1076

UNIT - I

MICROWAVE TRANSMISSION LINES

Introduction :-

Microwave frequencies lie in the range of 1GHz to 1000GHz

Main advantage is antenna size is reduced.

Frequency	Band designations
3 - 30 Hz	ultra low freq's (ULF)
30 - 300Hz	Extra low freq's (ELF)
300 - 3KHz	voice frequencies
3K - 30KHz	very low freq (VLF)
30K - 300KHz	Low freq (LF)
300K - 3MHz	Medium freq
3M - 30MHz	High freq
30M - 300 MHz	very high freq (VHF)
300M - 3GHz	ultra high freq (UHF)
3G - 30GHz	Super high freq (SHF)
30G - 300GHz	Extreme high freq (EHF)
300G - 3THz	
3T - 30THz	Infrared freq
30T - 300THz	

Micro wave freq's

Advantages of Microwaves :-

1. Increased bandwidth availability
2. Increased directivity $\quad G = D = \frac{4\pi A_e}{\lambda^2}$
3. Fading effect & Reliability
4. Tx & Rx power ($\downarrow\downarrow$ (mw))
5. Transparency property of microwaves (300 MHz - 10GHz)

[Fading - Fluctuation in signal strengths]

① **Increased bandwidth availability :-**

Microwave freq has large bandwidth when compared to short waves, medium waves and ultra waves

Microwave freq's consist of 1000 sections of freq. bands and any one of these 1000 sections may be used to transmit all radio, TV signals and other communication signals

② **Improved directivity :-**

At microwave freq's directivity is increased and beam width ~~bandwidth~~ is decreased $(\because \theta \propto \lambda/D)$

For parabolic reflector antenna. directivity

$$D = \frac{4\pi A_e}{\lambda^2}$$

At microwave freq's λ is decreased & D is increased

For parabolic reflector antenna $B = 140/(D/\lambda)$

At 30 GHz ($\lambda = 1$ cm) for 1° beam width $D = 140$ cm

At 30 MHz ($\lambda = 100$ cm) for 1° beam width $D = 140$ m

Beam width $BW = \frac{140\lambda}{d}$

where D is diameter of the reflector At microwave freq's Antenna size is very small

③ Fading Effect and reliability :—

Fading Effect due to variation in transmission media is more Effective at lower freq's. Due to line of sight (LOS) propagation at higher freq's there is less fading Effect ~~and~~ hence microwave communication is more reliable.

④ power Requirements :—

Tx/Rx power requirements are very low at microwave freq's compared to that of short waves

⑤ Transparency property of microwaves :—

Microwave freq band ranging from 300 MHz – 10 GHz. are freely propagat through the ionized layers surrounding the earth. The presence of such a transparent window in microwave region facilitates the study of microwave radiations from Sun and Stars.

Applications :—

Microwave frequencies have band range of applications in modern technology. Most important among them are in along distance communication, RADAR's, radio astronomy etc.

① Telecommunications :—

International telephones and TV, space Communication, telemetry Communication link for railways etc

② RADAR's (Radio detection and ranging)

These are used to detect aircrafts, track and guide super sonic missiles, observe weather Conditions, air traffic Control (ATC) police speed detectors etc

3. Commercial and Industrial Applications use heat property of microwaves.

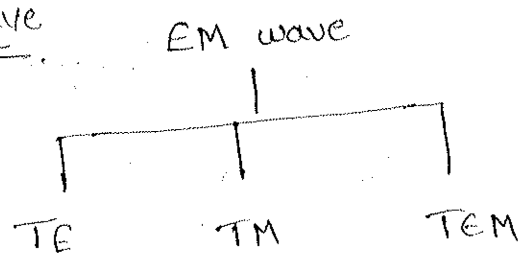
Microwave op oven, Drying machine, food processing industry, rubber industry, mining ores, dry links and biomedical applications

4. Electronic warfare ECM | ECCM system, spread spectrum system.

ECM - Electronic Counter Measurement

ECCM - Electronic Counter Counter measurement

Types of EM wave



1. Transverse electric (TE) wave :—

In TE wave, The Component of electric field vector lies in a plane transverse (or) perpendicular to the direction of propagation. where as Component of magnetic field vector lies in the direction of propogation. In TE waves $E_z = 0$, $H_z \neq 0$, if the wave is propagating in Z-direction.

② Transverse magnetic (TM) wave :—

In TM wave the component of magnetic field vector lies in a plane transverse or $\perp^{lar}$ to the direction of propagation where as the component of Electric field vector lies in the direction of propogation. In TM wave $H_z = 0$, $E_z \neq 0$

③ Transverse electromagnetic (TEM) wave :—

In TEM wave, both Electric and magnetic field vectors lies in a plane transverse or $\perp^{lr}$ to the direction of propagation ($E_z = H_z = 0$)

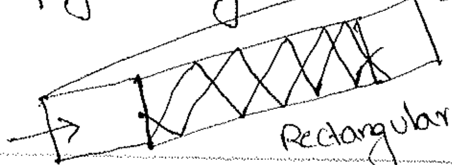
Note :— In waveguides, TEM wave does not Exist

WAVEGUIDES :—

If freq is greater than 3GHz, transmission of that Electromagnetic wave along Tx lines and coaxial cables is very difficult due to radiation losses and dielectric losses

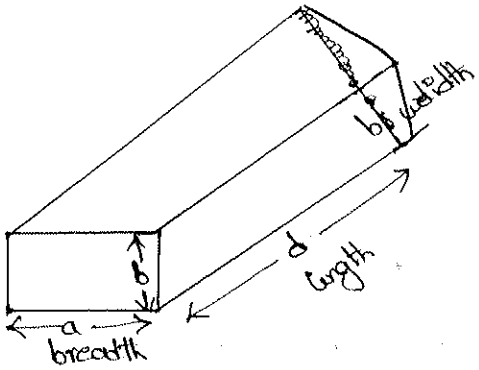
A hollow metalic tube is be used to transmit EM waves at higher freq's and that tube is called waveguide

In waveguide the wave is propogated by successive reflections from inner walls of waveguide

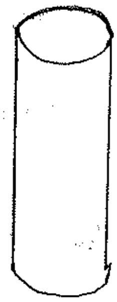


Rectangular waveguide

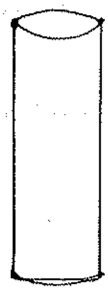
Types of waveguides :—



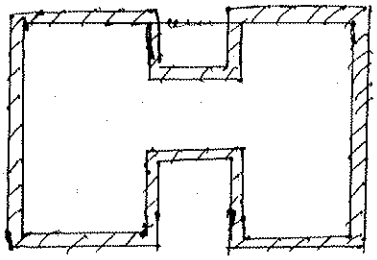
Rectangular waveguide



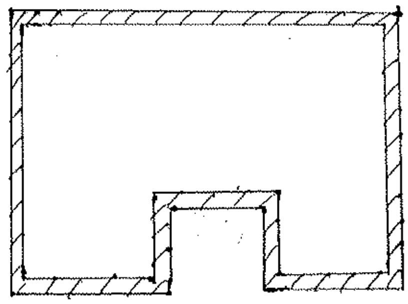
Circular wave guide



Elliptical waveguide



Double ridged wave guide



Single riged

Always a>> b

-Any shape of Crosssection of waveguide can support EM wave but irregular shapes are very difficult to analyse. In general rectangular and Circular waveguides are most popularly used.

Analysis of TE and TM waves in rectangular waveguides :-

TE waves :-

$$E_z = 0$$
$$H_z = A\cos\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y\ e^{-j\beta z}$$

$$E_x = \frac{-j\omega\mu}{k^2}\frac{\partial H_z}{\partial y} \Rightarrow E_x = \frac{j\omega\mu}{k^2} A\left(\frac{n\pi}{b}\right)\cos\left(\frac{m\pi}{a}\right)x\ \sin\left(\frac{n\pi}{b}\right)y \cdot e^{-j\beta z}$$

$$E_y = \frac{j\omega\mu}{k^2}\frac{\partial H_z}{\partial x} \Rightarrow E_y = \frac{-j\omega\mu}{k^2} A\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi}{a}\right)x\ \cos\left(\frac{n\pi}{b}\right)y \cdot e^{-j\beta z}$$

$$H_x = \frac{-j\beta}{k^2}\frac{\partial H_z}{\partial x} \Rightarrow H_x = \frac{j\beta}{k^2} A\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi}{a}\right)x\ \cos\left(\frac{n\pi}{b}\right)y \cdot e^{-j\beta z}$$

$$H_y = \frac{-j\beta}{k^2}\frac{\partial H_z}{\partial y} \Rightarrow H_y = \frac{j\beta}{k^2} A\left(\frac{n\pi}{b}\right)\cos\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \cdot e^{-j\beta z}$$

where a & b are dimensions of waveguide

m & n are mode no's. wave is designated as TE_{mn} (or) TM_{mn}

β is phase constant

(i) if $m=0$ & $n=0$, then

$E_x = 0$ $\quad H_x = 0$

$E_y = 0$ $\quad H_y = 0$

TE_{00} mode has does not Exist

(ii) If $m=1$, $n=0$, then $E_x = 0$, $H_y = 0$

E_y & $H_x \neq 0$

TE_{10} mode Exists

(iii) If $m=0$, $n=1$, then E_x & $H_y \neq 0$

E_y & $H_x = 0$

TE_{01} mode Exists

TM waves :—

$$H_z = 0$$

$$E_z = A \sin\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \cdot e^{-j\beta z}$$

$$E_x = \frac{-j\beta}{K^2}\frac{\partial E_z}{\partial x} \Rightarrow E_x = \frac{-j\beta}{K^2} A\left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \cdot e^{-j\beta z}$$

$$E_y = \frac{-j\beta}{K^2}\frac{\partial E_z}{\partial y} \Rightarrow E_y = \frac{-j\beta}{K^2} A\left(\frac{n\pi}{b}\right)\sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right) e^{-j\beta z}$$

$$H_x = \frac{j\omega\epsilon}{K^2}\frac{\partial E_z}{\partial y} \Rightarrow H_x = \frac{j\omega\epsilon}{K^2} A\left(\frac{n\pi}{b}\right)\sin\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y \cdot e^{-j\beta z}$$

$$H_y = \frac{-j\omega\epsilon}{K^2}\frac{\partial E_z}{\partial x} \Rightarrow H_y = \frac{-j\omega\epsilon}{K^2} A\left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \cdot e^{-j\beta z}$$

(i) If $m=0$, $n=0$ then

$H_z = E_z = 0$; $E_x = E_y = H_x = H_y = 0$

TM_{00} does not Exists

(ii) If $m=1$, $n=0$ then

$E_x = 0$, $E_y = 0$, $H_x = 0$, $H_y = 0$

TM_{10} does not Exists

(iii) If $m=0$, $n=1$ then

$E_x = 0$, $E_y = 0$, $H_x = 0$, $H_y = 0$

TM_{01} does not Exists

(iv) If $m=1$, $n=1$ then

$E_x = E_y = H_x = H_y \neq 0$

TM_{11} Exists and it is the starting mode.

Expression for cut off freq in rectangular wave guides

$$K^2 = P^2 + \omega^2\mu\varepsilon$$

$$= \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

In waveguides

$$K^2 = P^2 + \omega^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

It is characteristic equation.

$$P = \alpha + j\beta = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon}$$

where P is propagation constant $= \alpha + j\beta$

α = attenuation const

β = phase const

$\omega = 2\pi f$

m, n = mode no's

a, b = dimensions of rect waveguide

At lower freq's $\omega^2\mu\varepsilon < \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

The propagation const becomes +ve & real and is equal to attenuation const. i.e; wave is attenuated

At higher freq's $\omega^2\mu\varepsilon > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$, the propagation const becomes imaginary and is equal to phase const i.e wave is propagated

At some freq, $\omega^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$ and propagation const is zero and that freq is called cut off frequency (or) threshold frequency.

At cut off freq

$$p = 0$$

$$\Rightarrow \omega_c^2 \mu \varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\Rightarrow \omega_c^2 = \frac{1}{\mu\varepsilon} \cdot \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\Rightarrow \omega_c = \frac{1}{\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$\Rightarrow f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}} \sqrt{\pi^2\left[\left(\frac{m}{a}\right)^2 + \left(n/b\right)^2\right]}$$

$$\Rightarrow f_c = \frac{c}{2\pi} \cdot \pi \sqrt{\left(\frac{m}{a}\right)^2 + \left(n/b\right)^2} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\boxed{f_c = c/2 \sqrt{(m/a)^2 + (n/b)^2}}$$

$$\lambda_c = c/f_c$$

$$= \frac{c}{c/2\sqrt{(m/a)^2 + (n/b)^2}}$$

$$\boxed{\lambda_c = \frac{2}{\sqrt{(m/a)^2 + (n/b)^2}}}$$

This is the Expression for cut off wavelength

$$\lambda_c\, TE_{10} = \frac{2}{\sqrt{(1/a)^2}}$$

$$\lambda_c\, TE_{10} = 2a$$

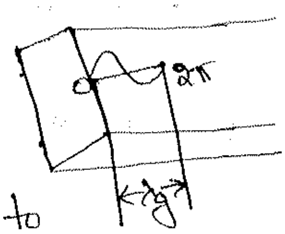
$$\lambda_c\, TE_{01} = 2b$$

$$\lambda_c\, TE_{20} = a$$

Note :— All the frequencies greater than f_c are propagated and freq's less than f_c are attenuated in the waveguide so the wave guide acts as highpass filter.

Guided wavelength (λ_g) :—

$$\lambda_g = \frac{2\pi}{\beta}$$



The guided wavelength is defined as the distance travelled by the wave in wG inorder to undergo a phaseshift of 2π radians

It is related to phase const β as

$$\lambda_g = \frac{2\pi}{\beta}$$

The guided wavelength λ_g is differ from freespace wavelength λ_0. The velocity of wave in waveguide (v) is always greater than the velocity of wave in freespace (c)

$$\therefore \lambda_g > \lambda_0$$

$$v = \lambda_g f$$
$$c = \lambda_0 f$$

Relation among λ_g, λ_0 and λ_c :—

The guided wave length is defined as

$$\lambda_g = \frac{2\pi}{\beta}$$

At higher freq's propagation const. = phase const

$$P = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon}$$

$$j\beta = \sqrt{\omega_c^2\mu\varepsilon - \omega^2\mu\varepsilon}$$

$$j\beta = \sqrt{-(\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon)}$$

$$= j\sqrt{(\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$$

$$\beta = \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$$

$$= \omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}$$

$$\beta = \omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$\lambda_g = \frac{2\pi}{\beta}$$

$$= \frac{2\pi}{\omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$= \frac{2\pi c}{2\pi f\sqrt{1-\left(f_c/f\right)^2}}$$

$$= \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

$\because\ f_c = c/\lambda_c$

$f_0 = c/\lambda_0$; $\frac{f_c}{f_0} = \frac{\lambda_0}{\lambda_c}$

$$\boxed{\therefore\ \lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}}$$

$$\frac{\lambda_g}{\lambda_0} = \frac{1}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

$$\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2} = \frac{\lambda_0}{\lambda_g} \Rightarrow 1-\frac{\lambda_0^2}{\lambda_c^2} = \frac{\lambda_0^2}{\lambda_g^2}$$

$$\frac{1}{\lambda_0^2} - \frac{1}{\lambda_c^2} = \frac{1}{\lambda_g^2}$$

$$\boxed{\frac{1}{\lambda_0^2} = \frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2}}$$

This is the relation b/w λ_0, λ_c and λ_g

Dominant mode in rectangular waveguide :—

TE waves :—

$$\lambda_c = \frac{2}{\sqrt{(m/a)^2 + (n/b)^2}}$$

$$= \frac{2ab}{\sqrt{(mb)^2 + (na)^2}}$$

Degenarative modes: when ever two or more modes have the same cut off frequency they are said to be degenarative modes.

In rectangular waveguides TE_{mn} and TM_{mn} modes are always degenarate.

TE_{10} $m=1, n=0$

$\lambda_c TE_{10} = 2a, \; f_c = c/2a$

for TE_{01}, $\lambda_c TE_{01} = 2b, \; f_c = c/2b$

for TE_{20}, $\lambda_c TE_{20} = a, \; f_c = c/a$

For TE_{02}, $\lambda_c TE_{02} = b, \; f_c = c/b$

For TE_{11}, $\lambda_c TE_{11} = \frac{2ab}{\sqrt{a^2+b^2}}, \; f_c = \frac{c\sqrt{a^2+b^2}}{2ab}$

For $a>b$, TE_{10} has lowest cut off frequency. ~~so it is the dominant mode TE_{10} also has~~ and highest cut off wavelength so it is called dominant mode in TE waves

TM waves :—

For TM_{11}, $\lambda_c TM_{11} = \frac{2ab}{\sqrt{b^2+a^2}}, \; f_c = \frac{c\sqrt{a^2+b^2}}{2ab}$

For TM_{21}, $\lambda_c TM_{21} = \frac{2ab}{\sqrt{4b^2+a^2}}, \; f_c = \frac{c\sqrt{4b^2+a^2}}{2ab}$

For TM_{12}, $\lambda_c TM_{12} = \frac{2ab}{\sqrt{b^2+4a^2}}, \; f_c = \frac{c\sqrt{4a^2+b^2}}{2ab}$

TM_{11} has lowest cutoff frequency, highest cutoff wavelength So it is dominant mode in TM waves

Note :— TE_{10} is dominant mode in rectangular waveguides.

***phase velocity (vp) :—

phase velocity is defined as the ~~ratio of~~ Rate at which wave changes its phase in terms of guided wave length

$$v_p = \frac{\lambda_g}{\text{unit time}} = \lambda_g f$$

$$= \frac{\lambda_g f \cdot 2\pi}{2\pi}$$

$$= \frac{\omega}{2\pi/\lambda_g} = \frac{\omega}{\beta}$$

$$\boxed{\therefore v_p = \frac{\omega}{\beta}}$$

* phase velocity is greater than velocity of high since

$$\boxed{\lambda_g > \lambda_0}$$

Any intelligence signal (or) modulation signal does not travels with velocity greater than velocity of light. So it is called phase velocity.

Expression for vp :—

$$P = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon}$$

For a propagating wave

$$P = j\beta = \sqrt{-(\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon)}$$

$$j\beta = j\sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$$

$$\beta = \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$$

$$\beta = \left(\sqrt{1-\frac{\omega_c^2}{\omega^2}}\right)\omega\sqrt{\mu\varepsilon}$$

$$\beta = \omega\sqrt{\mu\varepsilon}\sqrt{1-\frac{\omega_c^2}{\omega}}$$

phase velocity $v_p = \frac{\omega}{\beta}$

$$= \frac{\omega}{\omega\sqrt{\mu\varepsilon}\cdot\sqrt{1-\frac{\omega_c^2}{\omega^2}}}$$

$$= \frac{1}{\sqrt{\mu\varepsilon}\sqrt{1-\frac{f_c^2}{f^2}}}$$

$$\boxed{v_p = \frac{c}{\sqrt{1-\frac{f_c^2}{f^2}}}}$$

Group velocity (v_g) :—

The velocity of modulated wave in the waveguide is called group velocity and is given by

$$v_g = \frac{d\omega}{d\beta}$$

Expression for v_g :—

$$\beta = \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$$

$$= \sqrt{\mu\varepsilon}\sqrt{\omega^2-\omega_c^2}$$

$$\frac{d\beta}{d\omega} = \sqrt{\mu\varepsilon}\cdot\frac{1}{2\sqrt{\omega^2-\omega_c^2}}\cdot 2\omega$$

$$= \omega\sqrt{\mu\varepsilon}\cdot\frac{1}{\sqrt{\omega^2-\omega_c^2}}$$

$$\frac{d\omega}{d\beta} = \frac{\sqrt{\omega^2-\omega_c^2}}{\omega\sqrt{\mu\varepsilon}}$$

$$\frac{d\omega}{d\beta} = \frac{\omega\sqrt{1-(\omega_c/\omega)^2}}{\omega\sqrt{\mu\varepsilon}}$$

$$\therefore v_g = \frac{d\omega}{d\beta} = c\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$\boxed{\therefore v_g = c\cdot\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$v_p v_g = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \cdot c\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$= c^2$$

$$\boxed{v_p v_g = c^2}$$

problems :—

1. Determine cut off wavelength for dominant mode in a rectangular waveguide of breadth 10 cms for a 2.5 GHz signal calculate guided wavelength, phase velocity and group velocity

sol:- Given

$a = 10$ cms

$f = \cancel{25}\ 2.5$ GHz

$\lambda_0 = c/f$

$$= \frac{3\times 10^{10}}{2.5\times 10^9}$$

$\lambda_0 = 12$ cms

For TE_{10}, $\lambda_c = 2a$

$= 2\times 10$

$= 20$ cms

$$\lambda g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

$$= \frac{12}{\sqrt{1-\left(\frac{12}{20}\right)^2}}$$

$$= 15\text{cms}$$

$$v_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{c}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

$$= \frac{3\times10^{10}}{\sqrt{1-\left(\frac{12}{20}\right)^2}} = 3.75\times10^8 \text{ m/sec}$$

(or) $v_p = \lambda g f = 15\times2.5G = 3.75\times10^8$ m/sec

$$v_g = \frac{c^2}{v_p} = \frac{(3\times10^8)^2}{3.75\times10^8} = 2.4\times10^8 \text{ m/sec}$$

***

Wave Impedance :—

The wave impedance is defined as the ratio of the strength of the electric field in one transverse direction to the strength of magnetic field along other transverse direction. This ratio is termed as the wave impedance down a guide

$$\eta = \frac{E_x}{H_y} = \frac{-E_y}{H_x}$$

$$= \frac{\sqrt{E_x^2+E_y^2}}{\sqrt{H_x^2+H_y^2}}$$

Impedance of TE wave :—

$$\eta_{TE} = \frac{E_x}{H_y} \quad \left[\eta_{TE} = \frac{\frac{+j\omega\mu}{k^2}\frac{\partial H_z}{\partial y}}{+j\beta/k^2 \; \partial H_z/\partial y} = \cancel{\frac{E_x}{H_y}}\right]$$

$$\eta_{TE} = \frac{\frac{j\omega\mu}{k^2} A\left(\frac{n\pi}{b}\right)\cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \cdot e^{-j\beta z}}{\frac{j\beta}{k^2} A\left(\frac{n\pi}{b}\right)\cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \cdot e^{-j\beta z}}$$

$$= \frac{\omega\mu}{\beta}$$

$$\eta_{TE} = \frac{-E_y}{H_x} = \frac{+\omega\mu}{\beta}$$

$$\eta_{TE} = \frac{\omega\mu}{\beta}$$

$$\beta = \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$$

$$\eta_{TE} = \frac{\omega\mu}{\sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}}$$

$$\eta_{TE} = \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon}\sqrt{1-(f_c/f)^2}}$$

$$= \sqrt{\frac{\mu}{\varepsilon}} \cdot \frac{1}{\sqrt{1-(f_c/f)^2}}$$

$$\boxed{\eta_{TE} = \eta_0 \cdot \frac{1}{\sqrt{1-(f_c/f)^2}}}$$

Impedance of a TM wave :—

$$\eta_{TM} = \frac{E_x}{H_y} = \frac{-E_y}{H_x} = \frac{\frac{j\beta}{k^2}\frac{\partial E_z}{\partial x}}{\frac{j\omega\varepsilon}{k^2}\frac{\partial E}{\partial x}}$$

$$= \frac{\beta}{\omega\varepsilon}$$

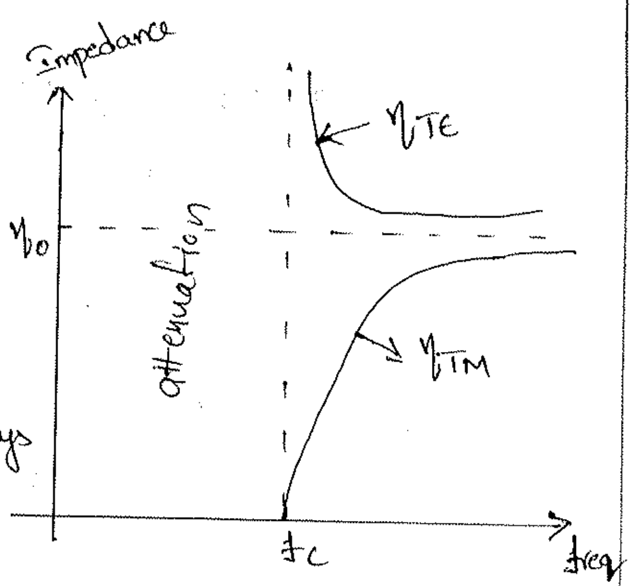
$$\eta_{TM} = \frac{\beta}{\omega\varepsilon}$$

$$\eta_{TM} = \frac{\sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}}{\omega\varepsilon}$$

$$= \frac{\omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c^2}{f^2}\right)}}{\omega\varepsilon}$$

$$= \sqrt{\frac{\mu}{\varepsilon}} \cdot \sqrt{1-\left(\frac{f_c^2}{f^2}\right)}$$

$$\boxed{\eta_{TM} = \eta_0\sqrt{1-\left(f_c/f\right)^2}}$$



$$\eta_{TE} \times \eta_{TM} = \eta_0^2$$

Impedance for TE wave is always greater than TM wave

$$\eta_{TE} > \eta_{TM}$$

* wave impedance for TM wave is always less than free space impedance

At $f = f_c$, $\eta_{TM} = 0$, $\eta_{TE} = \infty$

If $f < f_c$, the impedance is very high

problems :—

2. An air filled rectangular waveguide has dimensions of 0.9" x 0.4" supporting TE_{10} mode at a freq of 9800 MHz. calculate the percentage of change in the impedance for 10% increase in the operating freq

Sol:— Given

$f = 9800$ MHz

$$\eta_{TE} = \frac{\eta_0}{\sqrt{1-\left(f_c/f\right)^2}}$$

$$f_c = \frac{c/2}{\sqrt{(n/a)^2+(n/b)^2}} \cdot \frac{c}{2} \cdot \sqrt{(m/a)^2+(n/b)^2}$$

For TE_{10} mode, $f_c = \frac{c}{\cancel{2}\ 2}\left(\sqrt{(1/a)^2}\right)$

$$= \frac{c}{2a}$$

$1'' = 2.54$ cm

$$= \frac{c}{2(0.4)''}$$

$a = 0.9'' = 0.9 \times 2.54 = 2.286$ cm

$$f_c = \frac{c}{2(2.286)\text{cm}} = \frac{3\times10^{10}}{2(2.286)}$$

$$f_c = 6.56\,\text{GHz}$$

$$\eta_{TE} = \frac{377}{\sqrt{1-\left(\frac{6.56G}{9.8G}\right)^2}}$$

$$\eta_{TE} = 507.46\,\Omega$$

New freq $f' = 9800 + 9800 \times \frac{10}{100}$

$$= 10.78\,\text{GHz}$$

$$\therefore \eta_{TE} = \frac{377}{\sqrt{1-\left(\frac{6.56G}{10.78G}\right)^2}}$$

$$\eta_{TE} = 475\,\Omega$$

change in impedance $= 507.46 - 475$

$$= 32.46\,\Omega$$

$$= 32.46\,\Omega$$

% change $= \frac{32.46}{507.46} \times 100$

$$= 6.39\% \cong 6.4\%$$

power loss in a rectangular waveguide

If the wave freq is less than cutoff freq losses are exist in rectangular waveguide, That attenuation loss is due to power dissipation within the waveguide walls and dielectric within the waveguide

If freq 'f' is less than f_c, the propagation const will have only attenuation const

$$P = \alpha + j\beta = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon}$$

$$= \sqrt{\omega_c^2\mu\varepsilon - \omega^2\mu\varepsilon}$$

$$P = \alpha = \sqrt{\omega_c^2\mu\varepsilon - \omega^2\mu\varepsilon}$$

$$P = \alpha = \omega_c\sqrt{\mu\varepsilon}\sqrt{1 - (\omega/\omega_c)^2}$$

$$\alpha = \frac{8\pi f_c}{c}\sqrt{1 - (f/f_c)^2}$$

$$\boxed{\alpha = \frac{8\pi}{\lambda_c}\sqrt{1 - (f/f_c)^2}} \text{ Neper/meter}$$

$$\alpha = \frac{54.6}{\lambda_c}\sqrt{1 - (f/f_c)^2} \text{ dB/~~length~~meter}$$

1 NP = 8.686

*** power transmission in rectangular waveguide :-

power transmission in rectangular waveguide can be calculated by complex poynting theorem

According to apply apoynting theorem, power transmitted through waveguide is given by

$$P_{tr} = \frac{1}{2}\int_s (E \times H^*)\, ds$$

For lossless dielectric medium in waveguide the average power following through a rectangular waveguide is given by

$$P_{tr} = \frac{1}{2}\int_S \frac{|E|^2}{\eta}\,ds = \frac{1}{2}\eta\int_S |H|^2\,ds$$

For TM wave

$$\eta_{TM} = \eta_0\sqrt{1-\left(f_c/f\right)^2}$$

$$P_{trTM} = \frac{1}{2}\eta_0\sqrt{1-\left(f_c/f\right)^2}\int_0^a\int_0^b |H|^2\,dx\,dy$$

$$P_{trTM} = \frac{1}{2\eta_0\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\int_0^a\int_0^b |E|^2\,dx\,dy$$

For TE waves

$$\eta_{TE} = \eta_0 \Big/ \sqrt{1-\left(f_c/f\right)^2}$$

$$P_{trTE} = \frac{1}{2}\frac{\eta_0}{\sqrt{1-\left(f_c/f\right)^2}}\int_0^a\int_0^b |H|^2\,dx\,dy$$

$$P_{trTE} = \frac{1}{2\eta_0}\sqrt{1-\left(f_c/f\right)^2}\int_0^a\int_0^b |E|^2\,dx\,dy$$

power transmission is more in TE waves compared to TM waves. So generally we prefer TE waves

Impossibility of TEM wave in rectangular and circular wave guide :-

The wave that will propagate in hollow rectangular waveguide (or) cylinders have been divided into two sets

1. Transverse Electric wave which has no Z-component of E $(E_z=0)$
2. Transverse magnetic wave which has no Z-component of H $(H_z=0)$

TE and TM can propagate within rectangular (or) Circular (or) in cylindrical waveguides of any crossection but TEM wave has no axial Component of either E or H since TEM wave cannot propagate within single Conductor waveguide.

If TEM wave Exists inside the waveguide the lines of H will be a closed loops. $(\nabla \cdot H = 0)$ and lies in a plane $\perp^{lar}$ to the z-axis. Now by Maxwell's Equations, magnetomotive force around Each of these closed loop must be equal to axial current. axial current through H loop will be ~~condition~~ conduction Current or displacement ~~in the inner Conductor~~, Conduction current require inner conductor But there will be no inner Conductor in hallow waveguides so the axial current must be displacement current

and ~~But~~ an axial displacement current require an axial Component of E. It is not present TEM wave

Therefore TEM wave Cannot Exist in a single Conductor waveguide

** problems :—

3. A rectangular waveguide has Cross section of 1.5cm x 0.8 cm $\sigma = 0$, $\mu = \mu_0$, $\varepsilon = 4\epsilon_0$, the magnetic field Component is given as $H_x = 2 \sin\left(\frac{\pi x}{a}\right) \cos\left(\frac{3\pi y}{b}\right) \cdot \sin\left(\pi \times 10^{11} t - \beta z\right)$ determine mode of propagation, cut off freq phase Constant, propagation Const and wave impedance

Sol:— In TE and TM waves, common factor of magnetic field Component is

$$H_x = \sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \cdot e^{-j\beta z}$$

Here $m=1$, $n=3$

So the mode is TE_{13} or TM_{13}

Cut off freq $f_c = ?$

$$f_c = c/2 \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$= \frac{1}{2\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(n/b\right)^2}$$

$$= \frac{1}{2\sqrt{\mu_0 \cdot 4\varepsilon_0}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(n/b\right)^2}$$

$$= \frac{1}{4\sqrt{\mu_0\varepsilon_0}} \sqrt{\left(\frac{1}{1.5}\right)^2 + \left(\frac{3}{0.8}\right)^2} \qquad c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

$$= \frac{3\times10^{10}}{4} \sqrt{\left(\frac{1}{1.5}\right)^2 + \left(\frac{3}{0.8}\right)^2}$$

$$= 28.57\ GHz$$

phase constant, $\beta = ?$

$$2\pi f t = \pi \times 10^{11} t$$

$$f = 5\times10^{10}\ Hz$$

where 'f' is freq of wave

$$\beta = \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$$

$$= \omega\sqrt{\mu\varepsilon}\sqrt{1-\left(f_c/f\right)^2}$$

$$= 2\pi f\sqrt{\mu_0 \cdot 4\varepsilon_0}\sqrt{1-\left(f_c/f\right)^2}$$

$$= 4\pi f\sqrt{\mu_0\varepsilon_0}\sqrt{1-\left(f_c/f\right)^2}$$

$$= \frac{4\pi f}{c}\sqrt{1-\left(f_c/f\right)^2}$$

$$= 1718.81\ rad/m$$

propagation constant, $p = \alpha + j\beta$

$f > f_c$

$\therefore \; p = j\beta$

$= j\,1718.81$ rad/m

$$\eta_{TM} = \eta_0 \sqrt{1-(f_c/f)^2}$$

$$= \sqrt{\frac{\mu}{\varepsilon}} \sqrt{1-(f_c/f)^2}$$

$$= \sqrt{\frac{\mu_0}{4\varepsilon_0}} \sqrt{1-(f_c/f)^2}$$

$$= \frac{1}{2}\eta_0 \sqrt{1-(f_c/f)^2}$$

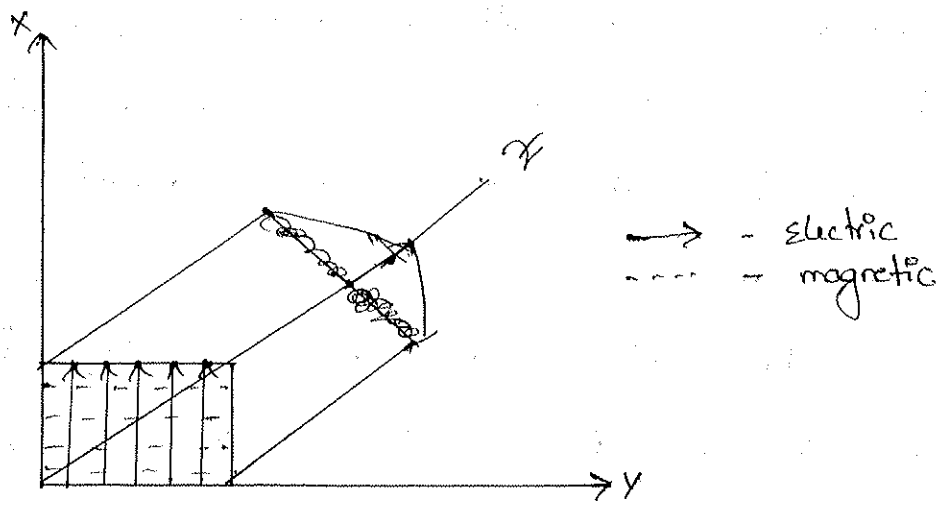
$$= 154.69\,\Omega$$

$$\eta_{TE} = \frac{\eta_0}{\sqrt{1-(f_c/f)^2}}$$

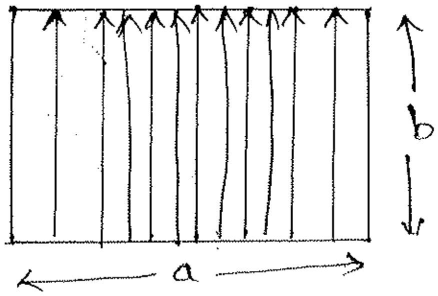
$$= \frac{\sqrt{\frac{\mu_0}{4\varepsilon_0}}}{\sqrt{1-(f_c/f)^2}} = \frac{1}{2}\eta_0 \frac{1}{\sqrt{1-(f_c/f)^2}}$$

$$= 229.68\,\Omega$$

*** Field patterns in rectangular waveguide :-

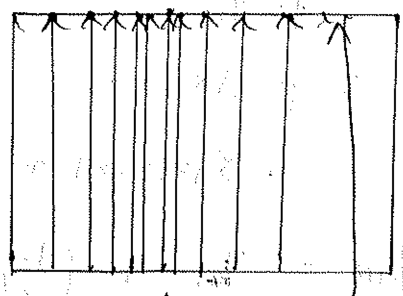


TE10

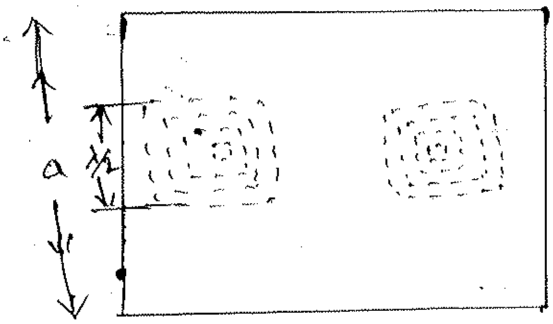


Front view

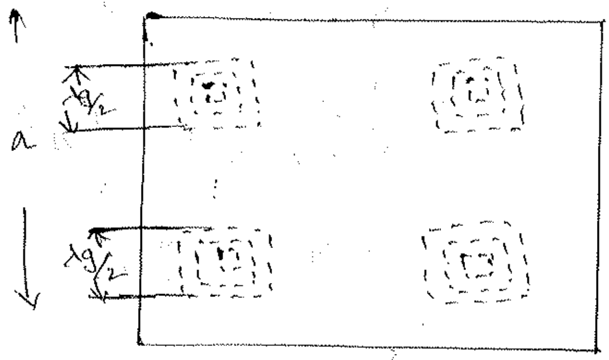
TE20



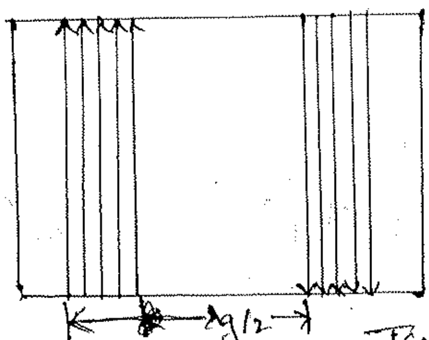
Front view



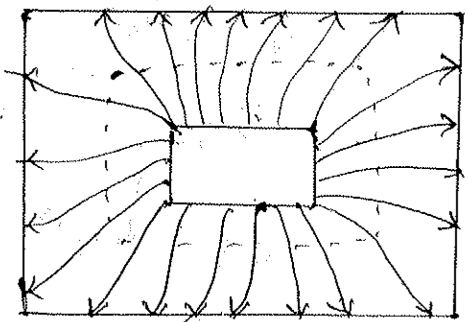
Top view



Top view



side view TE10, TE20



TE11 front view

we designate particular mode as TE_{mn} and TM_{mn} where m indicates no. of half wave ~~radiations~~ variations of Electric field (or) magnetic field across wider dimension 'a' lly 'n' indicates no of half wave variations of electric field (or) magnetic field across narrow dimension 'b'

The Electric field & magnetic field patterns in dominant mode TE_{10} is show in fig a) the Electric field lines Exists

(Continuation in 14th page)

only at right angles to the direction of propagation where as magnetic field ~~has~~ has a component in the direction of propagation As well as $\perp^{lr}$ ~~to~~ (or) normal to Electric field

The H field is in the form of closed loops ($\nabla \cdot H = 0$) which lies in a plane normal to E field i.e; parallel to ~~to~~ top & bottom of waveguide walls

The field pattern for TE_{20} mode is very similar to TE_{10} mode; but difference is two half wave ~~radiations~~ variations of E field & H field.

In TE_{11} mode, the E field & H field patterns are shown in fig : c)

problem :—

4. A rectangular waveguide has $a = 4cm$, $b = 3cm$ as it's sectional dimensions. Find all the modes which will propagate at 5000 MHz

Sol:-

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

For TE_{10}, $f_c = \frac{3\times10^{10}}{2}\sqrt{\left(\frac{1}{a}\right)^2} = \frac{3\times10^{10}}{2\times4}$

$$f_c = 3.7 \text{ GHz}$$

$$f_c < f$$

For TE_{01}, $f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$

$$= \frac{3\times10^{10}}{2\times3} = 5 \text{ GHz}$$

$$f = f_c$$

TE_{01} mode is not propagated

For TE_{11} & TM_{11}

$$f_c = \frac{3\times10^{10}}{2}\sqrt{(1/a)^2+(1/b)^2}$$

$$= \frac{3\times10^{10}}{2}\sqrt{(1/4)^2+(1/3)^2}$$

$$= 6.25\ GHz$$

$f_c > f$

TM_{11} and TE_{11} does not propagated

For TE_{20}, $f_c = \frac{3\times10^{10}}{2}\sqrt{(2/4)^2} = 7.5\ GHz$

TE_{20} does not propagated

(3) The dimensions of waveguide are 2.5cm x 1cm the freq is 8.6 GHz find possible modes

Sol:- Given a = 2.5 cm, b = 1cm

$f = 8.6\ GHz$

For TE_{10} mode

$$f_c = \frac{c}{2}\sqrt{(m/a)^2+(n/b)^2}$$

$$= \frac{3\times10^{10}}{2}\sqrt{\left(\frac{1}{2.5}\right)^2}$$

$$= 6\ GHz$$

$f_c < f$

TE_{10} mode is propagated

For TE_{01} mode

$$f_c = \frac{c}{2b} = \frac{3\times10^{10}}{2\times1} = 15\ GHz$$

$f_c > f$

TE_{01} mode is not propagated

For TE_{11} & TM_{11}

$$f_c = \frac{c}{2ab}\sqrt{a^2+b^2}$$

$$= \frac{3\times10^{10}}{2\times2.5\times1}\sqrt{(2.5)^2+1^2}$$

$$= 16.15 \text{ GHz}$$

$$f_c > f$$

TE_{11} & TM_{11} modes does not propagate

For TE_{20}, $f_c = \frac{c}{2}\sqrt{\left(\frac{2}{a}\right)^2} = \frac{3\times10^{10}}{2}\sqrt{\frac{4}{2.5^2}}$

$$= 12 \text{ GHz}$$

TE_{20} does not propagate

Only TE_{10} mode is propagated.

(P) A rectangular wave guide ($a = 2$cm, $b = 1$cm) filled with de-ionized water ($\varepsilon_r = 81, \mu_r = 1$) operates at 3 GHz. Determine all propagating modes and the corresponding cut off frequencies.