Homework - 7
(Written)

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Problem 1:- Using the optical flow contraint equation show how two different illuminations can be used to obtain a unique solution for (u, v) at each image point.

~~ ~~ Now, we know that, the Contraint Equation is given by, $I_x u + I_y v + I_t = 0$, Here (u, v) is the optical flow. From this equation we can clearly say that there are 2 unknowns & 1 equation. $\because (I_x, I_y, I_t)$ can be easily computed from 2 frames.

Now, we are given a structured environment, where illumination can be controlled at a speed much faster than the motion of objects in the scene.

First, let's consider a scenario, where the illumination is set to I_1, from this the contraint Equation that will eventually result be, $I_{1x} u + I_{1y} v + I_{1t} = 0$ at point (x, y)

Now, before the object moves lets change the illumination to I_2 & lets maintain this illumination onto the next frame. This will result in the following contraint Equation, $I_{2x} u + I_{2y} v + I_{2t} = 0$ at point (x, y).

Again, in the frame 2, before the object moves again, get the illumination back to I_1, will ~~ ~~ is necessary to obtain the first equation $[I_{1x} u + I_{1y} v + I_{1t} = 0]$.

Now, using this specialized setup described above, we have obtained 2 optical flow constraint equations for a single point (x,y).
Which is another way of saying we reduced the problem of 2 unknowns 1 equation, to 2 unknowns 2 equations. Solving this system of equations will give us the unique solution for (u,v) at each image point. Which is shown below,

Eq 1:- $I_{1x}U + I_{1y}V + I_{1t} = 0 \qquad \times I_{2x}$

Eq 2:- $I_{2x}U + I_{2y}V + I_{2t} = 0 \qquad \times I_{1x}$

$$\Rightarrow I_{1y}I_{2x}V + I_{1t}I_{2x} - I_{2y}I_{1x}V - I_{2t}I_{1x} = 0.$$

$$\Rightarrow V = \frac{I_{2t}I_{1x} - I_{1t}I_{2x}}{[I_{1y}I_{2x} - I_{2y}I_{1x}]}$$

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$$\Rightarrow I_{1x}I_{2y}U + I_{1t}I_{2y} - I_{2x}I_{1y}U - I_{2t}I_{1y} = 0$$

$$\Rightarrow U = \frac{I_{2t}I_{1y} - I_{1t}I_{2y}}{[I_{1x}I_{2y} - I_{2x}I_{1y}]}$$