate the motion of a gantry crane with a load attached. The controller must be capable of operating on a set of spatial constraints, which fall under 2 shape categories.

II. Formulation of Optimal Control Problem

The controller operates on a state variable **x**, containing positions, angles and rates for the crane object. The quadratic cost function $J(u^{*}, x_0)$ as described in the lecture notes is given by

$$J(u^{*}, x_0) = \min_{\bar{u}} ||x_N - x_e||_P^2 + \sum_{k=0}^{N-1} (||x_k - x_e||_Q^2 + ||u_k - u_e||_R^2 + ||u_k - u_{k-1}||_T^2)$$

Where $u_e = 0$ and the last term is a rate penalty applied to the input. The introduction of this element allows for a weaker penalty R to be applied to the control variables u. Consequently, the controller is given more degrees of freedom in selecting an optimal input. This is further expounded on in section IV.

III. Constraint Parameterisation

The previously described cost function is solved subject to a set of inequality constraints, the first of which is the angle constraint. Given that the position of the pendulum is not explicitly given, in order to ensure the pendulum is within bounds, a combination of this angle(using small angle approximation) and the position of the cart must be checked against each boundary. Alternatively, a sufficiently small angle constraint($\leq 8\,\mathrm{deg}$) can be used, and the boundaries can be tightened slightly. Both avenues were implemented, and it was found that the latter provided faster performance, at no cost to robustness.

Input hard constraints are set to $-1 \leq u_k \leq 1$. The linear equations of each boundary line are evaluated, and manipulated in order to obtain a set of upper bounds on the input. Lines which are classified as lower bounds(determined by the relative coordinates of the first and second point parameterising it) are converted to upper bound constraints for simplicity.

Lastly, a unique condition was placed on shape 2. The constraints are at any single point in time, one of a pair of overlapping rectangles. The decision of which set of constraints to use is made based on which side of a dividing line the cart is on. The dividing line is defined by a pair of points found using the sign of the cross products of adjacent lines.

IV. Implementation

All strategies in this paper were evaluated against a *test bench of 10 images for both shape 1 and 2*, varying thickness and rotation. Three main strategies were implemented in this control problem. As mentioned in section II, input rate penalties were incorporated into the cost function. The optimal set of parameters for this modification was found to be an extremely small R($10^{-4} I_{2x2}$) and comparable T. This produced an improvement in time from 9s to 3.5s for shape 1 and 20 to 11s for shape 2, under default conditions. Reducing the penalty on input magnitude introduced jagged oscillations of the cart around the target point, in some cases preventing the system from settling. In order to overcome this, the controller is forced to produce zero control input when the cart is sufficiently close to the target, and the angles have decayed below some ϵ.

A further problem was observed with shape 2, where the state would settle on the intermediate target point, without moving toward the final target. A simple state machine was implemented in the target calculator, wherein if the state was within range of the first target for sufficiently long, the target generator would update to the final destination regardless.

Secondly, a *smart choice of P* was made to provide better stability properties. Specifically, the functions `dlqr` and `dlyap` were used to find a P that provides global asymptotic stability. This modification also provided fractional improvements in speed of convergence to the target state.

Two further methods were investigated, *Offset Blocking*[1], and *Disturbance Rejection*[2]. The latter inculcated longer settling times, with no perceivable improvement against the test bench. To this extent, it was deemed unnecessary for addressing plant-model mismatch. Offset blocking was attempted, however the base version was unstable, and proved difficult to stabilise through parameter tuning. Moving Window Blocking is a potential future solution to this problem as, unlike Offset Blocking, it guarantees feasibility and stability.

Lastly, all matrices that don't require the state or output were generated offline, and a persistent variable was used to store the initial active inequalities for `mpcqpsolver` in order to give it a *'warm start'* on successive calls.

V. Addressing Infeasibility

The primary causes of infeasibility in this MPC problem are the hard constraints imposed on the QP. The natural solution to this was to introduce soft constraints, along with its corresponding modification to the cost function. However, it was found that soft constraints noticeably reduced speed performance. In order to achieve a similar effect at diminished cost to speed, the angle constraint was relaxed and a higher penalty was placed on angle. This configuration successfully completed non-linear simulations along paths of track width as low as 0.008m for both shape 1 and 2, without exceeding the input hard constraints of ± 1, suggesting that the solutions are highly feasible on actual hardware.

References

[1] R. Cagienard, P. Grieder, E. C. Kerrigan, and M. Morari. Move blocking strategies in receding horizon control. In *2004 43rd IEEE Conference on Decision and Control (CDC) (IEEE Cat. No.04CH37601)*, volume 2, pages 2023–2028 Vol.2, Dec 2004.

[2] G. Pannocchia and J. B. Rawlings. Disturbance models for offset-free model-predictive control. 49:426–437, 02 2003.