Homework - 3

Name: VINAY. PATIL
NetID :- Vpatil3

Written Assignment.

Problem 1: Show that if you use the line $x\sin\theta - y\cos\theta + \rho = 0$, each point (x,y) - image space results in a sinusoid in (ρ, θ).

Given: $x\sin\theta - y\cos\theta + \rho = 0$.

$\Rightarrow \rho = -x\sin\theta + y\cos\theta$.

Since, we are considering (x,y) points one by one, we can assume them to be constants.

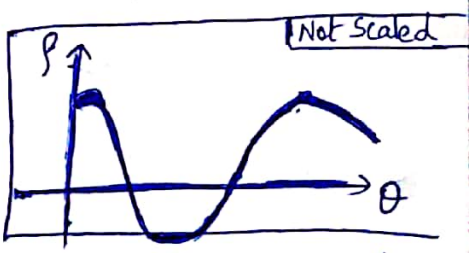
Now, lets divide both sides by $\sqrt{x^2+y^2}$.

$$\Rightarrow \frac{\rho}{\sqrt{x^2+y^2}} = \frac{-x}{\sqrt{x^2+y^2}}\sin\theta + \frac{y}{\sqrt{x^2+y^2}}\cos\theta$$

Let, $M = \frac{-x}{\sqrt{x^2+y^2}}$, $N = \frac{y}{\sqrt{x^2+y^2}}$, [M & N are guranteed to be in the range -1 to 1].

$\therefore$ Clearly $M^2+N^2=1$, so there exists an unique angle ϕ, such that $\cos\phi = M$ & $\sin\phi = N$.

$$\Rightarrow \frac{\rho}{\sqrt{x^2+y^2}} = \cos\phi\sin\theta + \sin\phi\cos\theta.$$



$$\therefore \rho = \sqrt{x^2+y^2}\,\sin(\theta+\phi)$$

$\because \sin(x+y) = \sin x\cos y + \cos x\sin y$

Which is clearly a sinusoid, with its amplitude $= \sqrt{x^2+y^2}$ & phase ϕ, where $\phi = \cos^{-1}\left(\frac{-x}{\sqrt{x^2+y^2}}\right)$ or $\phi = \sin^{-1}\left(\frac{y}{\sqrt{x^2+y^2}}\right)$.

Also, since period/frequency of the sinusoid is independent of (x,y), they don't vary with (x,y). i.e, the horizontal distance before repeat is same.